Corporate Finance

Compendium

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Contents

1. Introduction

This compendium provides a comprehensive overview of the most important topics covered in a corporate finance course at the Bachelor, Master or MBA level. The intension is to supplement renowned corporate finance textbooks such as Brealey, Myers and Allen's "Corporate Finance", Damodaran's "Corporate Finance - Theory and Practice", and Ross, Westerfield and Jordan's "Corporate Finance Fundamentals".

The compendium is designed such that it follows the structure of a typical corporate finance course. Throughout the compendium theory is supplemented with examples and illustrations.

2. The objective of the firm

Corporate Finance is about decisions made by corporations. Not all businesses are organized as corporations. Corporations have three distinct characteristics:

1. Corporations are legal entities, i.e. legally distinct from it owners and pay their own taxes
2. Corporations have limited liability, which means that shareholders can only loose their initial investment in case of bankruptcy
3. Corporations have separated ownership and control as owners are rarely managing the firm

The objective of the firm is to maximize shareholder value by increasing the value of the company's stock. Although other potential objectives (survive, maximize market share, maximize profits, etc.) exist these are consistent with maximizing shareholder value.

Most large corporations are characterized by separation of ownership and control. Separation of ownership and control occurs when shareholders not actively are involved in the management. The separation of ownership and control has the advantage that it allows share ownership to change without influencing with the day-to-day business. The disadvantage of separation of ownership and control is the agency problem, which incurs agency costs.

Agency costs are incurred when:

1. Managers do not maximize shareholder value
2. Shareholders monitor the management

In firms without separation of ownership and control (i.e. when shareholders are managers) no agency costs are incurred.

In a corporation the financial manager is responsible for two basic decisions:

1. The investment decision
2. The financing decision

The investment decision is what real assets to invest in, whereas the financing decision deals with how these investments should be financed. The job of the financial manager is therefore to decide on both such that shareholder value is maximized.

3. Present value and opportunity cost of capital

Present and future value calculations rely on the principle of time value of money.

> **Time value of money**
>
> One dollar today is worth more than one dollar tomorrow.

The intuition behind the time value of money principle is that one dollar today can start earning interest immediately and therefore will be worth more than one dollar tomorrow. Time value of money demonstrates that, all things being equal, it is better to have money now than later.

3.1 Compounded versus simple interest

When money is moved through time the concept of compounded interest is applied. Compounded interest occurs when interest paid on the investment during the first period is added to the principal. In the following period interest is paid on the new principal. This contrasts simple interest where the principal is constant throughout the investment period. To illustrate the difference between simple and compounded interest consider the return to a bank account with principal balance of €100 and an yearly interest rate of 5%. After 5 years the balance on the bank account would be:

- €125.0 with simple interest: €100 + 5 · 0.05 · €100 = €125.0
- €127.6 with compounded interest: €100 · 1.05^5 = €127.6

Thus, the difference between simple and compounded interest is the interest earned on interests. This difference is increasing over time, with the interest rate and in the number of sub-periods with interest payments.

3.2 Present value

Present value (PV) is the value today of a future cash flow. To find the present value of a future cash flow, C_t, the cash flow is multiplied by a discount factor:

(1) $$\text{PV} = \text{discount factor} \cdot C_t$$

The discount factor (DF) is the present value of €1 future payment and is determined by the rate of return on equivalent investment alternatives in the capital market.

(2) $$\text{DF} = \frac{1}{(1+r)^t}$$

Where r is the discount rate and t is the number of years. Inserting the discount factor into the present value formula yields:

$$\text{(3)} \qquad \text{PV} = \frac{C_t}{(1+\text{r})^{\text{t}}}$$

Example:

- What is the present value of receiving €250,000 two years from now if equivalent investments return 5%?

$$\text{PV} = \frac{C_t}{(1+\text{r})^{\text{t}}} = \frac{€250{,}000}{1.05^2} = €\,226{,}757$$

- Thus, the present value of €250,000 received two years from now is €226,757 if the discount rate is 5 percent.

From time to time it is helpful to ask the inverse question: How much is €1 invested today worth in the future?. This question can be assessed with a future value calculation.

3.3 Future value

The future value (FV) is the amount to which an investment will grow after earning interest. The future value of a cash flow, C_0, is:

(4) $$FV = C_0 \cdot (1+r)^t$$

Example:

– What is the future value of €200,000 if interest is compounded annually at a rate of 5% for three years?

$$FV = €200{,}000 \cdot (1+.05)^3 = €231{,}525$$

- Thus, the future value in three years of €200,000 today is €231,525 if the discount rate is 5 percent.

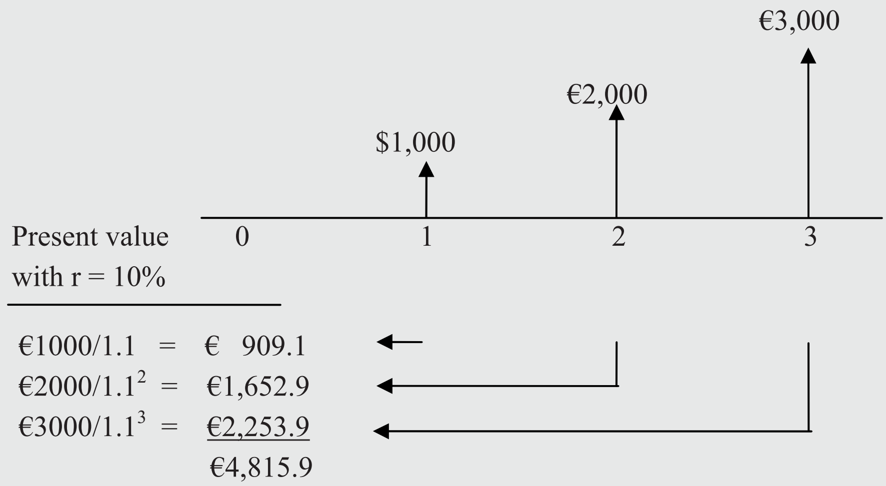
3.4 Principle of value additivity

The principle of value additivity states that present values (or future values) can be added together to evaluate multiple cash flows. Thus, the present value of a string of future cash flows can be calculated as the sum of the present value of each future cash flow:

(5) $$PV = \frac{C_1}{(1+r)^1} + \frac{C_2}{(1+r)^2} + \frac{C_3}{(1+r)^3} + ... = \sum \frac{C_t}{(1+r)^t}$$

Example:

- The principle of value additivity can be applied to calculate the present value of the income stream of €1,000, €2000 and €3,000 in year 1, 2 and 3 from now, respectively.



- The present value of each future cash flow is calculated by discounting the cash flow with the 1, 2 and 3 year discount factor, respectively. Thus, the present value of €3,000 received in year 3 is equal to €3,000 / 1.1^3 = €2,253.9.
- Discounting the cash flows individually and adding them subsequently yields a present value of €4,815.9.

3.5 Net present value

Most projects require an initial investment. Net present value is the difference between the present value of future cash flows and the initial investment, C_0, required to undertake the project:

$$\text{(6)} \qquad \text{NPV} = C_0 + \sum_{i=1}^{n} \frac{C_i}{(1+r)^i}$$

Note that if C_0 is an initial investment, then $C_0 < 0$.

3.6 Perpetuities and annuities

Perpetuities and annuities are securities with special cash flow characteristics that allow for an easy calculation of the present value through the use of short-cut formulas.

Perpetuity

Security with a constant cash flow that is (theoretically) received forever. The present value of a perpetuity can be derived from the annual return, r, which equals the constant cash flow, C, divided by the present value (PV) of the perpetuity:

$$r = \frac{C}{PV}$$

Solving for PV yields:

(7) $$\text{PV of perpetuity} = \frac{C}{r}$$

Thus, the present value of a perpetuity is given by the constant cash flow, C, divided by the discount rate, r.

In case the cash flow of the perpetuity is growing at a constant rate rather than being constant, the present value formula is slightly changed. To understand how, consider the general present value formula:

$$PV = \frac{C_1}{(1+r)} + \frac{C_2}{(1+r)^2} + \frac{C_3}{(1+r)^3} + \cdots$$

Since the cash flow is growing at a constant rate g it implies that $C_2 = (1+g) \cdot C_1$, $C_3 = (1+g)^2 \cdot C_1$, etc. Substituting into the PV formula yields:

$$PV = \frac{C_1}{(1+r)} + \frac{(1+g)C_1}{(1+r)^2} + \frac{(1+g)^2 C_1}{(1+r)^3} + \cdots$$

Utilizing that the present value is a geometric series allows for the following simplification for the present value of growing perpetuity:

(8) $$\text{PV of growing perpetituity} = \frac{C_1}{r-g}$$

Annuity

An asset that pays a fixed sum each year for a specified number of years. The present value of an annuity can be derived by applying the principle of value additivity. By constructing two perpetuities, one with cash flows beginning in year 1 and one beginning in year t+1, the cash flow of the annuity beginning in year 1 and ending in year t is equal to the difference between the two perpetuities. By calculating the present value of the two perpetuities and applying the principle of value additivity, the present value of the annuity is the difference between the present values of the two perpetuities.

Asset	**Year of Payment** 0 1 2……….t t +1…………...	**Present Value**
Perpetuity 1 (first payment in year 1)		$\frac{C}{r}$
Perpetuity 2 (first payment in year t + 1)		$\left(\frac{C}{r}\right)\frac{1}{(1+r)^t}$
Annuity from (year 1 to year t)		$\left(\frac{C}{r}\right)-\left(\frac{C}{r}\right)\left(\frac{1}{(1+r)^t}\right)$

(9) $$\text{PV of annuity} = C\underbrace{\left[\frac{1}{r} - \frac{1}{r(1+r)^t}\right]}_{\text{Annuity factor}}$$

Note that the term in the square bracket is referred to as the annuity factor.

Example: Annuities in home mortgages

- When families finance their consumption the question often is to find a series of cash payments that provide a given value today, e.g. to finance the purchase of a new home. Suppose the house costs €300,000 and the initial payment is €50,000. With a 30-year loan and a monthly interest rate of 0.5 percent what is the appropriate monthly mortgage payment?

  The monthly mortgage payment can be found by considering the present value of the loan. The loan is an annuity where the mortgage payment is the constant cash flow over a 360 month period (30 years times 12 months = 360 payments):

  PV(loan) = mortgage payment · 360-monthly annuity factor

  Solving for the mortgage payment yields:
  Mortgage payment = PV(Loan)/360-monthly annuity factor
  = €250K / (1/0.005 – 1/(0.005 · 1.005^{360})) = €1,498.87

  Thus, a monthly mortgage payment of €1,498.87 is required to finance the purchase of the house.

3.7 Nominal and real rates of interest

Cash flows can either be in current (nominal) or constant (real) dollars. If you deposit €100 in a bank account with an interest rate of 5 percent, the balance is €105 by the end of the year. Whether €105 can buy you more goods and services that €100 today depends on the rate of inflation over the year.

Inflation is the rate at which prices as a whole are increasing, whereas nominal interest rate is the rate at which money invested grows. The real interest rate is the rate at which the purchasing power of an investment increases.

The formula for converting nominal interest rate to a real interest rate is:

$$(10) \qquad 1 + \text{real interest rate} = \frac{1+\text{nominal interest rate}}{1+\text{inflation rate}}$$

For small inflation and interest rates the real interest rate is approximately equal to the nominal interest rate minus the inflation rate.

Investment analysis can be done in terms of real or nominal cash flows, but discount rates have to be defined consistently

– Real discount rate for real cash flows
– Nominal discount rate for nominal cash flows

3.8 Valuing bonds using present value formulas

A bond is a debt contract that specifies a fixed set of cash flows which the issuer has to pay to the bondholder. The cash flows consist of a coupon (interest) payment until maturity as well as repayment of the par value of the bond at maturity.

The value of a bond is equal to the present value of the future cash flows:

(11) Value of bond = PV(cash flows) = PV(coupons) + PV(par value)

Since the coupons are constant over time and received for a fixed time period the present value can be found by applying the annuity formula:

(12) PV(coupons) = coupon · annuity factor

Example:

- Consider a 10-year US government bond with a par value of $1,000 and a coupon payment of $50. What is the value of the bond if other medium-term US bonds offered a 4% return to investors?

  Value of bond = PV(Coupon) + PV(Par value)
  $= \$50 \cdot [1/0.04 - 1/(0.04 \cdot 1.04^{10})] + \$1{,}000 \cdot 1/1.04^{10}$
  $= \$50 \cdot 8.1109 + \$675.56 = \$1{,}081.1$

  Thus, if other medium-term US bonds offer a 4% return to investors the price of the 10-year government bond with a coupon interest rate of 5% is $1,081.1.

The rate of return on a bond is a mix of the coupon payments and capital gains or losses as the price of the bond changes:

(13) $$\text{Rate of return on bond} = \frac{\text{coupon income} + \text{price change}}{\text{investment}}$$

Because bond prices change when the interest rate changes, the rate of return earned on the bond will fluctuate with the interest rate. Thus, the bond is subject to interest rate risk. All bonds are not equally affected by interest rate risk, since it depends on the sensitivity to interest rate fluctuations.

The interest rate required by the market on a bond is called the bond's yield to maturity. Yield to maturity is defined as the discount rate that makes the present value of the bond equal to its price. Moreover, yield to maturity is the return you will receive if you hold the bond until maturity. Note that the yield to maturity is different from the rate of return, which measures the return for holding a bond for a specific time period.

To find the yield to maturity (rate of return) we therefore need to solve for r in the price equation.

Example:

- What is the yield to maturity of a 3-year bond with a coupon interest rate of 10% if the current price of the bond is 113.6?

  Since yield to maturity is the discount rate that makes the present value of the future cash flows equal to the current price, we need to solve for r in the equation where price equals the present value of cash flows:

$$\text{PV(Cash flows)} = \text{Price on bond}$$

$$\frac{10}{(1+r)} + \frac{10}{(1+r)^2} + \frac{110}{(1+r)^3} = 113.6$$

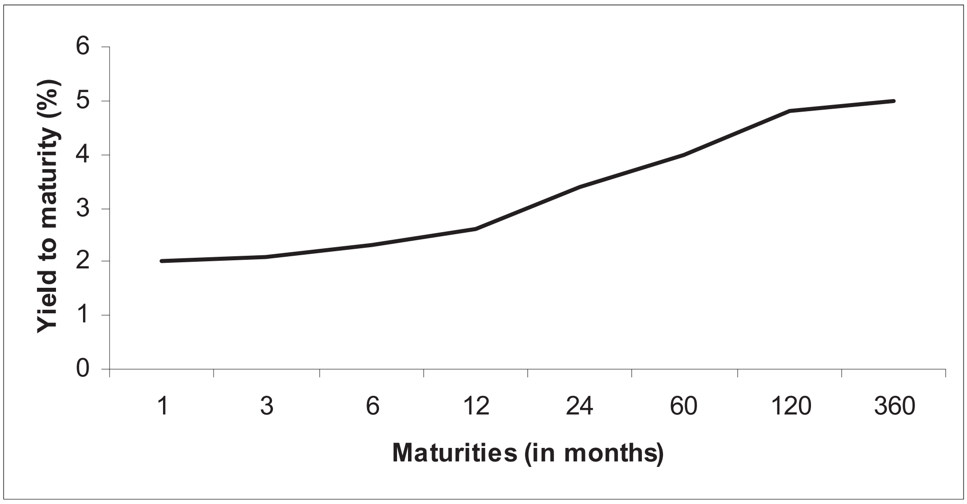
  The yield to maturity is the found by solving for r by making use of a spreadsheet, a financial calculator or by hand using a trail and error approach.

$$\frac{10}{1.05} + \frac{10}{1.05^2} + \frac{110}{1.05^3} = 113.6$$

  Thus, if the current price is equal to 113.6 the bond offers a return of 5 percent if held to maturity.

The yield curve is a plot of the relationship between yield to maturity and the maturity of bonds.

Figure 1: Yield curve



As illustrated in Figure 1 the yield curve is (usually) upward sloping, which means that long-term bonds have higher yields. This happens because long-term bonds are subject to higher interest rate risk, since long-term bond prices are more sensitive to changes to the interest rate.

The yield to maturity required by investors is determined by

1. Interest rate risk
2. Time to maturity
3. Default risk

The default risk (or credit risk) is the risk that the bond issuer may default on its obligations. The default risk can be judged from credit ratings provided by special agencies such as Moody's and Standard and Poor's. Bonds with high credit ratings, reflecting a strong ability to repay, are referred to as investment grade, whereas bonds with a low credit rating are called speculative grade (or junk bonds).

In summary, there exist five important relationships related to a bond's value:

1. The value of a bond is **reversely** related to changes in the interest rate
2. Market value of a bond will be **less** than par value if investor's required rate is **above** the coupon interest rate

3. As maturity approaches the market value of a bond approaches par value
4. Long-term bonds have **greater** interest rate risk than do short-term bonds
5. Sensitivity of a bond's value to changing interest rates depends **not only** on the length of time to maturity, but also on the patterns of cash flows provided by the bond

3.9 Valuing stocks using present value formulas

The price of a stock is equal to the present value of all future dividends. The intuition behind this insight is that the cash payoff to owners of the stock is equal to cash dividends plus capital gains or losses. Thus, the expected return that an investor expects from a investing in a stock over a set period of time is equal to:

$$(14) \qquad \text{Expected return on stock} = \text{r} = \frac{\text{dividend} + \text{capital gain}}{\text{investment}} = \frac{Div_1 + P_1 - P_0}{P_0}$$

Where Div_t and P_t denote the dividend and stock price in year t, respectively. Isolating the current stock price P_0 in the expected return formula yields:

$$(15) \qquad P_0 = \frac{Div_1 + P_1}{1+r}$$

The question then becomes "What determines next years stock price P_1?". By changing the subscripts next year's price is equal to the discounted value of the sum of dividends and expected price in year 2:

$$P_1 = \frac{Div_2 + P_2}{1+r}$$

Inserting this into the formula for the current stock price P_0 yields:

$$P_0 = \frac{Div_1 + P_1}{1+r} = \frac{1}{1+r}\left(Div_1 + P_1\right) = \frac{1}{1+r}\left(Div_1 + \frac{Div_2 + P_2}{1+r}\right) = \frac{Div_1}{1+r} + \frac{Div_2 + P_2}{(1+r)^2}$$

By recursive substitution the current stock price is equal to the sum of the present value of all future dividends plus the present value of the horizon stock price, P_H.

$$\begin{aligned} P_0 &= \frac{Div_1}{1+r} + \frac{Div_2}{(1+r)^2} + \frac{Div_3 + P_3}{(1+r)^3} \\ &\vdots \\ P_0 &= \frac{Div_1}{1+r} + \frac{Div_2}{(1+r)^2} + \cdots + \frac{Div_H + P_H}{(1+r)^H} \\ &= \sum_{t=1}^{H} \frac{Div_t}{(1+r)^t} + \frac{P_H}{(1+r)^H} \end{aligned}$$

The final insight is that as H approaches zero, $[P_H / (1+r)^H]$ approaches zero. Thus, in the limit the current stock price, P_0, can be expressed as the sum of the present value of all future dividends.

Discounted dividend model

(16) $$P_0 = \sum_{t=1}^{\infty} \frac{Div_t}{(1+r)^t}$$

In cases where firms have constant growth in the dividend a special version of the discounted dividend model can be applied. If the dividend grows at a constant rate, g, the present value of the stock can be found by applying the present value formula for perpetuities with constant growth.

Discounted dividend growth model

(17) $$P_0 = \frac{Div_1}{r-g}$$

The discounted dividend growth model is often referred to as the Gordon growth model.

Some firms have both common and preferred shares. Common stockholders are residual claimants on corporate income and assets, whereas preferred shareholders are entitled only to a fixed dividend (with priority over common stockholders). In this case the preferred stocks can be valued as a perpetuity paying a constant dividend forever.

(18) $$P_0 = \frac{Div}{r}$$

The perpetuity formula can also be applied to value firms in general if we assume no growth and that all earnings are paid out to shareholders.

(19) $$P_0 = \frac{Div_1}{r} = \frac{EPS_1}{r}$$

If a firm elects to pay a lower dividend, and reinvest the funds, the share price may increase because future dividends may be higher.

Growth can be derived from applying the return on equity to the percentage of earnings ploughed back into operations:

(20) g = return on equity · plough back ratio

Where the plough back ratio is the fraction of earnings retained by the firm. Note that the plough back ratio equals (1 - payout ratio), where the payout ratio is the fraction of earnings paid out as dividends.

The value of growth can be illustrated by dividing the current stock price into a non-growth part and a part related to growth.

(21) $$P_{\text{With growth}} = P_{\text{No growth}} + PVGO$$

Where the growth part is referred to as the present value of growth opportunities (PVGO). Inserting the value of the no growth stock from (22) yields:

(22) $$P_0 = \frac{EPS_1}{r} + PVGO$$

Firms in which PVGO is a substantial fraction of the current stock price are referred to as growth stocks, whereas firms in which PVGO is an insignificant fraction of the current stock prices are called income stocks.

4. The net present value investment rule

Net present value is the difference between a project's value and its costs. The net present value investment rule states that firms should only invest in projects with positive net present value.

When calculating the net present value of a project the appropriate discount rate is the opportunity cost of capital, which is the rate of return demanded by investors for an equally risky project. Thus, the net present value rule recognizes the time value of money principle.

To find the net present value of a project involves several steps:

How to find the net present value of a project

1. Forecast cash flows
2. Determinate the appropriate opportunity cost of capital, which takes into account the principle of time value of money and the risk-return trade-off
3. Use the discounted cash flow formula and the opportunity cost of capital to calculate the present value of the future cash flows
4. Find the net present value by taking the difference between the present value of future cash flows and the project's costs

There exist several other investment rules:

- Book rate of return
- Payback rule
- Internal rate of return

To understand why the net present value rule leads to better investment decisions than the alternatives it is worth considering the desirable attributes for investment decision rules. The goal of the corporation is to maximize firm value. A shareholder value maximizing investment rule is:

- Based on cash flows
- Taking into account time value of money
- Taking into account differences in risk

The net present value rule meets all these requirements and directly measures the value for shareholders created by a project. This is fare from the case for several of the alternative rules.

The book rate of return is based on accounting returns rather than cash flows:

Book rate of return

Average income divided by average book value over project life

(23) $$\text{Book rate of return} = \frac{\text{book income}}{\text{book value of assets}}$$

The main problem with the book rate of return is that it only includes the annual depreciation charge and not the full investment. Due to time value of money this provides a negative bias to the cost of the investment and, hence, makes the return appear higher. In addition no account is taken for risk. Due to the risk return trade-off we might accept poor high risk projects and reject good low risk projects.

Payback rule

The payback period of a project is the number of years it takes before the cumulative forecasted cash flow equals the initial outlay.

The payback rule only accepts projects that "payback" in the desired time frame.

This method is flawed, primarily because it ignores later year cash flows and the present value of future cash flows. The latter problem can be solved by using a payback rule based on discounted cash flows.

Internal rate of return (IRR)

Defined as the rate of return which makes NPV=0. We find IRR for an investment project lasting T years by solving:

(24) $$NPV = C_o + \frac{C_1}{1+IRR} + \frac{C_2}{(1+IRR)^2} + \cdots + \frac{C_T}{(1+IRR)^T} = 0$$

The IRR investment rule accepts projects if the project's IRR exceeds the opportunity cost of capital, i.e. when IRR > r.

Finding a project's IRR by solving for NPV equal to zero can be done using a financial calculator, spreadsheet or trial and error calculation by hand.

Mathematically, the IRR investment rule is equivalent to the NPV investment rule. Despite this the IRR investment rule faces a number of pitfalls when applied to projects with special cash flow characteristics.

1. Lending or borrowing?
   - With certain cash flows the NPV of the project ***increases*** if the discount rate ***increases***. This is contrary to the normal relationship between NPV and discount rates
2. Multiple rates of return
   - Certain cash flows can generate NPV=0 at multiple discount rates. This will happen when the cash flow stream changes sign. Example: Maintenance costs. In addition, it is possible to have projects with ***no*** IRR and a ***positive*** NPV
3. Mutually exclusive projects
   - Firms often have to choose between mutually exclusive projects. IRR sometimes ignores the magnitude of the project. Large projects with a lower IRR might be preferred to small projects with larger IRR.
4. Term structure assumption
   - We assume that discount rates are constant for the term of the project. What do we compare the IRR with, if we have different rates for each period, *r1, r2, r3*, …? It is not easy to find a traded security with equivalent risk and the same time pattern of cash flows

Finally, note that both the IRR and the NPV investment rule are discounted cash flow methods. Thus, both methods possess the desirable attributes for an investment rule, since they are based on cash flows and allows for risk and time value of money. Under careful use both methods give the same investment decisions (whether to accept or reject a project). However, they may not give the same ranking of projects, which is a problem in case of mutually exclusive projects.

5. Risk, return and opportunity cost of capital

Opportunity cost of capital depends on the risk of the project. Thus, to be able to determine the opportunity cost of capital one must understand how to measure risk and how investors are compensated for taking risk.

5.1 Risk and risk premia

The risk premium on financial assets compensates the investor for taking risk. The risk premium is the difference between the return on the security and the risk free rate.

To measure the average rate of return and risk premium on securities one has to look at very long time periods to eliminate the potential bias from fluctuations over short intervals.

Over the last 100 years U.S. common stocks have returned an average annual nominal compounded rate of return of 10.1% compared to 4.1% for U.S. Treasury bills. As U.S. Treasury bill has short maturity and there is no risk of default, short-term government debt can be considered risk-free. Investors in common stocks have earned a risk premium of 7.0 percent (10.1 - 4.1 percent.). Thus, on average investors in common stocks have historically been compensated with a 7.0 percent higher return per year for taking on the risk of common stocks.

Table 1: Average nominal compounded returns, standard deviation and risk premium on U.S. securities, 1900-2000.

	Annual return	Std. variation	Risk premium
U.S. Treasury Bills	4.1%	4.7%	0.0%
U.S. Government Bonds	4.8%	10.0%	0.7%
U.S. Common Stocks	10.1%	20.2%	7.0%

Source: E. Dimson, P.R. Mash, and M Stauton, Triumph of the Optimists: 101 Years of Investment returns, Princeton University Press, 2002.

Across countries the historical risk premium varies significantly. In Denmark the average risk premium was only 4.3 percent compared to 10.7 percent in Italy. Some of these differences across countries may reflect differences in business risk, while others reflect the underlying economic stability over the last century.

The historic risk premium may overstate the risk premium demanded by investors for several reasons. First, the risk premium may reflect the possibility that the economic development could have turned out to be less fortunate. Second, stock returns have for several periods outpaced the underlying growth in earnings and dividends, something which cannot be expected to be sustained.

The risk of financial assets can be measured by the spread in potential outcomes. The variance and standard deviation on the return are standard statistical measures of this spread.

Variance

Expected (average) value of squared deviations from mean. The variance measures the return volatility and the units are percentage squared.

(25) $$\text{Variance}(r) = \sigma^2 = \frac{1}{N-1}\sum_{t=1}^{N}(r_t - \bar{r})^2$$

Where $\bar{r}$ denotes the average return and N is the total number of observations.

Standard deviation

Square root of variance. The standard deviation measures the return volatility and units are in percentage.

(26) $$\text{Std.dev.}(r) = \sqrt{\text{variance}(r)} = \sigma$$

Using the standard deviation on the yearly returns as measure of risk it becomes clear that U.S. Treasury bills were the least variable security, whereas common stock were the most variable. This insight highlights the risk-return tradeoff, which is key to the understanding of how financial assets are priced.

Risk-return tradeoff
Investors will not take on additional risk unless they expect to be compensated with additional return

The risk-return tradeoff relates the expected return of an investment to its risk. Low levels of uncertainty (low risk) are associated with low expected returns, whereas high levels of uncertainty (high risk) are associated with high expected returns.

It follows from the risk-return tradeoff that rational investors will when choosing between two assets that offer the same expected return prefer the less risky one. Thus, an investor will take on increased risk only if compensated by higher expected returns. Conversely, an investor who wants higher returns must accept more risk. The exact trade-off will differ by investor based on individual risk aversion characteristics (i.e. the individual preference for risk taking).

5.2 The effect of diversification on risk

The risk of an individual asset can be measured by the variance on the returns. The risk of individual assets can be reduced through diversification. Diversification reduces the variability when the prices of individual assets are not perfectly correlated. In other words, investors can reduce their exposure to individual assets by holding a diversified portfolio of assets. As a result, diversification will allow for the same portfolio return with reduced risk.

Example:

- A classical example of the benefit of diversification is to consider the effect of combining the investment in an ice-cream producer with the investment in a manufacturer of umbrellas. For simplicity, assume that the return to the ice-cream producer is +15% if the weather is sunny and -10% if it rains. Similarly the manufacturer of umbrellas benefits when it rains (+15%) and looses when the sun shines (-10%). Further, assume that each of the two weather states occur with probability 50%.

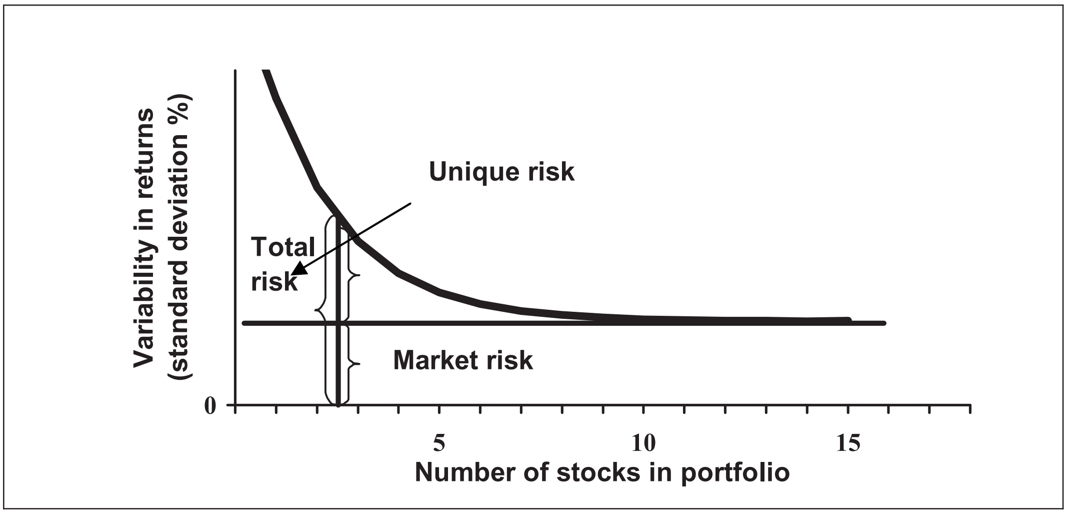
	Expected return	**Variance**
Ice-cream producer	$0.5 \cdot 15\% + 0.5 \cdot -10\% = 2.5\%$	$0.5 \cdot [15-2.5]^2 + 0.5 \cdot [-10-2.5]^2 = 12.5^2\%$
Umbrella manufacturer	$0.5 \cdot -10\% + 0.5 \cdot 15\% = 2.5\%$	$0.5 \cdot [-10-2.5]^2 + 0.5 \cdot [15-2.5]^2 = 12.5^2\%$

- Both investments offer an expected return of +2.5% with a standard deviation of 12.5 percent

- Compare this to the portfolio that invests 50% in each of the two stocks. In this case, the expected return is +2.5% both when the weather is sunny and rainy (0.5*15% + 0.5*-10% = 2.5%). However, the standard deviation drops to 0% as there is no variation in the return across the two states. Thus, by diversifying the risk related to the weather could be hedged. This happens because the returns to the ice-cream producer and umbrella manufacturer are perfectly negatively correlated.

Obviously the prior example is extreme as in the real world it is difficult to find investments that are perfectly negatively correlated and thereby diversify away all risk. More generally the standard deviation of a portfolio is reduced as the number of securities in the portfolio is increased. The reduction in risk will occur if the stock returns within our portfolio are not perfectly positively correlated. The benefit of diversification can be illustrated graphically:

Figure 2: How portfolio diversification reduces risk



As the number of stocks in the portfolio increases the exposure to risk decreases. However, portfolio diversification cannot eliminate all risk from the portfolio. Thus, total risk can be divided into two types of risk: (1) Unique risk and (2) Market risk. It follows from the graphically illustration that unique risk can be diversified way, whereas market risk is non-diversifiable. Total risk declines until the portfolio consists of around 15-20 securities, then for each additional security in the portfolio the decline becomes very slight.

Portfolio risk

Total risk = Unique risk + Market risk

Unique risk

- Risk factors affecting only a single assets or a small group of assets
- Also called
    - Idiosyncratic risk
    - Unsystematic risk
    - Company-unique risk
    - Diversifiable risk
    - Firm specific risk
- Examples:
    - A strike among the workers of a company, an increase in the interest rate a company pays on its short-term debt by its bank, a product liability suit.

Market risk

- Economy-wide sources of risk that affects the overall stock market. Thus, market risk influences a large number of assets, each to a greater or lesser extent.
- Also called
    - Systematic risk
    - Non-diversifiable risk
- Examples:
    - Changes in the general economy or major political events such as changes in general interest rates, changes in corporate taxation, etc.

As diversification allows investors to essentially eliminate the unique risk, a well-diversified investor will only require compensation for bearing the market risk of the individual security. Thus, the expected return on an asset depends only on the market risk.

5.3 Measuring market risk

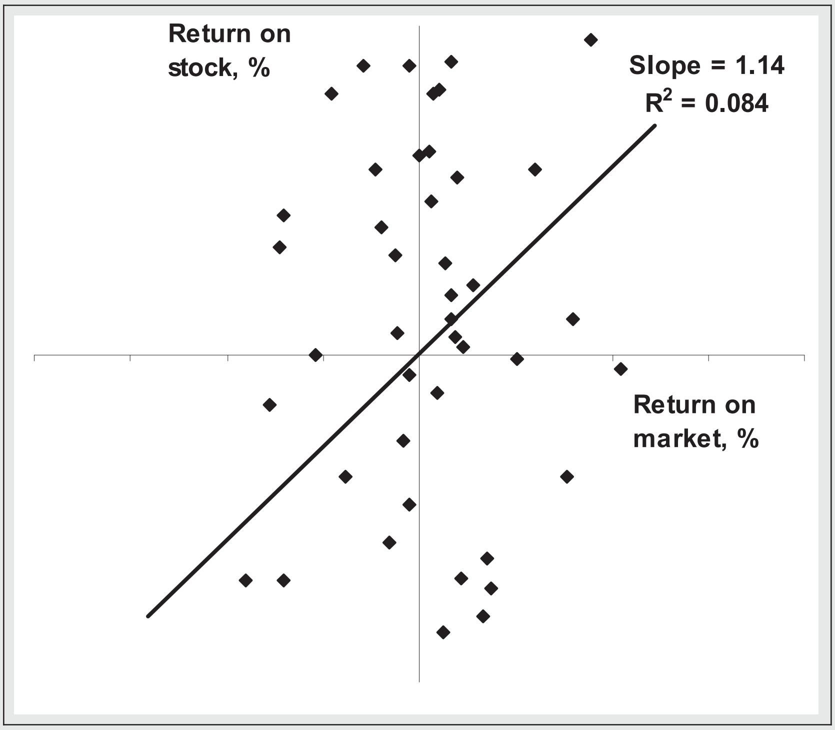
Market risk can be measured by beta, which measures how sensitive the return is to market movements. Thus, beta measures the risk of an asset relative to the average asset. By definition the average asset has a beta of one relative to itself. Thus, stocks with betas below 1 have lower than average market risk; whereas a beta above 1 means higher market risk than the average asset.

Estimating beta

Beta is measuring the individual asset's exposure to market risk. Technically the beta on a stock is defined as the covariance with the market portfolio divided by the variance of the market:

(27) $$\beta_i = \frac{\text{covariance with market}}{\text{variance of market}} = \frac{\sigma_{im}}{\sigma_m^2}$$

In practise the beta on a stock can be estimated by fitting a line to a plot of the return to the stock against the market return. The standard approach is to plot monthly returns for the stock against the market over a 60-month period.



Intuitively, beta measures the average change to the stock price when the market rises with an extra percent. Thus, beta is the slope on the fitted line, which takes the value 1.14 in the example above. A beta of 1.14 means that the stock amplifies the movements in the stock market, since the stock price will increase with 1.14% when the market rise an extra 1%. In addition it is worth noticing that r-square is equal to 8.4%, which means that only 8.4% of the variation in the stock price is related to market risk.

5.4 Portfolio risk and return

The expected return on a portfolio of stocks is a weighted average of the expected returns on the individual stocks. Thus, the expected return on a portfolio consisting of n stocks is:

$$\text{(28)} \qquad \text{Portfolio return} = \sum_{i=1}^{n} w_i r_i$$

Where w_i denotes the fraction of the portfolio invested in stock *i* and r_i is the expected return on stock *i*.

> **Example**:
>
> - Suppose you invest 50% of your portfolio in Nokia and 50% in Nestlé. The expected return on your Nokia stock is 15% while Nestlé offers 10%. What is the expected return on your portfolio?
> - $\text{Portfolio return} = \sum_{i=1}^{n} w_i r_i = 0.5 \cdot 15\% + 0.5 \cdot 10\% = 12.5\%$
> - A portfolio with 50% invested in Nokia and 50% in Nestlé has an expected return of 12.5%.

5.4.1 Portfolio variance

Calculating the variance on a portfolio is more involved. To understand how the portfolio variance is calculated consider the simple case where the portfolio only consists of two stocks, stock 1 and 2. In this case the calculation of variance can be illustrated by filling out four boxes in the table below.

Table 2: Calculation of portfolio variance

	Stock 1	Stock 2
Stock 1	$w_1^2\sigma_1^2$	$w_1 w_2 \sigma_{12} = w_1 w_2 \rho_{12}\sigma_1\sigma_2$
Stock 2	$w_1 w_2 \sigma_{12} = w_1 w_2 \rho_{12}\sigma_1\sigma_2$	$w_2^2\sigma_2^2$

In the top left corner of Table 2, you weight the variance on stock 1 by the square of the fraction of the portfolio invested in stock 1. Similarly, the bottom left corner is the variance of stock 2 times the square of the fraction of the portfolio invested in stock 2. The two entries in the diagonal boxes depend on the covariance between stock 1 and 2. The covariance is equal to the correlation coefficient times the product of the two standard deviations on stock 1 and 2. The portfolio variance is obtained by adding the content of the four boxes together:

$$\text{Portfolio variance} = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1 w_2 \rho_{12}\sigma_1\sigma_2$$

The benefit of diversification follows directly from the formula of the portfolio variance, since the portfolio variance is increasing in the covariance between stock 1 and 2. Combining stocks with a low correlation coefficient will therefore reduce the variance on the portfolio.

Example:

- Suppose you invest 50% of your portfolio in Nokia and 50% in Nestlé. The standard deviation on Nokia's and Nestlé's return is 30% and 20%, respectively. The correlation coefficient between the two stocks is 0.4. What is the portfolio variance?

$$\begin{aligned}\text{Portfolio variance} &= w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1 w_2 \rho_{12}\sigma_1\sigma_2 \\ &= 0.5^2 \cdot 30^2 + 0.5^2 20^2 + 2 \cdot 0.5 \cdot 0.5 \cdot 0.4 \cdot 30 \cdot 20 \\ &= 445 = 21.1^2\end{aligned}$$

- A portfolio with 50% invested in Nokia and 50% in Nestlé has a variance of 445, which is equivalent to a standard deviation of 21.1%.

For a portfolio of n stocks the portfolio variance is equal to:

(29) $$\text{Portfolio variance} = \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \sigma_{ij}$$

Note that when $i=j$, σ_{ij} is the variance of stock *i*, σ_i^2. Similarly, when $i\neq j$, σ_{ij} is the covariance between stock i and j as $\sigma_{ij} = \rho_{ij}\sigma_i\sigma_j$.

5.4.2 Portfolio's market risk

The market risk of a portfolio of assets is a simple weighted average of the betas on the individual assets.

(30) $$\text{Portfolio beta} = \sum_{i=1}^{n} w_i \beta_i$$

Where w_i denotes the fraction of the portfolio invested in stock *i* and β_i is market risk of stock *i*.

Example:

- Consider the portfolio consisting of three stocks A, B and C.

	Amount invested	Expected return	Beta
Stock A	1000	10%	0.8
Stock B	1500	12%	1.0
Stock C	2500	14%	1.2

- What is the beta on this portfolio?
- As the portfolio beta is a weighted average of the betas on each stock, the portfolio weight on each stock should be calculated. The investment in stock A is \$1000 out of the total investment of \$5000, thus the portfolio weight on stock A is 20%, whereas 30% and 50% are invested in stock B and C, respectively.
- The expected return on the portfolio is:

$$r_P = \sum_{i=1}^{n} w_i r_i = 0.2 \cdot 10\% + 0.3 \cdot 12\% + 0.5 \cdot 14\% = 12.6\%$$

- Similarly, the portfolio beta is:

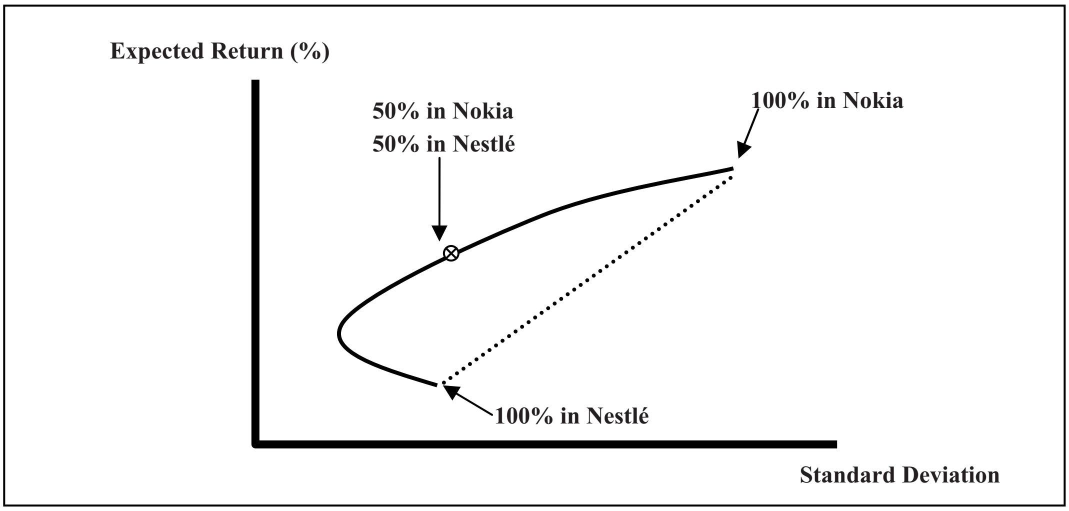
$$\beta_P = \sum_{i=1}^{n} w_i \beta_i = 0.2 \cdot 0.8 + 0.3 \cdot 1 + 0.5 \cdot 1.2 = 1.06$$

- The portfolio investing 20% in stock A, 30% in stock B, and 50% in stock C has an expected return of 12.6% and a beta of 1.06. Note that a beta above 1 implies that the portfolio has greater market risk than the average asset.

5.5 Portfolio theory

Portfolio theory provides the foundation for estimating the return required by investors for different assets. Through diversification the exposure to risk could be minimized, which implies that portfolio risk is less than the average of the risk of the individual stocks. To illustrate this consider Figure 3, which shows how the expected return and standard deviation change as the portfolio is comprised by different combinations of the Nokia and Nestlé stock.

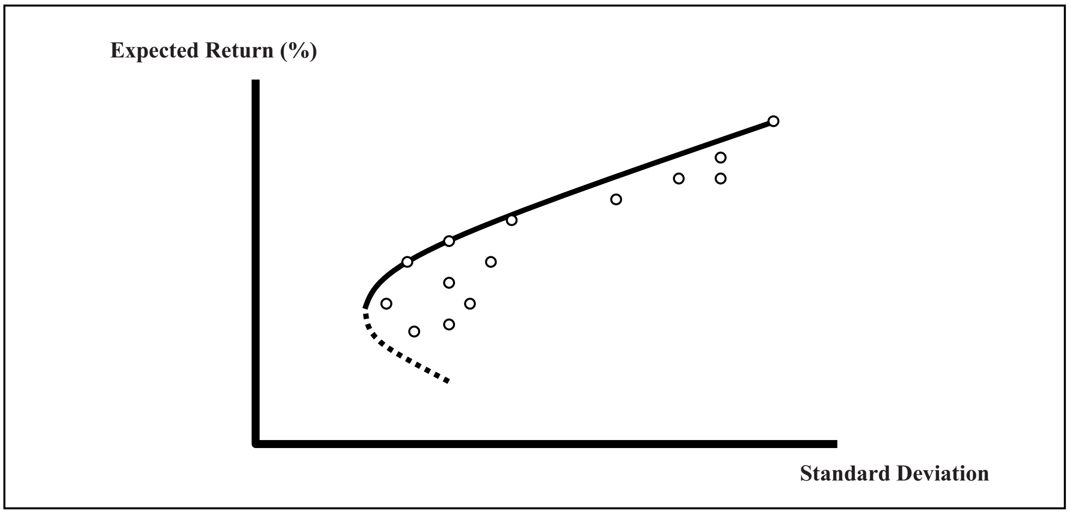
Figure 3: Portfolio diversification



If the portfolio invested 100% in Nestlé the expected return would be 10% with a standard deviation of 20%. Similarly, if the portfolio invested 100% in Nokia the expected return would be 15% with a standard deviation of 30%. However, a portfolio investing 50% in Nokia and 50% in Nestlé would have an expected return of 12.5% with a standard deviation of 21.1%. Note that the standard deviation of 21.1% is less than the average of the standard deviation of the two stocks (0.5 · 20% + 0.5 · 30% = 25%). This is due to the benefit of diversification.

In similar vein, every possible asset combination can be plotted in risk-return space. The outcome of this plot is the collection of all such possible portfolios, which defines a region in the risk-return space. As the objective is to minimize the risk for a given expected return and maximize the expected return for a given risk, it is preferred to move up and to the left in Figure 4.

Figure 4: Portfolio theory and the efficient frontier



The solid line along the upper edge of this region is known as the *efficient frontier.* Combinations along this line represent portfolios for which there is lowest risk for a given level of return. Conversely, for a given amount of risk, the portfolio lying on the efficient frontier represents the combination offering the best possible return. Thus, the efficient frontier is a collection of portfolios, each one optimal for a given amount of risk.

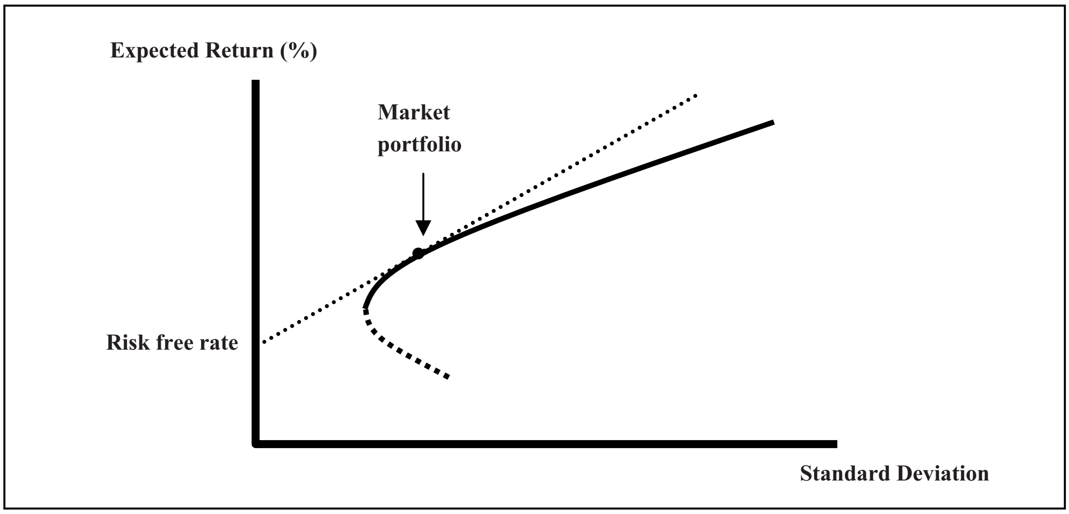
The Sharpe-ratio measures the amount of return above the risk-free rate a portfolio provides compared to the risk it carries.

(31) $$\text{Sharpe ratio on portfolio i} = \frac{r_i - r_f}{\sigma_i}$$

Where r_i is the return on portfolio *i*, r_f is the risk free rate and σ_i is the standard deviation on portfolio *i*'s return. Thus, the Sharpe-ratio measures the risk premium on the portfolio per unit of risk.

In a well-functioning capital market investors can borrow and lend at the same rate. Consider an investor who borrows and invests fraction of the funds in a portfolio of stocks and the rest in short-term government bonds. In this case the investor can obtain an expected return from such an allocation along the line from the risk free rate r_f through the tangent portfolio in Figure 5. As lending is the opposite of borrowing the line continues to the right of the tangent portfolio, where the investor is borrowing additional funds to invest in the tangent portfolio. This line is known as the capital allocation line and plots the expected return against risk (standard deviation).

Figure 5: Portfolio theory



The tangent portfolio is called the market portfolio. The market portfolio is the portfolio on the efficient frontier with the highest Sharpe-ratio. Investors can therefore obtain the best possible risk return trade-off by holding a mixture of the market portfolio and borrowing or lending. Thus, by combining a risk-free asset with risky assets, it is possible to construct portfolios whose risk-return profiles are superior to those on the efficient frontier.

5.6 Capital assets pricing model (CAPM)

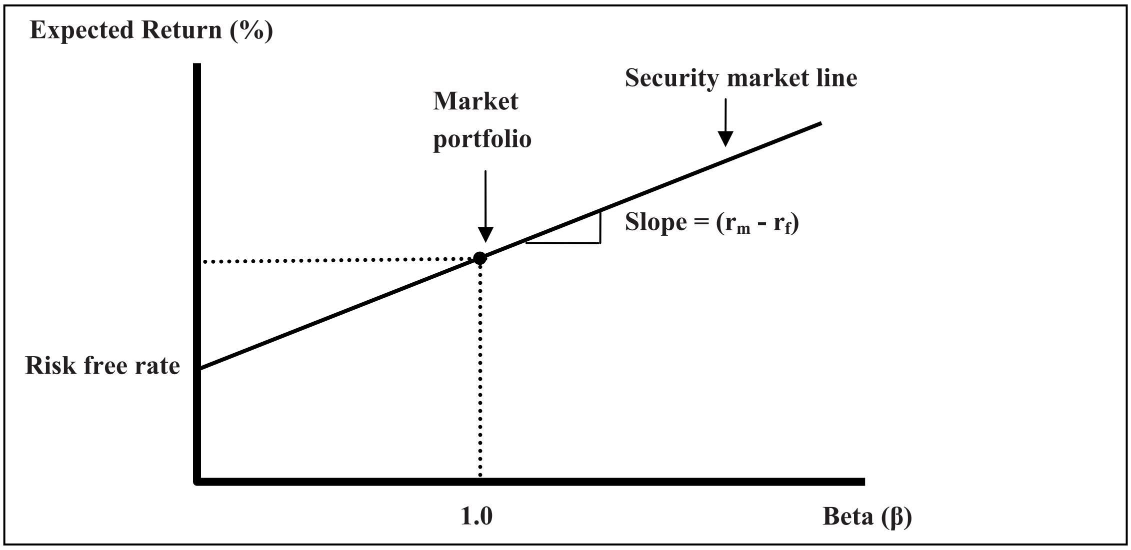
The **C**apital **A**ssets **P**ricing **M**odel (CAPM) derives the expected return on an assets in a market, given the risk-free rate available to investors and the compensation for market risk. CAPM specifies that the expected return on an asset is a linear function of its beta and the market risk premium:

(32) $$\text{Expected return on stock i} = r_i = r_f + \beta_i (r_m - r_f)$$

Where r_f is the risk-free rate, β_i is stock *i*'s sensitivity to movements in the overall stock market, whereas $(r_m - r_f)$ is the market risk premium per unit of risk. Thus, the expected return is equal to the risk free-rate plus compensation for the exposure to market risk. As β_i is measuring stock *i*'s exposure to market risk in units of risk, and the market risk premium is the compensations to investors per unit of risk, the compensation for market risk of stock *i* is equal to the $\beta_i (r_m - r_f)$.

Figure 6 illustrates CAPM:

Figure 6: Portfolio expected return



The relationship between β and required return is plotted on the *securities market line*, which shows expected return as a function of β. Thus, the security market line essentially graphs the results from the CAPM theory. The *x*-axis represents the risk (beta), and the *y*-axis represents the expected return. The intercept is the risk-free rate available for the market, while the slope is the market risk premium $(r_m - r_f)$

CAPM is a simple but powerful model. Moreover it takes into account the basic principles of portfolio selection:

1. Efficient portfolios (Maximize expected return subject to risk)
2. Highest ratio of risk premium to standard deviation is a combination of the market portfolio and the risk-free asset
3. Individual stocks should be selected based on their contribution to portfolio risk
4. Beta measures the marginal contribution of a stock to the risk of the market portfolio

CAPM theory predicts that all assets should be priced such that they fit along the security market line one way or the other. If a stock is priced such that it offers a higher return than what is predicted by CAPM, investors will rush to buy the stock. The increased demand will be reflected in a higher stock price and subsequently in lower return. This will occur until the stock fits on the security market line. Similarly, if a stock is priced such that it offers a lower return than the return implied by CAPM, investor would hesitate to buy the stock. This will provide a negative impact on the stock price and increase the return until it equals the expected value from CAPM.

5.7 Alternative asset pricing models

5.7.1 Arbitrage pricing theory

Arbitrage pricing theory (APT) assumes that the return on a stock depends partly on macroeconomic factors and partly on noise, which are company specific events. Thus, under APT the expected stock return depends on an unspecified number of macroeconomic factors plus noise:

(33) $$\text{Expected return} = a + b_1 \cdot r_{factor\ 1} + b_2 \cdot r_{factor\ 2} + \ldots + b_n \cdot r_{factor\ n} + noise$$

Where $b_1, b_2, \ldots, b_n$ is the sensitivity to each of the factors. As such the theory does not specify what the factors are except for the notion of pervasive macroeconomic conditions. Examples of factors that might be included are return on the market portfolio, an interest rate factor, GDP, exchange rates, oil prices, etc.

Similarly, the expected risk premium on each stock depends on the sensitivity to each factor ($b_1, b_2, \ldots, b_n$) and the expected risk premium associated with the factors:

(34) $$\text{Expected risk premium} = b_1 \cdot (r_{factor\,1} - r_f) + b_2 \cdot (r_{factor\,2} - r_f) + \ldots + b_n \cdot (r_{factor\,n} - r_f)$$

In the special case where the expected risk premium is proportional only to the portfolio's market beta, APT and CAPM are essentially identical.

APT theory has two central statements:

1. A diversified portfolio designed to eliminate the macroeconomic risk (i.e. have zero sensitivity to each factor) is essentially risk-free and will therefore be priced such that it offers the risk-free rate as interest.
2. A diversified portfolio designed to be exposed to e.g. factor 1, will offer a risk premium that varies in proportion to the portfolio's sensitivity to factor 1.

5.7.2 Consumption beta

If investors are concerned about an investment's impact on future consumption rather than wealth, a security's risk is related to its sensitivity to changes in the investor's consumption rather than wealth. In this case the expected return is a function of the stock's consumption beta rather than its market beta. Thus, under the consumption CAPM the most important risks to investors are those the might cutback future consumption.

5.7.3 Three-Factor Model

The three factor model is a variation of the arbitrage pricing theory that explicitly states that the risk premium on securities depends on three common risk factors: a market factor, a size factor, and a book-to-market factor:

$$(35)\ \text{Expected risk premium} = b_{market} \cdot (r_{market\ factor}) + b_{size} \cdot (r_{size\ factor}) + b_{book-to-market} \cdot (r_{book-to-market})$$

Where the three factors are measured in the following way:

- Market factor is the return on market portfolio minus the risk-free rate
- Size factor is the return on small-firm stocks minus the return on large-firm stocks (small minus big)
- Book-to-market factor is measured by the return on high book-to-market value stocks minus the return on low book-value stocks (high minus low)

As the three factor model was suggested by Fama and French, the model is commonly known as the Fama-French three-factor model.

6. Capital budgeting

The firms cost of capital is equal to the expected return on a portfolio of all the company's existing securities. In absence of corporate taxation the company cost of capital is a weighted average of the expected return on debt and equity:

(36) $$\text{Company cost of capital} = \mathrm{r}_{\text{assets}} = \frac{debt}{debt + equity} r_{debt} + \frac{equity}{debt + equity} r_{equity}$$

The firm's cost of capital can be used as the discount rate for the average-risk of the firm's projects.

Cost of capital in practice

Cost of capital is defined as the weighted average of the expected return on debt and equity

$$\text{Company cost of capital} = \mathrm{r}_{\text{assets}} = \frac{debt}{debt + equity} r_{debt} + \frac{equity}{debt + equity} r_{equity}$$

To estimate company cost of capital involves four steps:

1. Determine cost of debt
   - Interest rate for bank loans
   - Yield to maturity for bonds

2. Determine cost of equity
   - Find beta on the stock and determine the expected return using CAPM:
     $r_{equity} = r_{risk\ free} + \beta_{equity} \left(r_{market} - r_{risk\ free} \right)$
   - Beta can be estimated by plotting the return on the stock against the return on the market, and, fit a regression line to through the points. The slope on this line is the estimate of beta.

3. Find the debt and equity ratios
   - Debt and equity ratios should be calculated by using market value (rather than book value) of debt and equity.

4. Insert into the weighted average cost of capital formula

6.1 Cost of capital with preferred stocks

Some firm has issued preferred stocks. In this case the required return on the preferred stocks should be included in the company's cost of capital.

$$\text{(37) Company cost of capital} = \frac{debt}{firm\ value} r_{debt} + \frac{common\ equity}{firm\ value} r_{common} + \frac{preferred\ equity}{firm\ value} r_{preferred}$$

Where firm value equals the sum of the market value of debt, common, and preferred stocks.

The cost of preferred stocks can be calculated by realising that a preferred stock promises to pay a fixed dividend forever. Hence, the market value of a preferred share is equal to the present value of a perpetuity paying the constant dividend:

$$\text{Price of preferred stocks} = \frac{DIV}{r}$$

Solving for r yields the cost of preferred stocks:

(38) $$\text{Cost of preferred stocks} = r_{preferred} = \frac{DIV}{P}$$

Thus, the cost of a preferred stock is equal to the dividend yield.

6.2 Cost of capital for new projects

A new investment project should be evaluated based on its risk, not on company cost of capital. The company cost of capital is the average discount rate across projects. Thus, if we use company cost of capital to evaluate a new project we might:

- Reject good low-risk projects
- Accept poor high-risk projects

True cost of capital depends on project risk. However, many projects can be treated as average risk. Moreover, the company cost of capital provide a good starting reference to evaluate project risk

6.3 Alternative methods to adjust for risk

An alternative way to eliminate risk is to convert expected cash flows to certainty equivalents. A certainty equivalent is the (certain) cash flow which you are willing to swap an expected but uncertain cash flow for. The certain cash flow has exactly the same present value as an expected but uncertain cash flow. The certain cash flow is equal to

(39) $$\textit{Certain cash flow} = PV \cdot (1 + r)$$

Where PV is the present value of the uncertain cash flow and r is the interest rate.

6.4 Capital budgeting in practise

Capital budgeting consists of two parts; 1) Estimate the cash flows, and 2) Estimate opportunity cost of capital. Thus, knowing which cash flows to include in the capital budgeting decision is as crucial as finding the right discount factor.

6.4.1 What to discount?

1. Only cash flows are relevant
    - Cash flows are not accounting profits

2. Relevant cash flows are incremental
    - Include all incidental effects
    - Include the effect of imputation
    - Include working capital requirements
    - Forget sunk costs
    - Include opportunity costs
    - Beware of allocated overhead costs

6.4.2 Calculating free cash flows

Investors care about free cash flows as these measures the amount of cash that the firm can return to investors after making all investments necessary for future growth. Free cash flows differ from net income, as free cash flows are

- Calculated before interest
- Excluding depreciation
- Including capital expenditures and investments in working capital

Free cash flows can be calculated using information available in the income statement and balance sheet:

$$\begin{aligned} \textit{Free cash flow} = {} & \textit{profit after tax} + \textit{depreciation} + \textit{investment in fixed assets} \\ & + \textit{investment in working capital} \end{aligned} \tag{40}$$

6.4.3 Valuing businesses

The value of a business is equal to the present value of all future (free) cash flows using the after-tax WACC as the discount rate. A project's free cash flows generally fall into three categories

1. Initial investment
    – Initial outlay including installation and training costs
    – After-tax gain if replacing old machine
2. Annual free cash flow
    – Profits, interest, and taxes
    – Working capital
3. Terminal cash flow
    – Salvage value
    – Taxable gains or losses associated with the sale

For long-term projects or stocks (which last forever) a common method to estimate the present value is to forecast the free cash flows until a valuation horizon and predict the value of the project at the horizon. Both cash flows and the horizon values are discounted back to the present using the after-tax WACC as the discount rate:

$$PV = \frac{FCF_1}{(1+WACC)} + \frac{FCF_2}{(1+WACC)^2} + \cdots + \frac{FCF_t}{(1+WACC)^t} + \frac{PV_t}{(1+WACC)^t} \tag{41}$$

Where FCF_i denotes free cash flows in year i, WACC the after-tax weighted average cost of capital and PV_t the horizon value at time t.

There exist two common methods of how to estimate the horizon value

1. Apply the constant growth discounted cash flow model, which requires a forecast of the free cash flow in year t+1 as well as a long-run growth rate (g):

$$PV_t = \frac{FCF_{t+1}}{WACC - g}$$

2. Apply multiples of earnings, which assumes that the value of the firm can be estimated as a multiple on earnings before interest, taxes (EBIT) or earnings before interest, taxes, depreciation, and amortization (EBITDA):

$$PV_t = EBIT\ Multiple \cdot EBIT$$
$$PV_t = EBITDA\ Multiple \cdot EBITDA$$

Example:

- If other firms within the industry trade at 6 times EBIT and the firm's EBIT is forecasted to be €10 million, the terminal value at time t is equal to 6·10 = €60 million.

Capital budgeting in practice

Firms should invest in projects that are worth more than they costs. Investment projects are only worth more than they cost when the net present value is positive. The net present value of a project is calculated by discounting future cash flows, which are forecasted. Thus, projects may appear to have positive NPV because of errors in the forecasting. To evaluate the influence of forecasting errors on the estimated net present value of the projects several tools exists:

- Sensitivity analysis
  - Analysis of the effect on estimated NPV when a underlying assumption changes, e.g. market size, market share or opportunity cost of capital.
  - Sensitivity analysis uncovers how sensitive NPV is to changes in key variables.

- Scenario analysis
  - Analyses the impact on NPV under a particular combination of assumptions. Scenario analysis is particular helpful if variables are interrelated, e.g. if the economy enters a recession due to high oil prices, both the firms cost structure, the demand for the product and the inflation might change. Thus, rather than analysing the effect on NPV of a single variable (as sensitivity analysis) scenario analysis considers the effect on NPV of a consistent combination of variables.
  - Scenario analysis calculates NPV in different states, e.g. pessimistic, normal, and optimistic.

- Break even analysis
  - Analysis of the level at which the company breaks even, i.e. at which point the present value of revenues are exactly equal to the present value of total costs. Thus, break-even analysis asks the question how much should be sold before the production turns profitable.

- Simulation analysis
  - Monte Carlo simulation considers all possible combinations of outcomes by modelling the project. Monte Carlo simulation involves four steps:
    1. Modelling the project by specifying the project's cash flows as a function of revenues, costs, depreciation and revenues and costs as a function of market size, market shares, unit prices and costs.
    2. Specifying probabilities for each of the underlying variables, i.e. specifying a range for e.g. the expected market share as well as all other variables in the model
    3. Simulate cash flows using the model and probabilities assumed above and calculate the net present value

6.5 Why projects have positive NPV

In addition to performing a careful analysis of the investment project's sensitivity to the underlying assumptions, one should always strive to understand why the project earns economic rent and whether the rents can be sustained.

Economic rents are profits than more than cover the cost of capital. Economic rents only occur if one has

- Better product
- Lower costs
- Another competitive edge

Even with a competitive edge one should not assume that other firms will watch passively. Rather one should try to identify:

- How long can the competitive edge be sustained?
- What will happen to profits when the edge disappears?
- How will rivals react to my move in the meantime?
  - Will they cut prices?
  - Imitate the product?

Sooner or later competition is likely to eliminate economic rents.

7. Market efficiency

In an efficient market the return on a security is compensating the investor for time value of money and risk. The efficient market theory relies on the fact that stock prices follow a random walk, which means that price changes are independent of one another. Thus, stock prices follow a random walk if

- The movement of stock prices from day to day do not reflect any pattern
- Statistically speaking
  - The movement of stock prices is random
  - Time series of stock returns has low autocorrelation

In an efficient market competition ensures that

- New information is quickly and fully assimilated into prices
- All available information is reflected in the stock price
- Prices reflect the known and expected, and respond only to new information
- Price changes occur in an unpredictable way

The efficient market hypothesis comes in three forms: weak, semi-strong and strong efficiency

Weak form efficiency
- Market prices reflect all historical price information

Semi-strong form efficiency
- Market prices reflect all publicly available information

Strong form efficiency
- Market prices reflect all information, both public and private

Efficient market theory has been subject to close scrutiny in the academic finance literature, which has attempted to test and validate the theory.

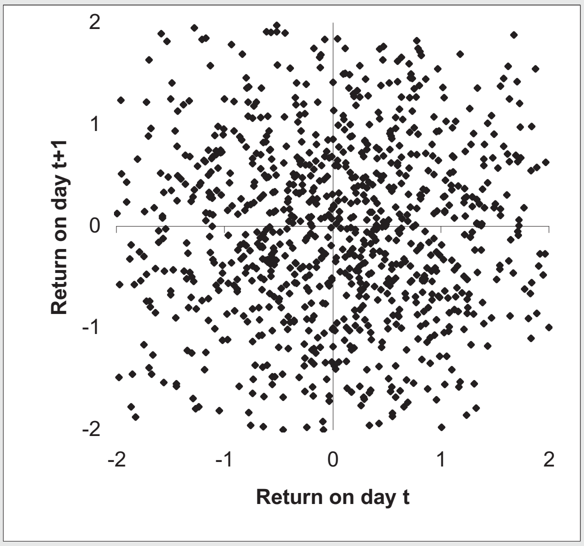
7.1 Tests of the efficient market hypothesis

7.1.1 Weak form

The weak form of market efficiency has been tested by constructing trading rules based on patterns in stock prices. A very direct test of the weak form of market efficient is to test whether a time series of stock returns has zero autocorrelation. A simple way to detect autocorrelation is to plot the return on a stock on day t against the return on day t+1 over a sufficiently long time period. The time series of returns will have zero autocorrelation if the scatter diagram shows no significant relationship between returns on two successive days.

Example:

- Consider the following scatter diagram of the return on the FTSE 100 index on London Stock Exchange for two successive days in the period from 2005-6.



- As there is no significant relationship between the return on successive days, the evidence is supportive of the weak form of market efficiency.

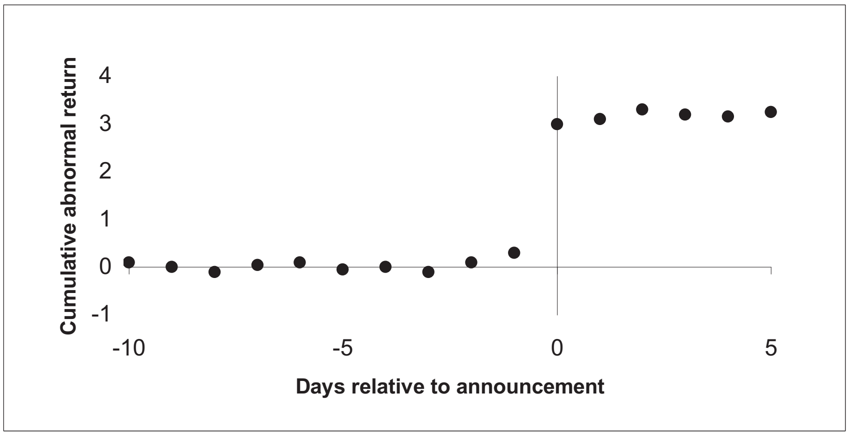
7.1.2 Semi-strong form

The semi-strong form of market efficiency states that all publicly available information should be reflected in the current stock price. A common way to test the semi-strong form is to look at how rapid security prices respond to news such as earnings announcements, takeover bids, etc. This is done by examining how releases of news affect abnormal returns where

- Abnormal stock return = actual stock return - expected stock return

As the semi-strong form of market efficiency predicts that stocks prices should react quickly to the release of new information, one should expect the abnormal stock return to occur around the news release. Figure 7 illustrates the stock price reaction to a news event by plotting the abnormal return around the news release. Prior to the news release the actual stock return is equal to the expected (thus zero abnormal return), whereas at day 0 when the new information is released the abnormal return is equal to 3 percent. The adjustment in the stock price is immediate. In the days following the release of information there is no further drift in the stock price, either upward or downward.

Figure 7: Stock price reaction to news announcement

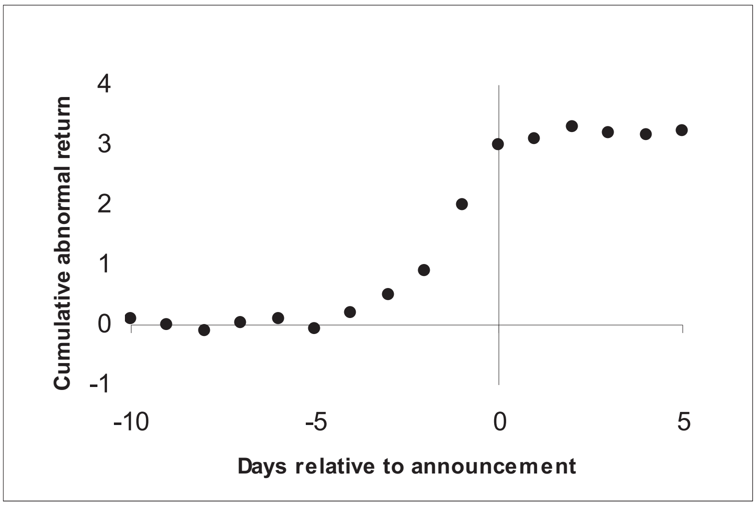


7.1.3 Strong form

Tests of the strong form of market efficiency have analyzed whether professional money managers can consistently outperform the market. The general finding is that although professional money managers on average slightly outperform the market, the outperformance is not large enough to offset the fees paid for their services. Thus, net of fees the recommendations from security analysts, and the investment performance of mutual and pension funds fail to beat the average. Taken at face value, one natural recommendation in line with these findings is to follow a passive investment strategy and "buy the index". Investing in the broad stock index would both maximize diversification and minimize the cost of managing the portfolio.

Another, perhaps more simple, test for strong form of market efficiency is based upon price changes close to an event. The strong form predicts that the release of private information should not move stock prices. For example, consider a merger between two firms. Normally, a merger or an acquisition is known about by an "inner circle" of lawyers and investment bankers and firm managers before the public release of the information. If these insiders trade on the private information, we should see a pattern close to the one illustrated in Figure 8. Prior to the announcement of the merger a price run-up occurs, since insiders have an incentive to take advantage of the private information.

Figure 8: Stock price reaction to news announcement



Although there is ample empirical evidence in support of the efficient market hypotheses, several anomalies have been discovered. These anomalies seem to contradict the efficient market hypothesis.

7.1.4 Classical stock market anomalies

January-effect
Small poor-performing smallcap stocks have historically tended to go up in January, whereas strong-performing largecaps have tended to rally in December. The difference in performance of smallcap and largecap stock around January has be coined the January-effect.

New-issue puzzle
Although new stock issues generally tend to be underpriced, the initial capital gain often turns into losses over longer periods of e.g. 5 years.

S&P-Index effect
Stocks generally tend to rise immediately after being added to an index (e.g. S&P 500, where the index effect was originally documented)

Weekend effect
Smallcap stocks have historically tended to rise on Fridays and fall on Mondays, perhaps because sellers are afraid to hold short positions in risky stocks over the weekend, so they buy back and re-initiate.

While the existence of these anomalies is well accepted, the question of whether investors can exploit them to earn superior returns in the future is subject to debate. Investors evaluating anomalies should keep in mind that although they have existed historically, there is no guarantee they will persist in the future. Moreover, there seem to be a tendency that anomalies disappear as soon as the academic papers discovering them get published.

7.2 Behavioural finance

Behavioural finance applies scientific research on cognitive and emotional biases to better understand financial decisions. Cognitive refers to how people think. Thus, behavioural finance emerges from a large psychology literature documenting that people make systematic errors in the way that they think: they are overconfident, they put too much weight on recent experience, etc.

In addition, behavioural finance considers limits to arbitrage. Even though misevaluations of financial assets are common, not all of them can be arbitraged away. In the absence of such limits a rational investor would arbitrage away price inefficiencies, leave prices in a non-equilibrium state for protracted periods of time.

Behavioural finance might help us to understand some of the apparent anomalies. However, critics say it is too easy to use psychological explanations whenever there something we do not understand. Moreover, critics contend that behavioural finance is more a collection of anomalies than a true branch of finance and that these anomalies will eventually be priced out of the market or explained by appealing to market microstructure arguments.

8. Corporate financing and valuation

How corporations choose to finance their investments might have a direct impact on firm value. Firm value is determined by discounting all future cash flows with the weighted average cost of capital, which makes it important to understand whether the weighted average cost of capital can be minimized by selecting an optimal capital structure (i.e. mix of debt and equity financing). To facilitate the discussion consider first the characteristics of debt and equity.

8.1 Debt characteristics

Debt has the unique feature of allowing the borrowers to walk away from their obligation to pay, in exchange for the assets of the company. "Default risk" is the term used to describe the likelihood that a firm will walk away from its obligation, either voluntarily or involuntarily. "Bond ratings" are issued on debt instruments to help investors assess the default risk of a firm.

Debt maturity

- Short-term debt is due in less than one year
- Long-term debt is due in more than one year

Debt can take many forms:

- Bank overdraft
- Commercial papers
- Mortgage loans
- Bank loans
- Subordinated convertible securities
- Leases
- Convertible bond

8.2 Equity characteristics

Ordinary shareholders:

- Are the owners of the business
- Have limited liability
- Hold an equity interest or residual claim on cash flows
- Have voting rights

Preferred shareholders:

- Shares that take priority over ordinary shares in regards to dividends
- Right to specified dividends
- Have characteristics of both debt (fixed dividend) and equity (no final repayment date)
- Have no voting privileges

8.3 Debt policy

The firm's debt policy is the firm's choice of mix of debt and equity financing, which is referred to as the firm's capital structure. The prior section highlighted that this choice is not just a simple choice between to financing sources: debt or equity. There exists several forms of debt (accounts payable, bank debt, commercial paper, corporate bonds, etc.) and two forms of equity (common and preferred), not to mention hybrids. However, for simplicity capital structure theory deals with which combination of the two overall sources of financing that maximizes firm value.

8.3.1 Does the firm's debt policy affect firm value?

The objective of the firm is to maximize shareholder value. A central question regarding the firm's capital structure choice is therefore whether the debt policy changes firm value?

The starting point for any discussion of debt policy is the influential work by Miller and Modigliani (MM), which states the firm's debt policy is irrelevant in perfect capital markets. In a perfect capital market no market imperfections exists, thus, alternative capital structure theories take into account the impact of imperfections such as taxes, cost of bankruptcy and financial distress, transaction costs, asymmetric information and agency problems.

8.3.2 Debt policy in a perfect capital market

The intuition behind Miller and Modigliani's famous proposition I is that in the absence of market imperfections it makes no difference whether the firm borrows or individual shareholders borrow. In that case the market value of a company does not depend on its capital structure.

To assist their argument Miller and Modigliani provides the following example:

Consider two firms, firm U and firm L, that generate the same cash flow

- Firm U is all equity financed (i.e. firm U is unlevered)
- Firm L is financed by a mix of debt and equity (i.e. firm L is levered)

Letting D and E denote debt and equity, respectively, total value V is comprised by

- $V_U = E_U$ for the unlevered Firm U
- $V_L = D_L + E_L$ for the levered Firm L

Then, consider buying 1 percent of either firm U or 1 percent of L. Since Firm U is wholly equity financed the investment of 1% of the value of U would return 1% of the profits. However, as Firm L is financed by a mix of debt and equity, buying 1 percent of Firm L is equivalent to buying 1% of the debt and 1% of the equity. The investment in debt returns 1% of the interest payment, whereas the 1% investment in equity returns 1% of the profits after interest. The investment and returns are summarized in the following table.

	Investment	Return
1% of Firm U	$1\% \cdot V_U$	$1\% \cdot$ Profits
1% of Firm L		
- 1% of debt	$1\% \cdot D_L$	$1\% \cdot$ Interest
- 1% of equity	$1\% \cdot E_L$	$1\% \cdot$ (Profits - Interest)
	$= 1\% (D_L + E_L) = 1\% \cdot V_L$	$= 1\% \cdot$ Profits

Thus, investing 1% in the unlevered Firm U returns 1% of the profits. Similarly investing 1% in the levered firm L also yields 1% of the profits. Since we assumed that the two firms generate the same cash flow it follows that profits are identical, which implies that the value of Firm U must be equal to the value of Firm L. In summary, firm value is independent of the debt policy.

Consider an alternative investment strategy where we consider investing only in 1 percent of L's equity. Alternatively, we could have borrowed 1% of firm L's debt, D_L, in the bank and purchased 1 percent of Firm U.

The investment in 1% of Firm L's equity yields 1% of the profits after interest payment in return. Similarly, borrowing 1% of L's debt requires payment of 1% of the interest, whereas investing in 1% of U yields 1% of the profits.

	Investment	Return
1% of Firm L's equity	$1\% \cdot E_L = 1\% \cdot (V_L - D_L)$	$1\% \cdot$ (Profits - Interest)
Borrow 1% of Firm L's debt and purchase 1% of Firm U		
- Borrow 1% of L's debt	$-1\% \cdot D_L$	$-1\% \cdot$ Interest
- 1% of U's equity	$1\% \cdot E_U = 1\% \cdot V_U$	$1\% \cdot$ Profits
	$= 1\% (V_U - D_L)$	$= 1\% \cdot$ (Profits - Interest)

It follows from the comparison that both investments return 1% of the profits after interest payment. Again, as the profits are assumed to be identical, the value of the two investments must be equal. Setting the value of investing 1% in Firm L's equity equal to the value of borrowing 1% of L's debt and investing in 1% of U's equity, yields that the value of Firm U and L must be equal

- $1\% \cdot (V_L - D_L) = 1\% \cdot (V_U - D_L) \quad \leftrightarrow \quad V_L = V_U$

The insight from the two examples above can be summarized by MM's proposition I:

> **Miller and Modigliani's Proposition I**
>
> In a perfect capital market firm value is independent of the capital structure

MM-theory demonstrates that if capitals markets are doing their job firms cannot increase value by changing their capital structure. In addition, one implication of MM-theory is that expected return on assets is independent of the debt policy.

The expected return on assets is a weighted average of the required rate of return on debt and equity,

$$(42) \qquad r_A = \frac{D}{D+E} r_D + \frac{E}{D+E} r_E$$

Solving for expected return on equity, r_E, yields:

$$(43) \qquad r_E = r_A + (r_A - r_D)\frac{D}{E}$$

This is known as MM's proposition II.

Miller and Modigliani's Proposition II

In a perfect capital market the expected rate of return on equity is increasing in the debt-equity ratio.

$$r_E = r_A + (r_A - r_D)\frac{D}{E}$$

At first glance MM's proposition II seems to be inconsistent with MM's proposition I, which states that financial leverage has no effect on shareholder value. However, MM's proposition II is fully consistent with their proposition I as any increase in expected return is exactly offset by an increase in financial risk borne by shareholders.

The financial risk is increasing in the debt-equity ratio, as the percentage spreads in returns to shareholders are amplified: If operating income falls the percentage decline in the return is larger for levered equity since the interest payment is a fixed cost the firm has to pay independent of the operating income.

Finally, notice that even though the expected return on equity is increasing with the financial leverage, the expected return on assets remains constant in a perfect capital market. Intuitively, this occurs because when the debt-equity ratio increases the relatively expensive equity is being swapped with the cheaper debt. Mathematically, the two effects (increasing expected return on equity and the substitution of equity with debt) exactly offset each other.

8.4 How capital structure affects the beta measure of risk

Beta on assets is just a weighted-average of the debt and equity beta:

(44) $$\beta_A = \left(\beta_D \cdot \frac{D}{V}\right) + \left(\beta_E \cdot \frac{E}{V}\right)$$

Similarly, MM's proposition II can be expressed in terms of beta, since increasing the debt-equity ratio will increase the financial risk, beta on equity will be increasing in the debt-equity ratio.

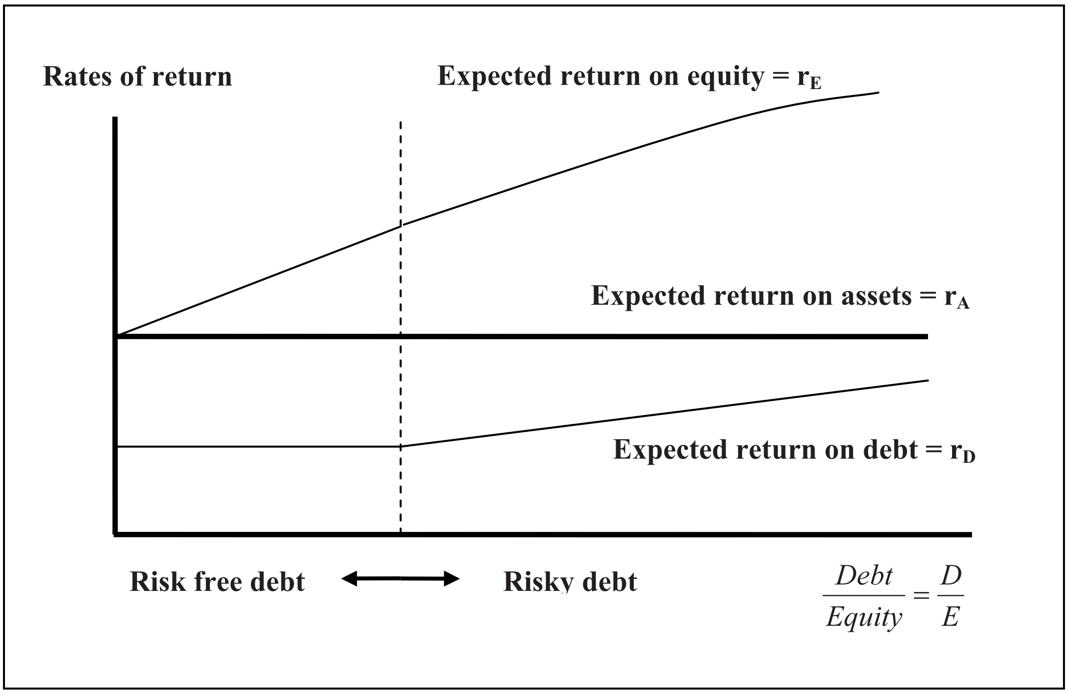
(45) $$\beta_E = \beta_A + (\beta_A - \beta_D)\frac{D}{E}$$

Again, notice MM's proposition I translates into no effect on the beta on assets of increasing the financial leverage. The higher beta on equity is exactly being offset by the substitution effect as we swap equity with debt and debt has lower beta than equity.

8.5 How capital structure affects company cost of capital

The impact of the MM-theory on company cost of capital can be illustrated graphically. Figure 9 assumes that debt is essentially risk free at low levels of debt, whereas it becomes risky as the financial leverage increases. The expected return on debt is therefore horizontal until the debt is no longer risk free and then increases linearly with the debt-equity ratio. MM's proposition II predicts that when this occur the rate of increase in, r_E, will slow down. Intuitively, as the firm has more debt, the less sensitive shareholders are to further borrowing.

Figure 9, Cost of capital: Miller and Modigliani Proposition I and II



The expected return on equity, r_E, increases linearly with the debt-equity ratio until the debt no longer is risk free. As leverage increases the risk of debt, debt holders demand a higher return on debt, this causes the rate of increase in r_E to slow down.

8.6 Capital structure theory when markets are imperfect

MM-theory conjectures that in a perfect capital market debt policy is irrelevant. In a perfect capital market no market imperfections exists. However, in the real world corporations are taxed, firms can go bankrupt and managers might be self-interested. The question then becomes what happens to the optimal debt policy when the market imperfections are taken into account. Alternative capital structure theories therefore address the impact of imperfections such as taxes, cost of bankruptcy and financial distress, transaction costs, asymmetric information and agency problems.

8.7 Introducing corporate taxes and cost of financial distress

When corporate income is taxed, debt financing has one important advantage: Interest payments are tax deductible. The value of this tax shield is equal to the interest payment times the corporate tax rate, since firms effectively will pay (1-corporate tax rate) per dollar of interest payment.

$$(46) \qquad \text{PV(Tax shield)} = \frac{\text{interest payment} \cdot \text{corporate tax rate}}{\text{expeced return on debt}} = \frac{r_D D \cdot T_C}{\text{r}_\text{D}} = D \cdot T_C$$

Where T_C is the corporate tax rate.

After introducing taxes MM's proposition I should be revised to include the benefit of the tax shield:

Value of firm = Value if all-equity financed + PV(tax shield)

In addition, consider the effect of introducing the cost of financial distress. Financial distress occurs when shareholders exercise their right to default and walk away from the debt. Bankruptcy is the legal mechanism that allows creditors to take control over the assets when a firm defaults. Thus, bankruptcy costs are the cost associated with the bankruptcy procedure.

The corporate finance literature generally distinguishes between direct and indirect bankruptcy costs:

– Direct bankruptcy costs are the legal and administrative costs of the bankruptcy procedure such as
    • Legal expenses (lawyers and court fees)
    • Advisory fees
– Indirect bankruptcy costs are associated with how the business changes as the firm enters the bankruptcy procedure. Examples of indirect bankruptcy costs are:

    • Debt overhang as a bankruptcy procedure might force the firm to pass up valuable investment projects due to limited access to external financing.
    • Scaring off costumers. A prominent example of how bankruptcy can scare off customers is the Enron scandal. Part of Enron's business was to sell gas futures (i.e. a contract that for a payment today promises to deliver gas next year). However, who wants to buy a gas future from a company that might not be around tomorrow? Consequently, all of Enron's futures business disappeared immediately when Enron went bankrupt.
    • Agency costs of financial distress as managers might be tempted to take excessive risk to recover from bankruptcy. Moreover, there is a general agency problem between debt and shareholders in bankruptcy, since shareholders are the residual claimants.

Moreover, cost of financial distress varies with the type of the asset, as some assets are transferable whereas others are non-transferable. For instance, the value of a real estate company can easily be auctioned off, whereas it is significantly more involved to transfer the value of a biotech company where value is related to human capital.

The cost of financial distress will increase with financial leverage as the expected cost of financial distress is the probability of financial distress times the actual cost of financial distress. As more debt will increase the likelihood of bankrupt, it follows that the expected cost of financial distress will be increasing in the debt ratio.

In summary, introducing corporate taxes and cost of financial distress provides a benefit and a cost of financial leverage. The trade-off theory conjectures that the optimal capital structure is a trade-off between interest tax shields and cost of financial distress.

8.8 The Trade-off theory of capital structure

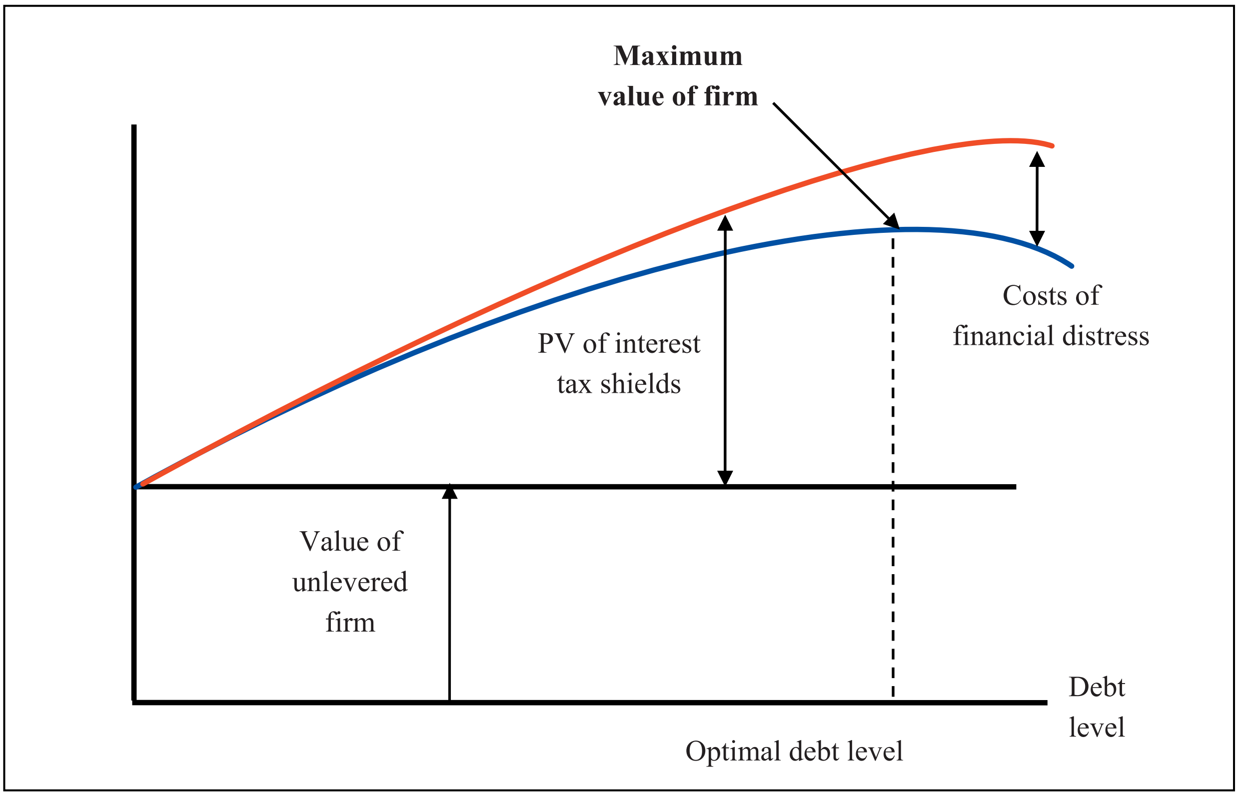
The trade-off theory states that the optimal capital structure is a trade-off between interest tax shields and cost of financial distress:.

(47) Value of firm = Value if all-equity financed + PV(tax shield) - PV(cost of financial distress)

The trade-off theory can be summarized graphically. The starting point is the value of the all-equity financed firm illustrated by the black horizontal line in Figure 10. The present value of tax shields is then added to form the red line. Note that PV(tax shield) initially increases as the firm borrows more, until additional borrowing increases the probability of financial distress rapidly. In addition, the firm cannot be sure to benefit from the full tax shield if it borrows excessively as it takes positive earnings to save corporate taxes. Cost of financial distress is assumed to increase with the debt level.

The cost of financial distress is illustrated in the diagram as the difference between the red and blue curve. Thus, the blue curve shows firm value as a function of the debt level. Moreover, as the graph suggest an optimal debt policy exists which maximized firm value.

Figure 10, Trade-off theory of capital structure



In summary, the trade-off theory states that capital structure is based on a trade-off between tax savings and distress costs of debt. Firms with safe, tangible assets and plenty of taxable income to shield should have high target debt ratios. The theory is capable of explaining why capital structures differ between industries, whereas it cannot explain why profitable companies within the industry have lower debt ratios (trade-off theory predicts the opposite as profitable firms have a larger scope for tax shields and therefore subsequently should have higher debt levels).

8.9 The pecking order theory of capital structure

The pecking order theory has emerged as alternative theory to the trade-off theory. Rather than introducing corporate taxes and financial distress into the MM framework, the key assumption of the pecking order theory is asymmetric information. Asymmetric information captures that managers know more than investors and their actions therefore provides a signal to investors about the prospects of the firm.

The intuition behind the pecking order theory is derived from considering the following string of arguments:

- If the firm announces a stock issue it will drive down the stock price because investors believe managers are more likely to issue when shares are overpriced.
- Therefore firms prefer to issue debt as this will allow the firm to raise funds without sending adverse signals to the stock market. Moreover, even debt issues might create information problems if the probability of default is significant, since a pessimistic manager will issue debt just before bad news get out.

This leads to the following ***pecking order*** in the financing decision:

1. Internal cash flow
2. Issue debt
3. Issue equity

The pecking order theory states that internal financing is preferred over external financing, and if external finance is required, firms should issue debt first and equity as a last resort. Moreover, the pecking order seems to explain why profitable firms have low debt ratios: This happens not because they have low target debt ratios, but because they do not need to obtain external financing. Thus, unlike the trade-off theory the pecking order theory is capable of explaining differences in capital structures within industries.

8.10 A final word on Weighted Average Cost of Capital

All variables in the weighted average cost of capital (WACC) formula refer to the firm as a whole.

$$(48) \qquad WACC = r_D(1-Tc)\left(\frac{D}{V}\right) + r_E\left(\frac{E}{V}\right)$$

Where T_C is the corporate tax rate.

The after-tax WACC can be used as the discount rate if

1. The project has the same business risk as the average project of the firm
2. The project is financed with the same amount of debt and equity

If condition 1 is violated the right discount factor is the required rate of return on an equivalently risky investment, whereas if condition 2 is violated the WACC should be adjusted to the right financing mix. This adjustment can be carried out in three steps:

- Step 1: Calculate the opportunity cost of capital
    - Calculate the opportunity cost of capital without corporate taxation.
    - $r = \frac{D}{V} r_D + \frac{E}{V} r_E$
- Step 2: Estimate the cost of debt, r_D, and cost of equity, r_E, at the new debt level
    - $r_E = r + (r - r_D)\frac{D}{E}$
- Step 3: Recalculate WACC
    - "Relever the WACC" by estimating the WACC with the new financing weights

Example:

- Consider a firm with a debt and equity ratio of 40% and 60%, respectively. The required rate of return on debt and equity is 7% and 12.5%, respectively. Assuming a 30% corporate tax rate the after-tax WACC of the firm is:
    - $$WACC = r_D(1-Tc)\left(\frac{D}{V}\right) + r_E\left(\frac{E}{V}\right) = 7\% \cdot (1-0.3) \cdot 0.4 + 12.5\% \cdot 0.6 = 9.46\%$$

- The firm is considering investing in a new project with a perpetual stream of cash flows of \$11.83 million per year pre-tax. The project has the same risk as the average project of the firm.
- Given an initial investment of \$125 million, which is financed with 20% debt, what is the value of the project?
- The first insight is that although the business risk is identical, the project is financed with lower financial leverage. Thus, the WACC cannot be used as the discount rate for the project. Rather, the WACC should be adjusted using the three step procedure.
- Step 1: Estimate opportunity cost of capital, i.e. estimate r using a 40% debt ratio, 60% equity ration as well as the firm's cost of debt and equity
    - $$r = \frac{D}{V}r_D + \frac{E}{V}r_E = 0.4 \cdot 7\% + 0.6 \cdot 12.5\% = 10.3\%$$

- Step 2: Estimate the expected rate of return on equity using the project's debt-equity ratio. As the debt ratio is equal to 20%, the debt-equity ratio equals 25%.
    - $$r_E = r + (r - r_D)\frac{D}{E} = 10.3\% + (10.3\% - 7\%) \cdot 0.25 = 11.1\%$$

- Step 3: Estimate the project's WACC
    - $$WACC = r_D(1-Tc)\left(\frac{D}{V}\right) + r_E\left(\frac{E}{V}\right) = 7\% \cdot (1-0.3) \cdot 0.2 + 11.1\% \cdot 0.8 = 9.86\%$$

- The adjusted WACC of 9.86% can be used as the discount rate for the new project as it reflects the underlying business risk and mix of financing. As the project requires an initial investment of \$125 million and produced a constant cash flow of \$11.83 per year for ever, the projects NPV is:
    - $$NPV = -125 + \frac{11.83}{0.0986} = \text{-\$5.02 million}$$

- In comparison the NPV is equal to \$5.03 if the company WACC is used as the discount rate. In this case we would have invested in a negative NPV project if we ignored that the project was financed with a different mix of debt and equity.

8.11 Dividend policy

Dividend policy refers to the firm's decision whether to plough back earnings as retained earnings or payout earnings to shareholders. Moreover, in case the latter is preferred the firm has to decide how to payback the shareholders: As dividends or capital gains through stock repurchase.

Dividend policy in practice

Earnings can be returned to shareholders in the form of either dividends or capital gain through stock repurchases. For each of the two redistribution channels there exists several methods:

Dividends can take the form of
- Regular cash dividend
- Special cash dividend

Stock repurchase can take the form of
- Buy shares directly in the market
- Make a tender offer to shareholders
- Buy shares using a declining price auction (i.e. Dutch auction)
- Through private negotiation with a group of shareholders

8.11.1 Dividend payments in practise

The most common type of dividend is a regular cash dividend, where "regular" refers to expectation that the dividend is paid out in regular course of business. Regular dividends are paid out on a yearly or quarterly basis. A special dividend is a one-time payment that most likely will not be repeated in the future.

When the firm announces the dividend payment it specifies a date of payment at which they are distributed to shareholders. The announcement date is referred to as the declaration date. To make sure that the dividends are received by the right people the firm establishes an ex-dividend date that determines which shareholders are entitled to the dividend payment. Before this date the stock trades with dividend, whereas after the date it trades without. As dividends are valuable to investors, the stock price will decline around the ex-dividend date.

8.11.2 Stock repurchases in practise

Repurchasing stock is an alternative to paying out dividends. In a stock repurchase the firm pays cash to repurchase shares from its shareholders with the purpose of either keeping them in the treasury or reducing the number of outstanding shares.

Over the last two decades stock repurchase programmes have increased sharply: Today the total value exceeds the value of dividend payments. Stock repurchases compliment dividend payments as most companies with a stock repurchase programme also pay dividends. However, stock repurchase programmes are temporary and do therefore (unlike dividends) not serve as a long-term commitment to distribute excess cash to shareholders.

In the absence of taxation, shareholders are indifferent between dividend payments and stock repurchases. However, if dividend income is taxed at a higher rate than capital gains it provides a incentive for stock repurchase programmes as it will maximize the shareholder's after-tax return. In fact, the large surge in the use of stock repurchase around the world can be explained by higher taxation of dividends. More recently, several countries, including the United States, have reformed the tax system such that dividend income and capital gains are taxed at the same rate.

8.11.3 How companies decide on the dividend policy

In the 1950'ties the economist John Lintner surveyed how corporate managers decide the firm's dividend policy. The outcome of the survey can be summarized in five stylized facts that seem to hold even today.

Lintner's "Stylized Facts": How dividends are determined

1. Firms have longer term target dividend payout ratios
2. Managers focus more on dividend changes than on absolute levels
3. Dividends changes follow shifts in long-run, sustainable levels of earnings rather than short-run changes in earnings
4. Managers are reluctant to make dividend changes that might have to be reversed
5. Firms repurchase stocks when they have accumulated a large amount of unwanted cash or wish to change their capital structure by replacing equity with debt.

8.11.4 Does the firm's dividend policy affect firm value?

The objective of the firm is to maximize shareholder value. A central question regarding the firm's dividend policy is therefore whether the dividend policy changes firm value?

As the dividend policy is the trade-off between retained earnings and paying out cash, there exist three opposing views on its effect on firm value:

1. Dividend policy is irrelevant in a competitive market
2. High dividends increase value
3. Low dividends increase value

The first view is represented by the Miller and Modigliani dividend-irrelevance proposition.

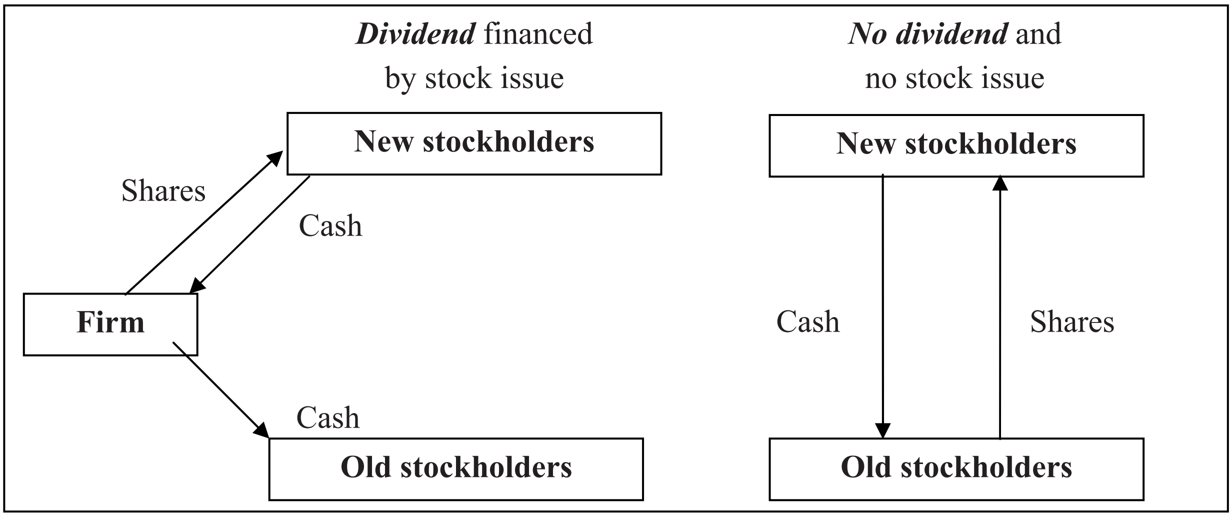
Miller and Modigliani Dividend-Irrelevance Proposition

In a perfect capital market the dividend policy is irrelevant.
Assumptions
- No market imperfections
  - No taxes
  - No transaction costs

The essence of the Miller and Modigliani (MM) argument is that investor do not need dividends to covert their shares into cash. Thus, as the effect of the dividend payment can be replicated by selling shares, investors will not pay higher prices for firms with higher dividend payouts.

To understand the intuition behind the MM-argument, suppose that the firm has settled its investment programme. Thus, any surplus from the financing decision will be paid out as dividend. As case in point, consider what happens to firm value if we decide to increase the dividends without changing the debt level. In this case the extra dividends must be financed by equity issue. New shareholders contribute with cash in exchange for the issued shares and the generated cash is subsequently paid out as dividends. However, as this is equivalent to letting the new shareholders buy existing shares (where cash is exchanged as payment for the shares), there is not effect on firm value. **Figure 11** illustrates the argument:

Figure 11: Illustration of Miller and Modigliani's dividend irrelevance proposition



The left part of **Figure 11** illustrates the case where the firm finances the dividend with the new equity issue and where new shareholders buy the new shares for cash, whereas the right part illustrates the case where new shareholders buy shares from existing shareholders. As the net effect for both new and existing shareholders are identical in the two cases, firm value must be equal. Thus, in a world with a perfect capital market dividend policy is irrelevant.

8.11.5 Why dividend policy may increase firm value

The second view on the effect of the dividend policy on firm value argues that high dividends will increase firm value. The main argument is that there exists natural clienteles for dividend paying stocks, since many investors invest in stocks to maintain a steady source of cash. If paying out dividends is cheaper than letting investors realise the cash by selling stocks, then the natural clientele would be willing to pay a premium for the stock. Transaction costs might be one reason why its comparatively cheaper to payout dividends. However, it does not follow that any particular firm can benefit by increasing its dividends. The high dividend clientele already have plenty of high dividend stock to choose from.

8.11.6 Why dividend policy may decrease firm value

The third view on dividend policy states that low dividends will increase value. The main argument is that dividend income is often taxed, which is something MM-theory ignores. Companies can convert dividends into capital gains by shifting their dividend policies. Moreover, if dividends are taxed more heavily than capital gains, taxpaying investors should welcome such a move. As a result firm value will increase, since total cash flow retained by the firm and/or held by shareholders will be higher than if dividends are paid. Thus, if capital gains are taxed at a lower rate than dividend income, companies should pay the lowest dividend possible.

9. Options

An option is a contractual agreement that gives the buyer the right but not the obligation to buy or sell a financial asset on or before a specified date. However, the seller of the option is obliged to follow the buyer's decision.

Call option
Right to buy an financial asset at a specified exercise price (strike price) on or before the exercise date

Put option
Right to sell an financial asset at a specified exercise price on or before the exercise date

Exercise price (Striking price)
The price at which you buy or sell the security

Expiration date
The last date on which the option can be exercised

The rights and obligations of the buyer and seller of call and put options are summarized below.

	Buyer	Seller
Call option	Right to buy asset	Obligation to sell asset if option is exercised
Put option	Right to sell asset	Obligation to buy asset if option is exercised

The decision to buy a call option is referred to as taking a long position, whereas the decision to sell a call option is a short position.

If the exercise price of a option is equal to the current price on the asset the option is said to be at the money. A call (put) option is in the money when the current price on the asset is above (below) the exercise price. Similarly, a call (put) option is out of the money if the current price is below (above) the exercise price.

With respect to the right to exercise the option there exist two general types of options:

- American call which can be exercised on or before the exercise date
- European call which can only be exercised at the exercise date

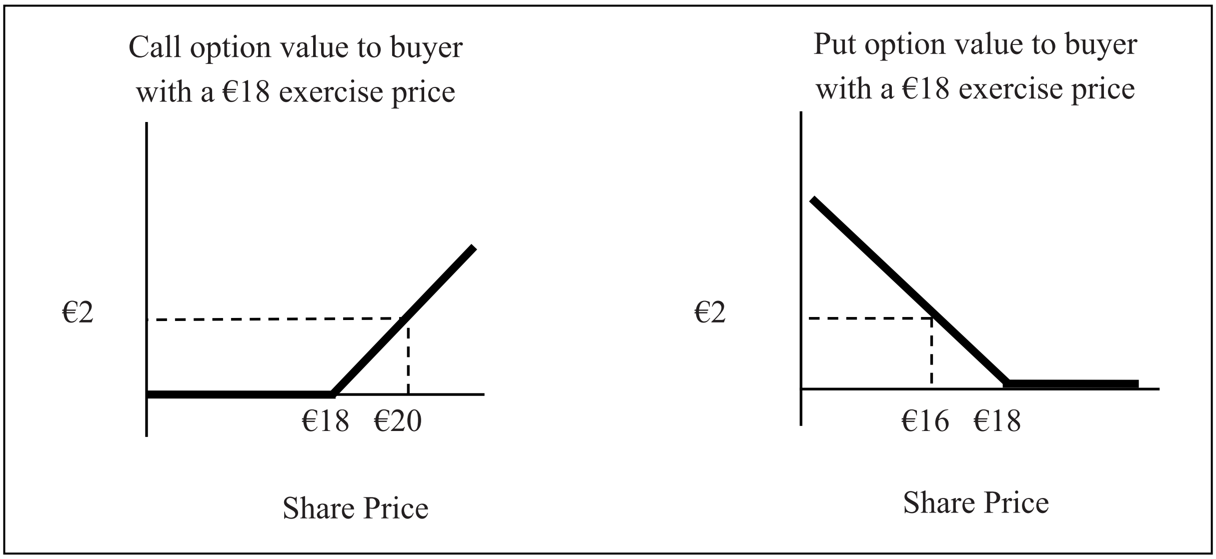
9.1 Option value

The value of an option at expiration is a function of the stock price and the exercise price. To see this consider the option value to the buyer of a call and put option with an exercise price of €18 on the Nokia stock.

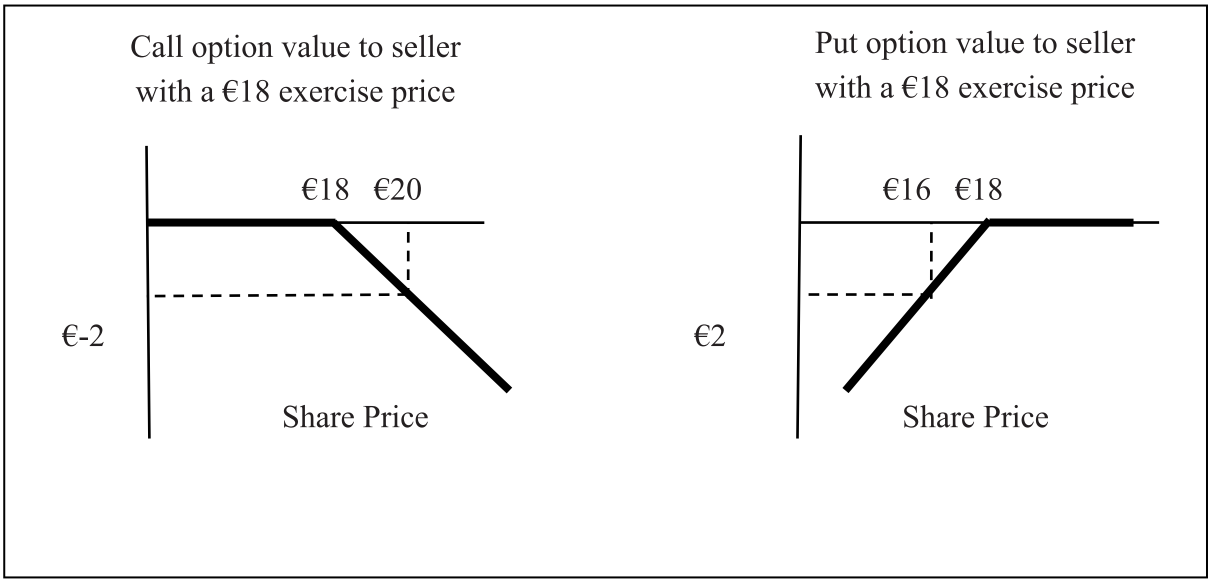
Stock price	€15	€16	€17	€18	€19	€20	€21
Call value	0	0	0	0	1	2	3
Put value	3	2	1	0	0	0	0

If the stock price is 18, both the call and the put option are worth 0 as the exercise price is equal to the market value of the Nokia stock. When the stock price raises above €18 the buyer of the call option will exercise the option and gain the difference between the stock price and the exercise price. Thus, the value of the call option is €1, €2, and €3 if the stock price rises to €19, €20, and €21, respectively. When the stock price is lower than the exercise price the buyer will not exercise and, hence, the value is equal to 0. Vice versa with the put option.

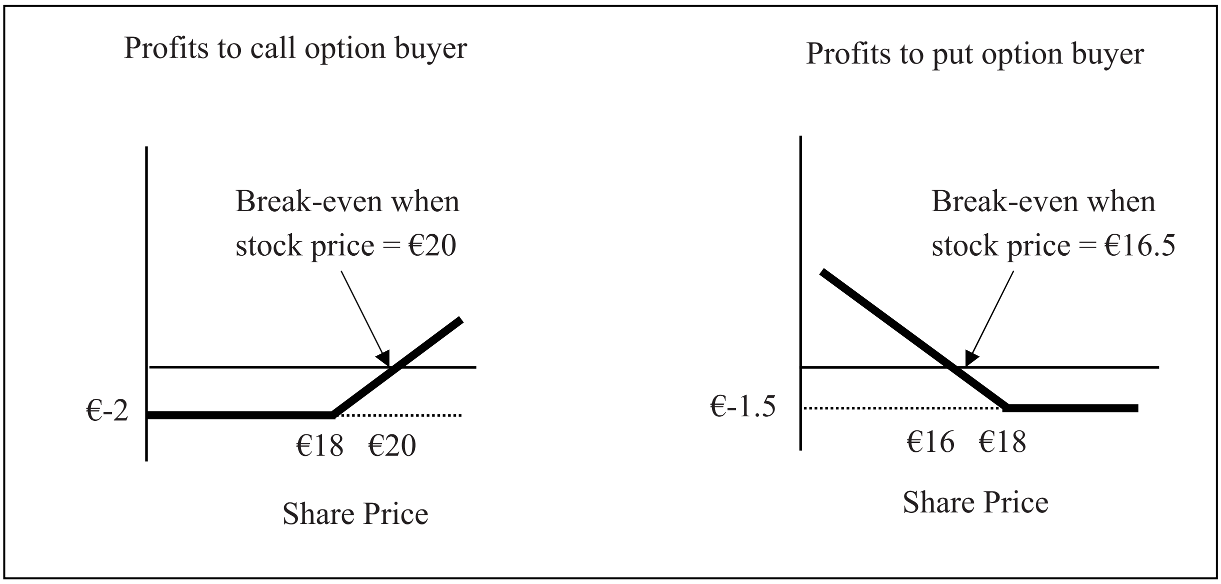
The value to the buyer of a call and a put options can be graphically illustrated in a position diagram:



As the seller of a call and a put option takes the opposite position of the buyer, the value of a call and put option can be illustrated as:



The total payoff of a option is the sum of the initial price and the value of the option when exercised. The following diagram illustrates the profits to buying a call option with an exercise price of €18 priced at €2 and a put option with an exercise price of €18 priced at €1.5.



Note that although the profits to the call option buyer is negative when the difference between the share price and exercise price is between 0 and €2 it is still optimal to exercise the option as the value of the option is positive. The same holds for the buyer of the put option: its optimal to exercise the put whenever the share price is below the exercise price.

9.2 What determines option value?

The following table summarizes the effect on the expected value of call and put option of an increase in the underlying stock price, exercise price, volatility of the stock price, time to maturity and discount rate.

The impact on the … option price of an increase in…	Call	Put
1. Underlying stock price (P)	Positive	Negative
2. Exercise price (EX)	Negative	Positive
3. Volatility of the stock price (σ)	Positive	Positive
4. Time to option expiration (t)	Positive	Positive
5. Discount rate (r)	Positive	Negative

1. Underlying stock price

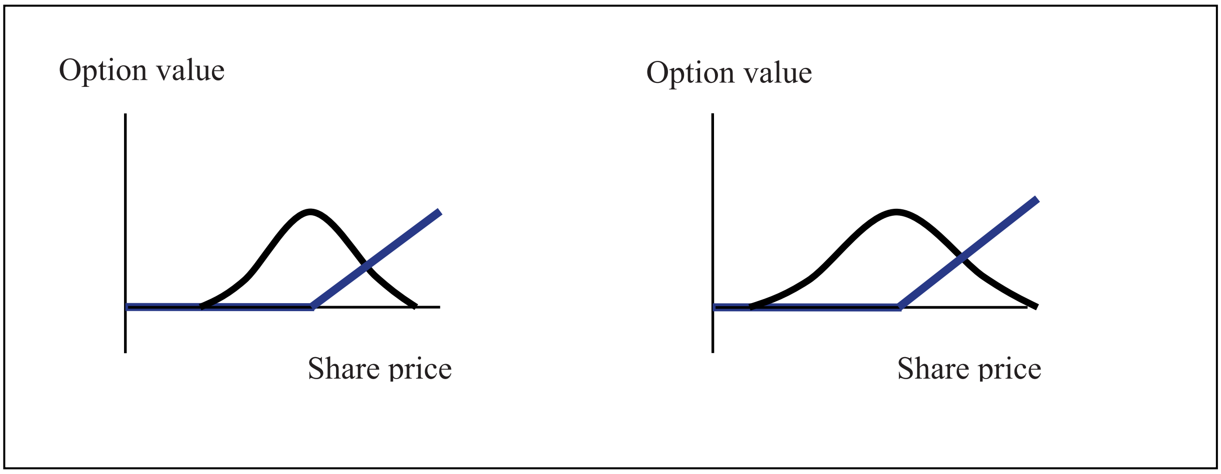
The effect on the option price of an increase in the underlying stock price follows intuitively from the position diagram. If the underlying stock price increases the value of the call (put) option for a given exercise price increases (decreases).

2. Exercise price

This follows directly from the position diagram as the value of the call (put) option is the difference between the underlying stock price and the exercise price (the exercise price and underlying stock price). For a given underlying stock price the value of the call decreases (put increases) when the exercise price increases

3. Volatility of the underlying stock price

Consider call options on two stocks. The only difference between the two call options is the volatility in the underlying stock price: One stock has low stock price volatility, whereas the other has high. This difference is illustrated in the position diagrams where the bell-shaped line depicts the probability distribution of future stock prices.



For both stocks there is a 50% probability that the stock price exceeds the exercise price, which implies that the option value is positive. However, for the option to the right the probability of observing large positive option values is significantly higher compared to the option to the left. Thus, it follows that the expected option value is increasing in the underlying stock price volatility.

4. Time to option expiration

If volatility in the underlying stock price is positively related to option value and volatility, σ^2, is measured per period, it follows that the cumulative volatility over t sub periods is $t \cdot \sigma^2$. Thus, option value is positively related to the time to expiration.

5. Discount rate

If the discount rate increases the present value of the exercise price decreases. Everything else equal, the option value increases when the present value of the exercise price decreases.

9.3 Option pricing

As with all financial assets the price of an option should equal the expected value of the option. However, unlike other financial assets it is impossible to figure out expected cash flows and discount them using the opportunity cost of capital as discount rate. In particular the latter is impossible, as the risk of an option changes every time the underlying stock price moves.

Black and Scholes solved this problem by introducing a simple option valuation model, which applies the principle of value additivity to create an ***option equivalent***. The option equivalent is combining stocks and borrowing, such that they yield the same payoff as the option. As the value of stocks and borrowing arrangements is easily assessed and they yield the same payoff as the option, the price of the option must equal the combined price on the stock and borrowing arrangement.

Example:

- How to set up an ***option equivalent***
- Consider a 3-month Google call option issued at the money with an exercise price of $400.
- For simplicity, assume that the stock can either fall to $300 or rise to $500.

- Consider the payoff to the option given the two possible outcomes:
  - Stock price = $300 → Payoff = $0
  - Stock price = $500 → Payoff = $500 – $400 = $100

- Compare this to the alternative: Buy 0.5 stock & borrow $150
  - Stock price = $300 → Payoff = 0.5 · $300-$150 = $0
  - Stock price = $500 → Payoff = 0.5 · $500-$150 = $100

- As the payoff to the option equals the payoff to the alternative of buying 0.5 stock and borrowing $150 (i.e. the option equivalent), the price must be identical. Thus, the value of the option is equal to the value of 0.5 stocks minus the present value of the $150 bank loan.

- If the 3-month interest rate is 1%, the value of the call option on the Google stock is:
  - Value of call = Value of 0.5 shares – PV(Loan)
    = 0.5 · $400–$150/1.01= 51.5

The option equivalent approach uses a hedge ratio or option delta to construct a replicating portfolio, which can be priced. The option delta is defined as the spread in option value over the spread in stock prices:

$$\text{Option delta} = \frac{\text{spread in option value}}{\text{spread in stock price}} \tag{49}$$

Example:

- In the prior example with the 3-month option on the Google stock the option delta is equal to:

$$\text{Option delta} = \frac{\text{spread in option value}}{\text{spread in stock price}} = \frac{[100-0]}{[500-300]} = 0.5$$

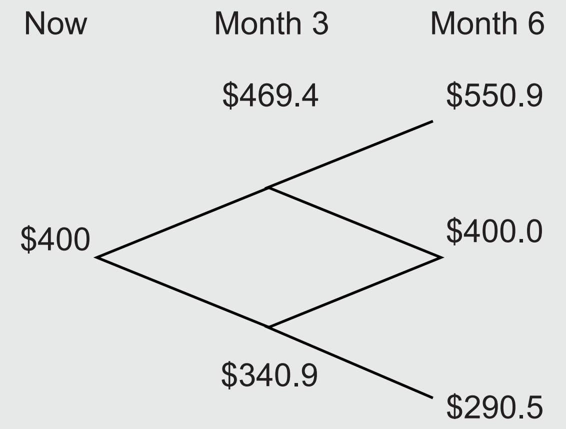
- Thus, the options equivalent buys 0.5 shares in Google and borrow $150 to replicate the payoffs from the option on the Google stock.

9.3.1 Binominal method of option pricing

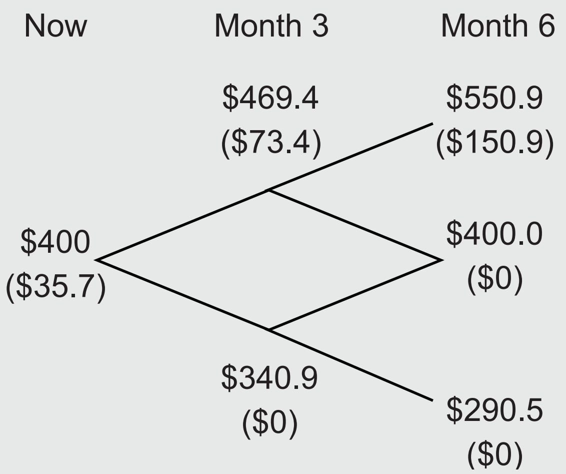
The binominal model of option pricing is a simple way to illustrate the above insights. The model assumes that in each period the stock price can either go up or down. By increasing the number of periods in the model the number of possible stock prices increases.

Example:

- Two-period binominal method for a 6-month Google call-option with a exercise price of $400 issued at the money.



- In the first 3-month period the stock price of Google can either increase to $469.4 or decrease to $340.9. In the second 3-month period the stock price can again either increase or decrease. If the stock price increased in the first period, then the stock price in period two will either be $550.9 or $400. Moreover, if the stock price decreased in the first period it can either increase to $400 or decrease to $290.5.
- To find the value of the Google call-option, start in month 6 and work backwards to the present. Number in parenthesis reflects the value of the option.



- In Month 6 the value of the option is equal to Max[0, Stock price - exercise price]. Thus, when the stock price is equal to $550.9 the option is worth $150.9 (i.e. $550.9 - $400) when exercised. When the stock price is equal to $400 the value of the option is 0, whereas if the stock price falls below the exercise price the option is not exercised and, hence, the value is equal to zero.

- In Month 3 suppose that the stock price is equal to \$469.4. In this case, investors would know that the future stock price in Month 6 will be \$550.9 or \$400 and the corresponding option prices are \$150.9 and \$0, respectively. To find the option value, simply set up the option equivalent by calculating the option delta, which is equal to the spread of possible option prices over the spread of possible stock prices. In this case the option delta equals 1 as (\$150.9-\$0)/(\$550.9-\$400) = 1. Given the option delta find the amount of bank loan needed:

	Month 6 stock price equal to	
	\$400	\$550.9
Buy 1 share	\$400.0	\$550.9
Borrow PV(X)	-\$400.0	-\$400.0
Total payoff	\$0.0	\$150.9

- Since the above portfolio has identical cash flows to the option, the price on the option is equal to the sum of market values.
    - Value of Google call option in month 3 = \$469.4 - \$400/1.01 = \$73.4
- If the stock price in Month 3 has fallen to \$340.9 the option will not be exercised and the value of the option is equal to \$0.
- Option value today is given by setting up the option equivalent (again). Thus, first calculate the option equivalent. In this case the option delta equals 0.57 as (\$73.4-\$0)/(\$469.4-\$340.9) = 0.57.

	Month 3 stock price equal to	
	\$340.9	\$469.4
Buy 0.57 share	\$194.7	\$268.1
Borrow PV(X)	-\$194.7	-\$194.7
Total payoff	\$0.0	\$73.4

- As today's value of the option is the equal to the present value of the option equivalent, the option price = \$400 · 0.57 - \$194.7/1.01 =\$35.7.

To construct the binominal three, the binominal method of option prices relates the future value of the stock to the standard deviation of stock returns, σ, and the length of period, h, measured in years:

(50)
$$1 + \text{upside change} = \mathrm{u} = \mathrm{e}^{\sigma\sqrt{h}}$$
$$1 + \text{upside change} = \mathrm{d} = 1/\mathrm{u}$$

In the prior example the upside and downside change to the Google stock price was +17.35% (469.4/400 - 1= 0.1735) and -14.78% (340.9/400 - 1 = -0.1478), respectively. The percentage upside and downside change is determined by the standard deviation on return to the Google stock, which is equal to 32%. Since each period is 3 month (i.e. 0.25 year) the changes must equal:

$$1 + \text{upside change} = \text{u} = \text{e}^{\sigma\sqrt{h}} = \text{e}^{0.32\sqrt{0.35}} = 1.1735$$
$$1 + \text{upside change} = \text{d} = 1/\text{u} = 1/1.1735 = 0.8522$$

Multiplying the current stock price, $400, with the upside and downside change yields the stock prices of $469.4 and $340.9 in Month 3. Similarly, the stock prices in Month 6 is the current stock price conditional on whether the stock price increased or decreased in the first period.

9.3.2 Black-Scholes' Model of option pricing

The starting point of the Black-Scholes model of option pricing is the insight from the binominal model: If the option's life is subdivided into an infinite number of sub-periods by making the time intervals shorter, the binominal three would include a continuum of possible stock prices at maturity.

The Black-Scholes formula calculates the option value for an infinite number of sub-periods.

> **Black-Scholes Formula for Option Pricing**
>
> (51) Value of call option = [delta · share price] – [bank loan]
>
> = [N(d1) · P] – [N(d2) · PV(EX)]
>
> where
>
> - **N(d1) =** Cumulative normal density function of (d1)
> - $d_1 = \frac{\log[P/PV(EX)]}{\sigma\sqrt{t}} + \frac{\sigma\sqrt{t}}{2}$
> - **P =** Stock Price
> - **N(d2) =** Cumulative normal density function of (d2)
> - $d_2 = d_1 - \sigma\sqrt{t}$
> - **PV(EX) =** Present Value of Strike or Exercise price = EX · e^{-rt}

The Black-Scholes formula has four important assumptions:

- Price of underlying asset follows a lognormal random walk
- Investors can hedge continuously and without costs
- Risk free rate is known
- Underlying asset does not pay dividend

> **Example**
>
> - Use Black-Scholes' formula to value the 6-month Google call-option
> - Current stock price (P) is equal to 400
> - Exercise price (EX) is equal to 400
> - Standard deviation (σ) on the Google stock is 0.32
> - Time to maturity (t) is 0.5 (measured in years, hence 6 months = 0.5 years)
> - 6-month interest rate is 2 percent
>
> - Find option value in five steps:
>   - Step 1: Calculate the present value of the exercise price
>   - PV(EX) = EX · e^{-rt} = 400 · $e^{-0.04 \cdot 0.5}$ = 392.08
>   - Step 2: Calculate d_1:
>   - $d_1 = \frac{\log[P/PV(EX)]}{\sigma\sqrt{t}} + \frac{\sigma\sqrt{t}}{2} = \frac{\log[400/392.08]}{0.32\sqrt{0.5}} + \frac{0.32\sqrt{0.5}}{2} = 0.2015$
>   - Step 3: Calculate d_2:
>   - $d_2 = d_1 - \sigma\sqrt{t} = 0.2015 - 0.32\sqrt{0.5} = -0.025$

- Step 4: Find $N(d_1)$ and $N(d_2)$:
  - N(X) is the probability that a normally distributed variable is less than X. The function is available in Excel (the Normdist function) as well as on most financial calculators.
  - $N(d_1) = N(0.2015) = 0.5799$
  - $N(d_2) = N(-0.025) = 0.4901$
- Step 5: Plug into the Black-Scholes formula
  - Option value = [delta · share price] – [bank loan]

$$= [N(d1) \cdot P] - [N(d2) \cdot PV(EX)]$$
$$= [0.5799 \cdot 400] - [0.4901 \cdot 392.08]$$
$$= 39.8$$

- Thus, the value of the 6-month call on the Google stock is equal to $39.8

10. Real options

In many investment projects the firm faces one or more options to make strategic changes during its lifetime. A classical example is mining firm's option to suspend extraction of natural resources if the price falls below the extraction costs. Such strategic options are known as real options, and, can significantly increase the value of a project by eliminating unfavourable outcomes.

Generally there exist four types of “real options”:

1. The opportunity to expand and make follow-up investments
2. The opportunity to “wait” and invest later
3. The opportunity to shrink or abandon a project
4. The opportunity to vary the mix of the firm’s output or production methods

10.1 Expansion option

The option to expand is often imbedded in investment projects. The value of follow-on investments can be significant and in some case even trigger the project to have positive NPV.

Examples of options to expand:

- Provide extra land and space for a second production line when designing a production facility.
- A pharmaceutical company acquiring a patent that gives the right, but not the obligation to market a new drug.
- Building 6-lane bridges when building a 4-lane highway.

10.2 Timing option

An investment opportunity with positive NPV does not mean that we should go ahead today. In particular if we can delay the investment decision we have an option to wait. The optimal timing is a trade-off between cash flows today and cash flows in the future.

Examples of timing options:

- The decision when to harvest a forest

10.3 Abandonment option

Traditional capital budgeting assumes that a project will operate in each year during its lifetime. However, in reality firms may have the option to cease a project during its life. An option to abandon a project is valuable: If bad news arrives you will exercise the option to abandon the project if the value recovered from the project's assets is greater than the present value of continuing the project. Abandonment options can usually be evaluated using the binominal method.

Examples of abandonment options:

- Airlines routinely close routes where the demand is insufficient to make the connection profitable.
- Natural resource companies

10.4 Flexible production option

Firms often have an option to vary inputs to the production or change the output from production. Such options are known as flexible production options. Flexible production options are in particular valuable within industries where the lead time (time between an order and delivery) can extend for years.

Examples of flexible production options:

- In agriculture, a beef producer will value the option to switch between various feed sources to use the cheapest alternative.
- Airlines and shipping lines can switch capacity from one route to another.

10.5 Practical problems in valuing real options

Option pricing models can help to value the real options in capital investment decisions, but when we price options we rely on the trick, where we construct an option equivalent of the underlying asset and a bank loan. Real options are often complex and have lack of a formal structure, which makes it difficult to estimate cash flows. In addition, competitors might have real options as well that needs to be taken into account when the economic rent of the project is assessed.

11. Appendix: Overview of formulas

Present value (PV) of single cash flow

(1) $$\text{PV} = \text{discount factor} \cdot C_t$$

Discount factor (DF)

(2) $$\text{DF} = \frac{1}{(1+r)^t}$$

Present value formula for single cash flow

(3) $$\text{PV} = \frac{C_t}{(1+\text{r})^\text{t}}$$

Future value formula for single cash flow

(4) $$FV = C_0 \cdot (1+r)^t$$

Present value formula for multiple cash flow

(5) $$PV = \frac{C_1}{(1+r)^1} + \frac{C_2}{(1+r)^2} + \frac{C_3}{(1+r)^3} + ... = \sum \frac{C_t}{(1+r)^t}$$

Net present value

(6) $$\text{NPV} = \text{C}_0 + \sum_{i=1}^{n} \frac{C_i}{(1+r)^i}$$

Present value of a perpetuity

(7) $$\text{PV of perpetuity} = \frac{C}{r}$$

Present value of a perpetuity with constant growth g

(8) $$\text{PV of growing perpetituity} = \frac{C_1}{r-g}$$

Present value of annuity

(9) $$\text{PV of annuity} = C \underbrace{\left[\frac{1}{r} - \frac{1}{r(1+r)^t} \right]}_{\text{Annuity factor}}$$

Real interest rate formula

(10) $$1 + \text{real interest rate} = \frac{1+\text{nominal interest rate}}{1+\text{inflation rate}}$$

Present value of bonds

(11) Value of bond = PV(cash flows) = PV(coupons) + PV(par value)

Present value of coupon payments

(12) PV(coupons) = coupon · annuity factor

Expected return on bonds

$$\text{(13)} \quad \text{Rate of return on bond} = \frac{\text{coupon income} + \text{price change}}{\text{Investment}}$$

Expected return on stocks

$$\text{(14)} \quad \text{Expected return} = \text{r} = \frac{\text{dividend} + \text{capital gain}}{\text{investment}} = \frac{Div_1 + P_1 - P_0}{P_0}$$

Stock price

$$\text{(15)} \quad P_0 = \frac{Div_1 + P_1}{1 + r}$$

Discounted dividend model:

(16) $$P_0 = \sum_{t=1}^{\infty} \frac{Div_t}{(1+r)^t}$$

Discounted dividend growth model

(17) $$P_0 = \frac{Div_1}{r-g}$$

Stock price of preferred share paying a constant dividend

(18) $$P_0 = \frac{Div}{r}$$

Stock price with no growth (i.e. all earnings are paid out to shareholders as dividends)

(19) $$P_0 = \frac{Div_1}{r} = \frac{EPS_1}{r}$$

Expected growth calculation

(20) g = return on equity · plough back ratio

Stock price with growth

(21) $$P_{\text{With growth}} = P_{\text{No growth}} + PVGO$$

(22) $$P_0 = \frac{EPS_1}{r} + PVGO$$

Book rate of return

(23) $$\text{Book rate of return} = \frac{\text{book income}}{\text{book value of assets}}$$

Internal rate of return (IRR) calculation

(24) $$NPV = C_o + \frac{C_1}{1+IRR} + \frac{C_2}{(1+IRR)^2} + \cdots + \frac{C_T}{(1+IRR)^T} = 0$$

Return variance

(25) $$\text{Variance}(r) = \sigma^2 = \frac{1}{N-1}\sum_{t=1}^{N}(r_t - \bar{r})^2$$

Return standard deviation

(26) $$\text{Std.dev.}(r) = \sqrt{\text{variance}(r)} = \sigma$$

Stock beta

(27) $$\beta_i = \frac{\text{covariance with market}}{\text{variance of market}} = \frac{\sigma_{im}}{\sigma_m^2}$$

Portfolio return

(28) $$\text{Portfolio return} = \sum_{i=1}^{n} \mathrm{w}_i r_i$$

Portfolio variance

(29) $$\text{Portfolio variance} = \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j \sigma_{ij}$$

Portfolio beta

(30) $$\text{Portfolio beta} = \sum_{i=1}^{n} \mathrm{w}_i \beta_i$$

Sharpe ratio

(31) $$\text{Sharpe ratio on portfolio i} = \frac{r_i - r_f}{\sigma_i}$$

Capital Assets Pricing Model (CAPM)

(32) $$\text{Expected return on stock i} = r_i = r_f + \beta_i (r_m - r_f)$$

Arbitrage pricing theory (APM)

(33) $$\text{Expected return} = a + b_1 \cdot r_{factor\,1} + b_2 \cdot r_{factor\,2} + \ldots + b_n \cdot r_{factor\,n} + noise$$

(34) $$\text{Expected risk premium} = b_1 \cdot (r_{factor\,1} - r_f) + b_2 \cdot (r_{factor\,2} - r_f) + \ldots + b_n \cdot (r_{factor\,n} - r_f)$$

Fama-French Three-factor Model

(35) $$\text{Expected risk premium} = b_{market} \cdot (r_{market\ fa\cot r}) + b_{size} \cdot (r_{size\ factor}) + b_{book-to-market} \cdot (r_{book-to-market})$$

Company cost of capital

(36) $$\text{Company cost of capital} = \mathrm{r}_{\text{assets}} = \frac{debt}{debt + equity} r_{debt} + \frac{equity}{debt + equity} r_{equity}$$

Company cost of capital with preferred stocks

(37) $$\text{Company cost of capital} = \frac{debt}{firm\ value} r_{debt} + \frac{common\ equity}{firm\ value} r_{common} + \frac{preferred\ equity}{firm\ value} r_{preferred}$$

Cost of preferred stocks

(38) $$\text{Cost of preferred stocks} = r_{preferred} = \frac{DIV}{P}$$

Certain cash flow

$$(39)\qquad Certain\ cash\ flow = PV \cdot (1+r)$$

Free cash flow

$$(40)\qquad \begin{aligned} Free\ cash\ flow = {} & profit\ after\ tax + depreciation + investment\ in\ fixed\ assets \\ & + investment\ in\ working\ capital \end{aligned}$$

Present value of project using free cash flows and weighted average cost of capital

$$(41)\qquad PV = \frac{FCF_1}{(1+WACC)} + \frac{FCF_2}{(1+WACC)^2} + \cdots + \frac{FCF_t}{(1+WACC)^t} + \frac{PV_t}{(1+WACC)^t}$$

Weighted average cost of capital (no corporate taxation)

$$(42)\qquad r_A = \frac{D}{D+E} r_D + \frac{E}{D+E} r_E$$

Miller and Modigliani Proposition II

$$(43)\qquad r_E = r_A + (r_A - r_D)\frac{D}{E}$$

Beta on assets

(44) $$\beta_A = \left(\beta_D \cdot \frac{D}{V}\right) + \left(\beta_E \cdot \frac{E}{V}\right)$$

Beta on equity

(45) $$\beta_E = \beta_A + (\beta_A - \beta_D)\frac{D}{E}$$

Present value of tax shield

(46) $$\text{PV(Tax shield)} = \frac{\text{interest payment} \cdot \text{corporate tax rate}}{\text{expeced return on debt}} = \frac{r_D D \cdot T_C}{\mathrm{r_D}} = D \cdot T_C$$

Value of firm with corporate taxes and cost of financial distress

(47) Value of firm = Value if all-equity financed + PV(tax shield) - PV(cost of financial distress)

Weighted average cost of capital with corporate taxation

(48) $$WACC = r_D(1 - Tc)\left(\frac{D}{V}\right) + r_E\left(\frac{E}{V}\right)$$

Option delta

(49) $$\text{Option delta} = \frac{\text{spread in option val ue}}{\text{spread in stock price}}$$

Up- and downside change in the binominal model

(50) $$1 + \text{upside change} = \mathrm{u} = \mathrm{e}^{\sigma\sqrt{h}}$$
$$1 + \text{upside change} = \mathrm{d} = 1/\mathrm{u}$$

Black-Scholes Formula

(51) Value of call option = [delta · share price] – [bank loan]
= [N(d1) · P] – [N(d2) · PV(EX)]

where

- **N(d1) =** Cumulative normal density function of (d1)
- $d_1 = \dfrac{\log[P/PV(EX)]}{\sigma\sqrt{t}} + \dfrac{\sigma\sqrt{t}}{2}$
- **P =** Stock Price
- **N(d2) =** Cumulative normal density function of (d2)
- $d_2 = d_1 - \sigma\sqrt{t}$
- **PV(EX) =** Present Value of Strike or Exercise price = EX · e^{-rt}

Index