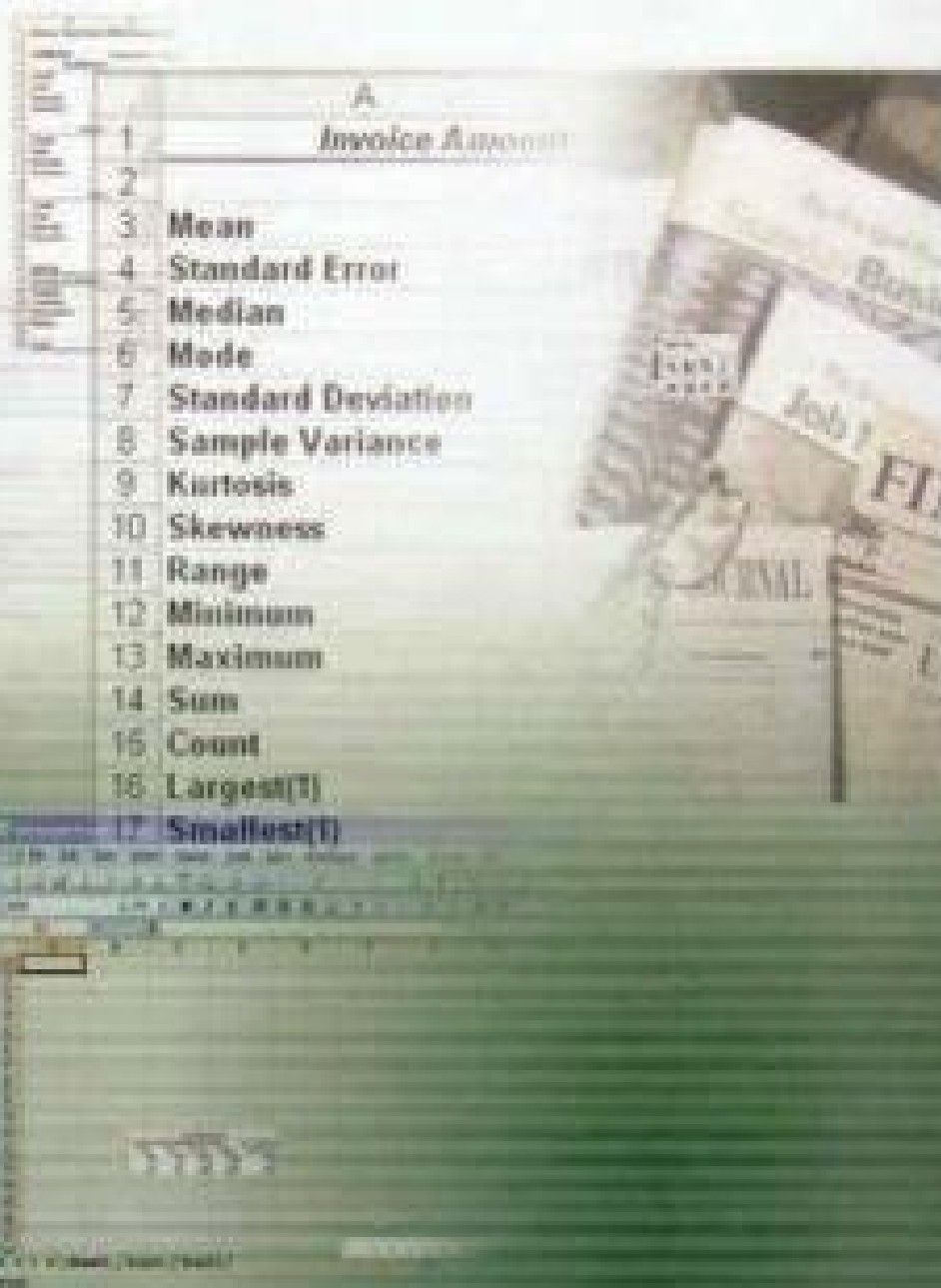


# STATISTICS *for* MANAGERS

USING MICROSOFT® EXCEL

FIFTH EDITION



LEVINE STEPHAN KREHBIEL BERENSON



Business Data Analysis  
SCH-MGMT 650

# STATISTICS FOR MANAGERS USING Microsoft® Excel

David M. Levine • David F. Stephan  
Timothy C. Krehbiel • Mark L. Berenson

Custom Edition for  
UMASS-Amherst  
Professor Robert Nakosteen

Taken from:

*Statistics for Managers: Using Microsoft® Excel*, Fifth Edition  
by David M. Levine, David F. Stephan, Timothy C. Krehbiel, and Mark L. Berenson



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*To our wives,  
Marilyn L., Mary N., Patti K., and Rhoda B.,*

*and to our children  
Sharyn, Mark, Ed, Rudy, Rhonda, Kathy, and Lori*

# ABOUT THE AUTHORS

The textbook authors meet to discuss statistics at Shea Stadium for a Mets v. Phillies game. Shown left to right, Mark Berenson, David Stephan, David Levine, Tim Krehbiel.



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He also recently wrote *Even You Can Learn Statistics* and *Statistics for Six Sigma Green Belts* published by Financial Times-Prentice-Hall. He is coauthor of *Six Sigma for Green Belts and Champions* and *Design for Six Sigma for Green Belts and Champions*, also published by Financial Times-Prentice-Hall, and *Quality Management Third Ed.*, McGraw-Hill-Irwin (2005). He is also the author of *Video Review of Statistics* and *Video Review of Probability*, both published by Video Aided Instruction. He has published articles in various journals including *Psychometrika*, *The American Statistician*, *Communications in Statistics*, *Multivariate Behavioral Research*, *Journal of Systems Management*, *Quality Progress*, and *The American Anthropologist* and given numerous talks at Decision Sciences, American Statistical Association, and Making Statistics More Effective in Schools of Business conferences. While at Baruch College, Dr. Levine received several awards for outstanding teaching and curriculum development.

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Berenson's research has been published in *Decision Sciences Journal of Innovative Education*, *Review of Business Research*, *The American Statistician*, *Communications in Statistics*, *Psychometrika*, *Educational and Psychological Measurement*, *Journal of Management Sciences and Applied Cybernetics*, *Research Quarterly*, *Stats Magazine*, *The New York Statistician*, *Journal of Health Administration Education*, *Journal of Behavioral Medicine*, and *Journal of Surgical Oncology*. His invited articles have appeared in *The Encyclopedia of Measurement & Statistics* and in *Encyclopedia of Statistical Sciences*. He is co-author of 11 statistics texts published by Prentice Hall, including *Statistics for Managers using Microsoft Excel*, *Basic Business Statistics: Concepts and Applications*, and *Business Statistics: A First Course*.

Over the years, Berenson has received several awards for teaching and for innovative contributions to statistics education. In 2005 he was the first recipient of The Catherine A. Becker Service for Educational Excellence Award at Montclair State University.





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# PREFACE

## Educational Philosophy

In our many years of teaching business statistics, we have continually searched for ways to improve the teaching of these courses. Our active participation in a series of Making Statistics More Effective in Schools and Business (MSMESB), Decision Sciences Institute (DSI), and American Statistical Association conferences as well as the reality of serving a diverse group of students at large universities have shaped our vision for teaching these courses. Over the years, our vision has come to include these key principles:

- 1. Students need to be shown the relevance of statistics.**
  - Students need a frame of reference when learning statistics, especially when statistics is not their major. That frame of reference for business students should be the functional areas of business—that is, accounting, finance, information systems, management, and marketing. Each statistical topic needs to be presented in an applied context related to at least one of these functional areas.
  - The focus in teaching each topic should be on its application in business, the interpretation of results, the presentation of assumptions, the evaluation of the assumptions, and the discussion of what should be done if the assumptions are violated.
- 2. Students need to be familiar with the software used in the business world.**
  - Integrating spreadsheet software into all aspects of an introductory statistics course allows the course to focus on interpretation of results instead of computations.
  - Introductory business statistics courses should recognize that in business, spreadsheet software is typically available on a decision maker's desktop.
- 3. Students need to be given sufficient guidance on using software.**
  - Textbooks should provide enough instructions so that students can effectively use the software integrated with the study of statistics, without having the software instruction dominate the course.
- 4. Students need ample practice in order to understand how statistics is used in business.**
  - Both classroom examples and homework exercises should involve actual or realistic data as much as possible.
  - Students should work with data sets, both small and large, and be encouraged to look beyond the statistical analysis of data to the interpretation of results in a managerial context.

## New to This Edition: Statistics Coverage

This new fifth edition of *Statistics for Managers Using Microsoft Excel* enhances the statistical coverage of previous editions in a number of ways:

- Every chapter has been rewritten to use a more engaging, conversational writing style that students will appreciate. Complex topics are discussed in simple, straightforward sentences.
- “From the Authors’ Desktop” essays provide greater background for the topic just covered and raise important issues.
- This edition includes many more examples from everyday life. Notable examples include what you would do with \$1,000 (Chapter 2), time to get ready in the morning (Chapter 3), and waiting time at a fast-food restaurant (Chapter 9).
- Many new applied examples and exercises with data from *The Wall Street Journal*, *USA Today*, *Consumer Reports*, and other sources have been added to the book.
- Many problems have been restructured to contain no more than four parts, allowing students to break down the concepts and apply the material more easily.
- A “Key Equations” list at the end of each chapter lists the equations used in the chapter.
- Worked-out solutions to self-test questions are provided at the back of the book.

- A roadmap for selecting the proper statistical method is included at the front of the text to help students select the proper technique and to make connections between topics.
- Student surveys are included as an integrating theme for exercises across many chapters.

## New to This Edition: Excel Coverage

This new fifth edition of *Statistics for Managers Using Microsoft Excel* enhances the Excel coverage of previous editions in a number of ways:

- Totally rewritten Excel sections have been organized into end-of-chapter Excel Companions for easy reference.
- Wherever possible, Excel Companions present step-by-step instructions and Excel command sequences that are compatible across all current versions of Excel, including Excel 2007.
- Clearly marked separate Excel 97–2003 and Excel 2007 instructions are provided for those Excel techniques that are fundamentally different in Excel 2007.
- “Basic Excel” sections allow the use of Excel without any outside enhancement, and “PHStat2” sections describe the use of the PHStat2 add-in included on the student CD-ROM.
- Margin notes link worksheet and chart illustrations to the instructions of Excel Companion sections.
- Worksheet illustrations, like the following example, display underlying cell formulas that show how results are computed:

	A	B	
1	<b>Estimate for the Mean Sales Invoice Amount</b>		
2			
3	<b>Data</b>		
4	<b>Sample Standard Deviation</b>	28.95	
5	<b>Sample Mean</b>	110.27	
6	<b>Sample Size</b>	100	
7	<b>Confidence Level</b>	95%	
8			
9	Intermediate Calculations		
10	Standard Error of the Mean	2.8950	=B4/SQRT(B6)
11	Degrees of Freedom	99	=B6 - 1
12	t Value	1.9842	=TINV(1 - B7, B11)
13	Interval Half Width	5.7443	=B12 * B10
14			
15	<b>Confidence Interval</b>		
16	<b>Interval Lower Limit</b>	104.53	=B5 - B13
17	<b>Interval Upper Limit</b>	116.01	=B5 + B13

## Chapter-by-Chapter Changes in the Fifth Edition

Each chapter includes a new opening page that displays the chapter sections and subsections. Accompanying each chapter is an Excel Companion that discusses how to apply Microsoft Excel to the statistical techniques of the chapter. In addition, the Excel Companion includes completely new material for using Excel in most chapters. The following changes have been made to this fifth edition:

- *Chapter 1* has rewritten Sections 1.1 (Why Learn Statistics), 1.2 (Statistics for Managers), and 1.3 (Basic Vocabulary of Statistics) and a completely new Section 1.6 (Microsoft Excel Worksheets). The sections on survey sampling have been moved to Chapter 7.
- *Chapter 2* includes a new data set concerning mutual fund returns for 2001–2005. Graphs for a single variable are covered prior to graphs for two variables. Graphs for categorical variables are covered prior to graphs for numerical variables. The examples in this chapter refer to what to do with \$1,000 and the cost of restaurant meals in addition to mutual fund returns.
- *Chapter 3* includes a new data set concerning mutual fund returns for 2001–2005. The examples in this chapter refer to the time to get ready in the morning as well as mutual fund

returns.  $Z$  scores for detecting outliers are now included. The sample covariance is now included, as is the coefficient of correlation.

- *Chapter 4* now includes additional examples.
- *Chapter 5* covers the Poisson distribution prior to the hypergeometric distribution.
- *Chapter 6* has a simplified section on the normal probability plot. Coverage of sampling distributions has been moved to a new Chapter 7.
- *Chapter 7* now covers sampling distributions along with types of survey sampling methods and survey worthiness.
- *Chapter 8* has 28 new problems.
- *Chapter 9* uses a simple, six-step method to perform hypothesis tests using the critical value approach and a straightforward five-step method to perform hypothesis tests using the  $p$ -value.
- *Chapter 10* (formerly Chapter 13) now includes computations for the regression coefficients and sum of squares in chapter examples.
- *Chapter 11* (formerly Chapter 14) now covers  $R^2$ , and adjusted  $R^2$  prior to residual analysis.

## Hallmark Features

We have continued many of the traditions of past editions and have highlighted some of those features below:

- **Using Statistics business scenarios**—Each chapter begins with a Using Statistics example that shows how statistics is used in accounting, finance, information systems, management, or marketing. Each scenario is used throughout the chapter to provide an applied context for the concepts.
- **Emphasis on data analysis and interpretation of Excel results**—We believe that the use of computer software is an integral part of learning statistics. Our focus emphasizes analyzing data by interpreting the results from Microsoft Excel while reducing emphasis on doing computations. For example, in the coverage of tables and charts in Chapter 2, the focus is on the interpretation of various charts, not on their construction by hand. In our coverage of hypothesis testing in Chapter 9, extensive computer results have been included so that the  $p$ -value approach can be emphasized.
- **Pedagogical aides**—An active writing style, boxed numbered equations, set-off examples to provide reinforcement for learning concepts, problems divided into “Learning the Basics” and “Applying the Concepts,” key equations, and key terms are included.
- **Answers**—Most answers to the even-numbered exercises are provided in an appendix at the end of the book.
- **PHStat2**—This add-in, which is included on the student CD-ROM, extends the statistical capabilities of Microsoft Excel and executes the low-level menu selection and worksheet entry tasks associated with implementing statistical analysis in Excel. When combined with the Analysis ToolPak add-in, virtually all statistical methods taught in an introductory statistics course can be demonstrated using Microsoft Excel.
- **Web Cases**—A chapter-ending Web Case is included for each of the first 11 chapters. By visiting Web sites related to the companies and researching the issues raised in the Using Statistics scenarios that start each chapter, students learn to identify misuses of statistical information. The Web Cases require students to sift through claims and assorted information in order to discover the data most relevant to the case. Students then determine whether the conclusions and claims are supported by the data. (Instructional tips for using the Web Cases and solutions to the Web Cases are included in the Instructor’s Solutions Manual.)
- **Case studies and team projects**—Detailed case studies are included in numerous chapters. A *Springville Herald* case is included at the end of most chapters as an integrating theme. A team project relating to mutual funds is included in many chapters as an integrating theme.
- **Visual Explorations**—Microsoft Excel add-in workbook that allows students to interactively explore important statistical concepts in descriptive statistics, the normal distribution,

sampling distributions, and regression analysis. For example, in descriptive statistics, students observe the effect of changes in the data on the mean, median, quartiles, and standard deviation. With the normal distribution, students see the effect of changes in the mean and standard deviation on the areas under the normal curve. In sampling distributions, students use simulation to explore the effect of sample size on a sampling distribution. In regression analysis, students have the opportunity of fitting a line and observing how changes in the slope and intercept affect the goodness of fit.

## Supplement Package

The supplement package that accompanies this text includes the following:

- **Instructor’s Solutions Manual**—This manual includes solutions for end-of-section and end-of-chapter problems, answers to case questions, where applicable, and teaching tips for each chapter. Electronic solutions are provided in Excel and Word formats.
- **Student Solutions Manual**—This manual provides detailed solutions to virtually all the even-numbered exercises and worked-out solutions to the self-test problems.
- **Test Item File**—The Test Item File contains true/false, multiple-choice, fill-in, and problem-solving questions based on the definitions, concepts, and ideas developed in each chapter of the text.
- **TestGen software**—A test bank has been designed for use with the TestGen test-generating software. This computerized package allows instructors to custom design, save, and generate classroom tests. The test program permits instructors to edit, add, or delete questions from the test bank; edit existing graphics and create new graphics; analyze test results; and organize a database of tests and student results. This software allows for flexibility and ease of use. It provides many options for organizing and displaying tests, along with a search and sort feature. The program is available on the instructor’s CD-ROM, and associated conversion files can be found online at the Instructor’s Resource Center.
- **Instructor’s Resource Center**—The Instructor’s Resource Center contains the electronic files for the complete Instructor’s Solutions Manual, the Test Item File, and Lecture PowerPoint presentations ([www.prenhall.com/levine](http://www.prenhall.com/levine)).
- **Course and Homework Management Tools**
  - **Prentice Hall’s OneKey**—This tool offers the best teaching and learning resources, all in one place. OneKey for *Statistics for Managers Using Microsoft Excel*, is all an instructor needs to plan and administer a course and is all students need for anytime, anywhere access to course materials. Conveniently organized by textbook chapter, the compiled resources include links to quizzes, PowerPoint presentations, data files, links to Web Cases, a PHStat2 download, a Visual Explorations download, the Student Solutions Manual, and additional instructor resources.
  - **WebCT and Blackboard**—With a local installation of either course management system, Prentice Hall provides content designed especially for this textbook to create a complete course suite, tightly integrated with the system’s course management tools.
  - **PH GradeAssist**—This online homework and assessment system allows the instructor to assign problems for student practice, homework, or quizzes. The problems, taken directly from the text, are algorithmically generated, so each student gets a slightly different problem with a different answer. This feature allows students multiple attempts for more practice and improved competency. PH GradeAssist grades the results and can export them to Microsoft Excel worksheets.
- **Companion Web site**—[www.prenhall.com/levine](http://www.prenhall.com/levine) contains the following:
  - An online study guide with true/false, multiple-choice, and essay questions designed to test students’ comprehension of chapter topics
  - PowerPoint presentation files with chapter outlines and key equations
  - Student data files for text problems in Excel
- **PHStat2 Web site**—PHStat2 has a home page at [www.prenhall.com/phstat](http://www.prenhall.com/phstat).

- **Index page Web site**—An index page for the supporting material for all the Web Cases included in the text can be found at [www.prenhall.com/Springville/SpringvilleSFM5e.htm](http://www.prenhall.com/Springville/SpringvilleSFM5e.htm).

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Finally, we would like to thank our parents, wives, and children for their patience, understanding, love, and assistance in making this book a reality. It is to them that we dedicate this book.

## Concluding Remarks

We have gone to great lengths to make this text both pedagogically sound and error free. If you have any suggestions or require clarification about any of the material, or if you find any errors, please contact us at [David\\_Levine@baruch.cuny.edu](mailto:David_Levine@baruch.cuny.edu) or [KREHBITC@muohio.edu](mailto:KREHBITC@muohio.edu). Include the phrase “SMUME edition 5” in the subject line of your email. For more information about using PHStat2, see Appendix F, review the PHStat2 readme file on the student CD-ROM, and visit the PHStat2 Web site, at [www.prenhall.com/phstat](http://www.prenhall.com/phstat).

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# CHAPTER 1

## Introduction and Data Collection

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### 1.1 WHY LEARN STATISTICS

### 1.2 STATISTICS FOR MANAGERS

How This Text Is Organized

**USING STATISTICS @** Good Tunes

### 1.3 BASIC VOCABULARY OF STATISTICS

### 1.4 DATA COLLECTION

### 1.5 TYPES OF VARIABLES

Levels of Measurement and Measurement Scales

### 1.6 MICROSOFT EXCEL WORKSHEETS

Worksheet Cells

Designing Effective Worksheets

### EXCEL COMPANION TO CHAPTER 1

E1.1 Preliminaries: Basic Computing Skills

E1.2 Basic Workbook Operations

E1.3 Worksheet Entries

E1.4 Worksheet Formatting

E1.5 Copy-and-Paste Operations

E1.6 Add-ins: Making Things Easier for You

### LEARNING OBJECTIVES

This chapter will help you learn:

- How statistics is used in business
- The sources of data used in business
- The types of data used in business
- The basics of Microsoft Excel

## 1.1 WHY LEARN STATISTICS

The reality TV series *The Apprentice* stars the real estate developer Donald Trump. When it premiered several years ago, Trump assigned two teams of contestants the task of setting up and running a lemonade stand. At the time, a number of businesspeople criticized that task as not being a *realistic* business task. They saw the task of selling lemonade as a simple act of salesmanship that was more dependent on the persuasive skills of the seller than anything else.

If you have ever sold lemonade or held other childhood jobs such as selling cookies or delivering daily newspapers, you know your task was fairly simple. For example, to deliver newspapers, you need only to keep track of a list of addresses and perhaps record the weekly or monthly payments. In contrast, sales and marketing managers of the newspaper need to keep track of much more data—including the incomes, education levels, lifestyles, and buying preferences of their subscribers—in order to make appropriate decisions about increasing circulation and attracting advertisers. But unless that newspaper has a tiny circulation, those managers are probably not looking at data directly. Instead, they are looking at summaries, such as the percentage of subscribers who attended at least some college, or trying to uncover useful patterns, such as whether more subscriptions are delivered to single-family homes in areas associated with heavy sales of luxury automobiles. That is to say, the managers at the newspaper are using statistics, the subject of this text.

**Statistics** is the branch of mathematics that transforms data into useful information for decision makers. These transformations often require complex calculations that are practical only if done by computer, so using statistics usually means also using computers. This is especially true when dealing with the large volumes of data that a typical business collects. Attempting to do statistics using manual calculations for such data would be too time-consuming to benefit a business.

When you learn statistics, you learn a set of methods and the conditions under which it is appropriate for you to use those methods. And because so many statistical methods are practical only when you use computers, learning statistics also means learning more about using computer programs that perform statistical analyses.

## 1.2 STATISTICS FOR MANAGERS

Today, statistics plays an ever increasing important role for business managers. These decision makers use statistics to:

- Present and describe business data and information properly
- Draw conclusions about large populations, using information collected from samples<sup>1</sup>
- Make reliable forecasts about a business activity
- Improve business processes

“Statistics for managers” means knowing more than just how to perform these tasks. Managers need a conceptual understanding of the principles behind each statistical analysis they undertake in order to have confidence that the information produced is correct and appropriate for a decision-making situation.

To help you master these necessary skills, every chapter of *Statistics for Managers Using Microsoft Excel* has a *Using Statistics* scenario. While the scenarios are fictional, they represent realistic situations in which you will be asked to make decisions while using Microsoft Excel to transform data into statistical information. For example, in one chapter, you will be asked to decide the location in a supermarket that best enhances sales of a cola drink, and in another chapter, you will be asked to forecast sales for a clothing store. (You will not be asked, as the television apprentices were asked, to decide how best to sell lemonade on a New York City street corner.)

<sup>1</sup>The statistical terms population and sample are formally defined in Section 1.3, on page 5.

## How This Text Is Organized

Table 1.1 shows the chapters of *Statistics for Managers Using Microsoft Excel* organized according to the four activities for which decision makers use statistics.

**TABLE 1.1**  
Organization of This Text

### Presenting and Describing Information

Data Collection (Chapter 1)  
Presenting Data in Tables and Charts (Chapter 2)  
Numerical Descriptive Measures (Chapter 3)

### Drawing Conclusions About Populations Using Sample Information

Basic Probability (Chapter 4), a prerequisite for the rest of the chapters of this group  
Some Important Discrete Probability Distributions (Chapter 5)  
The Normal Distribution and Other Continuous Distributions (Chapter 6) and Sampling and Sampling Distributions (Chapter 7), which lead to Confidence Interval Estimation (Chapter 8) and Hypothesis Testing (Chapter 9)

### Making Reliable Forecasts

Simple Linear Regression (Chapter 10)  
Introduction to Multiple Regression (Chapter 11)

Methods presented in the Chapters 1–3 are all examples of **descriptive statistics**, the branch of statistics that collects, summarizes, and presents data. Methods discussed in Chapters 7 through 9 are examples of **inferential statistics**, the branch of statistics that uses sample data to draw conclusions about an entire population. (Chapters 4–6 provide the foundation in probability and probability distributions needed for Chapters 7–9.) The definition of inferential statistics uses the terms *sample* and *population*, the second time you have encountered these words in this section. You can probably figure out that you cannot learn much about statistics until you learn the basic vocabulary of statistics. Continue now with the first Using Statistics scenario, which will help introduce you to several important terms used in statistics.

## USING STATISTICS @ Good Tunes



Good Tunes, a growing four-store home entertainment systems retailer, seeks to double their number of stores within the next three years. The managers have decided to approach local area banks for the cash needed to underwrite this expansion. They need to prepare an electronic slide show and a formal prospectus that will argue that Good Tunes is a thriving business that is a good candidate for expansion.

You have been asked to assist in the process of preparing the slide show and prospectus. What data would you include that will convince bankers to extend the credit it needs to Good Tunes? How would you present that data?

In this scenario, you need to identify the most relevant data for the bankers. Because Good Tunes is an ongoing business, you can start by reviewing the company's records, which show both its current and recent past status. Because Good Tunes is a retailer, presenting data about the company's sales seems a reasonable thing to do. You could include the details of every sales transaction that has occurred for the past few years as a way of demonstrating that Good Tunes is a thriving business.

However, presenting the bankers with the thousands of transactions would overwhelm them and not be very useful. As mentioned in Section 1.1, you need to transform the transactions data into information by summarizing the details of each transaction in some useful way that would allow the bankers to (perhaps) uncover a favorable pattern about the sales over time.

One piece of information that the bankers would presumably want to see is the dollar sales totals by year. Tallying and totaling sales is a common process of transforming data into information and a very common statistical analysis. When you tally sales—or any other relevant data about Good Tunes you choose to use—you follow normal business practice and tally by a business period such as by month, quarter, or year. When you do so, you end up with multiple values: sales for this year, sales for last year, sales for the year before that, and so on. How best to refer to these multiple values requires learning the basic vocabulary of statistics.

### 1.3 BASIC VOCABULARY OF STATISTICS

Variables are characteristics of items or individuals and are what you analyze when you use a statistical method. For the Good Tunes scenario, sales, expenses by year, and net profit by year are variables that the bankers would want to analyze.

#### VARIABLE

A **variable** is a characteristic of an item or individual.

When used as an adjective in everyday speech, *variable* suggests that something changes or varies, and you would expect the sales, expenses, and net profit to have different values from year to year. These different values are the **data** associated with a variable, and more simply, the “data” to be analyzed. In later sections, you will be sometimes asked to enter the cell range of a variable in Excel. When you see such an instruction, you should enter the cell range of the different values that collectively are the data to be analyzed. (Section 1.6 on page 11 explains what a cell range is and further discusses how to enter data in Excel.)

Variables can differ for reasons other than time. For example, if you conducted an analysis of the composition of a large lecture class, you would probably want to include the variables class standing, gender, and major field of study. Those variables would vary, too, because each student in the class is different. One student might be a freshman male Economics major, while another may be a sophomore female Finance major.

You also need to remember that values are meaningless unless their variables have **operational definitions**. These definitions are universally accepted meanings that are clear to all associated with an analysis. While the operational definition for sales per year might seem clear, miscommunication could occur if one person was referring to sales per year for the entire chain of stores and another to sales per year per store. Even individual values for variables sometimes need definition—for the class standing variable, for example, what *exactly* is meant by the words *sophomore* and *junior*? (Perhaps the most famous example of vague definitions was the definition of a valid vote in the state of Florida during the 2000 U.S. presidential election. Vagueness about the operational definitions there ultimately required a U.S. Supreme Court ruling.)

Understanding the distinction between variables and their values helps in learning four other basic vocabulary terms, two of which you have already encountered in previous sections.

### POPULATION

A **population** consists of all the items or individuals about which you want to draw a conclusion.

### SAMPLE

A **sample** is the portion of a population selected for analysis.

### PARAMETER

A **parameter** is a numerical measure that describes a characteristic of a population.

### STATISTIC

A **statistic** is a numerical measure that describes a characteristic of a sample.

All the Good Tunes sales transactions for a specific year, all the customers who shopped at Good Tunes this weekend, all the full-time students enrolled in a college, and all the registered voters in Ohio are examples of populations. Examples of samples from these four populations would be 200 Good Tunes sales transactions randomly selected by an auditor for study, 30 Good Tunes customers asked to complete a customer satisfaction survey, 50 full-time students selected for a marketing study, and 500 registered voters in Ohio contacted via telephone for a political poll. In each sample, the transactions or people in the sample represent a portion of the items or individuals that make up the population.

“The average amount spent by all customers who shopped at Good Tunes this weekend” is an example of a parameter because the amount spent in the entire population is needed. In contrast, “the average amount spent by the 30 customers completing the customer satisfaction survey” is an example of a statistic because the amount spent from only the sample of 30 people is required.

## Using and Learning Microsoft Excel

**A**lthough we have talked a lot about statistics to this point, we haven't mentioned much about using Microsoft Excel.

Using any computer program is a two-step process that begins with learning to operate the program and then advances to mastering how to apply the program to a decision-making task—and Excel is no exception.

The Excel Companion to this chapter will help you become familiar with operating Excel. In writing that Companion, we have assumed that you have operated a personal computer in the past to do something such as surf the Web, send an instant message, play music or games, or write homework assignments. If you have never used a personal computer for any of these or similar activities, you should ask a friend to introduce you to personal computers before you read the Companion for this chapter. While this Companion was primarily written for novices, experienced Excel users will benefit from learning the words used to describe the Excel operations in this book.

The Excel Companion to Chapter 2 and those for later chapters will help you understand how you can apply the statistical methods discussed in this book by using Microsoft Excel. For each method discussed, you will typically learn two ways that you can use Excel. One way, labeled **Basic Excel**, uses Excel without any outside enhancements to the program. The other way, labeled **PHStat2**, uses the free PHStat2 statistics add-in\* that is included on this book's CD.

These two ways are truly interchangeable. The Excel solutions you create using either way will be identical (or nearly so) to each other and the example worksheets and charts you see in this book. You can switch between the two ways at any time as you use this text without losing any comprehension of the Excel material. That this book includes two complementary ways of learning Microsoft Excel is a distinctive feature of the book.

### *Which way is best for you?*

Unless your instructor requires you to use one way, you may want to choose a way that best

suits how you like to learn. Do you like to learn by building things from scratch, one step at a time? If so, using the Basic Excel way would be the best way for you. Do you worry about the time it takes to build things and the typing errors you might make? Or do you like to learn by closely examining a solution to discover its details, a discovery process some call reverse engineering? In either of these cases, using PHStat2 would be your best choice.

For a few statistical methods, you will not find either a Basic Excel way or a PHStat2 way, due to the limitations of Excel. For such methods, you will find Excel workbook files on the book's CD that you can open and use as a template for creating your own solutions. (Actually, you will find files that contain template examples for every statistical method discussed in the Excel Companions and for every Excel worksheet or chart illustrated in this book. A good starting point for these examples are the Excel workbook files named for the chapters of this text, such as Chapter 2.xls.)

Regardless of the way you use and learn Microsoft Excel, you are invited to go online and

explore the Web site for this book. There, you will find supplementary material about using Excel, including discussions of Excel techniques that the authors were not able to include in the book because of space limitations.

#### Postscript: Isn't Using an Add-in a Bad Thing?

If you are an experienced Microsoft Excel user, you may have concerns about using an add-in such as PHStat2. You may be concerned that you will become dependent on something you would not be able to use in business or think that using PHStat2 somehow means that you

are not really using and learning Microsoft Excel.

Both of these concerns are unfounded. PHStat2 is a learning tool whose sole purpose is to help you understand how Excel can be used to support specific statistical methods. PHStat2 is designed to make using Microsoft Excel more convenient, by doing the “busy work” activities of creating an Excel solution, such as cell formatting, for you. When you read the Excel Companions, you will understand what PHStat2 is doing for you in a generalized way as well as what it is doing specifically for a problem (following the Basic Excel instructions).

It is true that other add-ins, including add-ins for other introductory business statistics textbooks, can obscure Microsoft Excel by not building an Excel-based solution and only reporting outcomes and statistical information that the add-in has internally (and invisibly) computed. Using such add-ins would not be truly “learning” Microsoft Excel, and using such add-ins would leave you dependent on their use. In contrast, PHStat2 creates model Excel solutions that you can examine and incorporate into your own Excel solutions.

\*Section E1.6 that begins on page 28 explains what an add-in is.

## 1.4 DATA COLLECTION

The managers at Good Tunes believe that they will have a stronger argument for expansion if they can show the bankers that the customers of Good Tunes are highly satisfied with the service they received. How could the managers demonstrate that good service was the typical customer experience at Good Tunes?

Unlike the earlier Good Tunes scenario, in which sales per year was automatically collected as part of normal business activities, the managers now face the twin challenges to first identify relevant variables for a customer satisfaction study and then devise a method for *data collection*—that is, collecting the values for those variables.

Many different types of circumstances, such as the following, require data collection:

- A marketing research analyst needs to assess the effectiveness of a new television advertisement.
- A pharmaceutical manufacturer needs to determine whether a new drug is more effective than those currently in use.
- An operations manager wants to monitor a manufacturing process to find out whether the quality of product being manufactured is conforming to company standards.
- An auditor wants to review the financial transactions of a company in order to determine whether the company is in compliance with generally accepted accounting principles.

In each of these examples, and for the Good Tunes managers as well, collecting data from every item or individual in the population would be too difficult or too time-consuming. Because this is the typical case, data collection almost always involves collecting data from a sample. (Chapter 7 discusses methods of sample selection.)

Unlike the Good Tunes example that begins this section, the source of the data to be collected is not always obvious. Data sources are classified as being either **primary sources** or **secondary sources**. When the data collector is the one using the data for analysis, the source is primary. When the person performing the statistical analysis is not the data collector, the source is secondary. Sources of data fall into one of four categories:

- Data distributed by an organization or an individual
- A designed experiment
- A survey
- An observational study

Organizations and individuals that collect and publish data typically use that data as a primary source and then let others use it as a secondary source. For example, the United States federal government collects and distributes data in this way for both public and private purposes. The Bureau of Labor Statistics collects data on employment and also distributes the monthly consumer price index. The Census Bureau oversees a variety of ongoing surveys

regarding population, housing, and manufacturing and undertakes special studies on topics such as crime, travel, and health care.

Market research firms and trade associations also distribute data pertaining to specific industries or markets. Investment services such as Mergent's provide financial data on a company-by-company basis. Syndicated services such as AC Nielsen provide clients with data that enables the comparison of client products with those of their competitors. Daily newspapers are filled with numerical information regarding stock prices, weather conditions, and sports statistics.

Outcomes of a designed experiment are another data source. These outcomes are the results of an experiment, such as a test of several laundry detergents to compare how well each detergent removes a certain type of stain. Developing proper experimental designs is a subject mostly beyond the scope of this text because such designs often involve sophisticated statistical procedures.

Conducting a survey is a third type of data source. People being surveyed are asked questions about their beliefs, attitudes, behaviors, and other characteristics. For example, people could be asked their opinion about which laundry detergent best removes a certain type of stain. (This could lead to a result different from a designed experiment seeking the same answer.)

Conducting an observational study is the fourth important data source. A researcher collects data by directly observing a behavior, usually in a natural or neutral setting. Observational studies are a common tool for data collection in business. Market researchers use *focus groups* to elicit unstructured responses to open-ended questions posed by a moderator to a target audience. Other, more structured types of studies involve group dynamics and consensus building. Observational study techniques are also used in situations in which enhancing teamwork or improving the quality of products and service is a management goal.

Identifying the most appropriate source is a critical task because if biases, ambiguities, or other types of errors flaw the data being collected, even the most sophisticated statistical methods will not produce useful information. For the Good Tunes example, variables relevant to the customer experience could take the form of survey questions related to various aspects of the customer experience, examples of which are shown in Figure 1.1.

**FIGURE 1.1**  
Questions about the  
Good Tunes customer  
experience

1. How many days did it take from the time you ordered your merchandise to the time you received it? \_\_\_\_\_
2. Did you buy any merchandise that was featured in the Good Tunes Sunday newspaper sales flyer for the week of your purchase? Yes \_\_\_\_\_ No \_\_\_\_\_
3. Was this your first purchase at Good Tunes? Yes \_\_\_\_\_ No \_\_\_\_\_
4. Are you likely to buy additional merchandise from Good Tunes in the next 12 months? Yes \_\_\_\_\_ No \_\_\_\_\_
5. How much money (in U.S.dollars) do you expect to spend on stereo and consumer electronics equipment in the next 12 months? \_\_\_\_\_
6. How do you rate the overall service provided by Good Tunes with respect to your recent purchase?  
Excellent  Very good  Fair  Poor
7. How do you rate the selection of products offered by Good Tunes with respect to other retailers of home entertainment systems?  
Excellent  Very good  Fair  Poor
8. How do you rate the quality of the items you recently purchased from Good Tunes?  
Excellent  Very good  Fair  Poor

The survey might also ask questions that seek to classify customers into groups for later analysis.

One good way for Good Tunes to avoid data-collection flaws would be to distribute the questionnaire to a random sample of customers (as discussed in Chapter 7). A poor way would be to rely on a business rating Web site that allows online visitors to rate a merchant. Such Web sites cannot provide assurance that those who do the rating are customers.

## Web Surveys

From the Authors' Desktop

**W**eb-based surveys and ratings seem to be of growing importance for many marketers. Their use and misuse raise many concerns. By coincidence, while writing Section 1.4, one of us received an email requesting that he rate the Marriott Rewards travel loyalty program a perfect “10” in the voting for the InsideFlyer Freddie Awards. The author had never heard of those awards, but soon he received other emails from various other travel loyalty programs, also asking that the same high rating be submitted. He even got an email for a program for which he had just signed up in the prior month (and for a travel company of which he was not yet a customer).

At the same time, another one of us found an article in *The New York Times* that reported

that Internet travel sites had to closely monitor submitted reviews to avoid fraudulent claims (C. Elliott, “Hotel Reviews Online: In Bed with Hope, Half-Truths and Hype,” *The New York Times*, February 7, 2006, pp. C1, C8). The article also reported that a hotel in Key West, Florida, offered its guests a 10% discount if they published a rave review of that hotel on a particular travel Web site! Our co-author with all the “Freddie” emails felt cheated.

Have you ever received an email asking you to rate an online merchant? Many of us have, especially when we have just purchased something from an online merchant. Often, such emails come with an incentive, not unlike the Key West hotel’s discount. Would an incentive cause you to rate the merchant? Would the incentive affect your opinion?

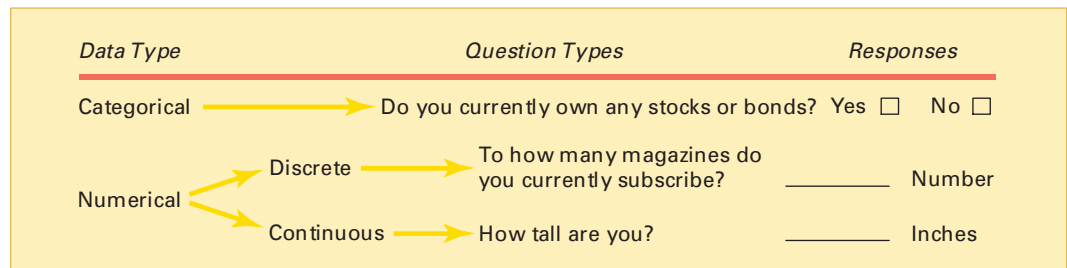
What would you say about a ratings Web site that accepts advertising from merchants that are rated on the site? What would you say about a ratings Web site that gets paid a commission if a visitor first views a rating and then clicks on a link for the merchant? These are among the several practices that may raise ethical concerns for some.

If you do use a ratings Web site, be sure to check out the “fine print” on your next visit. Although you will find a privacy statement that explains how the Web site uses data that can personally identify you, most likely you will not find a “data collection statement” that explains the methods the Web site uses to collect its data. Perhaps you should find such a statement.

### 1.5 TYPES OF VARIABLES

Statisticians classify **variables** as either being categorical or numerical and further classify numerical variables as having either discrete or continuous values. Figure 1.2 shows the relationships and provides examples of each type of variable.

**FIGURE 1.2**  
Types of variables



**Categorical variables** (also known as **qualitative variables**) have values that can only be placed into categories, such as “yes” and “no”. Questions 2–4 in Figure 1.1 are examples of categorical variables, all of which have “yes” or “no” as their values. Categorical variables can also result in more than two possible responses. An example of this type of variable is asking customers to indicate the day of the week on which they made their purchases. Questions 6–8 result in one of four possible responses.

**Numerical variables** (also known as **quantitative variables**) have values that represent quantities. For example, Questions 1 and 5 in Figure 1.1 are numerical variables. Numerical variables are further subdivided as discrete or continuous variables.



**Discrete variables** have numerical values that arise from a counting process. “The number of magazines subscribed to” is an example of a discrete numerical variable because the response is one of a finite number of integers. You subscribe to zero, one, two, and so on magazines. The number of days it takes from the time you ordered your merchandise to the time you receive it is a discrete numerical variable because you are counting the number of days.

**Continuous variables** produce numerical responses that arise from a measuring process. The time you wait for teller service at a bank is an example of a continuous numerical variable because the response takes on any value within a *continuum*, or interval, depending on the precision of the measuring instrument. For example, your waiting time could be 1 minute, 1.1 minutes, 1.11 minutes, or 1.113 minutes, depending on the precision of the measuring device you use.

Theoretically, with sufficient precision of measurement, no two continuous values will be identical. As a practical matter, however, most measuring devices are not precise enough to detect small differences, and tied values for a continuous variable (i.e., two or more items or individuals with the same value) are often found in experimental or survey data.

### Levels of Measurement and Measurement Scales

Using levels of measurement is another way of classifying data. There are four widely recognized levels of measurement: nominal, ordinal, interval, and ratio scales.

**Nominal and Ordinal Scales** Data from a categorical variable are measured on a nominal scale or on an ordinal scale. A **nominal scale** (see Figure 1.3) classifies data into distinct categories in which no ranking is implied. In the Good Tunes customer satisfaction survey, the answer to the question “Are you likely to buy additional merchandise from Good Tunes in the next 12 months?” is an example of a nominal scaled variable, as are your favorite soft drink, your political party affiliation, and your gender. Nominal scaling is the weakest form of measurement because you cannot specify any ranking across the various categories.

<i>Categorical Variable</i>	<i>Categories</i>
Personal Computer Ownership	Yes <input type="checkbox"/> No <input type="checkbox"/>
Type of Stocks Owned	Growth <input type="checkbox"/> Value <input type="checkbox"/> Other <input type="checkbox"/> None <input type="checkbox"/>
Internet Provider	Microsoft Network <input type="checkbox"/> AOL <input type="checkbox"/> Other <input type="checkbox"/>

**FIGURE 1.3** Examples of nominal scales

An **ordinal scale** classifies data into distinct categories in which ranking is implied. In the Good Tunes survey, the answers to the question “How do you rate the overall service provided by Good Tunes with respect to your recent purchase?” represent an ordinal scaled variable because the responses “excellent, very good, fair, and poor” are ranked in order of satisfaction level. Figure 1.4 lists other examples of ordinal scaled variables.

<i>Categorical Variable</i>	<i>Ordered Categories</i>
Student class designation	Freshman Sophomore Junior Senior
Product satisfaction	Very Unsatisfied Fairly Unsatisfied Neutral Fairly Satisfied Very Satisfied
Faculty rank	Professor Associate Professor Assistant Professor Instructor
Standard & Poor’s bond ratings	AAA AA A BBB BB B CCC CC C DDD DD D
Student grades	A B C D F

**FIGURE 1.4** Examples of ordinal scales

Ordinal scaling is a stronger form of measurement than nominal scaling because an observed value classified into one category possesses more of a property than does an observed value classified into another category. However, ordinal scaling is still a relatively weak form of measurement because the scale does not account for the amount of the differences *between* the categories. The ordering implies only *which* category is “greater,” “better,” or “more preferred”—not by *how much*.

**Interval and Ratio Scales** Data from a numerical variable are measured on an interval or a ratio scale. An **interval scale** (see Figure 1.5) is an ordered scale in which the difference between measurements is a meaningful quantity but does not involve a true zero point. For example, a noon-time temperature reading of 67 degrees Fahrenheit is 2 degrees warmer than a noontime reading of 65 degrees. In addition, the 2 degrees Fahrenheit difference in the noontime temperature readings is the same as if the two noontime temperature readings were 74 and 76 degrees Fahrenheit because the difference has the same meaning anywhere on the scale.

**FIGURE 1.5**

Examples of interval and ratio scales

Numerical Variable	Level of Measurement
Temperature (in degrees Celsius or Fahrenheit)	Interval
Standardized exam score (e.g., ACT or SAT)	Interval
Height (in inches or centimeters)	Ratio
Weight (in pounds or kilograms)	Ratio
Age (in years or days)	Ratio
Salary (in American dollars or Japanese yen)	Ratio

A **ratio scale** is an ordered scale in which the difference between the measurements involves a true zero point, as in height, weight, age, or salary measurements. In the Good Tunes customer satisfaction survey, the amount of money (in U.S. dollars) you expect to spend on stereo equipment in the next 12 months is an example of a ratio scaled variable. As another example, a person who weighs 240 pounds is twice as heavy as someone who weighs 120 pounds. Temperature is a trickier case: Fahrenheit and Celsius (centigrade) scales are interval but not ratio scales; the “zero” value is arbitrary, not real. You cannot say that a noontime temperature reading of 4 degrees Fahrenheit is twice as hot as 2 degrees Fahrenheit. But a Kelvin temperature reading, in which zero degrees means no molecular motion, is ratio scaled. In contrast, the Fahrenheit and Celsius scales use arbitrarily selected zero-degree beginning points.

Data measured on an interval scale or on a ratio scale constitute the highest levels of measurement. They are stronger forms of measurement than an ordinal scale because you can determine not only which observed value is the largest but also by how much.

## PROBLEMS FOR SECTION 1.5

### Learning the Basics

**PH Grade ASSIST** **1.1** Three different beverages are sold at a fast-food restaurant—soft drinks, tea, and coffee.

- Explain why the type of beverage sold is an example of a categorical variable.
- Explain why the type of beverage sold is an example of a nominal scaled variable.

**1.2** Soft drinks are sold in three sizes at a fast-food restaurant—small, medium, and large. Explain why the size of the soft drink is an example of an ordinal scaled variable.

**1.3** Suppose that you measure the time it takes to download an MP3 file from the Internet.

- Explain why the download time is a continuous numerical variable.
- Explain why the download time is a ratio scaled variable.

### Applying the Concepts

**SELF Test** **1.4** For each of the following variables, determine whether the variable is categorical or numerical.

If the variable is numerical, determine whether the variable is discrete or continuous. In addition, determine the level of measurement for each of the following.

- Number of telephones per household
- Length (in minutes) of the longest long-distance call made per month

- c. Whether someone in the household owns a cell phone
- d. Whether there is a high-speed Internet connection in the household

**PH Grade ASSIST** 1.5 The following information is collected from students upon exiting the campus bookstore during the first week of classes:

- a. Amount of time spent shopping in the bookstore
- b. Number of textbooks purchased
- c. Academic major
- d. Gender

Classify each of these variables as categorical or numerical. If the variable is numerical, determine whether the variable is discrete or continuous. In addition, determine the level of measurement for these variables.

**PH Grade ASSIST** 1.6 For each of the following variables, determine whether the variable is categorical or numerical. If the variable is numerical, determine whether the variable is discrete or continuous. In addition, determine the level of measurement for each of the following.

- a. Name of Internet provider
- b. Amount of time spent surfing the Internet per week
- c. Number of emails received in a week
- d. Number of online purchases made in a month

1.7 For each of the following variables, determine whether the variable is categorical or numerical. If the variable is numerical, determine whether the variable is discrete or continuous. In addition, determine the level of measurement for each of the following.

- a. Amount of money spent on clothing in the past month
- b. Favorite department store
- c. Most likely time period during which shopping for clothing takes place (weekday, weeknight, or weekend)
- d. Number of pairs of winter gloves owned

1.8 Suppose the following information is collected from Robert Keeler on his application for a home mortgage loan at the Metro County Savings and Loan Association:

- a. Monthly payments: \$1,427
- b. Number of jobs in past 10 years: 1
- c. Annual family income: \$86,000
- d. Marital status: Married

Classify each of the responses by type of data and level of measurement.

1.9 One of the variables most often included in surveys is income. Sometimes the question is phrased “What is your income (in thousands of dollars)?” In other surveys, the respondent is asked to “Place an X in the circle corresponding to your income level” and given a number of income ranges to choose from.

- a. In the first format, explain why income might be considered either discrete or continuous.
- b. Which of these two formats would you prefer to use if you were conducting a survey? Why?
- c. Which of these two formats would likely bring you a greater rate of response? Why?

**PH Grade ASSIST** 1.10 If two students score a 90 on the same examination, what arguments could be used to show that the underlying variable—test score—is continuous?

1.11 The director of market research at a large department store chain wanted to conduct a survey throughout a metropolitan area to determine the amount of time working women spend shopping for clothing in a typical month.

- a. Describe both the population and the sample of interest, and indicate the type of data the director might want to collect.
- b. Develop a first draft of the questionnaire needed in (a) by writing a series of three categorical questions and three numerical questions that you feel would be appropriate for this survey.

## 1.6 MICROSOFT EXCEL WORKSHEETS

When you use Microsoft Excel, you place the data you have collected in **worksheets**. Worksheets appear as pages containing gridlines that separate individually lettered columns from numbered rows. While worksheets look like the simple tables you can create in a word processing program, worksheets have special features that are particularly suited to data analysis. Understanding the special features of worksheets will help you to better understand the interplay of data and results in Microsoft Excel.

### Worksheet Cells

The intersections of the columns and rows of worksheets form boxes called **cells**. You refer to a cell by its column letter and row number. For example, you refer to the cell in the first column and second row as cell A2 and the cell in the fifth column and first row as cell E1. You enter in a cell a single value or an expression that can include a reference to another cell. This flexibility, as explained further in Section E1.3 of the Excel Companion to this chapter, is one of the special features that makes Microsoft Excel more than just a fancy table-oriented word processor.

You can refer to more than one cell in a cell reference. If you want to refer to a group of cells that forms a contiguous rectangular area, you can use a **cell range** in which references to the upper leftmost cell and the lower rightmost cell are joined with a colon. For example, the cell range A1:C2 refers to the six cells found in the first two rows and three columns of a worksheet. Excel also allows ranges such as A:A or 4:4, as a shorthand way of referring to all the cells in a column or a row. Later in this text, you will see cell ranges such as D1:D8,F1:F8 that refer to cells from two non-adjacent areas of a worksheet.

Worksheets exist inside a **workbook**, a collection of worksheets and other types of sheets, including **chart sheets** that help visualize data. Usually, you will use only one sheet at any given time and open to a worksheet by clicking its sheet tab (see Section E1.1). If someone says that they are opening an “Excel file,” they are most likely opening a workbook file. All versions of Excel can open workbook files saved using the **.xls** file format (and all Excel files on the Student CD are in this format). Excel 2007 can also open workbooks saved in the newer **.xlsx** format discussed in Appendix F.

## Designing Effective Worksheets

Because thousands of cells are available on individual worksheets, you will never have to worry about running out of cells to use. This spaciousness of worksheets invites careless use and causes some to ignore the important process of effectively arranging worksheet data. Poor arrangements can increase the chance of user errors, create confusing results, lead to unattractive printouts, or worse.

To be consistent with standard business usage, you should associate column cell ranges with variables. In this arrangement, you use the first (row 1) cell of a column for a name label for a variable and place the data for the variable in the subsequent cells of the column. You do not skip any rows as you enter data, so column cell ranges will never contain any empty cells. (Empty cells can interfere with Excel ability’s to process your data and can lead to inaccurate results.)

This standard practice is always used in this text and in all of the Excel files on the student CD. Because all of the Excel instructions assume this data arrangement, you should never deviate from this practice when you use this book.

Another good practice is to place all the variables on a worksheet that is *separate* from the worksheet containing the results. Such separation will increase the reusability of your results worksheet and minimize the chance of inadvertent changes to the values of your variables as you construct your results. In the workbooks found on the book’s CD as well as the workbooks produced by PHStat2, you will generally find a Results worksheet showing the results separate from the worksheet containing the variables.

Sometimes, worksheets used in this book require only the values of certain parameters or statistics and not the values associated with a variable. For such worksheets, good practice is to place the parameters and statistics at the top of the worksheet so that a user can easily perform what-if analyses, changing values to see their effects on the results. In this book, these values always appear in bold, in cells tinted a shade Excel calls light turquoise and under the heading Data. When you see such tinted cells, you know that you can change the values in those cells to perform what-if analyses and solve other, similar problems.

Another good design practice is to allow the user to be able to explicitly see the chain of calculations from the starting data, through any intermediate calculations, to the results. This practice is particularly advantageous when preparing statistical worksheets because most “intermediate calculations” are statistics themselves. Showing the chain of calculations helps you review your worksheet for errors and helps others better understand what your worksheet does.

In the worksheets of this book, intermediate calculations appear under the heading Intermediate Calculations and are in a cell range that immediately precedes the cell range containing the results. The results appear in cells that are tinted a light yellow and contain boldfaced text. There is also a heading over the results cells that varies with the type of statistical analysis performed.

Whether you use the worksheet design of this book or your own design, do not overlook the importance of skipping rows or columns to create white space to separate different regions

of the worksheet that present results. In this book, worksheets tend to skip only a single row or a single column. This choice is due more to making all illustrations compact than any hard or fast rule. You should experiment with your own worksheets with an eye to making them easy to follow on both the display screen and the printed page. Do not hesitate to create two copies of your worksheets—one optimized for the screen, the other for the printer, if you have anything but the simplest worksheet to produce.

## When to Excel

### From the Authors' Desktop

Perhaps you have heard from some people that Microsoft Excel shouldn't be used for statistics or you have searched the Internet and discovered that statistics educators have had a long-running discussion over the use of Excel in the classroom.

As authors of a text whose title includes the phrase *Using Microsoft Excel*, we believe that Microsoft Excel provides a good way to introduce you to basic statistical methods and demonstrate how to apply these methods in business decision making. Many managers, noting the prevalence of Microsoft Excel on the computers in their businesses, have similarly considered using Excel, rather than a specialized statistical program, for statistical analysis. Microsoft Excel seems like an attractive choice because:

- Using Excel means not having to incur the extra costs of using specialized statistical programs.
- Most business users already have some familiarity with Excel.
- Excel is easy to use and easy to learn, at least for casual users.
- Excel graphical and statistical functions can use the same worksheet-based data that users have created for other business purposes.
- Some Excel graphical functions produce more vivid visual outputs than some specialized statistical programs.

While these traits are attractive, those who have chosen Microsoft Excel have not necessarily considered the accuracy and completeness of the statistical results that Excel

produces. Unfortunately, some investigators have determined that certain Microsoft Excel statistical capabilities contain flaws that can lead to invalid results, especially when data sets are very large or have unusual statistical properties (see reference 1, 2, and 4). Even using Microsoft Excel with small data sets to produce the relatively simple descriptive statistics can lead to nonstandard results. (As an example, see the discussion for creating histograms in the Excel Companion to chapter 2 on page 86.) Clearly, when you use Microsoft Excel, you must be careful about the data and the method you are using. Whether this complication outweighs the benefits of Excel's attractive features is still an unanswered question in business today.

## SUMMARY

In this chapter, you have been introduced to the role of statistics in turning data into information and the importance of using computer programs such as Microsoft Excel. In addition, you have studied data collection and the various types of data used in business. In conjunction with the Using Statistics scenario, you were asked to review the customer survey used by the Good Tunes company (see page 7). The first and fifth questions of the survey shown will pro-

duce numerical data. Questions 2, 3, and 4 will produce nominal categorical data. Questions 6–8 will produce ordinal categorical data. The responses to the first question (number of days) are discrete, and the responses to the fifth question (amount of money spent) are continuous. In the next two chapters, tables and charts and a variety of descriptive numerical measures that are useful for data analysis are developed.

## KEY TERMS

categorical variable 8  
cell 11  
cell range 12  
chart sheet 12  
continuous variable 9  
data 4  
descriptive statistics 3  
discrete variable 9  
inferential statistics 3

interval scale 10  
nominal scale 9  
numerical variable 8  
operational definition 4  
ordinal scale 9  
parameter 5  
population 5  
primary source 6  
qualitative variable 8

quantitative variable 8  
ratio scale 10  
sample 5  
secondary source 6  
statistic 5  
statistics 2  
variable 4  
workbook 12  
worksheet 11

# CHAPTER REVIEW PROBLEMS

## Checking Your Understanding

**PH Grade ASSIST** 1.12 What is the difference between a sample and a population?

**PH Grade ASSIST** 1.13 What is the difference between a statistic and a parameter?

**PH Grade ASSIST** 1.14 What is the difference between descriptive and inferential statistics?

**PH Grade ASSIST** 1.15 What is the difference between a categorical variable and a numerical variable?

1.16 What is the difference between a discrete variable and a continuous variable?

1.17 What is an operational definition and why is it so important?

1.18 What are the four levels of measurement scales?

## Applying the Concepts

1.19 The Data and Story Library, [lib.stat.cmu.edu/DASL](http://lib.stat.cmu.edu/DASL), is an online library of data files and stories that illustrate the use of basic statistical methods. The stories are classified by method and by topic. Go to this site and click on **List all topics**. Pick a story and summarize how statistics were used in the story.

1.20 Go to the official Microsoft Excel Web site, [www.microsoft.com/office/excel](http://www.microsoft.com/office/excel). Explain how you think Microsoft Excel could be useful in the field of statistics.

1.21 The Gallup organization releases the results of recent polls at its Web site, [www.gallup.com](http://www.gallup.com). Go to this site and read today's top analysis.

- Give an example of a categorical variable found in the poll.
- Give an example of a numerical variable found in the poll.
- Is the variable you selected in (b) discrete or continuous?

1.22 The U.S. Census Bureau site, [www.census.gov](http://www.census.gov), contains survey information on people, business, geography, and other topics. Go to the site and click on **Housing** in the "People and Households" section. Then click on **American Housing Survey**.

- Briefly describe the American Housing Survey.
- Give an example of a categorical variable found in this survey.
- Give an example of a numerical variable found in this survey.
- Is the variable you selected in (c) discrete or continuous?

1.23 On the U.S. Census Bureau site, [www.census.gov](http://www.census.gov), click on **Survey of Business Owners** in the "Business & Industry" section and read about The Survey of Business Owners. Click on **Sample SBO-1 Form** to view a survey form.

- Give an example of a categorical variable found in this survey.
- Give an example of a numerical variable found in this survey.
- Is the variable you selected in (b) discrete or continuous?

1.24 An online survey of almost 53,000 people (N. Hellmich, "Americans Go for the Quick Fix for Dinner," *USA Today*, February 14, 2005, p. B1) indicated that 37% decide what to make for dinner at home at the last minute and that the amount of time to prepare dinner averages 12 minutes, while the amount of time to cook dinner averages 28 minutes.

- Which of the four categories of data sources listed in Section 1.4 on page 6 do you think were used in this study?
- Name a categorical variable discussed in this article.
- Name a numerical variable discussed in this article.

1.25 According to a Harris Interactive survey of 502 senior human resource executives, 58% responded that referrals were one of the methods for finding the best candidates. ("USA Snapshots," *USA Today*, February 9, 2006, p. A1).

- Describe the population for the Harris Interactive survey.
- Is a response to the question "By which methods do you feel you find the best candidates?" categorical or numerical?
- Fourteen percent of the senior human resources executives polled indicated that professional associations were one of the methods for finding the best candidates. Is this a parameter or a statistic?

1.26 A manufacturer of cat food was planning to survey households in the United States to determine purchasing habits of cat owners. Among the questions to be included are those that relate to

- where cat food is primarily purchased.
  - whether dry or moist cat food is purchased.
  - the number of cats living in the household.
  - whether the cat is pedigreed.
- Describe the population.
  - For each of the four items listed, indicate whether the variable is categorical or numerical. If it is numerical, is it discrete or continuous?
  - Develop five categorical questions for the survey.
  - Develop five numerical questions for the survey.

## Student Survey Data Base

**1.27** A sample of 50 undergraduate students answered the following survey.

1. What is your gender? Female \_\_\_ Male \_\_\_
2. What is your age (*as of last birthday*)? \_\_\_
3. What is your height (*in inches*)? \_\_\_
4. What is your current registered class designation?  
Freshman \_\_\_ Sophomore \_\_\_ Junior \_\_\_ Senior \_\_\_
5. What is your major area of study?  
Accounting \_\_\_ Economics/Finance \_\_\_  
Information Systems \_\_\_ International Business \_\_\_  
Management \_\_\_ Marketing/Retailing \_\_\_ Other \_\_\_  
Undecided \_\_\_
6. At the present time, do you plan to attend graduate school?  
Yes \_\_\_ No \_\_\_ Not sure \_\_\_
7. What is your current cumulative grade point average? \_\_\_
8. What would you expect your starting annual salary (*in \$000*) to be if you were to seek employment immediately after obtaining your bachelor's degree? \_\_\_
9. What do you anticipate your salary to be (*in \$000*) after five years of full-time work experience? \_\_\_
10. What is your current employment status?  
Full-time \_\_\_ Part-time \_\_\_ Unemployed \_\_\_
11. How many clubs, groups, organizations, or teams are you currently affiliated with on campus? \_\_\_
12. How satisfied are you with the student advisement services on campus? \_\_\_  
Extremely 1 2 3 4 5 6 7 Extremely  
unsatisfied Neutral satisfied
13. About how much money did you spend this semester for textbooks and supplies? \_\_\_

The results of the survey are in the file [undergradsurvey.xls](#).

- a. Which variables in the survey are categorical?
- b. Which variables in the survey are numerical?
- c. Which variables are discrete numerical variables?

**1.28** A sample of 50 MBA students answered the following survey:

1. What is your gender? Female \_\_\_ Male \_\_\_
2. What is your age (*as of last birthday*)? \_\_\_
3. What is your height (*in inches*)? \_\_\_
4. What is your current major area of study?  
Accounting \_\_\_ Economics/Finance \_\_\_  
Information Systems \_\_\_ International Business \_\_\_  
Management \_\_\_ Marketing/Retailing \_\_\_ Other \_\_\_  
Undecided \_\_\_
5. What is your graduate cumulative grade point index? \_\_\_
6. What was your undergraduate area of specialization?  
Biological Sciences \_\_\_ Business Administration \_\_\_  
Computers or Math \_\_\_ Education \_\_\_  
Engineering \_\_\_ Humanities \_\_\_ Performing Arts \_\_\_  
Physical Sciences \_\_\_ Social Sciences \_\_\_  
Other \_\_\_
7. What was your undergraduate cumulative grade point average? \_\_\_
8. What was your GMAT score? \_\_\_
9. What is your current employment status? \_\_\_  
Full-time \_\_\_ Part-time \_\_\_ Unemployed \_\_\_
10. How many different full-time jobs have you held in the past 10 years? \_\_\_
11. What do you expect your annual salary (*in \$000*) to be immediately after completion of the MBA program? \_\_\_
12. What do you anticipate your salary to be (*in \$000*) after five years of full-time work experience following the completion of the MBA program? \_\_\_
13. How satisfied are you with the student advisement services on campus?  
Extremely 1 2 3 4 5 6 7 Extremely  
unsatisfied Neutral satisfied
14. About how much money did you spend this semester for textbooks and supplies? \_\_\_

The results of the survey are in the file [gradsurvey.xls](#).

- a. Which variables in the survey are categorical?
- b. Which variables in the survey are numerical?
- c. Which variables are discrete numerical variables?

## End-of-Chapter Cases

At the end of most chapters, you will find a continuing case study that allows you to apply statistics to problems faced by the management of the *Springville Herald*, a daily

newspaper. Complementing this case are a series of Web Cases that extend many of the Using Statistics scenarios that begin each chapter.

## Learning with the Web Cases

*People use statistical techniques to help communicate and present important information to others both inside and outside their businesses. Every day, as in these examples, people misuse these techniques:*

- A sales manager working with an “easy-to-use” charting program chooses an inappropriate chart that obscures data relationships.
- The editor of an annual report presents a chart of revenues with an abridged *Y*-axis that creates the false impression of greatly rising revenues.
- An analyst generates meaningless statistics about a set of categorical data, using analyses designed for numerical data.

Identifying and preventing misuses of statistics, whether intentional or not, is an important responsibility for all managers. The Web Cases help you develop the skills necessary for this important task.

Web Cases send you to Web sites that are related to the Using Statistics scenarios that begin each chapter. You review internal documents as well as publicly stated claims, seeking to identify and correct the misuses of statistics. Unlike a traditional case study, but much like real-world situations, not all of the information you encounter will be relevant to your task, and you may occasionally discover conflicting information that you need to resolve before continuing with the case.

To assist your learning, the Web Case for each chapter begins with the learning objective and a summary of the problem or issue at hand. Each case directs you to one or more Web pages where you can discover information to answer case questions that help guide your exploration. If you prefer, you can view these pages by opening corresponding HTML files that can be found on this **Web Case** folder on the Student CD. You can find an index of all files/pages by opening the **SpringvilleCC.htm** file in the **Web Case** folder or by visiting the Springville Chamber of Commerce page, at [www.prenhall.com/Springville/SpringvilleCC.htm](http://www.prenhall.com/Springville/SpringvilleCC.htm).

### Web Case Example

To illustrate how to learn from a Web Case, open a Web browser and link to [www.prenhall.com/Springville/Good\\_Tunes.htm](http://www.prenhall.com/Springville/Good_Tunes.htm), or open the **Good\_Tunes.htm** file in the **WebCase** folder on the book’s CD. This Web page represents the home page of Good Tunes, the online retailer

mentioned in the Using Statistics scenario in this chapter. Recall that the privately held Good Tunes is seeking financing to expand its business by opening retail locations. Since it is in management’s interest to show that Good Tunes is a thriving business, it is not too surprising to discover the “our best sales year ever” claim in the “Good Times at Good Tunes” entry at the top of their home page.

The claim is also a hyperlink, so click on **our best sales year ever** to display the page that supports the claim. How would you support such a claim? with a table of numbers? a chart? remarks attributed to a knowledgeable source? Good Tunes has used a chart to present “two years ago” and “latest twelve months” sales data by category. Are there any problems with the choices made on this Web page? *Absolutely!*

First, note that there are no scales for the symbols used, so it is impossible to know what the actual sales volumes are. In fact, as you will learn in Section 2.6, charts that incorporate symbols in this way are considered examples of *chart junk* and would never be used by people seeking to properly use graphs.

This important point aside, another question that arises is whether the sales data represent the number of units sold or something else. The use of the symbols creates the impression that unit sales data are being presented. If the data are unit sales, does such data best support the claim being made—or would something else, such as dollar volumes—be a better indicator of sales at Good Tunes?

Then there are those curious chart labels. “Latest twelve months” is ambiguous; it could include months from the current year as well as months from one year ago and therefore may not be an equivalent time period to “two years ago.” Since the business was established in 1997, and the claim being made is “best sales year ever,” why hasn’t management included sales figures for *every* year?

Is Good Tunes management hiding something, or are they just unaware of the proper use of statistics? Either way, they have failed to properly communicate a vital aspect of their story.

In subsequent Web Cases, you will be asked to provide this type of analysis, using the open-ended questions of the case as your guide. Not all the cases are as straightforward as this sample, and some cases include perfectly appropriate applications of statistics.



## REFERENCES

1. McCullough, B. D., and B. Wilson, "On the Accuracy of Statistical Procedures in Microsoft Excel 97," *Computational Statistics and Data Analysis*, 31 (1999), 27–37.
2. McCullough, B. D. and B. Wilson, "On the Accuracy of Statistical Procedures in Microsoft Excel 2003," *Computational Statistics and Data Analysis*, 49, (2005), 1244–1252.
3. *Microsoft Excel 2007* (Redmond, WA: Microsoft Corporation, 2007).
4. Nash, J. C., "Spreadsheets in Statistical Practice—Another Look," *The American Statistician*, 60, (2006), 287–289.

# Excel Companion

## to Chapter 1

This Excel Companion serves as a primer for Microsoft Excel, helping you become familiar with the concepts and commands that everyday use of Microsoft Excel requires. Before you intensely study this Companion, you should first consider how you plan to use Excel as you learn statistics. If you skipped reading the “From the Authors’ Desktop: Using and Learning Microsoft Excel” on page 5, you may want to review it now so that you have a better understanding of the choices you have in using Excel with this text.

How you plan to use Excel will affect which Excel skills you *immediately* need to know. If you plan to use the Basic Excel instructions in later companions, you should know, or at least have some awareness of, just about every skill discussed in the rest of this Companion. If you plan to use PHStat2, you will have a less immediate need for the skills related to worksheet entries and formulas discussed in Sections E1.3, E1.4, and E1.5. (You can master such skills later, as you read through this book.)

The rest of this companion presents skills in an increasing order of difficulty. Make sure you have mastered the skills presented in the first sections before going on to the later sections. If you consider yourself an experienced Excel user, you might want to take the time to scan this companion if only to become familiar with the terms used throughout the book to describe Excel objects and operations.

### E1.1 PRELIMINARIES: BASIC COMPUTING SKILLS

If you have ever surfed the Web, sent an instant message, played music or games, or written word-processed assignments, you have already mastered the skills necessary in order to use Microsoft Excel. However, if you are new to computing, you should use the following countdown of skills—on your mark, get ready, get set, go—to master the basic computing skills needed with this book. (If you are an experienced computer user, you will want to skim this section and note the definitions of the boldfaced terms.)

#### On Your Mark!

Like many other programs, Microsoft Excel makes frequent use of a keyboard and a mouse-type pointing device. Microsoft Excel expects your pointing device to have two

buttons: a primary button (typically the left button) and a secondary button (typically the right). You move your pointing device and use one of the buttons to execute one of these six basic operations<sup>1</sup>:

**Click** Move the mouse pointer over an object and press the primary button. You click links on a Web page and many of the user interface elements identified in the “Get Set!” part of this section. The book also uses the verb **clear** when telling you to click on a check box to remove its check mark.

**Select** Similar to click, but when you press the primary button, another list of menu commands or choices appear. You click the Microsoft Windows Start menu and then select Programs or All Programs (depending on the version) to display a list of programs and program folders installed on your system.

**Double-click** Move the **mouse pointer** over an object and click the primary button twice in rapid succession. You double-click Desktop program icons to open programs, and you double-click the icons that represent files to open and use those files.

**Right-click** Move the mouse pointer over an object and click the secondary button. You typically right-click an object to reveal a **shortcut menu** of commands that apply to that object.

**Drag** A multipart mouse operation. First, you move the mouse pointer over an object and then, while pressing and holding down the primary button, you move the mouse pointer somewhere else on the screen and release the primary button. You use drag to resize windows and, in Microsoft Excel, to select a group of adjacent worksheet cells.

**Drag-and-drop** A multipart mouse operation. First, you move the mouse pointer over an object and then, while pressing and holding down the primary button, you move the mouse pointer over another onscreen object and release the primary button. Drag-and-drop has many applications, but in this text, you mostly use this operation when defining PivotTables (discussed in the Excel Companion to Chapter 2).

<sup>1</sup>Alternate methods of interaction, such as using speech recognition input, are possible on suitably equipped systems. If you are using an alternate method, make sure you know the equivalents to the mouse operations defined in this section.

If these operations are new to you, you can practice them by opening the **Mousing Practice.xls** file on the Student CD. (If you do not know how to open a file, ask a friend or use the instructions available in Section E1.2.) Otherwise, you are ready to advance to the “Get Ready!” stage.

## Get Ready!

When you start Microsoft Excel, you see a window that contains the Excel user interface and a workspace area that displays open workbooks. If you start any Excel version other than 2007, you will see a window very much like the one shown in Figure E1.1, the actual window for Excel 2003. If you start Excel 2007, you will see the very different-looking window, similar to the one shown in Figure E1.2 on page 20.

In designing Excel 2007, Microsoft tried to minimize the find-and-seek process that many users of earlier versions experience trying to find a particular command or Excel feature. Generally, Excel 2007 displays all relevant commands for a particular task at the top of the window. If you are new to Microsoft Excel and have a choice of using Excel 2007 or an earlier version, you will most likely benefit from choosing Excel 2007 because displaying all relevant commands lessens the initial training you need. However, Excel 2007 is functionally equivalent to earlier Excel versions, and every Excel activity in this text can be done with any version of Excel, starting with Excel 97, although sometimes there are special instructions specific to Excel 2007.

## Get Set!

To get started using Microsoft Excel, you need to be familiar with the objects you will commonly see when using

Excel. When opening Excel, you see a window that is either similar to Figure E1.1 if you use Excel 97, 2000, 2002, or 2003<sup>2</sup> or Figure E1.2 if you use Excel 2007. (Should your Excel window not contain some of the elements shown in the figure appropriate for the Excel version you use, see the “Microsoft Excel FAQs” section in Appendix F. As you begin using Excel, you will encounter dialog boxes, special windows that display messages to allow you to make entries or selections. Because you will be frequently interacting with dialog boxes, you should be familiar with the objects you will commonly see in these special windows (see Figure E1.3 on page 20).

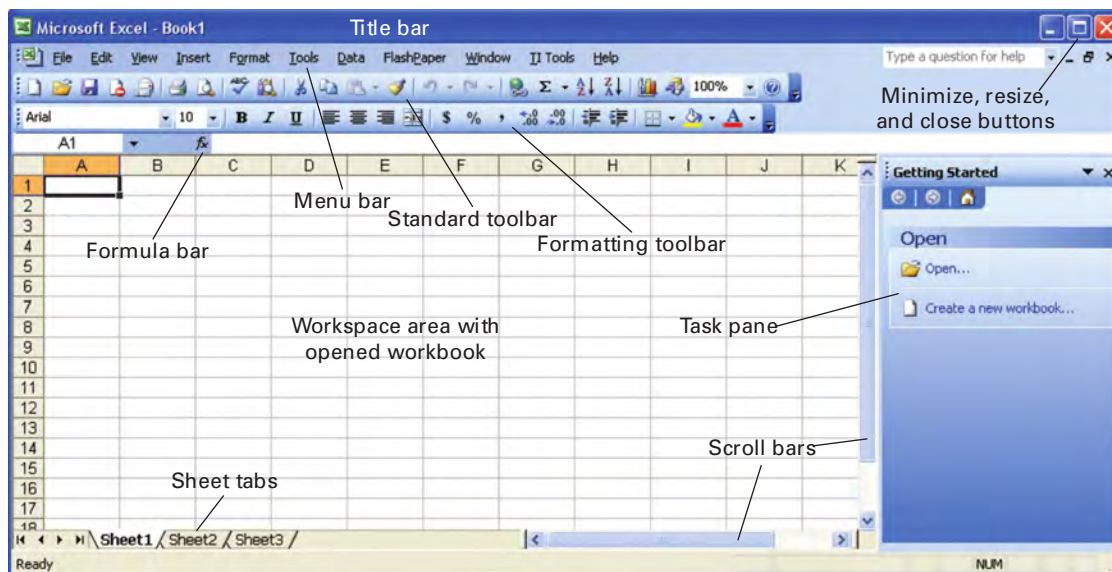
The rest of this section defines the objects labeled in Figures E1.1, E1.2, and E1.3. You should be familiar with the objects that apply to the Excel version you use before continuing to the next section.

### Excel Window Elements (all versions)

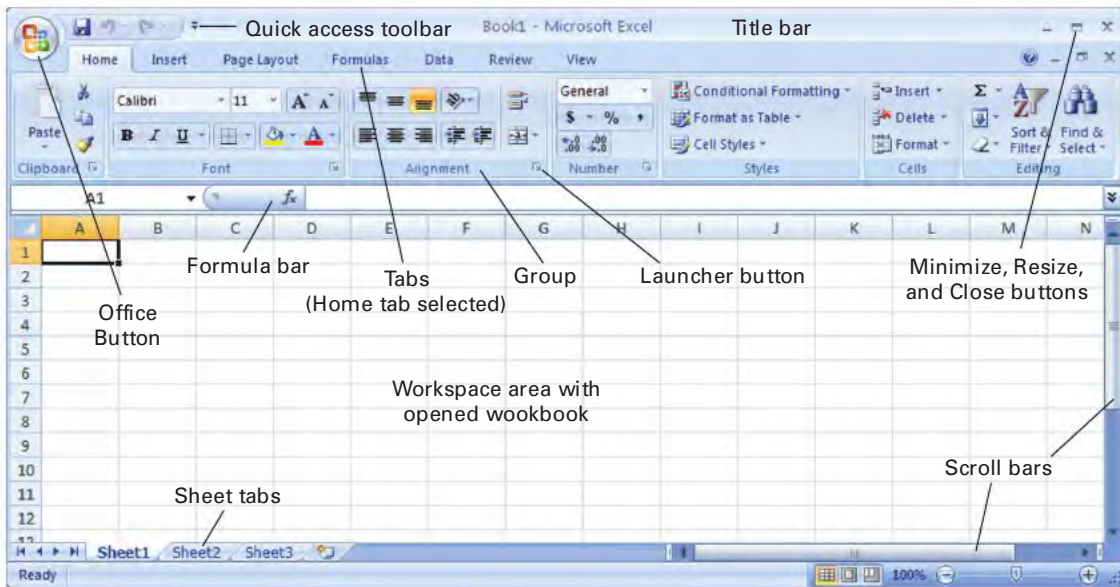
**Minimize, resize, and close buttons** minimize (that is, hide without closing), resize, and close windows. When you click the sets of labeled buttons in Figures E1.1 and E1.2, you affect the Excel window itself; other, similar buttons operate on other elements in the Excel window, such as the currently opened workbook.

**Workspace area** displays the currently opened workbook or workbooks. (Although you will not need to open more than one workbook at a time when using this text, you can open multiple workbooks and view all of them by resizing them to fit in the workspace area.)

<sup>2</sup>In the remainder of the book, the labels “Excel 97–2003” or “97–2003” are used when collectively referring to one of these Excel versions.



**FIGURE E1.1** The Excel 2003 window



**FIGURE E1.2** The Excel 2007 window

**Formula bar** shows the name of the currently selected worksheet cell (see Section E1.3) and the current contents of that cell.

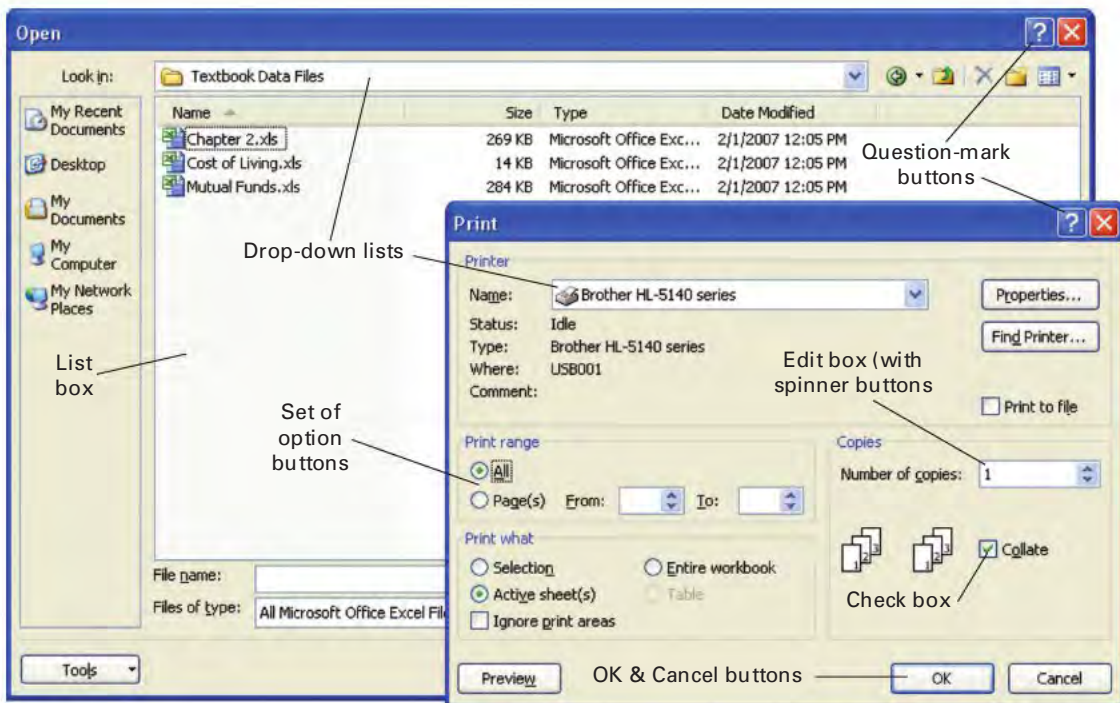
**Sheet tabs** display the name of each sheet in the opened workbook. You click a sheet tab to select a sheet and make it the currently active sheet. You double-click a sheet tab to rename the sheet.

**Scroll bars** allow you to travel horizontally or vertically through parts of a worksheet that are offscreen (for example, row 100 or column T in Figures E1.1 or E1.2).

**Title bar** displays the name of the currently active workbook and contains the minimize, resize, and close buttons for the Excel window. You drag the title bar to reposition the Excel window on your screen.

**Additional Window Objects (Excel 97–2003)**

**Menu Bar** The horizontal list of words at the top of the window that represent sets of commands. You click a menu bar word and pull down lists of command choices, some of which lead to further menu choices.



**FIGURE E1.3** Commonly encountered elements

**Standard Toolbar** The toolbar that contains shortcuts to many file-oriented commands, including the common workbook operations discussed in Section E1.4.

**Formatting Toolbar** The toolbar that contains shortcuts to many common worksheet cell formatting commands (see Section E1.4).

**Task Pane** A closable window that contains clickable links that represent shortcuts to menu and toolbar operations (Excel 2002 and 2003 only).

### Additional Windows Objects Excel 2007

**Office Button** displays a menu of commonly issued commands similar to the File menu in Excel 97–2003. The Office Button also gives you access to many Excel options settings.

**Quick Access Toolbar** (to the right of the office button) Displays buttons that are shortcuts to commonly used commands. When you first see Excel 2007, the **Save**, **Undo**, and **Redo** buttons are displayed, but you can add or remove buttons in this toolbar (see “Microsoft Excel 2007-Specific FAQs” in Appendix F to add or remove buttons.)

**Tabs** Displays groups of commands and features associated with a single type of Excel task. The **Home Tab** (seen in Figure E1.2) displays all commands and features associated with making worksheet cell entries.

**Tab Groups** Display named collections of related commands and features. Some tab groups, such as the Font, Alignment, and Number groups shown in Figure E1.2 contain **launcher buttons** that open related dialog boxes or task panes.

**Contextual Tabs** Additional tabs that appear only when you are doing a specific task (that is, working in a specific “context”), such as creating charts. Contextual tabs are displayed with a title (for example, “Chart Tools”) that appears in the title bar.

**Ribbon** is the collective name for the tab-and-group user interface exclusive to Excel 2007. (See “Microsoft Excel 2007-Specific FAQs” in Appendix F if your Excel 2007 window does not display the ribbon.)

### Common Dialog Box Objects

**List boxes** Display lists of choices available to you. Should a list exceed the dimensions of a list box, you will see **scroll buttons** and a **slider** that you can click in order to see the other choices available.

**Drop-down lists** Display lists of commands or choices when you click over them. Many drop-downs in Excel 2007 display **galleries**, which are illustrated (and sometimes annotated) sets of choices.

**Edit boxes** Areas into which you type entries. Some edit boxes also contain drop-down lists or **spinner buttons** that you can use to complete the entry. Cell range edit boxes typically include a button that allows you to point to a cell range instead of typing the range.

**Option buttons** Present a set of mutually exclusive choices. When you click one option button, all the other option buttons in the set are cleared.

**Check boxes** Present optional actions that are not mutually exclusive choices. Unlike with option buttons, clicking a check box does not affect the status of other check boxes, and more than one check box can be checked at a time. If you click an already checked check box, you clear the check from the box.

**OK buttons** (in dialog boxes) Allow you to execute tasks using the current values and settings of the currently displayed dialog box. Sometimes the “OK button” will have a different legend, such as Finish, Create, Open, or Save.

**Cancel buttons** Close dialog boxes and cancel operations represented by the dialog boxes. In most contexts, clicking the Cancel button is equivalent to clicking the Close button in the title bar of the dialog box.

**Question-mark buttons** Display Excel help messages. Many dialog boxes contain a button with the legend **Help** that also displays help messages.

## Go!

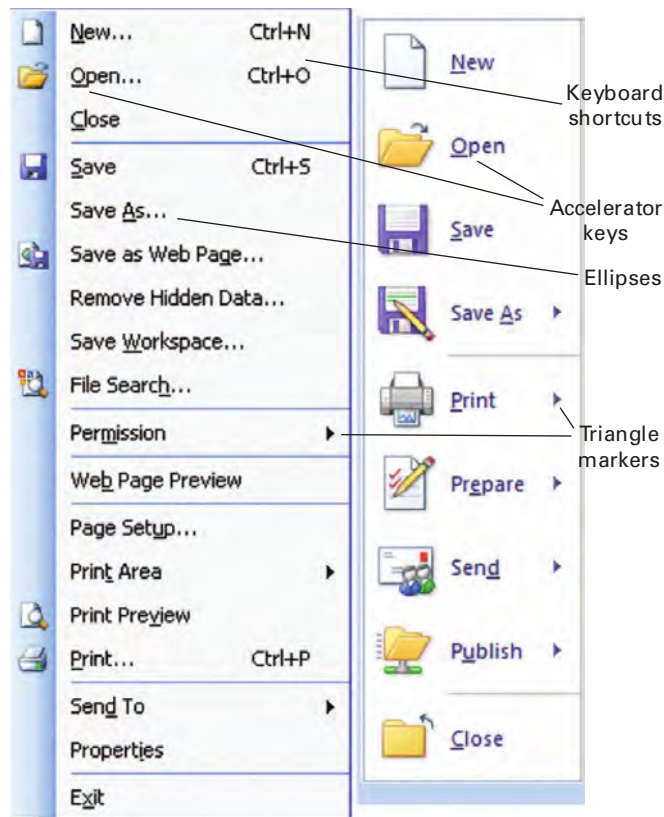
To start using Microsoft Excel, you need to understand the conventions used in Excel menus and be familiar with commonly used special keys and keystroke combinations.

Excel menus use these conventions (labeled in Figure E1.4):

- An underlined letter in a menu choice represents an **accelerator key**, a keystroke that is equivalent to selecting the choice with your mouse.
- An **ellipsis** indicates that when you select a menu choice, a dialog box will appear.
- A triangle marker indicates that when you select that menu choice, you will see either another menu of choices or a gallery of choices.

You can execute a number of frequently used menu choices or Excel operations by using a **combination keystroke**—that is, holding down one or more keys while pressing another key. In Excel versions other than Excel 2007, many of these combinations appear next to the menu choices they represent. You can print your currently active worksheet by pressing the combination **Ctrl+P** (that is, while holding down the **Ctrl** key, press the **P** key) and save your currently active workbook by pressing **Ctrl+S**. You can copy (or cut) and paste worksheet cell entries by pressing **Ctrl+C** (or **Ctrl+X**) and **Ctrl+V**.

You can use some of the special keys to execute frequently used operations. When you type an entry, pressing the **Escape** key usually cancels that entry. Pressing the **Backspace** key when typing an entry erases typed characters



**FIGURE E1.4** Excel menu conventions

to the left of the cursor, one character at a time. Pressing the **Delete** key when typing erases characters to the right of the cursor, one character at a time. Pressing either the **Enter** or **Tab** keys finalizes a typed worksheet cell entry.

When viewing a dialog box, pressing **Escape** usually cancels its operation. After clicking a workbook object such as a chart, pressing **Delete** deletes that object. If you get stuck while using Excel, you can move the mouse pointer over an object and then pause to see if Excel displays a **Tool Tip**, a pop-up help message, about the object. At any time, you can also press the **F1** key to display either a help message or help search box.

## Textbook Conventions

In this book, dialog box objects are usually referred to by their names or labels. You will find instructions such as “Click **Labels**” or “Click **OK**” when referring to check boxes and command buttons. You will find instructions such as “Select the **Lower-Tail** option” when referring to option boxes or drop-down lists. When object names or labels can vary due to context, the book uses italics, as in “Select *variable name*.”

To describe a sequence of menu (Excel 97–2003) or ribbon (Excel 2007) choices, the book uses an arrow sym-

bol to link selections. For example, in Excel versions other than 2007, “select **Tools** → **Data Analysis**” means that you would first select **Tools** from the Excel menu bar and then select **Data Analysis** from the submenu of choices that appears after you select **Tools**. The equivalent Excel 2007 sequence “select **Data** → **Data Analysis**” means select the Formulas tab and then select **Data Analysis** (from the Solutions group).

## E1.2 BASIC WORKBOOK OPERATIONS

As you work with Microsoft Excel, you will need to open workbooks to use data and results created by you or others at an earlier time. You will also need to save workbooks to ensure their future availability and to protect yourself against any computer system failures that might occur as you work with Excel. You may also need to create new workbooks and may want to print out individual sheets for your workbooks for later study or use in projects and assignments.

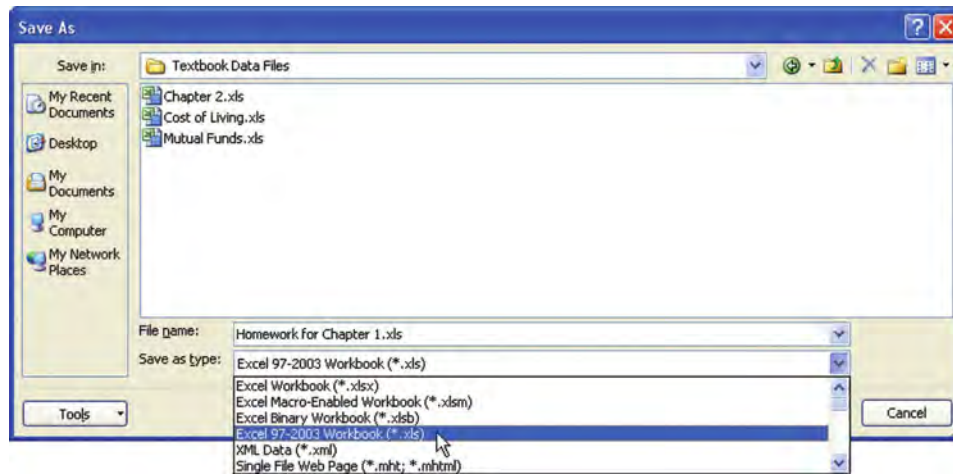
In all Excel versions, including Excel 2007, these operations involve dialog boxes that differ only in minor ways. While the dialog box of the version you are using may subtly differ from the ones shown in Figure E1.3 (see page 21), the instructions in this section apply to all Excel versions, except when otherwise noted.

### Opening and Saving Workbooks

You open and save workbooks by selecting the storage folder to use and then specifying the file name of the workbook. You begin the process by selecting **File** from the Excel menu bar in Excel 97–2003 or by clicking the **Office Button** in Excel 2007. In either case, a menu of commands appears, as partially shown in Figure E1.4. While the contents of the menus differ (the Excel 2007 menu is on the right in Figure E1.4), they both have **Open** and **Save As** choices. These choices lead to similar dialog boxes, shown in Figure E1.3 and Figure E1.5.

You select the storage folder using the drop-down list at the top of these dialog boxes. You enter (or select from the list box) a file name for the workbook in the **File name** box. You click **Open** (obscured in Figure E1.3) or **Save** to complete the task.

Sometimes when saving files, you will want to change the file type before you click **Save**. If you use Excel 2007 and want to save your workbook in the format used by earlier Excel versions, you select **Excel 97-2003 Workbook (\*.xls)** from the **Save as type** drop-down list before you click **Save** (shown in Figure E1.5). If you use any version of Excel and want to save data in a form that can be opened by programs that cannot open Excel workbooks, you might select either **Text (Tab delimited) (\*.txt)** or **CSV (Comma delimited) (\*.csv)** as the save type.



**FIGURE E1.5** Save As dialog box (Excel 2007 version)

Likewise, if you are opening a data file that is not an Excel workbook format, you can change the file type (labeled as **File of type** in Figure E1.3). If you cannot find a file that you are sure is in the current Look in folder, changing the file type to **All Files (\*.\*)** can reveal an inadvertent misspelling or missing file extension (the part of the file name after the period) that otherwise prevents the file from being displayed.

Although all versions of Microsoft Excel include a **Save** command, you should avoid this choice until you gain experience. Using **Save** makes it too easy for you to inadvertently overwrite your work, and in Excel 2007, it saves your workbook in the new **.xlsx** workbook format that cannot be used by Excel 97–2003. In contrast, using **Save As** always presents you with an opportunity to name your file and choose its file type, and using **Save As** is also the simplest way you can create a backup copy of your workbook as you work.

If you open a workbook from a nonmodifiable source, such as a CD-ROM, Excel marks the workbook “read-only.” You must use **Save As** to save a modified version of such a workbook, which is another good reason for always using this command.

## Creating New Workbooks

You create a new workbook through a straightforward process that varies depending on the version of Excel you are using. In Excel 97 or 2000, you select **File → New**. In Excel 2002 (also known as Excel XP) or 2003, you select **File → New** and then click **Blank workbook** in the New Workbook task pane. In Excel 2007, you click **Office button → New** and in the New Workbook dialog box, you first click **Blank workbook** and then **Create**.

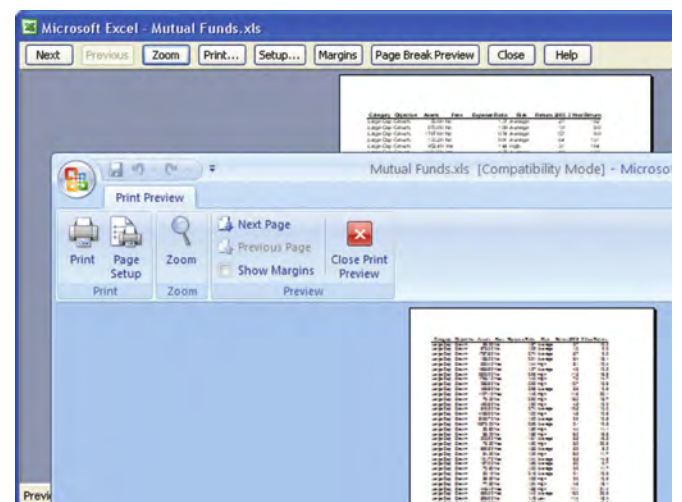
New workbooks are created with a fixed number of worksheets. You can delete extra worksheets or insert more sheets by right-clicking a sheet tab and clicking either **Delete** or **Insert**.

## Printing Worksheets

When you want to print the contents of a workbook, you should print one sheet at a time to get the best results. You print sheets by first previewing their printed form onscreen and then making any adjustments to the worksheet and/or to the print setup settings.

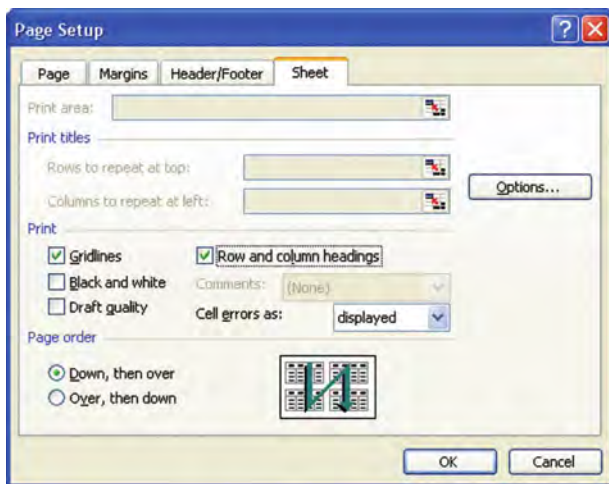
To print a specific worksheet, you first click on the sheet tab of that worksheet to make the worksheet the currently active one. Then you display the Print Preview window. If you use a version of Excel other than 2007, select **File → Print Preview**. If you use Excel 2007, click **Office Button**, move the mouse pointer over **Print** (do not click) and select **Print Preview** from the Preview and Print gallery.

The Print Preview windows for all Excel versions are similar to one another. Figure E1.6 shows a partial window for Excel 2003 (top) and Excel 2007 (bottom). If the preview contains errors or displays the worksheet in an unde-



**FIGURE E1.6** Partial Print Preview windows (Excel 2003 and 2007 versions)

sirable manner, click **Close** (or **Close Print Preview** in Excel 2007), make the changes necessary, and reselect the preview command. You can customize your printout by clicking **Setup** (or **Page Setup**) and making the appropriate entries in the Page Setup dialog box, which is similar for all Excel versions. For example, to print your worksheet with grid lines and numbered row and lettered column headings (similar to the appearance of the worksheet onscreen), you click the **Sheet** tab in the Page Setup dialog box and then click **Gridlines** and **Row and column headings** and click **OK** (see Figure E1.7). (You can find more information about using Page Setup in Section F4 of Appendix F.)



**FIGURE E1.7**

Sheet tab of the Page Setup dialog box (Excel 2003 version)

When you are ready to print, you can click **Print** in the Print Preview window, but to gain maximum control over your printout, click **Close** (or **Close Print Preview**) and then select **File** → **Print** (Excel 97–2003) or **Office Button** → **Print** (Excel 2007). In the Print dialog box that appears (see Figure E1.3 on page 20), you can select the printer to use, make sure you are printing only the currently active worksheet, or print multiple copies at once.

## E1.3 WORKSHEET ENTRIES

As first discussed in Section 1.6 (see page 11), you make entries into worksheet cells, the intersections of lettered columns and numbered rows. You use the cursor keys or your pointing device to move a **cell pointer** through a worksheet and to select a cell for entry. As you type an entry, it appears in the formula bar (see Figures E1.1 and E1.2 on pages 19 and 20), and you place that entry into the

cell by either pressing **Tab** or **Enter** or clicking the checkmark button in the formula bar.

You enter individual numeric and label (sometimes called text) values into cells. You can also enter **formulas**, which are instructions to perform a calculation or some other task. Usually, formulas use values found in other cells to produce a displayed result. Formulas can automatically change the displayed result when the values in the supporting cells change.

To refer to a cell in a formula, you use a cell address in the form *Sheetname!ColumnRow*. For example, *Data!A2* refers to the cell in the Data worksheet that is in column A and row 2. You can also use just the *ColumnRow* portion of a full address, for example *A2*, if you are referring to a cell on the same worksheet as the one into which you are entering a formula.

Sometimes you need to refer to a group of cells, called a **cell range**. If the group of cells forms a rectangular area—for example, a group composed of cells from two adjacent columns—you use an address in the form *Sheetname!Upperleftcell:Lowerrightcell*. For example, *Data!A1:B10* refers to the 20 cells that are in rows 1 through 10 in columns A and B of the Data worksheet. If your group forms two or more rectangular areas, you enter the range as a list of rectangular areas separated by commas, for example, *Data!A1:A10,Data!C1:C10*. If the name of the sheet contains spaces or special characters, such as “City Data” or “Figure-1.2”, you must enclose the sheet name in a pair of single quotes, as in ‘City Data’!A1:A10 or ‘Figure-1.2’!A1:A10.<sup>3</sup>

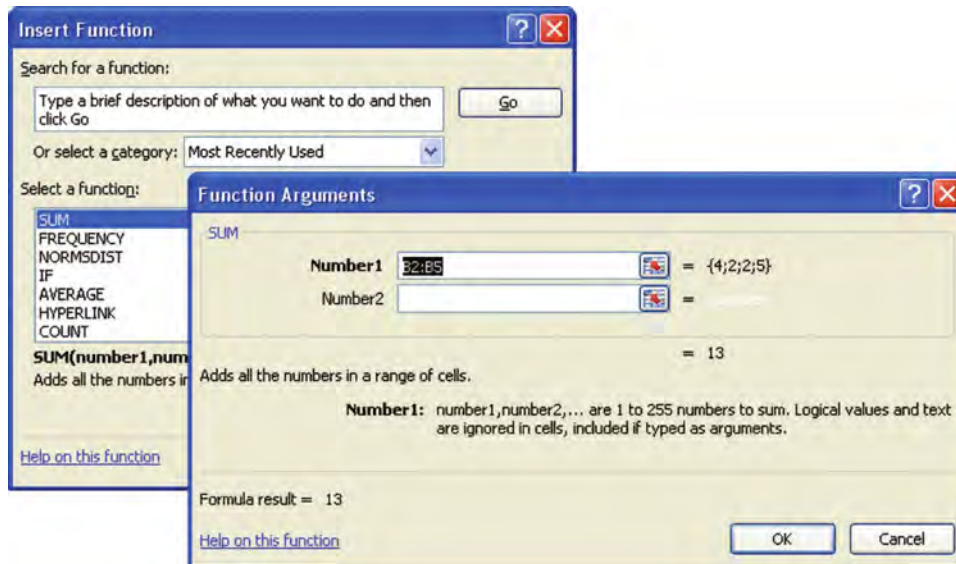
(Because you use names of sheets in formulas, you may want to rename sheets from their default names. As mentioned in Section E1.1, you can rename a sheet by double-clicking the **sheet tab** for the sheet, typing a new, more descriptive name, and then pressing **Enter**.)

## Entering Formulas

You enter formulas by typing the equal sign (=) followed by some combination of mathematical or other data processing operations. For simple formulas, the symbols +, −, \*, /, and ^ are used for the operations addition, subtraction, multiplication, division, and exponentiation (a number raised to a power), respectively. For example, the

<sup>3</sup>For cases in which you need to distinguish between two similarly located cells on two similarly named worksheets in two different workbooks, you use an address in the form *[Workbookname]Sheetname!ColumnRow*. For example, you use *[Chapter1]Data!A1* to refer to the first cell on the Data worksheet in the Chapter1 workbook.





**FIGURE E1.8** Insert Function dialog box of the Function Wizard

formula = **Data!B2 + Data!B3 + Data!B4 + Data!B5** adds the contents of cells B2, B3, B4, and B5 of the Data worksheet and displays the sum as the value of the cell containing the formula. You can also use **worksheet functions** to simplify formulas. For example, the formula =**SUM(Data!B2:B5)** uses the worksheet **SUM** function to create a shorter equivalent to the example in this paragraph.

When you are using worksheet functions, you can use the Function Wizard to minimize your typing. To use this wizard, which is similar in all Excel versions, you select **Insert → Function** (Excel 97–2003) or **Formulas → Function Wizard** (Excel 2007) and then make entries and selections in one or more dialog boxes. For example, to enter the formula =**SUM(Data!B2:B5)** using this wizard, you select the function **SUM** and click **OK** in the Insert Function dialog box (obscured in Figure E1.8). You then type or point to the cell range **B2:B5** in the **Number1** box and click **OK** in the Function Argument dialog box (also shown in Figure E1.8). The wizard then enters the completed formulas in the currently active cell.

As Figure E1.8 shows, the Function Wizard also previews the results of the **SUM** function (13) and displays the contents of the range B2:B5 (4, 2, 2, and 5).

## Verifying Formulas

Whether you enter formulas on your own in a new worksheet, open a workbook that contains formulas, or use commands that add formulas to worksheets, you should review and verify formulas before you use their results. To view the formulas in a worksheet, press **Ctrl+`** (backtick key).

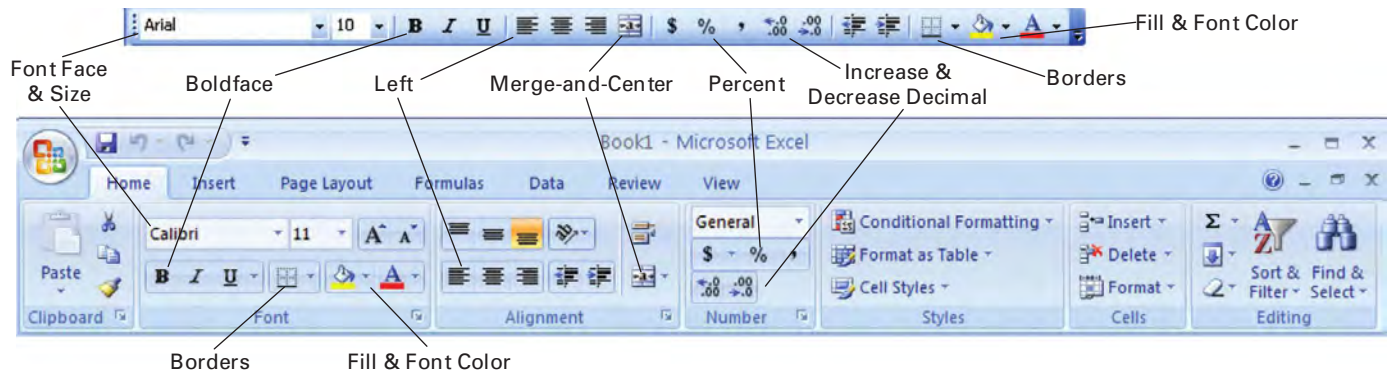
To restore the original view, the results of the formulas, press **Ctrl+\** a second time.

As you create and use more complicated worksheets, you may want to visually examine the relationships among a formula and the cells it uses (called the precedents) and the cells that use the results of the formula (the dependents). To display arrows that show these relationships, use the formula auditing feature of Excel. For Excel 97 or 2000, select **Tools → Auditing** and for Excel 2002–2003, select **Tools → Formula Auditing**. Then select one of the choices on the **auditing** submenu. To use this feature in Excel 2007, select **Formulas** and then select one of the choices from the **Formula Auditing** group. The **Remove All Arrows** choice restores your display by removing all auditing arrows when you are finished auditing the formulas.

## E1.4 WORKSHEET FORMATTING

You can use various formatting commands to enhance the appearance of your worksheets. If you use Excel 97–2003, you will find many of the common formatting operations on the Formatting Toolbar (see Figures E1.1 and E1.9) and the rest inside the Format Cells dialog box (select **Format → Cells** to view this dialog box). If you use Excel 2007, most formatting operations are visible on the Home tab (see Figures E1.2 and E1.9), and the rest are available through the launcher buttons of several groups or the **Format** choice of the Cells group.

Use Figure E1.9 as a visual guide for locating the common formatting operations.



**FIGURE E1.9** Formatting Toolbar (Excel 97–2003) and Home tab groups (Excel 2007)

## Common Formatting Operations

**Font Face & Size** Drop-down lists set the font face and font size for the currently selected cells. You will get best results if you select entire rows before changing the font face or size.

**Boldface** Displays the values of the currently selected cells, using boldface type. Consider using this for cells that contain titles, column headings, or important results. You can also use the **Italic** and **Underline** buttons to the immediate right of Boldface for additional type effects.

**Left** Displays the values of the currently selected cells left-justified in their cells. The **Center** and **Right** buttons next to Left, center, and right-justify values, respectively. In a worksheet with many filled columns, using Center for the values of categorical variables can sometimes improve readability.

**Merge-and-Center** Combines the currently selected cells, merges them into one, and displays the value in the first cell of the group, centered across the merged cells. Consider using this for a worksheet title or a title that applies to several contiguous rows or columns.

**Percent** Displays numeric values in the currently selected cells as percentages. The value 0.01 displays as 1%, the value 1 as 100%, and the value 100 as 10000%.

**Increase Decimal and Decrease Decimal** Increases/decreases the number of decimal places that will be used to display numeric values in the currently selected cells. Particularly useful to align the decimal points in a column of numeric values.

**Borders** Drops down a gallery of border effects from which you can choose to change the borders for the currently selected cells. Many worksheet examples in this text use one or more of these border effects: **All Borders**, **Outside Borders**, and **Top Border**.

**Fill Color and Font Color** Drop down a gallery from which you can choose to change the cell background color and the type color for the currently selected cells. Many

worksheet examples in this text use the color “Light Turquoise” to tint areas that contain user-changable data values and the color “Light Yellow” to tint areas that contain results. These colors are available in the Excel 2002–2003 galleries and by clicking **More Colors** in the Excel 2007 gallery.

In addition to these operations, you may want to adjust the width of a worksheet column so that its column heading and all its values are clearly visible. To do this, select the entire column and then select **Format** → **Column** → **AutoFit Selection** (Excel 97–2003) or **Home** → **Format** (in the Cells group) → **AutoFit Selection** (Excel 2007).

## E1.5 COPY-AND-PASTE OPERATIONS

There will be times that you will want to copy cell entries to another part of the same worksheet or to another worksheet as well as copy an entire worksheet for insertion into another workbook. Copying and pasting cell entries entirely composed of numeric or text values is fairly straightforward. You select the cells to be copied, press **Ctrl+C**, move to the first cell of the range in which you want to paste the copy, and press **Ctrl+V**. However, often you will copy cells that contain one or more formulas. Copying cells that contain formulas is not necessarily as straightforward as you might expect. Likewise, copying cells or entire worksheets between workbooks raises some issues you need to understand in order to get the best results possible.

### Copying Formulas

Copying entries that contain formulas requires extra attention because exact duplicates may or may not result, depending on how you have entered cell addresses. If you entered addresses using the *Sheetname!ColumnRow* or *ColumnRow* form, as introduced in Section E1.3, Excel

considers them **relative references** that will change to reflect the difference, or offset, between the original (source) cell and the cell into which you are pasting the formula (the target cell). For example, when you copy the formula  $=A2 + B2$  in cell C2 down to cell C3, an offset of one row, Excel pastes the formula  $=A3 + B3$  in C3 to reflect that one-row offset. Cell ranges also get changed, so if you copy the formula  $=SUM(A1:A4)$  from cell A5 to cell B5, the formula is changed to  $=SUM(B1:B4)$ .

You typically want Excel to make these changes when you are copying a formula down a column or across a row. This allows you to enter the formula once and then use copy-and-paste to fill in the similar formulas in the column or row. Sometimes, especially in cases in which you want to copy and paste a single formula, you may not want Excel to make a change. You can stop Excel from making changes in the column or row offset by inserting a dollar sign (\$) before either the column letter or row number (or both) of a cell address. Addresses written with inserted dollar signs, such as  $\$A\$2$ , are called **absolute references** and do not change during a copy operation. For example, if you copy the formula from one cell to another, the formula that appears in the target cell will be the same,  $=\$A\$2 + \$B\$2$ . You can also use addresses such as  $\$A2$  or  $A\$2$ , if you want Excel to change only the row or column, respectively. For example, when you copy the cell C2 formula  $=A2/B\$10$  to cell C3, the formula that appears in C3 is  $=A3/B\$10$ . However, if you copy the cell C2 formula to cell D3, the formula that would appear is  $=B3/C\$10$ .

Do not confuse the use of the dollar sign with formatting cell values for currency display. To format cell values for currency display (in dollars and cents, in the U.S. version of Microsoft Excel), you use the **Currency (\$)** format button, located to the left of the Percent button in either the Formatting toolbar or the Home tab group (see Figure E1.9).

## Copying Formulas Between Worksheets

When you copy and paste formulas between worksheets, you generally want to make sure that all cell addresses contain a worksheet name and are absolute references—for example,  $Data!\$A\$1:\$A\$12$ —in order to ensure consistent results.

If you only need to transfer the calculated results from one sheet to another (for example, to create your own summary or report worksheet), you should consider using one of two methods to transfer information between worksheets. If you need to transfer only one or a few cells' worth of information, consider entering formulas in the form  $=Source\sheetname!Cell\reference$  to transfer the information into the second worksheet. For example, if you want to transfer the results of a formula found in cell B10 of the

worksheet named Results to cell A5 in the worksheet named Summary, you can enter the formula  $=Results!B10$  in cell A5 of the Summary sheet instead of copying the original formula. You can also use this technique to transfer column headings or other labeling information between worksheets in order to maintain consistency across worksheets.

If you need to transfer information from a large range of cells, you should consider using the Paste Special command. To use this command, you first select the cell range to be copied and press **Ctrl+C**. Then select the first cell of the cell range in the second worksheet that is your target and right-click. Next, select **Paste Special** from the shortcut menu that appears. In the Excel Paste Special dialog box (similar for all Excel versions; Figure E1.10 shows the Excel 2003 dialog box), you select the **Values and number formats** option and click **OK**. Selecting this option places the current values of all formulas in the second worksheet, so if you change the underlying data, you need to repeat this procedure to update the values shown on the second worksheet.



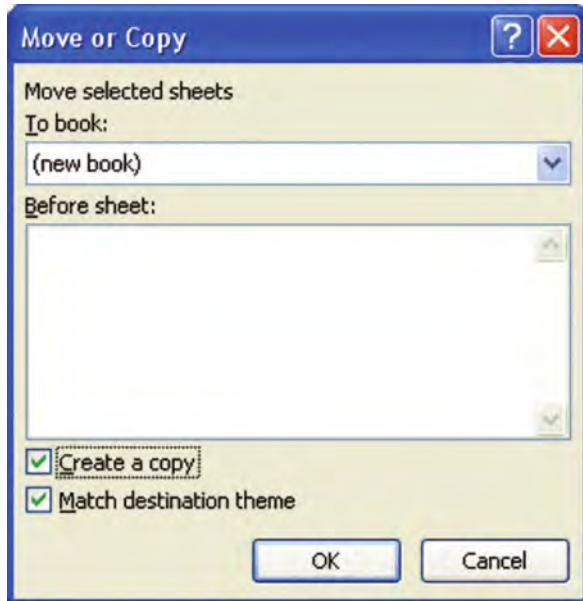
**FIGURE E1.10**

Paste Special dialog box (Excel 2003 version)

## Copying Worksheets

You can copy worksheets to a new workbook or to any opened workbook, including the source of the copy. To copy a worksheet to a new workbook, you first select the worksheet by clicking its sheet tab. Then you right-click the tab and select **Move or Copy** from the shortcut

menu that appears. In the Move or Copy dialog box (similar for all Excel versions; Figure E1.11 shows the Excel 2007 dialog box), select **(new book)** from the **To book** drop-down, click **Create a copy**, and click **OK**.



**FIGURE E1.11**  
Move or Copy dialog box (Excel 2007 version)

You use the same procedure, changing only your selection in the To book drop-down list, to copy a worksheet to any opened workbook, including the source of the copy. (You may want to duplicate a worksheet in the same workbook so that you can have a copy that shows the formulas contained in the worksheet or that presents the same information in an alternate format.) When you make a copy of a worksheet and place it in the same workbook, Excel assigns the copied worksheet the name of the original sheet, plus a number, in parentheses. For example, if you copy a sheet named Calculations, the copied sheet will be named Calculations (2). (You can and should consider renaming copied worksheets to give them more descriptive names.)

## E1.6 ADD-INS: MAKING THINGS EASIER FOR YOU

**Add-ins** can simplify the task of creating something to add to a workbook. Add-ins are programming components not included in the main Excel program and may need to be **installed**, or added, to your computer system separately. Add-ins are not always available for you to use because they can be disabled by other users or system security set-

tings. However, the little extra effort you need to ensure that the right add-ins are installed and enabled is well worth the features that add-ins bring to Excel.

In this book, you use the **Analysis ToolPak** add-in that is included with Microsoft Excel. You may make use of **PHStat2**, a Prentice Hall add-in that is included on the Student CD. The Analysis ToolPak add-in (which the book simply calls the “ToolPak” from this point forward) adds statistical procedures to Excel, but creates worksheets that contain only text and numbers and no formulas. This means that the results you create using the ToolPak will not change if you change the underlying data. (To get updated results, you would have to use the ToolPak a second time. This is in contrast to worksheets that contain formulas which update themselves automatically when data are changed.)

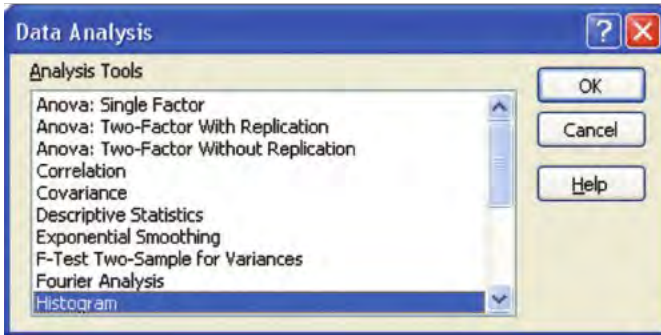
PHStat2 adds a PHStat menu of procedures to the Excel menu bar (Excel 97–2003) or the Add-ins tab (Excel 2007). Unlike the ToolPak, PHStat2 usually creates worksheets that contain formulas and that will produce new results as the underlying data changes. Sometimes, though, PHStat2 asks the ToolPak to create sheets on its behalf, and the resulting sheets are similar to the no-formulas sheets that the ToolPak creates. In many such cases, PHStat2 enhances the sheets it asks the ToolPak to create, correcting errors the ToolPak makes or adding new formula-based calculations.

Before you continue, you should check to see if the ToolPak is already installed and active in your copy of Microsoft Excel. If you are using an Excel version other than Excel 2007, select **Tools** and then see if **Data Analysis** appears on the **Tools** menu. If you see Data Analysis, the ToolPak is installed and enabled. If you are using Excel 2007, the best way to check the ToolPak status is to click the **Office Button** and then click **Excel Options** in the Office menu. Finally, you click **Add-Ins** in the Excel Options dialog box. If Analysis ToolPak appears in the **Active Applications Add-Ins** list, the ToolPak is installed and enabled.

If the ToolPak is not installed and enabled, you may need to use your original Microsoft Office or Excel CD or DVD and go through the Microsoft Office setup process to add the ToolPak add-in to your copy of Excel. If you plan to use the PHStat2 add in, you should review the appropriate part of Appendix F and read the PHStat2 readme file on the book’s CD.

### How to Use the ToolPak Add-in

Once you have determined that the ToolPak add-in is properly installed and active, you use the ToolPak by selecting **Tools → Data Analysis** (Excel 97–2003) or **Formulas → Data Analysis** (Excel 2007). In either case, you will see a Data Analysis dialog box similar to the one shown in Figure E1.12.



**FIGURE E1.12** Data Analysis dialog box

To use a ToolPak procedure, you select the name of the procedure that you want to use from the **Analysis Tools** list and then click **OK**. (The **Histogram** procedure has been selected in Figure E1.12.) A second dialog box then appears, in which you make entries and selections appropriate for the selected procedure. Click **OK** in this second dialog box to execute the procedure.

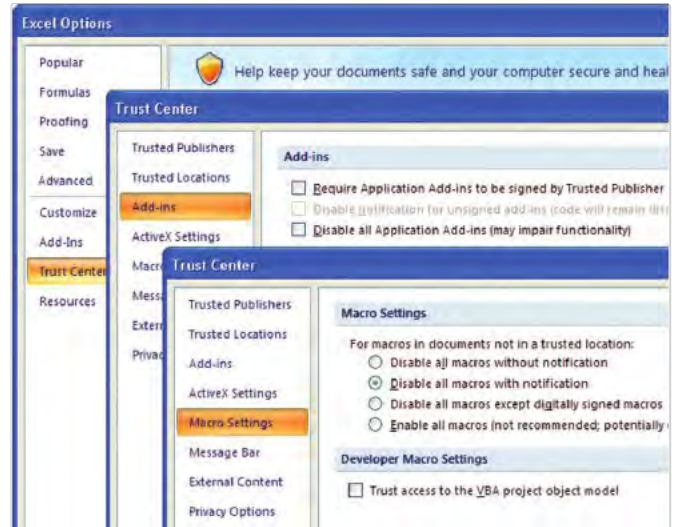
### Macro Security Issues

In all Excel versions other than Excel 97, all add-ins that you open will be screened by Microsoft Office security components. If you use Excel 2000, 2002, or 2003, you can review and change the security settings by selecting **Tools** → **Macros** → **Security** to display a Security dialog box (similar to the one shown in Figure E1.13 for Excel 2003). To use an add-in such as PHStat2 that is “not signed,” select the **Medium** option and click **OK**.



**FIGURE E1.13** Security dialog box (Excel 2003 version)

If you use Excel 2007, click the **Office Button**, then click **Excel Options** in the Office menu. In the Excel Options dialog box (see Figure E1.14) click **Trust Center** in the left pane to display information about security



**FIGURE E1.14** Excel Options and Trust Center panes (Excel 2007)

issues. Then click **Trust Center Settings** (obscured in Figure E1.14) to display the Trust Center dialog box. To use an add-in such as PHStat2 that is not signed, click **Add-Ins** in the left pane and then clear all checkboxes in the right pane (see Figure E1.15). Then click **Macro Settings** in the left pane and click **Disable all macros with notification** in the right pane (see Figure E1.14). (On some systems, you may also have to click **Trusted Locations** in the left pane and add the file location of the add-in.)

With the Security screen set to an appropriate level, you will see a macro virus warning dialog box when you open



**FIGURE E1.15** Macro warning dialog boxes (Excel 2003 and 2007)

an add-in such as PHStat2. Figure E1.15 shows the Excel 2003 and Excel 2007 warning boxes. You click the **Enable Macros** button to allow virus-free add-ins, such as PHStat2, to be opened and used.

**Forum:** Click the [SETUP AND INSTALLATION ISSUES](#) link if you need additional information setting up and using the add-ins.

## KEY TERMS

- absolute reference 27
- accelerator key 21
- add-in 28
- Cancel button 21
- cell 26
- cell pointer 24
- cell range 24
- check box 21
- clear 18
- click 18
- combination keystroke 21
- contextual tabs 21
- dialog box 21
- double-click 18
- drag 18
- drag-and-drop 18
- drop-down list 21
- edit box 21
- ellipsis 21
- Formatting toolbar 21
- formula 24
- formula bar 20
- gallery 21
- Help 21
- Home tab 21
- launch button 21
- list box 21
- menu bar 20
- minimize, resize, and close buttons 19
- mouse pointer 18
- Office Button 21
- OK button 21
- option buttons 21
- question-mark button 21
- quick access tool bar 21
- relative reference 27
- ribbon 21
- right-click 18
- scroll bars 20
- select 18
- sheet tab 20
- shortcut menus 18
- slider 21
- spinner button 21
- Standard toolbar 21
- tab 21
- tab groups 21
- task pane 21
- title bar 20
- Tool Tip 22
- workbook 22
- worksheet 24
- workspace area 19

# CHAPTER 2

## Presenting Data in Tables and Charts

USING STATISTICS @ Choice Is Yours, Part I

### 2.1 TABLES AND CHARTS FOR CATEGORICAL DATA

The Summary Table  
The Bar Chart  
The Pie Chart  
The Pareto Diagram

### 2.2 ORGANIZING NUMERICAL DATA

The Ordered Array  
The Stem-and-Leaf Display

### 2.3 TABLES AND CHARTS FOR NUMERICAL DATA

The Frequency Distribution  
The Relative Frequency Distribution and the Percentage Distribution  
The Cumulative Distribution  
The Histogram  
The Polygon  
The Cumulative Percentage Polygon (Ogive)

### 2.4 CROSS TABULATIONS

The Contingency Table  
The Side-by-Side Bar Chart

### 2.5 SCATTER PLOTS AND TIME-SERIES PLOTS

The Scatter Plot  
The Time-Series Plot

### 2.6 MISUSING GRAPHS AND ETHICAL ISSUES

Microsoft Excel Graphs

### EXCEL COMPANION TO CHAPTER 2

E2.1 Creating Summary Tables  
E2.2 Creating Charts  
E2.3 Creating Bar and Pie Charts from Summary Tables  
E2.4 Creating Pareto Diagrams from Summary Tables  
E2.5 Creating an Ordered Array  
E2.6 Creating Stem-and-Leaf Displays  
E2.7 Creating Frequency Distributions and Histograms  
E2.8 Creating a Histogram from Summarized Data  
E2.9 Creating Polygons  
E2.10 Creating Contingency Tables  
E2.11 Creating Side-by-Side Charts  
E2.12 Creating Scatter Plots  
E2.13 Creating Time-Series Plots

### LEARNING OBJECTIVES

In this chapter, you learn:

- To develop tables and charts for categorical data
- To develop tables and charts for numerical data
- The principles of properly presenting graphs

## USING STATISTICS @ Choice Is Yours, Part I



Choice Is Yours is a service that helps customers make wise investment choices. You've been hired to assist investors interested in mutual funds, a market basket of securities. According to investopedia.com, "A mutual fund is nothing more than a collection of stocks and/or bonds. You can think of a mutual fund as a company that brings together a group of people and invests their money in stocks, bonds, and other securities. Each investor owns shares, which represent a portion of the holdings of the fund." (You can learn more about mutual funds at [www.investopedia.com/university/mutualfunds/](http://www.investopedia.com/university/mutualfunds/).)

The Choice Is Yours company previously selected a sample of 838 mutual funds that it believes might be of interest to its customers. You have been asked to present data about these funds in a way that will help customers make good investment choices. What facts about each mutual fund would you collect to help customers compare and contrast the many funds?

A good starting point would be to collect data that would help customers classify mutual funds into various categories. You could research such things as the amount of risk involved in a fund's investment strategy and whether the fund focuses on growth securities, those companies that are expected to grow quickly in the next year, or, on value securities, those companies whose stock prices are currently considered undervalued. You might also investigate whether a mutual fund specializes in a certain size of company and whether the fund charges management fees that would reduce the percentage return earned by an investor.

Of course, you would want to know how well the fund performed in the past. You would also want to supply the customer with several measures of each fund's past performance. While past performance is no assurance of future performance, past data could give customers insight into how well each mutual fund has been managed.

As you further think about your task, you realize that all these data for all 838 mutual funds would be a lot for anyone to review. How could you "get your hands around" such data and explore them in a comprehensible manner?

To get your hands around the data described in this chapter's Using Statistics scenario, you need to use methods of descriptive statistics, defined in Chapter 1 as the branch of statistics that collects, summarizes, and presents data. In this scenario, you need to use descriptive techniques for both categorical variables (to help investors classify the mutual funds) and numerical variables (to help show the return each fund has achieved). Reading this chapter will help you to prepare tables and charts that are appropriate for both types of variables. You'll also learn techniques to help answer questions that require two variables, such as "Do growth-oriented mutual funds have lower returns than 'value' mutual funds?" and "Do growth funds tend to be riskier investments than value funds?"

*Many examples in this chapter use a sample of 838 real mutual funds, the data for which you can find in the Data worksheet of the [Mutual Funds.xls](#) file on the Student CD-ROM.*

### 2.1 TABLES AND CHARTS FOR CATEGORICAL DATA

When you have categorical data, you tally responses into categories and then present the frequency or percentage in each category in tables and charts.



## The Summary Table

A **summary table** indicates the frequency, amount, or percentage of items in a set of categories so that you can see differences between categories. A summary table lists the categories in one column and the frequency, amount, or percentage in a different column or columns. Table 2.1 illustrates a summary table based on a recent survey that asked people what they would do if they had an extra \$1,000 to spend (“If You Had an Extra \$1,000 to Spend, What Would You Do with the Money?” *USA Today*, January 11, 2006, p. A1). In Table 2.1, the most common choices are saving and paying debt, followed by buying a luxury item, vacation, or gift and spending on essentials. Very few respondents mentioned giving to charity or other uses.

**TABLE 2.1**

What People Would Do with an Extra \$1,000

What You Would Do with the Money	Percentage (%)
Buy a luxury item, vacation, or gift	20
Give it to charity	2
Pay debt	24
Save	31
Spend on essentials	16
Other	7

### EXAMPLE 2.1

#### SUMMARY TABLE OF LEVELS OF RISK OF MUTUAL FUNDS

The 838 mutual funds that are part of the Using Statistics scenario (see page 32) are classified according to their risk level, categorized as low, average, and high. Construct a summary table of the mutual funds, categorized by risk.

**SOLUTION** The mutual funds are fairly evenly divided by risk (see Table 2.2). There are more high-risk funds (346, or 41%) than low-risk or average-risk funds. There are about the same number of average-risk funds as low-risk funds (29%).

**TABLE 2.2**

Frequency and Percentage Summary Table Pertaining to Risk Level for 838 Mutual Funds

Fund Risk Level	Number of Funds	Percentage of Funds (%)
Low	247	29.47
Average	245	29.24
High	346	41.29
Total	838	100.00

## The Bar Chart

In a **bar chart**, a bar shows each category, the length of which represents the amount, frequency, or percentage of values falling into a category. Figure 2.1 on page 34 displays the bar chart for spending an extra \$1,000 presented in Table 2.1.

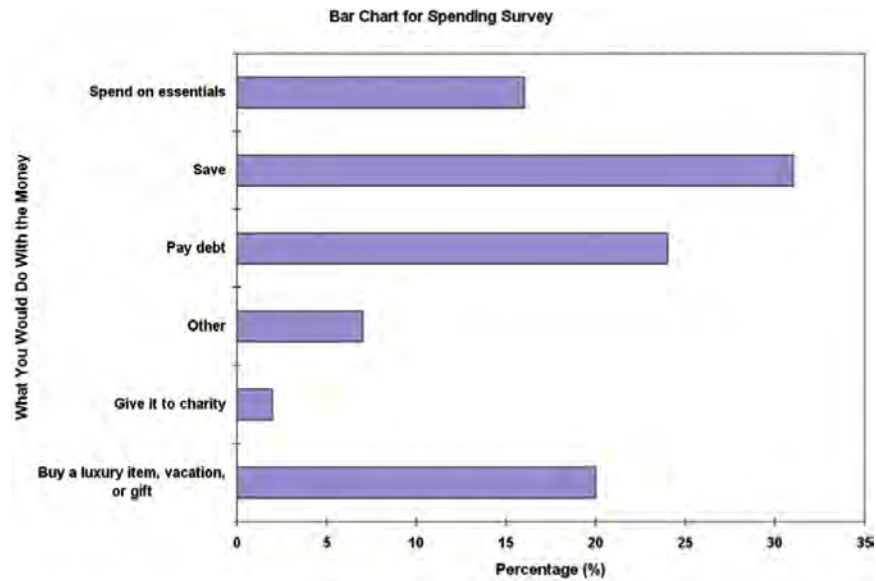
Bar charts allow you to compare percentages in different categories. In Figure 2.1, respondents are most likely to save or pay debt, followed by buy a luxury item, vacation, or gift and spend on essentials. Very few respondents mentioned giving to charity or other uses.

**FIGURE 2.1**

Microsoft Excel bar chart for spending an extra \$1,000



See Sections E2.2 and E2.3 to create this.

**EXAMPLE 2.2**

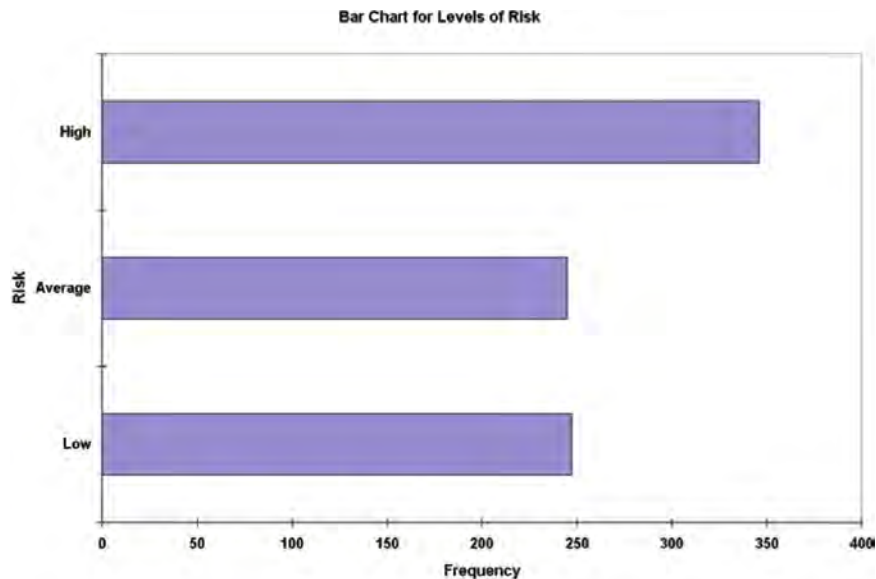
### BAR CHART OF LEVELS OF RISK OF MUTUAL FUNDS

Construct a bar chart for the levels of risk of mutual funds (based on the information in Table 2.2) and interpret the results.

**SOLUTION** The mutual funds are fairly evenly divided by risk (see Figure 2.2 on page 33). There are more high-risk funds (346, or 41%) than low-risk or average-risk funds. There are about the same number of average-risk funds as low-risk funds (29%).

**FIGURE 2.2**

Microsoft Excel bar chart of the levels of risk of mutual funds



### The Pie Chart

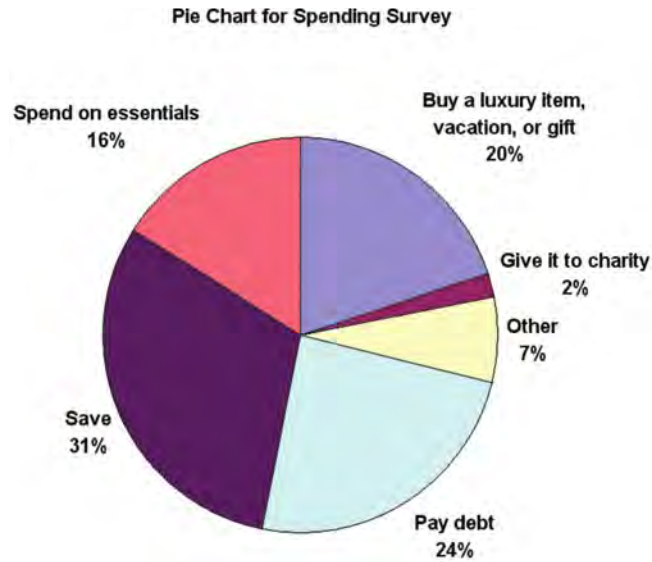
The **pie chart** is a circle broken up into slices that represent categories. The size of each slice of the pie varies according to the percentage in each category. In Table 2.1 on page 33, for example, 31% of the respondents stated that they would save the \$1,000. Thus, in constructing the pie chart, the 360 degrees that makes up a circle is multiplied by 0.31, resulting in a slice of the pie that takes up 111.6 degrees of the 360 degrees of the circle. From Figure 2.3, you can see that the pie chart lets you visualize the portion of the entire pie that is in each category. In this figure, saving takes 31% of the pie and giving to charity takes only 2%.

**FIGURE 2.3**

Microsoft Excel pie chart for spending an extra \$1,000



See Sections E2.2 and E2.3 to create this.



Which chart should you use—a bar chart or a pie chart? The selection of a particular chart often depends on your intention. If a comparison of categories is most important, you should use a bar chart. If observing the portion of the whole that is in a particular category is most important, you should use a pie chart.

**EXAMPLE 2.3**

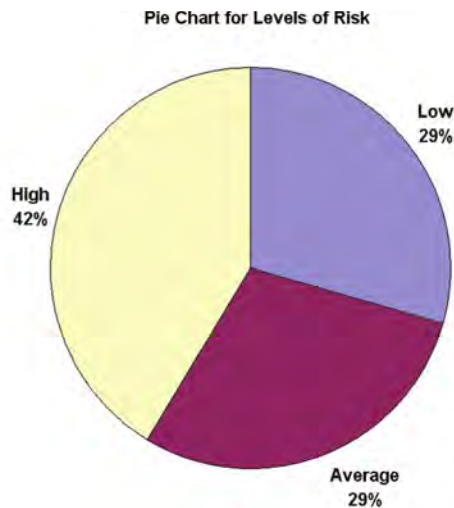
**PIE CHART OF LEVELS OF RISK OF MUTUAL FUNDS**

Construct a pie chart for the levels of risk of mutual funds (see Table 2.2 on page 33) and interpret the results.

**SOLUTION** (See Figure 2.4.) The mutual funds are fairly evenly divided by risk. There are more high-risk funds (346, or 41%) than low-risk or average-risk funds. There are about the same number of average-risk funds as low-risk funds (29%).

**FIGURE 2.4**

Microsoft Excel pie chart of the levels of risk of mutual funds



**The Pareto Diagram**

In a **Pareto diagram**, the categorized responses are plotted in descending order, according to their frequencies, and are combined with a cumulative percentage line on the same chart. The Pareto diagram can identify situations in which the Pareto principle occurs.

**PARETO PRINCIPLE**

The **Pareto principle** exists when the majority of items in a set of data occur in a small number of categories and the few remaining items are spread out over a large number of categories. These two groups are often referred to as the “vital few” and the “trivial many.”

The Pareto diagram has the ability to separate the “vital few” from the “trivial many,” enabling you to focus on the important categories. In situations in which the data involved consist of defective or nonconforming items, the Pareto diagram is a powerful tool for prioritizing improvement efforts.

Table 2.3 presents data for a large injection-molding company that manufactures plastic molded components used in computer keyboards, washing machines, automobiles, and television sets (see the [keyboarddefects.xls](#) file). The data presented in Table 2.3 consist of all computer keyboards with defects produced during a three-month period.

**TABLE 2.3**

Summary Table of Causes of Defects in Computer Keyboards in a Three-Month Period

Cause	Frequency	Percentage
Black spot	413	6.53
Damage	1,039	16.43
Jetting	258	4.08
Pin mark	834	13.19
Scratches	442	6.99
Shot mold	275	4.35
Silver streak	413	6.53
Sink mark	371	5.87
Spray mark	292	4.62
Warpage	1,987	31.42
Total	6,324	100.01*

\* Result differs slightly from 100.00 due to rounding.

Source: Extracted from U. H. Acharya and C. Mahesh, “Winning Back the Customer’s Confidence: A Case Study on the Application of Design of Experiments to an Injection-Molding Process,” *Quality Engineering*, 11, 1999, pp. 357–363.

Table 2.4 presents a summary table for the computer keyboard data in which the categories are ordered based on the percentage of defects present (rather than arranged alphabetically). The cumulative percentages for the ordered categories are also included as part of the table.

**TABLE 2.4**

Ordered Summary Table of Causes of Defects in Computer Keyboards in a Three-Month Period

Cause	Frequency	Percentage	Cumulative Percentage
Warpage	1,987	31.42	31.42
Damage	1,039	16.43	47.85
Pin mark	834	13.19	61.04
Scratches	442	6.99	68.03
Black spot	413	6.53	74.56
Silver streak	413	6.53	81.09
Sink mark	371	5.87	86.96
Spray mark	292	4.62	91.58
Shot mold	275	4.35	95.93
Jetting	258	4.08	100.00
Total	6,324	100.01*	

\* Result differs slightly from 100.00 due to rounding.

In Table 2.4, the first category listed is warpage (with 31.42% of the defects), followed by damage (with 16.43%), followed by pin mark (with 13.19%). The two most frequently occurring categories—warpage and damage—account for 47.85% of the defects; the three most frequently occurring categories—warpage, damage, and pin mark—account for 61.04% of the defects, and so on. Figure 2.5 is a Pareto diagram based on the results displayed in Table 2.4.

**FIGURE 2.5**

Microsoft Excel  
Pareto diagram for the  
keyboard defects data



See Sections E2.2 and E2.4  
to create this.

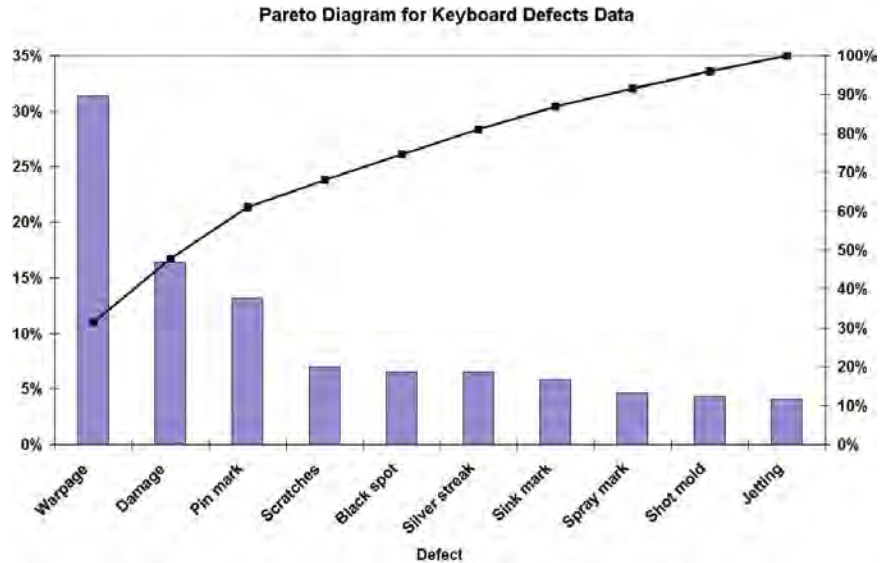


Figure 2.5 presents the bars vertically, along with a cumulative percentage line. The cumulative line is plotted at the midpoint of each bar, at a height equal to the cumulative percentage. If you follow the line, you see that these first three categories account for about 60% of the defects. Because the categories in the Pareto diagram are ordered by the frequency of occurrences, decision makers can see where to concentrate efforts to improve the process. Attempts to reduce defects due to warpage, damage, and pin marks should produce the greatest payoff. Then efforts can be made to reduce scratches, black spots, and silver streaks.

In order for a Pareto diagram to include all categories, even those with few defects, in some situations you need to include a category labeled *Other* or *Miscellaneous*. In these situations, the bar representing these categories is placed to the right of the other bars.

#### EXAMPLE 2.4

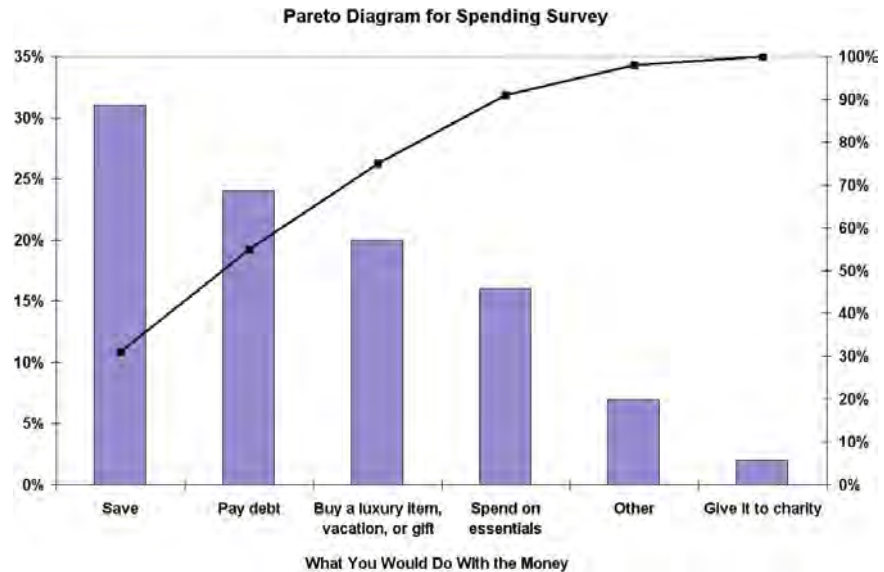
#### PARETO DIAGRAM OF SPENDING AN EXTRA \$1,000

Construct a Pareto diagram of what respondents would do with an extra \$1,000 (see Table 2.1 on page 33).

**SOLUTION** In Figure 2.6, saving and paying debt account for 55% of what respondents would do with the extra \$1,000; 91% of the respondents would save; pay debt; buy a luxury item, vacation, or gift; or spend on essentials.

**FIGURE 2.6**

Microsoft Excel Pareto diagram for spending an extra \$1,000



## Using Microsoft Excel *Properly*

Usually when you learn something, there is eventually some sort of feedback. Providing feedback is one reason instructors give tests. When you get the results of a test, you can reflect on how well you have properly learned the subject matter. Unfortunately, feedback opportunities are rare when you are learning to use a computer program such as Microsoft Excel as you study statistics. You either get the worksheet to calculate what you want or get a worksheet with misleading or inaccurate results. This good/bad, pass/fail type of feedback cannot help you refine your Microsoft Excel skills. We hope the following list will.

You are using Microsoft Excel *properly* if you can:

- 1. Understand how to operate the program.** Understanding how to operate the program does not mean that you need to know everything about Microsoft Excel. Don't waste your time teaching yourself every command or trying to memorize every function. Use the Excel Companion to Chapter 1 as your guide for what you need to know to get started using Excel effectively. Learn about additional features only just-in-time—when you need to use the feature. Excel features that you learn but don't immediately use are features that you will soon forget. However, if you learn an Excel feature when you need to

use it, such as discovering that you can use Data → Sort to sort data in Excel 97–2003, you are more likely to remember the proper command.

- 2. Understand the underlying statistical concepts.** Make sure you understand what a worksheet is doing as it produces its information. As a minimum, be informed about what each function you use calculates, or returns, as its value. Knowing this information will minimize your confusion and help you avoid making mistakes such as using the NORMSDIST function described in Chapter 6 when you meant to use the similar looking but different NORMDIST function.

- 3. Understand how to format and present information in a worksheet.** Section 1.6 on page 11 has already discussed how to organize your worksheets. With that as a starting point, you should plan how you are going to use the rows and columns of a worksheet to enhance the contents of your worksheets. For almost every worksheet illustrated in this text, the authors changed column widths, book font characteristics, and background tints of selected cells and added borders to highlight various cell ranges. Use the author's techniques (reviewed in the Excel Companion to Chapter 1) or invent your own, but always remember to have a plan that will present information effectively.

- 4. Know how to review formulas for errors.** Never assume that any worksheet you create or that was created for you is free of error. Always examine all formulas that have been entered into a worksheet. (See the Excel Companion to Chapter 1 for one way to do this.) You need to do this even if you use worksheets created by PHStat2 or retrieved from the Student CD-ROM.

- 5. Take control of your copy of Microsoft Excel.** Customize the settings of Microsoft Excel (and Microsoft Windows) to best suit your style, abilities, and needs. For example, many overlook increasing the default 10-point Arial font to a more readable size when examining the contents of a worksheet. If you own or control the computer on which you use Excel, you can make such customizations permanent so that they are automatically available to you every time you use Microsoft Excel.

In addition, if you own or control your own computer, you should regularly apply the security updates to Microsoft Office that Microsoft puts on its Web site. And if you plan to use PHStat2 on your own computer, you should read all the technical documentation including the PHStat readme file on the Student CD-ROM.

**Forum >** Visit the online forum to continue this discussion online. <<<

## PROBLEMS FOR SECTION 2.1

### Learning the Basics

**PH Grade ASSIST** 2.1 A categorical variable has three categories with the following frequencies of occurrence:

Category	Frequency
A	13
B	28
C	9

- Compute the percentage of values in each category.
- Construct a bar chart.
- Construct a pie chart.
- Construct a Pareto diagram.

**PH Grade ASSIST** 2.2 A categorical variable has four categories with the following percentages of occurrence:

Category	Percentage	Category	Percentage
A	12	C	35
B	29	D	24

- Construct a bar chart.
- Construct a pie chart.
- Construct a Pareto diagram.

### Applying the Concepts

2.3 A survey of 705 workers asked how much they used the Internet at work. The results (*USA Today Snapshots*, March 21, 2006) were as follows:

Use of the Internet at Work	%
Too much	5
More than I should	4
Within limits	60
Very little	5
Do not use	26

- Construct a bar chart, a pie chart, and a Pareto diagram.
- Which graphical method do you think is best to portray these data?
- Based on this survey, what conclusions can you reach about the use of the Internet at work?

**SELF Test** 2.4 An article (R. Richmond, "Anatomy of a Threat," *The Wall Street Journal*, February 13, 2006, pp. R5, R6) discussed the costs companies face in defending their networks from attack. The following table provides the breakdown in costs:

Cost	Percentage (%)
Consulting	7.6
Hardware tools	8.2
Labor	25.9
Lost business/revenue	23.6
Nonproductive employee time	15.5
Software tools	14.2
Other	5.0

- Construct a bar chart, a pie chart, and a Pareto diagram.
- Which graphical method do you think is best to portray these data?
- What conclusions can you reach concerning the costs companies face in defending their networks from attack?

2.5 When do Americans decide what to make for dinner? An online survey (N. Hellmich, "Americans Go for the Quick Fix for Dinner," *USA Today*, February 14, 2005, p. 1 B) indicated the following:

When Americans Decide What to Make for Dinner	%
At the last minute	37
Plan in advance	25
That day	37
Don't know	1

- Construct a bar chart, a pie chart, and a Pareto diagram.
- Which graphical method do you think is best to portray these data?

2.6 The following table represents the U.S. sources of electric energy in a recent year:

Source	%
Coal	51
Hydropower	6
Natural gas	16
Nuclear	21
Petroleum	3
Oil	3

Source: U.S. Department of Energy

- Construct a Pareto diagram.
- What percentage of electricity is derived from either coal, nuclear energy, or natural gas?
- Construct a pie chart.
- Which chart do you prefer to use for these data—the Pareto diagram or the pie chart? Why?

**2.7** An article (P. Kitchen, “Retirement Plan: To Keep Working,” *Newsday*, September 24, 2003) discussed the results of a sample of 2,001 Americans ages 50 to 70 who were employed full time or part time. The following table represents their plans for retirement:

Plans	Percentage (%)
Not work for pay at all	29
Start own business	10
Work full time	7
Work part time	46
Don’t know	3
Other	5

- a. Construct a bar chart and a pie chart.
- b. Which graphical method do you think is best to portray these data?

**2.8** U.S. companies spent more than \$250 billion in advertising in 2005 (K. Delaney, “In Latest Deal, Google Steps Further into World of Old Media,” *The Wall Street Journal*, January 18, 2006, pp. A1, A6). The spending was as follows:

Media	Amount (\$billions)	Percentage (%)
Cinema	0.4	0.16
Direct mail	44.5	17.35
Internet	10.0	3.90
Magazines	23.9	9.32
Newspapers	50.2	19.57
Outdoor	5.7	2.22
Radio	20.6	8.03
TV	55.4	21.60
Other	45.8	17.86

- a. Construct a bar chart and a pie chart.
- b. Which graphical method do you think is best to portray these data?

**2.9** Medication errors are a serious problem in hospitals. The following data represent the root causes of pharmacy errors at a hospital during a recent time period:

Reason for Failure	Frequency
Additional instructions	16
Dose	23
Drug	14
Duplicate order entry	22
Frequency	47
Omission	21
Order not discontinued when received	12
Order not received	52
Patient	5
Route	4
Other	8

- a. Construct a Pareto diagram.
- b. Discuss the “vital few” and “trivial many” reasons for the root causes of pharmacy errors.

**2.10** The following data represent complaints about hotel rooms:

Reason	Number
Room dirty	32
Room not stocked	17
Room not ready	12
Room too noisy	10
Room needs maintenance	17
Room has too few beds	9
Room doesn’t have promised features	7
No special accommodations	2

- a. Construct a Pareto diagram.
- b. What reasons for complaints do you think the hotel should focus on if it wants to reduce the number of complaints? Explain.

## 2.2 ORGANIZING NUMERICAL DATA

When the number of data values is large, you can organize numerical data into an ordered array or a stem-and-leaf display to help understand the information you have. Suppose you decide to undertake a study that compares the cost for a restaurant meal in a major city to the cost of a similar meal in the suburbs outside the city. The data file [restaurants.xls](#) contains the data for 50 city restaurants and 50 suburban restaurants, as shown in Table 2.5. The data are not arranged in order from lowest to highest. This arrangement makes it difficult to make conclusions about the price of meals in the two geographical areas.



**TABLE 2.5**

Price per Person at 50 City Restaurants and 50 Suburban Restaurants

City									
50	38	43	56	51	36	25	33	41	44
34	39	49	37	40	50	50	35	22	45
44	38	14	44	51	27	44	39	50	35
31	34	48	48	30	42	26	35	32	63
36	38	53	23	39	45	37	31	39	53
Suburban									
37	37	29	38	37	38	39	29	36	38
44	27	24	34	44	23	30	32	25	29
43	31	26	34	23	41	32	30	28	33
26	51	26	48	39	55	24	38	31	30
51	30	27	38	26	28	33	38	32	25

## The Ordered Array

An **ordered array** is a sequence of data, in rank order, from the smallest value to the largest value. Table 2.6 contains ordered arrays for the price of meals at city restaurants and suburban restaurants. From Table 2.6 you can see that the price of a meal at the city restaurants is between \$14 and \$63 and the price of a meal at the suburban restaurants is between \$23 and \$55.

**TABLE 2.6**

Ordered Array of Price per Person at 50 City Restaurants and 50 Suburban Restaurants

City									
14	22	23	25	26	27	30	31	31	32
33	34	34	35	35	35	36	36	37	37
38	38	38	39	39	39	39	40	41	42
43	44	44	44	44	45	45	48	48	49
50	50	50	50	51	51	53	53	56	63
Suburban									
23	23	24	24	25	25	26	26	26	26
27	27	28	28	29	29	29	30	30	30
30	31	31	32	32	32	33	33	34	34
36	37	37	37	38	38	38	38	38	38
39	39	41	43	44	44	48	51	51	55

## The Stem-and-Leaf Display

A **stem-and-leaf display** organizes data into groups (called stems) so that the values within each group (the leaves) branch out to the right on each row. The resulting display allows you to see how the data are distributed and where concentrations of data exist. To see how to construct a stem-and-leaf display, suppose that 15 students from your class eat lunch at a fast-food restaurant. The following data are the amounts spent for lunch:

5.40 4.30 4.80 5.50 7.30 8.50 6.10 4.80 4.90 4.90 5.50 3.50 5.90 6.30 6.60

To form the stem-and-leaf display, you use the units as the stems and round the decimals (the leaves) to one decimal place. For example, the first value is 5.40. Its stem (row) is 5, and its leaf is 4. The second value, is 4.30. Its stem (row) is 4, and its leaf is 3. You continue with the remainder of the 15 values and then reorder the leaves within each stem as follows;

3	5
4	38899
5	4559
6	136
7	3
8	5

**EXAMPLE 2.5** **STEM-AND-LEAF DISPLAY OF THE THREE-YEAR ANNUALIZED RETURN OF MUTUAL FUNDS**

In this chapter's Using Statistics scenario, you are interested in studying the past performance of mutual funds. One measure of past performance is the three-year annualized return (2003–2005)—that is, the average percentage return over the past three years. Construct a stem-and-leaf display of the three-year annualized returns.

**SOLUTION** From Figure 2.7, you can conclude that:

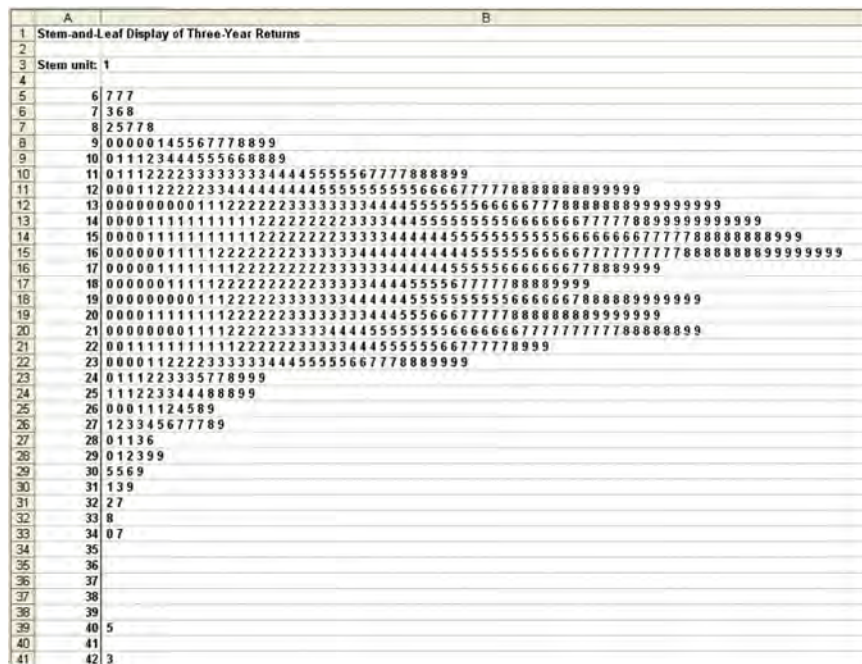
- The lowest three-year annualized return was 6.7.
- The highest three-year annualized return was 42.3.
- The three-year annualized returns were concentrated between 11 and 23.
- Only six mutual funds had three-year annualized returns below 8, and only two mutual funds had three-year annualized returns above 40.

**FIGURE 2.7**

Microsoft Excel stem-and-leaf display of the three-year annualized returns (2003–2005)



See Section E2.6 to create this.



**PROBLEMS FOR SECTION 2.2**

**Learning the Basics**

**PH Grade ASSIST** **2.11** Form an ordered array, given the following data from a sample of  $n = 7$  midterm exam scores in accounting:

68 94 63 75 71 88 64

**PH Grade ASSIST** **2.12** Form a stem-and-leaf display, given the following data from a sample of  $n = 7$  midterm exam scores in finance:

80 54 69 98 93 53 74

**PH Grade ASSIST** **2.13** Form an ordered array, given the following data from a sample of  $n = 7$  midterm exam scores in marketing:

88 78 78 73 91 78 85

**PH Grade ASSIST** **2.14** Form an ordered array, given the following stem-and-leaf display from a sample of  $n = 7$  midterm exam scores in information systems:

```

5 | 0
6 |
7 | 446
8 | 19
9 | 2

```

### Applying the Concepts

**PH Grade ASSIST** **2.15** The following is a stem-and-leaf display representing the amount of gasoline purchased, in gallons (with leaves in tenths of gallons), for a sample of 25 cars that use a particular service station on the New Jersey Turnpike:

```

9 | 147
10 | 02238
11 | 125566777
12 | 223489
13 | 02

```

- Place the data into an ordered array.
- Which of these two displays seems to provide more information? Discuss.
- What amount of gasoline (in gallons) is most likely to be purchased?
- Is there a concentration of the purchase amounts in the center of the distribution?

**PH Grade ASSIST** **2.16** The data in the file [bankcost1.xls](#) contains the bounced check fees, in dollars, for a sample of 23 banks for direct-deposit customers who maintain a \$100 balance:

26 28 20 20 21 22 25 25 18 25 15 20  
18 20 25 25 22 30 30 30 15 20 29

Source: Extracted from *The New Face of Banking*, June 2000  
Copyright © 2000 by Consumers Union of U.S., Inc., Yonkers, NY 10703-1057.

- Place the data into an ordered array.
- Construct a stem-and-leaf display for these data.
- Which of these two displays seems to provide more information? Discuss.
- Around what value, if any, are the bounced check fees concentrated? Explain.

**2.17** The file [movieprices.xls](#) contains data on the price for two tickets, with online service charges, large popcorn, and two medium soft drinks at a sample of six theater chains:

\$36.15 \$31.00 \$35.05 \$40.25 \$33.75 \$43.00

Source: Extracted from K. Kelly, "The Multiplex Under Siege," *The Wall Street Journal*, December 24–25, 2005, pp. P1, P5.

- Place the data into an ordered array.
- Construct a stem-and-leaf display for these data.
- Which of these two displays seems to provide more information? Discuss.
- Around what value, if any, are the movie prices concentrated? Explain.

**SELF Test** **2.18** The file [chicken.xls](#) contains data on the total fat, in grams per serving, for a sample of 20 chicken sandwiches from fast-food chains. The data are as follows:

7 8 4 5 16 20 20 24 19 30  
23 30 25 19 29 29 30 30 40 56

Source: Extracted from "Fast food: Adding Health to the Menu," *Consumer Reports*, September 2004, pp. 28–31.

- Place the data into an ordered array.
- Construct a stem-and-leaf display.
- Does the ordered array or the stem-and-leaf display provide more information? Discuss.

**2.19** The data in the file [batterylife.xls](#) represent the battery life, in shots, for three pixel digital cameras:

300 180 85 170 380 460 260 35 380 120 110 240

Source: Extracted from "Cameras: More Features in the Mix," *Consumer Reports*, July 2005, pp. 14–18.

- Place the data into an ordered array.
- Construct a stem-and-leaf display.
- Does the ordered array or the stem-and-leaf display provide more information? Discuss.
- Around what value, if any, is the battery life concentrated? Explain.

## 2.3 TABLES AND CHARTS FOR NUMERICAL DATA

When you have a data set that contains a large number of values, reaching conclusions from an ordered array or a stem-and-leaf display can be difficult. In such circumstances, you need to use tables and charts. There are many types of tables and charts that you can use to visually present numerical data. These include the frequency and percentage distributions, histogram, polygon, and cumulative percentage polygon (ogive).

### The Frequency Distribution

The **frequency distribution** is a summary table in which the data are arranged into numerically ordered class groupings. In constructing a frequency distribution, you must give attention to selecting the appropriate *number* of **class groupings** for the table, determining a suitable *width* of a class grouping, and establishing the *boundaries* of each class grouping to avoid overlapping.

The number of class groupings you use depends on the number of values in the data. Larger numbers of values allow for a larger number of class groupings. In general, the frequency distribution should have at least 5 class groupings but no more than 15. Having too few or too many class groupings provides little new information.

When developing a frequency distribution, you define each class grouping by class intervals of equal width. To determine the **width of a class interval**, you divide the **range** (highest value – lowest value) of the data by the number of class groupings desired.

#### DETERMINING THE WIDTH OF A CLASS INTERVAL

$$\text{Width of interval} = \frac{\text{Range}}{\text{Number of desired class groupings}} \quad (2.1)$$

Because the city restaurant data consist of a sample of 50 restaurants, 10 class groupings are acceptable. From the ordered array in Table 2.6 on page 41, the range of the data is  $\$63 - \$14 = \$49$ . Using Equation (2.1), you approximate the width of the class interval as follows:

$$\text{Width of interval} = \frac{49}{10} = 4.9$$

You should choose an interval width that simplifies reading and interpretation. Therefore, instead of using an interval width of \$4.90, you should select an interval width of \$5.00.

To construct the frequency distribution table, you should establish clearly defined **class boundaries** for each class grouping so that the values can be properly tallied into the classes. You place each value in one and only one class. You must avoid overlapping of classes.

Because you have set the width of each class interval for the restaurant data at \$5, you need to establish the boundaries of the various class groupings so as to include the entire range of values. Whenever possible, you should choose these boundaries to simplify reading and interpretation. Thus, for the city restaurants, because the cost ranges from \$14 to \$63, the first class interval ranges from \$10 to less than \$15, the second from \$15 to less than \$20, and so on, until they have been tallied into 11 classes. Each class has an interval width of \$5, without overlapping. The center of each class, the **class midpoint**, is halfway between the lower boundary of the class and the upper boundary of the class. Thus, the class midpoint for the class from \$10 to under \$15 is \$12.50, the class midpoint for the class from \$15 to under \$20 is \$17.50, and so on. Table 2.7 is a frequency distribution of the cost per meal for the 50 city restaurants and the 50 suburban restaurants.

**TABLE 2.7**

Frequency Distribution of the Cost per Meal for 50 City Restaurants and 50 Suburban Restaurants

Cost per Meal (\$)	City Frequency	Suburban Frequency
10 but less than 15	1	0
15 but less than 20	0	0
20 but less than 25	2	4
25 but less than 30	3	13
30 but less than 35	7	13
35 but less than 40	14	12
40 but less than 45	8	4
45 but less than 50	5	1
50 but less than 55	8	2
55 but less than 60	1	1
60 but less than 65	1	0
Total	<u>50</u>	<u>50</u>

The frequency distribution allows you to draw conclusions about the major characteristics of the data. For example, Table 2.7 shows that the cost of meals at city restaurants is concentrated between \$30 and \$55, and the cost of meals at suburban restaurants is clustered between \$25 and \$40.

If the data set does not contain many values, one set of class boundaries may provide a different picture than another set. For example, for the restaurant cost data, using a class-interval width of 4.0 instead of 5.0 (as was used in Table 2.7) may cause shifts in the way the values distribute among the classes.

You can also get shifts in data concentration when you choose different lower and upper class boundaries. Fortunately, as the sample size increases, alterations in the selection of class boundaries affect the concentration of data less and less.

**EXAMPLE 2.6**

### FREQUENCY DISTRIBUTION OF THE THREE-YEAR ANNUALIZED RETURN FOR GROWTH AND VALUE MUTUAL FUNDS

In the Using Statistics scenario, you are interested in comparing the three-year annualized return (2003–2005) of growth and value mutual funds. Construct frequency distributions for the growth funds and the value funds.

**SOLUTION** The three-year annualized returns of the growth funds and the value funds are highly concentrated between 10 and 25 (see Table 2.8). You should not directly compare the frequencies of the growth funds and the value funds because there are 480 growth funds and 358 value funds in the sample. On page 47, relative frequencies and percentages are introduced.

**TABLE 2.8**

Frequency Distribution of the Three-Year Annualized Return for Growth and Value Mutual Funds

Three-Year Annualized Return	Growth Frequency	Value Frequency
5 but less than 10	21	7
10 but less than 15	148	85
15 but less than 20	172	132
20 but less than 25	113	97
25 but less than 30	21	28
30 but less than 35	5	7
35 but less than 40	0	0
40 but less than 45	0	2
Total	<u>480</u>	<u>358</u>

## The Relative Frequency Distribution and the Percentage Distribution

Because you usually want to know the proportion or the percentage of the total that is in each group, the relative frequency distribution or the percentage distribution is preferred to the frequency distribution. When you are comparing two or more groups that differ in sample size, you must use either a relative frequency distribution or a percentage distribution.

You form the **relative frequency distribution** by dividing the frequencies in each class of the frequency distribution (see Table 2.7 on page 45) by the total number of values. You form the **percentage distribution** by multiplying each relative frequency by 100%. Thus, the relative frequency of meals at city restaurants that cost between \$30 and \$35 is 7 divided by 50, or 0.14, and the percentage is 14%. Table 2.9 presents the relative frequency distribution and percentage distribution of the cost of restaurant meals at city and suburban restaurants.

**TABLE 2.9**

Relative Frequency Distribution and Percentage Distribution of the Cost of Restaurant Meals at City and Suburban Restaurants

Cost per Meal (\$)	City		Suburban	
	Relative Frequency	Percentage	Relative Frequency	Percentage
10 but less than 15	0.02	2.0	0.00	0.0
15 but less than 20	0.00	0.0	0.00	0.0
20 but less than 25	0.04	4.0	0.08	8.0
25 but less than 30	0.06	6.0	0.26	26.0
30 but less than 35	0.14	14.0	0.26	26.0
35 but less than 40	0.28	28.0	0.24	24.0
40 but less than 45	0.16	16.0	0.08	8.0
45 but less than 50	0.10	10.0	0.02	2.0
50 but less than 55	0.16	16.0	0.04	4.0
55 but less than 60	0.02	2.0	0.02	2.0
60 but less than 65	<u>0.02</u>	<u>2.0</u>	<u>0.00</u>	<u>0.0</u>
Total	1.00	100.0	1.00	100.0

From Table 2.9, you conclude that meals cost more at city restaurants than at suburban restaurants—16% of the meals at city restaurants cost between \$40 and \$45 as compared to 8% of the suburban restaurants; 16% of the meals at city restaurants cost between \$50 and \$55 as compared to 4% of the suburban restaurants; and only 6% of the meals at city restaurants cost between \$25 and \$30 as compared to 26% of the suburban restaurants.

### EXAMPLE 2.7

#### RELATIVE FREQUENCY DISTRIBUTION AND PERCENTAGE DISTRIBUTION OF THE THREE-YEAR ANNUALIZED RETURN FOR GROWTH AND VALUE MUTUAL FUNDS

In the Using Statistics scenario, you are interested in comparing the three-year annualized return (2003–2005) for growth and value mutual funds. Construct relative frequency distributions and percentage distributions for the growth funds and the value funds.

**SOLUTION** You conclude (see Table 2.10) that the three-year annualized return (2003–2005) for the growth funds is slightly lower than for the value funds. 4.38% of growth funds have returns below 10, while only 1.96% of value funds have returns below 10. Of the growth funds, 30.83% have returns between 10 and 15 as compared to 23.74% of the value funds. Also, more of the value funds have higher returns. For example, 34.91% of the value funds made between 20 and 30, while 27.92% of the growth funds made between 20 and 30.

**TABLE 2.10**

Relative Frequency Distribution and Percentage Distribution of the Three-Year Annualized Return (2003–2005) for Growth and Value Mutual Funds

Three-Year Annualized Return	Growth		Value	
	Proportion	Percentage	Proportion	Percentage
5 but less than 10	0.0438	4.38	0.0196	1.96
10 but less than 15	0.3083	30.83	0.2374	23.74
15 but less than 20	0.3583	35.83	0.3687	36.87
20 but less than 25	0.2354	23.54	0.2709	27.09
25 but less than 30	0.0438	4.38	0.0782	7.82
30 but less than 35	0.0104	1.04	0.0196	1.96
35 but less than 40	0.0000	0.00	0.0000	0.00
40 but less than 45	0.0000	0.00	0.0056	0.56
Total	1.0000	100.00	1.0000	100.00

### The Cumulative Distribution

The **cumulative percentage distribution** provides a way of presenting information about the percentage of items that are less than a certain value. For example, you might want to know what percentage of the city restaurant meals cost less than \$20, less than \$30, less than \$50, and so on. The percentage distribution is used to form the cumulative percentage distribution. Table 2.11 illustrates how to develop the cumulative percentage distribution for the cost of meals at city restaurants. 0.00% of the meals cost less than \$10, 2% cost less than \$15, 2% also cost less than \$20 (because none of the meals cost between \$15 and \$20), 6% (2% + 4%) cost less than \$25, and so on, until all 100% of the meals cost less than \$65.

**TABLE 2.11**

Developing the Cumulative Percentage Distribution for the Cost of Meals at City Restaurants

Cost per Meal (\$)	Percentage	Percentage of Meals Less Than Lower Boundary of Class Interval
10 but less than 15	2	0
15 but less than 20	0	2
20 but less than 25	4	2 = 2 + 0
25 but less than 30	6	6 = 2 + 0 + 4
30 but less than 35	14	12 = 2 + 0 + 4 + 6
35 but less than 40	28	26 = 2 + 0 + 4 + 6 + 14
40 but less than 45	16	54 = 2 + 0 + 4 + 6 + 14 + 28
45 but less than 50	10	70 = 2 + 0 + 4 + 6 + 14 + 28 + 16
50 but less than 55	16	80 = 2 + 0 + 4 + 6 + 14 + 28 + 16 + 10
55 but less than 60	2	96 = 2 + 0 + 4 + 6 + 14 + 28 + 16 + 10 + 16
60 but less than 65	2	98 = 2 + 0 + 4 + 6 + 14 + 28 + 16 + 10 + 16 + 2
65 but less than 70	0	100 = 2 + 0 + 4 + 6 + 14 + 28 + 16 + 10 + 16 + 2 + 2

Table 2.12 on page 48 summarizes the cumulative percentages of the cost of city and suburban restaurant meals. The cumulative distribution clearly shows that the cost of meals is lower in suburban restaurants than in city restaurants—34% of the suburban restaurants cost less than \$30 as compared to only 12% of the city restaurants; 60% of the suburban restaurants cost less than \$35 as compared to only 26% of the city restaurants; 84% of the suburban restaurants cost less than \$40 as compared to only 54% of the city restaurants.

**TABLE 2.12**

Cumulative Percentage Distributions of the Cost of City and Suburban Restaurant Meals

Cost (\$)	City Percentage of Restaurants Less Than Indicated Value	Suburban Percentage of Restaurants Less Than Indicated Value
10	0	0
15	2	0
20	2	0
25	6	8
30	12	34
35	26	60
40	54	84
45	70	92
50	80	94
55	96	98
60	98	100
65	100	100

**EXAMPLE 2.8**

### CUMULATIVE PERCENTAGE DISTRIBUTION OF THE THREE-YEAR ANNUALIZED RETURN FOR GROWTH AND VALUE MUTUAL FUNDS

In the Using Statistics scenario, you are interested in comparing the three-year annualized return (2003–2005) of growth and value mutual funds. Construct cumulative percentage distributions for the growth funds and the value funds.

**SOLUTION** The cumulative distribution in Table 2.13 indicates that more of the growth funds have lower returns than the value funds—35.21% of the growth funds have returns below 15 as compared to 25.70% of the value funds; 71.04% of the growth funds have returns below 20 as compared to 62.57% of the value funds; 94.58% of the growth funds have returns below 25 as compared to 89.66% of the value funds.

**TABLE 2.13**

Cumulative Percentage Distributions of the Three-Year Annualized Return for Growth and Value Funds

Annual Return	Growth Fund Percentage Less Than Indicated Value	Value Fund Percentage Less Than Indicated Value
5	0.00	0.00
10	4.38	1.96
15	35.21	25.70
20	71.04	62.57
25	94.58	89.66
30	98.96	97.48
35	100.00	99.44
40	100.00	99.44
45	100.00	100.00

## The Histogram

A **histogram** is a bar chart for grouped numerical data in which the frequencies or percentages of each group of numerical data are represented as individual vertical bars. In a histogram, there are no gaps between adjacent bars as there are in a bar chart of categorical data. You display the variable of interest along the horizontal ( $X$ ) axis. The vertical ( $Y$ ) axis represents either the frequency or the percentage of values per class interval.



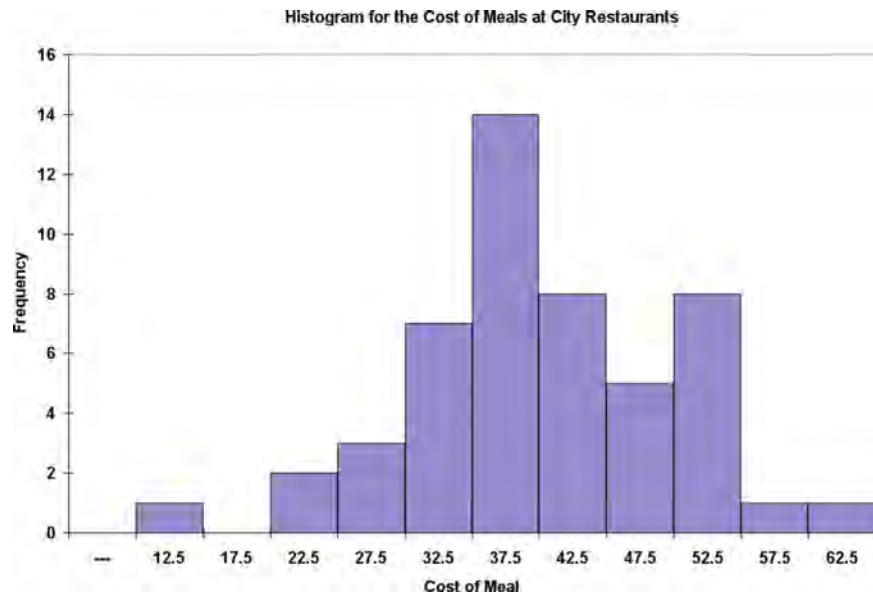
Figure 2.8 displays a Microsoft Excel frequency histogram for the cost of restaurant meals at city restaurants. The histogram indicates that the cost of restaurant meals at city restaurants is concentrated between approximately \$30 and \$50. Very few meals cost less than \$20 or more than \$60.

**FIGURE 2.8**

Microsoft Excel histogram for the cost of restaurant meals at city restaurants



See Sections E2.7 and E2.8 to create this.



**EXAMPLE 2.9**

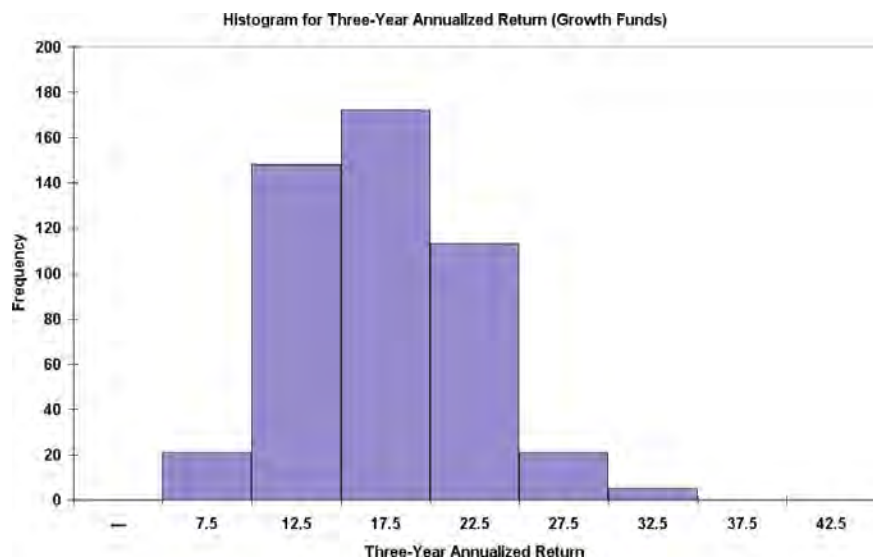
**HISTOGRAM OF THE THREE-YEAR ANNUALIZED RETURN FOR GROWTH AND VALUE MUTUAL FUNDS**

In the Using Statistics scenario, you are interested in comparing the three-year annualized return (2003–2005) of growth and value mutual funds. Construct histograms for the growth funds and the value funds.

**SOLUTION** Figure 2.9 shows that the distribution of the growth funds (shown below) has more low returns as compared to the value funds (shown on p. 50), which have more high returns.

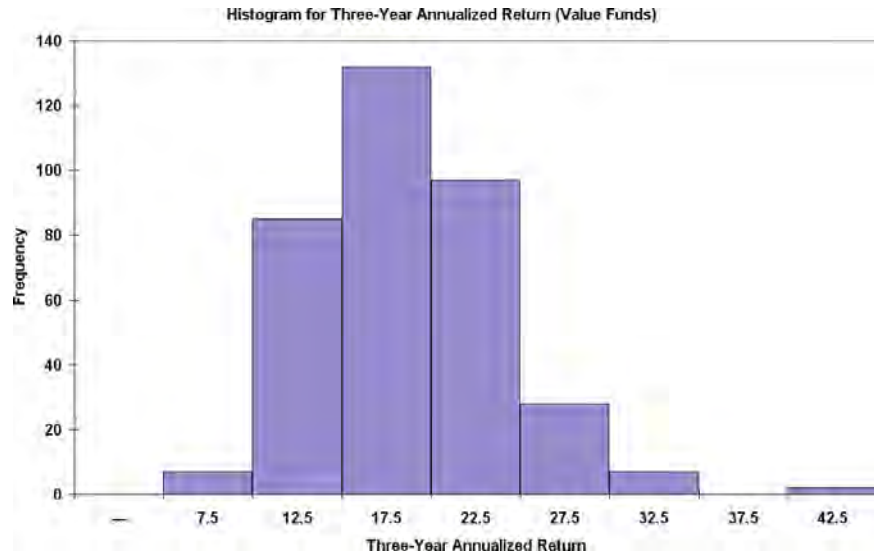
**FIGURE 2.9**

Histogram of the three-year annualized return (2003–2005) (Panel A—growth funds and Panel B—value funds)



Panel A

Panel B



### The Polygon

Constructing multiple histograms on the same graph when comparing two or more sets of data is confusing. Superimposing the vertical bars of one histogram on another histogram makes interpretation difficult. When there are two or more groups, you should use a percentage polygon.

#### PERCENTAGE POLYGON

A **percentage polygon** is formed by having the midpoint of each class represent the data in that class and then connecting the sequence of midpoints at their respective class percentages.

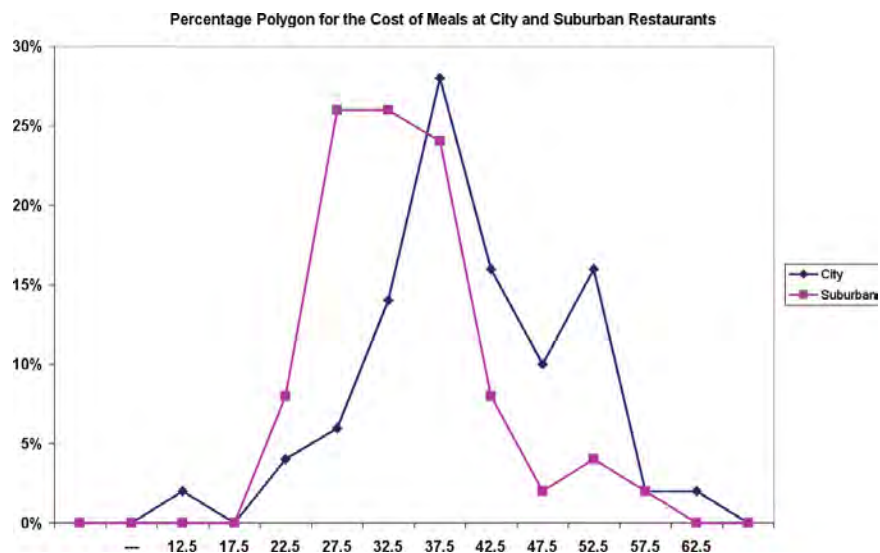
Figure 2.10 displays percentage polygons for the cost of restaurant meals for city and suburban restaurants. The polygon for the suburban restaurants is concentrated to the left of (corresponding to lower cost) the polygon for city restaurants. The highest percentages of cost for the suburban restaurants are for class midpoints of \$27.50 and \$32.50, while the highest percentages of cost for the city restaurants are for a class midpoint of \$37.50.

**FIGURE 2.10**

Microsoft Excel percentage polygons of the cost of restaurant meals for city and suburban restaurants



See Section E2.9 to create this.



The polygons in Figure 2.10 have points whose values on the  $X$  axis represent the mid-point of the class interval. For example, look at the points plotted on the  $Y$  axis at 22.5 (\$22.50). The point for the suburban restaurants (the higher one) represents the fact that 8% of these restaurants have meal costs between \$20 and \$25. The point for the city restaurants (the lower one) represents the fact that 4% of these restaurants have meal costs between \$20 and \$25.

When you construct polygons or histograms, the vertical ( $Y$ ) axis should show the true zero, or “origin,” so as not to distort the character of the data. The horizontal ( $X$ ) axis does not need to show the zero point for the variable of interest, although the range of the variable should constitute the major portion of the axis.

### EXAMPLE 2.10

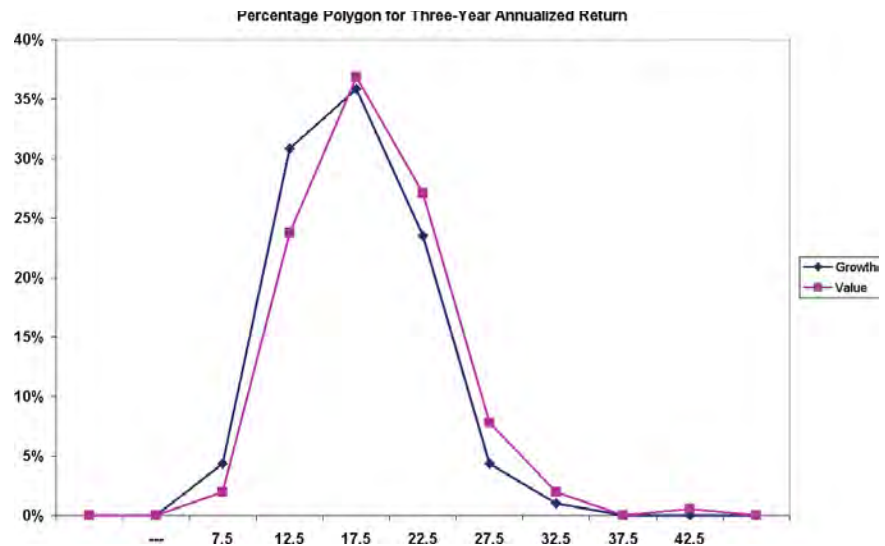
#### PERCENTAGE POLYGONS OF THE THREE-YEAR ANNUALIZED RETURN (2003–2005) FOR GROWTH AND VALUE MUTUAL FUNDS

In the Using Statistics scenario, you are interested in comparing the three-year annualized return (2003–2005) of growth and value mutual funds. Construct percentage polygons for the growth funds and the value funds.

**SOLUTION** Figure 2.11 shows that the distribution of the growth funds has slightly more low returns as compared to the distribution of value funds, which has more high returns.

**FIGURE 2.11**

Microsoft Excel percentage polygons of the three-year annualized return



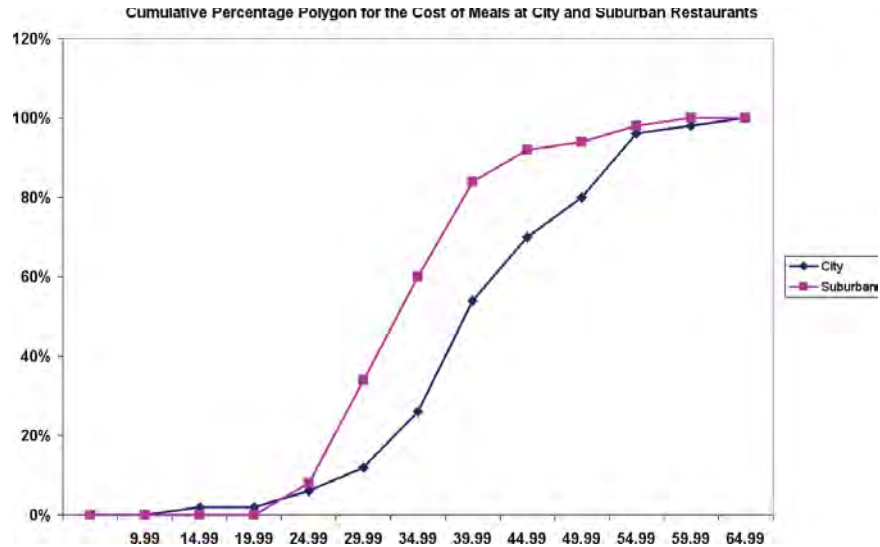
### The Cumulative Percentage Polygon (Ogive)

The **cumulative percentage polygon**, or **ogive**, displays the variable of interest along the  $X$  axis, and the cumulative percentages along the  $Y$  axis.

Figure 2.12 illustrates the Microsoft Excel cumulative percentage polygons of the cost of restaurant meals at city and suburban restaurants. Most of the curve for the city restaurants is located to the right of the curve for the suburban restaurants. This indicates that the city restaurants have fewer meals that cost below a particular value. For example, 12% of the city restaurant meals cost less than \$30 as compared to 34% of the suburban restaurant meals.

**FIGURE 2.12**

Microsoft Excel cumulative percentage polygons of the cost of restaurant meals at city and suburban restaurants



**EXAMPLE 2.11**

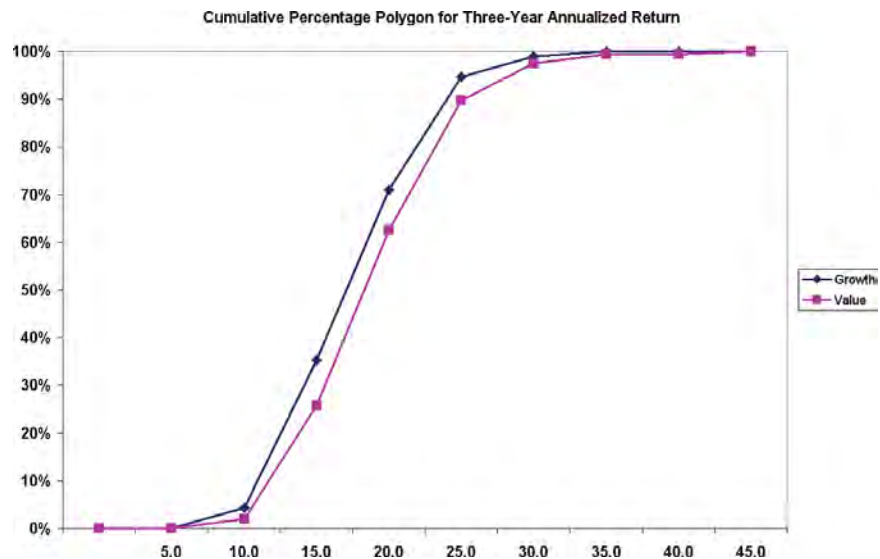
**CUMULATIVE PERCENTAGE POLYGONS OF THE THREE-YEAR ANNUALIZED RETURN (2003–2005) FOR GROWTH AND VALUE MUTUAL FUNDS**

In the Using Statistics scenario, you are interested in comparing the three-year annualized return (2003–2005) of growth and value mutual funds. Construct cumulative percentage polygons for the growth funds and the value funds.

**SOLUTION** Figure 2.13 illustrates the Microsoft Excel cumulative percentage polygons of the three-year annualized return for growth and value funds. The curve for the value funds is located slightly to the right of the curve for the growth funds. This indicates that the value funds have fewer returns below a particular value. For example, 62.57% of the value funds have returns less than 20 as compared to 71.04% of the growth funds.

**FIGURE 2.13**

Microsoft Excel cumulative percentage polygons of the three-year annualized return



## PROBLEMS FOR SECTION 2.3

### Learning the Basics

**PH Grade ASSIST 2.20** The values for a set of data vary from 11.6 to 97.8.

- If these values are grouped into nine classes, indicate the class boundaries.
- What class-interval width did you choose?
- What are the nine class midpoints?

**PH Grade ASSIST 2.21** The GMAT scores from a sample of 50 applicants to an MBA program indicate that none of the applicants scored below 450. A frequency distribution was formed by choosing class intervals 450 to 499, 500 to 549, and so on, with the last class grouping being 700 to 749. Two applicants scored in the interval 450 to 499, and 16 applicants scored in the interval 500 to 549. Construct a cumulative percentage polygon.

- What percentage of applicants scored below 500?
- What percentage of applicants scored between 500 and 549?
- What percentage of applicants scored below 550?
- What percentage of applicants scored below 750?

### Applying the Concepts

**PH Grade ASSIST 2.22** The following data (contained in the file [utility.xls](#)) represent the cost of electricity during July 2006 for a random sample of 50 one-bedroom apartments in a large city:

Raw Data on Utility Charges (\$)

96	171	202	178	147	102	153	197	127	82
157	185	90	116	172	111	148	213	130	165
141	149	206	175	123	128	144	168	109	167
95	163	150	154	130	143	187	166	139	149
108	119	183	151	114	135	191	137	129	158

- Form a frequency distribution and a percentage distribution that have class intervals with the upper class limits \$99, \$119, and so on.
- Construct a histogram and a percentage polygon.
- Form a cumulative percentage distribution and plot a cumulative percentage polygon.
- Around what amount does the monthly electricity cost seem to be concentrated?

**2.23** One operation of a mill is to cut pieces of steel into parts that will later be used as the frame for front seats in an automobile. The steel is cut with a diamond saw and requires the resulting parts to be within  $\pm 0.005$  inch of the length specified by the automobile company. The following table (contained in the file [steel.xls](#)) comes from a sample

of 100 steel parts. The measurement reported is the difference in inches between the actual length of the steel part, as measured by a laser measurement device, and the specified length of the steel part. For example, the first value,  $-0.002$ , represents a steel part that is 0.002 inch shorter than the specified length.

-0.002	0.002	0.0005	-0.0015	-0.001
0.0005	0.001	0.001	-0.0005	-0.001
0.0025	0.001	0.0005	-0.0015	0.0005
0.001	0.001	0.001	-0.0005	-0.0025
0.002	-0.002	0.0025	-0.0005	0.0025
0.001	-0.003	0.001	-0.001	0.002
0.005	-0.0015	0	-0.0015	0.0025
-0.002	-0.0005	-0.0025	0.0025	-0.002
0	0	-0.001	0.001	0
0.001	-0.0025	0.0035	0.0005	-0.0005
-0.0025	-0.003	0	0	-0.001
-0.003	-0.001	-0.003	0.002	0
0.001	0.002	-0.002	-0.0005	-0.002
-0.0005	-0.001	-0.001	0.0005	0
0	0	-0.0015	0.0005	0
-0.003	0.003	-0.0015	0	0.002
-0.001	0.0015	-0.002	-0.0005	-0.003
0.0005	0	0.001	0.002	-0.0005
0.0025	0	-0.0025	0.001	-0.002
-0.0025	-0.0025	-0.0005	-0.0015	-0.002

- Construct a frequency distribution and a percentage distribution.
- Construct a histogram and a percentage polygon.
- Plot a cumulative percentage polygon.
- Is the steel mill doing a good job in meeting the requirements set by the automobile company? Explain.



**2.24** A manufacturing company produces steel housings for electrical equipment. The main component part of the housing is a steel trough that is made out of a 14-gauge steel coil. It is produced using a 250-ton progressive punch press with a wipe-down operation that puts two 90-degree forms in the flat steel to make the trough. The distance from one side of the form to the other is critical because of weatherproofing in outdoor applications. The company requires that the width of the trough be between 8.31 inches and 8.61 inches. The following (contained in the

file [trough.xls](#)) are the widths of the troughs, in inches, for a sample of 49 troughs.

8.312 8.343 8.317 8.383 8.348 8.410 8.351 8.373  
 8.481 8.422 8.476 8.382 8.484 8.403 8.414 8.419  
 8.385 8.465 8.498 8.447 8.436 8.413 8.489 8.414  
 8.481 8.415 8.479 8.429 8.458 8.462 8.460 8.444  
 8.429 8.460 8.412 8.420 8.410 8.405 8.323 8.420  
 8.396 8.447 8.405 8.439 8.411 8.427 8.420 8.498  
 8.409

- Construct a frequency distribution and a percentage distribution.
- Construct a histogram and a percentage polygon.
- Plot a cumulative percentage polygon.
- What can you conclude about the number of troughs that will meet the company's requirements of troughs being between 8.31 and 8.61 inches wide?

**2.25** The manufacturing company in Problem 2.24 also produces electric insulators. If the insulators break when in use, a short circuit is likely to occur. To test the strength of the insulators, destructive testing in high-powered labs is carried out to determine how much *force* is required to break the insulators. Force is measured by observing how many pounds must be applied to the insulator before it breaks. The strengths of 30 insulators (contained in the file [force.xls](#)) are as follows:

1,870 1,728 1,656 1,610 1,634 1,784 1,522 1,696  
 1,592 1,662 1,866 1,764 1,734 1,662 1,734 1,774  
 1,550 1,756 1,762 1,866 1,820 1,744 1,788 1,688  
 1,810 1,752 1,680 1,810 1,652 1,736

- Construct a frequency distribution and a percentage distribution.
- Construct a histogram and a percentage polygon.
- Plot a cumulative percentage polygon.
- What can you conclude about the strength of the insulators if the company requires a force measurement of at least 1,500 pounds before breaking?

**2.26** The ordered arrays in the accompanying table (and contained in the file [bulbs.xls](#)) deal with the life (in hours) of a sample of 40 100-watt light bulbs produced by Manufacturer A and a sample of 40 100-watt light bulbs produced by Manufacturer B.

Manufacturer A					Manufacturer B				
684	697	720	773	821	819	836	888	897	903
831	835	848	852	852	907	912	918	942	943
859	860	868	870	876	952	959	962	986	992
893	899	905	909	911	994	1,004	1,005	1,007	1,015
922	924	926	926	938	1,016	1,018	1,020	1,022	1,034
939	943	946	954	971	1,038	1,072	1,077	1,077	1,082
972	977	984	1,005	1,014	1,096	1,100	1,113	1,113	1,116
1,016	1,041	1,052	1,080	1,093	1,153	1,154	1,174	1,188	1,230

- Form a frequency distribution and a percentage distribution for each manufacturer, using the following class-interval widths for each distribution:
  - Manufacturer A: 650 but less than 750, 750 but less than 850, and so on.
  - Manufacturer B: 750 but less than 850, 850 but less than 950, and so on.
- Construct percentage histograms on separate graphs and plot the percentage polygons on one graph.
- Form cumulative percentage distributions and plot cumulative percentage polygons on one graph.
- Which manufacturer has bulbs with a longer life—Manufacturer A or Manufacturer B? Explain.

**2.27** The following data (contained in the file [drink.xls](#)) represent the amount of soft drink in a sample of 50 2-liter bottles:

2.109 2.086 2.066 2.075 2.065 2.057 2.052 2.044  
 2.036 2.038 2.031 2.029 2.025 2.029 2.023 2.020  
 2.015 2.014 2.013 2.014 2.012 2.012 2.012 2.010  
 2.005 2.003 1.999 1.996 1.997 1.992 1.994 1.986  
 1.984 1.981 1.973 1.975 1.971 1.969 1.966 1.967  
 1.963 1.957 1.951 1.951 1.947 1.941 1.941 1.938  
 1.908 1.894

- Construct a frequency distribution and a percentage distribution.
- Construct a histogram and a percentage polygon.
- Form a cumulative percentage distribution and plot a cumulative percentage polygon.
- On the basis of the results of (a) through (c), does the amount of soft drink filled in the bottles concentrate around specific values?

## 2.4 CROSS TABULATIONS

The study of patterns that may exist between two or more categorical variables is common in business. These patterns are explained by cross-tabulating the data. You can present **cross tabulations** in tabular form (contingency tables) or graphical form (side-by-side charts).

## The Contingency Table

A **contingency table** presents the results of two categorical variables. The joint responses are classified so that the categories of one variable are located in the rows and the categories of the other variable are located in the columns. The values located at the intersections of the rows and columns are called **cells**. Depending on the type of contingency table constructed, the cells for each row–column combination contain the frequency, the percentage of the overall total, the percentage of the row total, or the percentage of the column total.

Suppose that in the Using Statistics scenario, you want to examine whether there is any pattern or relationship between the level of risk and the objective of the mutual fund (growth versus value). Table 2.14 summarizes this information for all 838 mutual funds.

**TABLE 2.14**

Contingency Table  
Displaying Fund  
Objective and  
Fund Risk



See Section E2.10 to create this.

OBJECTIVE	RISK LEVEL			Total
	High	Average	Low	
Growth	332	132	16	480
Value	14	113	231	358
Total	346	245	247	838

You construct this contingency table by tallying the joint responses for each of the 838 mutual funds with respect to objective and risk into one of the six possible cells in the table. The first fund listed in the [Mutual Funds.xls](#) file is classified as a growth fund with an average risk. Thus, you tally this joint response into the cell that is the intersection of the first row and second column. The remaining 837 joint responses are recorded in a similar manner. Each cell contains the frequency for the row–column combination.

In order to further explore any possible pattern or relationship between objective and fund risk, you can construct contingency tables based on percentages. You first convert these results into percentages based on the following three totals:

1. The overall total (i.e., the 838 mutual funds)
2. The row totals (i.e., 480 growth funds and 358 value funds)
3. The column totals (i.e., 346 high, 245 average, and 247 low)

Tables 2.15, 2.16, and 2.17 summarize these percentages.

**TABLE 2.15**

Contingency Table  
Displaying Fund  
Objective and Fund Risk,  
Based on Percentage  
of Overall Total

OBJECTIVE	RISK LEVEL			Total
	High	Average	Low	
Growth	39.62	15.75	1.91	57.28
Value	1.67	13.48	27.57	42.72
Total	41.29	29.23	29.48	100.00

**TABLE 2.16**

Contingency Table  
Displaying Fund  
Objective and Fund Risk,  
Based on Percentage  
of Row Total

OBJECTIVE	RISK LEVEL			Total
	High	Average	Low	
Growth	69.17	27.50	3.33	100.00
Value	3.91	31.56	64.53	100.00
Total	29.24	41.29	29.47	100.00

**TABLE 2.17**

Contingency Table  
Displaying Fund  
Objective and Fund Risk,  
Based on Percentage of  
Column Total

OBJECTIVE	RISK LEVEL			Total
	High	Average	Low	
Growth	95.95	53.88	6.48	57.28
Value	4.05	46.12	93.52	42.72
<b>Total</b>	<b>100.00</b>	<b>100.00</b>	<b>100.00</b>	<b>100.00</b>

Table 2.15 shows that 41.29% of the mutual funds sampled are high risk, 57.28% are growth funds, and 39.62% are high-risk funds that are growth funds. Table 2.16 shows that 69.17% of the growth funds are high risk and 3.33% are low risk. Table 2.17 shows that 95.95% of the high-risk funds and only 6.48% of the low-risk funds are growth funds. The tables reveal that growth funds are more likely to be high risk, while value funds are more likely to be low risk.

### The Side-by-Side Bar Chart

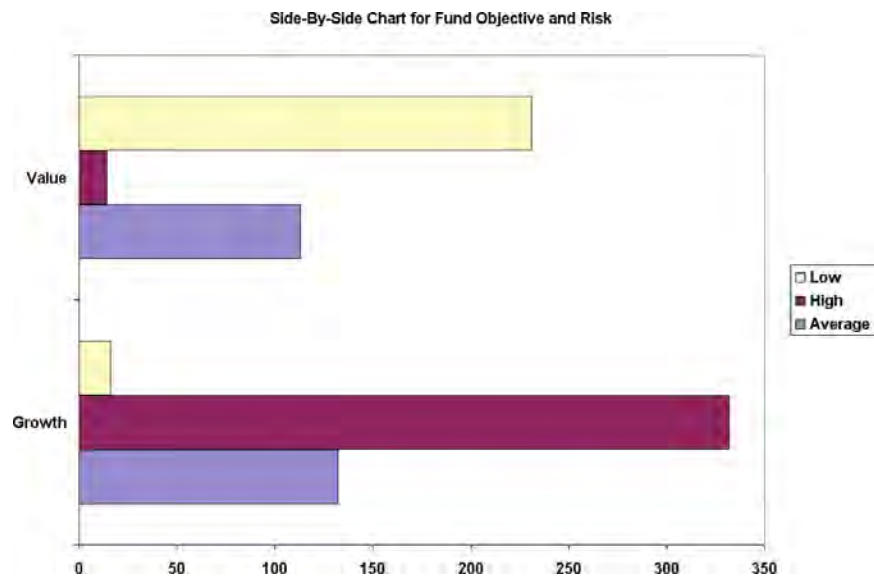
A useful way to visually display the results of cross-classification data is by constructing a **side-by-side bar chart**. Figure 2.14, which uses the data from Table 2.14, is a Microsoft Excel side-by-side bar chart that compares the three fund risk levels, based on their objectives. An examination of Figure 2.14 reveals results consistent with those of Tables 2.15, 2.16, and 2.17: Growth funds are more likely to be high risk, while value funds are more likely to be low risk.

**FIGURE 2.14**

Microsoft Excel side-by-side bar chart for fund objective and risk



See Section E2.11 to create this.



## PROBLEMS FOR SECTION 2.4

### Learning the Basics

**PH Grade ASSIST** **2.28** The following data represent the responses to two questions asked in a survey of 40 college

students majoring in business—What is your gender? (male = M; female = F) and What is your major? (accounting = A; computer information systems = C; marketing = M):

Gender: M M M F M F F M F M F M M M M F F M F F  
 Ma.jr: A C C M A C A A C C A A A M C M A A A C  
 Gender: M M M M F M F F M M F M M M M F M F M M  
 Ma.jr: C C A A M M C A A A C C A A A A C C A C



- Tally the data into a contingency table where the two rows represent the gender categories and the three columns represent the academic-major categories.
- Form contingency tables based on percentages of all 40 student responses, based on row percentages and based on column percentages.
- Using the results from (a), construct a side-by-side bar chart of gender based on student major.

**PH Grade ASSIST** **2.29** Given the following contingency table, construct a side-by-side bar chart comparing *A* and *B* for each of the three-column categories:

	1	2	3	Total
A	20	40	40	100
B	80	80	40	200

### Applying the Concepts

**PH Grade ASSIST** **2.30** The results of a study made as part of a yield improvement effort at a semiconductor manufacturing facility provided defect data for a sample of 450 wafers. The following table presents a summary of the responses to two questions: Was a particle found on the die that produced the wafer? and Is the wafer good or bad?

QUALITY OF WAFER	CONDITION OF DIE		Totals
	No Particles	Particles	
Good	320	14	334
Bad	80	36	116
Totals	400	50	450

Source: Extracted from S. W. Hall, "Analysis of Defectivity of Semiconductor Wafers by Contingency Table," Proceedings Institute of Environmental Sciences, Vol. 1 (1994), pp. 177–183.

- Construct contingency tables based on total percentages, row percentages, and column percentages.
- Construct a side-by-side bar chart of quality of wafers based on condition of die.
- What conclusions do you draw from these analyses?

**PH Grade ASSIST** **2.31** Each day at a large hospital, several hundred laboratory tests are performed. The rate at which these tests are done improperly (and therefore need to be redone) seems steady, at about 4%. In an effort to get to the root cause of these nonconformances (that is, tests that need to be redone), the director of the lab decided to keep records over a period of one week. The laboratory tests were subdivided by the shift of workers who performed the lab tests. The results are as follows:

LAB TESTS PERFORMED	SHIFT		Total
	Day	Evening	
Nonconforming	16	24	40
Conforming	654	306	960
Total	670	330	1,000

- Construct contingency tables based on total percentages, row percentages, and column percentages.
- Which type of percentage—row, column, or total—do you think is most informative for these data? Explain.
- What conclusions concerning the pattern of nonconforming laboratory tests can the laboratory director reach?

**SELF Test** **2.32** A sample of 500 shoppers was selected in a large metropolitan area to determine various information concerning consumer behavior. Among the questions asked was "Do you enjoy shopping for clothing?" The results are summarized in the following cross-classification table:

ENJOY SHOPPING FOR CLOTHING	GENDER		Total
	Male	Female	
Yes	136	224	360
No	104	36	140
Total	240	260	500

- Construct contingency tables based on total percentages, row percentages, and column percentages.
- Construct a side-by-side bar chart of enjoying shopping for clothing based on gender.
- What conclusions do you draw from these analyses?

**2.33** As more Americans use cell phones, they question where it is okay to talk on cell phones. The following is a table of results, in percentages, for 2000 and 2006 (extracted from W. Koch, "Business Put a Lid on Chatterboxes," *USAToday*, February 7, 2006, p. 3A):

OK TO TALK ON A CELL PHONE IN A RESTAURANT	YEAR	
	2000	2006
Yes	31	21
No	69	79
Total	100	100

- Construct a side-by-side bar chart.
- Discuss the changes in attitude concerning the use of cell phones in restaurants between 2000 and 2006.

**2.34** An experiment was conducted by James Choi, David Labson, and Brigitte Madrian to study the choices made in fund selection. When presented with four S&P 500 index funds that were identical except for their fees, undergraduate and MBA students chose the funds as follows (in percentages):

Fund	Student Group	
	Undergraduate	MBA
Lowest-cost fund	19	19
Second-lowest-cost fund	37	40
Third-lowest-cost fund	17	23
Highest-cost fund	27	18

Source: Extracted from J. Choi, D. Laibson, and B. Madrian, Why Does the Law of One Practice Fail? An Experiment on Mutual Funds [www.som.yale.edu/faculty/jj83/fes.pdf](http://www.som.yale.edu/faculty/jj83/fes.pdf)

a. Construct a side-by-side bar chart for the two student groups.

b. What do these results tell you about the differences between undergraduate and MBA students in their ability to choose S&P 500 index funds?

**2.35** Where people turn to for news is different for various age groups. A study indicated where different age groups primarily get their news.

Media	Age Group		
	Under 36	36–50	50+
Local TV	107	119	133
National TV	73	102	127
Radio	75	97	109
Local newspaper	52	79	107
Internet	95	83	76

a. Construct a side-by-side bar chart for the three age groups.

b. What differences are there in the age groups?

## 2.5 SCATTER PLOTS AND TIME-SERIES PLOTS

When analyzing a single numerical variable such as the cost of a restaurant meal or the three-year annualized return, you use histograms, polygons, and cumulative percentage polygons, developed in Section 2.3. This section discusses scatter plots and time series plots, which are used when you have two numerical variables.

### The Scatter Plot

You use a **scatter plot** to examine possible relationships between two numerical variables. For each observation, you plot one variable on the *X* axis and the other variable on the vertical *Y* axis. For example, a marketing analyst could study the effectiveness of advertising by comparing weekly sales volumes and weekly advertising expenditures. Or a human resources director interested in the salary structure of the company could compare the employees' years of experience with their current salaries.

To demonstrate a scatter plot, you can examine the relationship between the cost of different items in various cities (extracted from K. Spors, "Keeping Up with . . . Yourself," *The Wall Street Journal*, April 11, 2005, p. R4). Table 2.18 provides the cost of a fast-food hamburger meal and the cost of two movie tickets in 10 cities around the world. The data file [Cost of Living.xls](#) contains the complete data set.

**TABLE 2.18**

Cost of a Fast-Food Hamburger Meal and Cost of Two Movie Tickets in 10 Cities

City	Hamburger	Movie Tickets
Tokyo	5.99	32.66
London	7.62	28.41
New York	5.75	20.00
Sydney	4.45	20.71
Chicago	4.99	18.00
San Francisco	5.29	19.50
Boston	4.39	18.00
Atlanta	3.70	16.00
Toronto	4.62	18.05
Rio de Janeiro	2.99	9.90

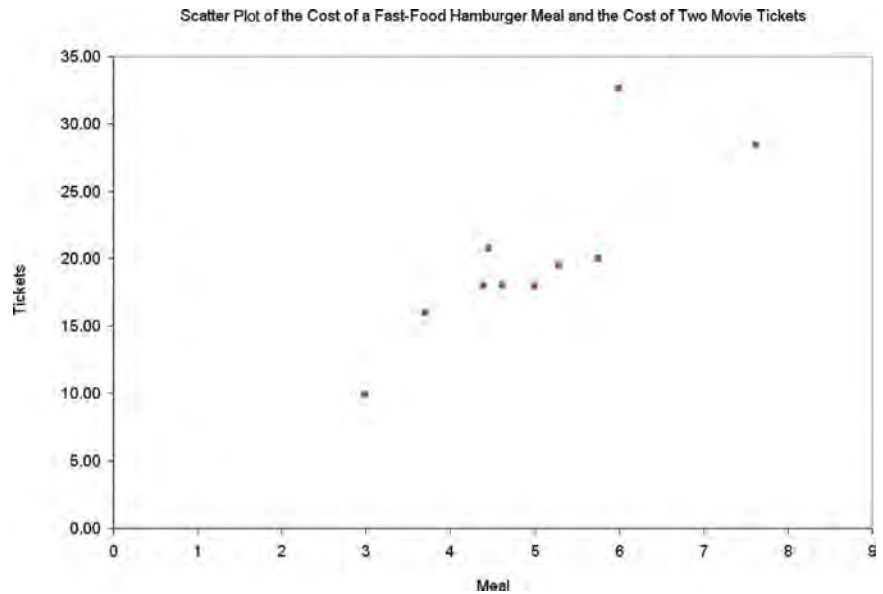
For each city, you plot the cost of a fast-food hamburger meal on the  $X$  axis, and the cost of two movie tickets on the  $Y$  axis. Figure 2.15 presents a Microsoft Excel scatter plot for these two variables.

**FIGURE 2.15**

Microsoft Excel scatter plot of the cost of a fast-food hamburger meal and the cost of two movie tickets



See Section E2.12 to create this.



Although there is some variation, there appears to be a clearly increasing (positive) relationship between the cost of a fast-food hamburger meal and the cost of two movie tickets. In other words, cities in which the cost of a fast-food hamburger meal is low seem to also have a low cost of two movie tickets. Other pairs of variables may have a decreasing (negative) relationship in which one variable decreases as the other increases. The scatter plot will be studied again in Chapter 3, when the coefficient of correlation and the covariance are presented, and in Chapter 13, when regression analysis is developed.

### The Time-Series Plot

A **time-series plot** is used to study patterns in the values of a numerical variable over time. Each value is plotted as a point in two dimensions with the time period on the horizontal  $X$  axis and the variable of interest on the  $Y$  axis.

To demonstrate a time-series plot, you can examine the yearly movie attendance, in billions, from 1999 to 2005 (extracted from C. Passy, “Good Night and Good Luck,” *Palm Beach Post*, February 5, 2006, p. 1J). Table 2.19 presents the data for the yearly movie attendance (see the file [movies.xls](#)). Figure 2.16 is a time-series plot of the movie attendance (in billions) from 1999 to 2005. You can see that although movie attendance increased from 1999 to 2002, it has declined since then. Attendance in 2005 was below attendance in 1999.

**TABLE 2.19**

Movie Attendance, in Billions, from 1999 to 2005

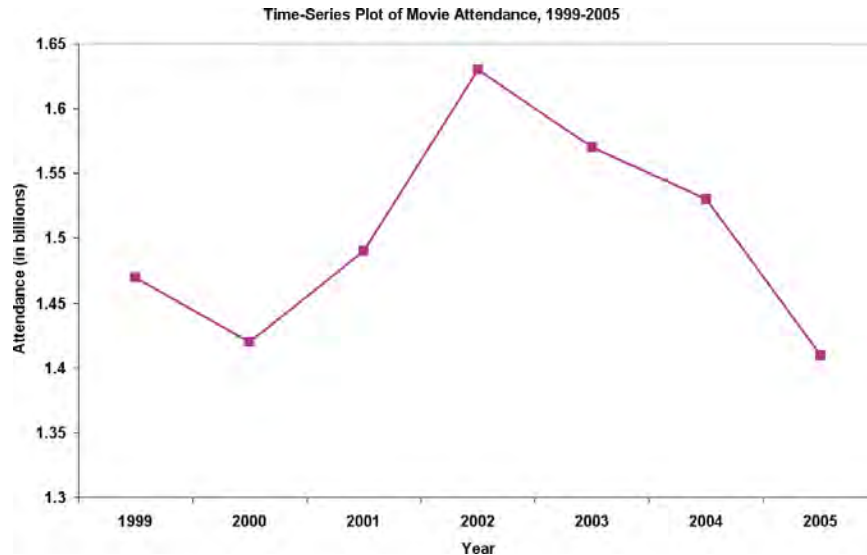
Year	Attendance
1999	1.47
2000	1.42
2001	1.49
2002	1.63
2003	1.57
2004	1.53
2005	1.41

**FIGURE 2.16**

Microsoft Excel time-series plot of movie attendance from 1999 to 2005



See Section E2.13 to create this.



## PROBLEMS FOR SECTION 2.5

### Learning the Basics

**PH Grade ASSIST 2.36** The following is a set of data from a sample of  $n = 11$  items:

X	7	5	8	3	6	10	12	4	9	15	18
Y	21	15	24	9	18	30	36	12	27	45	54

- Construct a scatter plot.
- Is there a relationship between  $X$  and  $Y$ ? Explain.

**PH Grade ASSIST 2.37** The following is a series of annual sales (in millions of dollars) over an 11-year period (1996 to 2006):

Year	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006
Sales	13.0	17.0	19.0	20.0	20.5	20.5	20.5	20.0	19.0	17.0	13.0

- Construct a time-series plot.
- Does there appear to be any change in real annual sales over time? Explain.

### Applying the Concepts

**SELF Test 2.38** There are several methods for calculating fuel economy. The following table (contained in the file [mileage.xls](#)) indicates the mileage as calculated by owners and by current government standards:

Car	Owner	Government
2005 Ford F-150	14.3	16.8
2005 Chevrolet Silverado	15.0	17.8
2002 Honda Accord LX	27.8	26.2
2002 Honda Civic	27.9	34.2

Car	Owner	Government
2004 Honda Civic Hybrid	48.8	47.6
2002 Ford Explorer	16.8	18.3
2005 Toyota Camry	23.7	28.5
2003 Toyota Corolla	32.8	33.1
2005 Toyota Prius	37.3	56.0

- Construct a scatter plot with owner mileage on the  $X$  axis and current government standards mileage on the  $Y$  axis.
- Does there appear to be a relationship between owner and current government standards mileage? If so, is the relationship positive or negative?

**2.39** The file [chicken.xls](#) contains data on the calories and total fat, in grams per serving, for a sample of 20 chicken sandwiches from fast-food chains:

Sandwich	Calories	Fat
Wendy's Ultimate Chicken Grill	360	7
Baja Fresh Original Baja Taco with Charbroiled Chicken	370	8
Burger King Smoky BBQ Fire Grilled Chicken Baguette	380	4
Quiznos Sub Honey Bourbon Chicken on wheat bread	400	5
McDonald's Chicken McGrill	400	16
Blimpie Grilled Chicken Hot Sub	470	20
Subway Oven Roasted Chicken Breast	470	20
Blimpie Buffalo Chicken Hot Sub	500	24
Wendy's Spicy Chicken Fillet	510	19

*(continued)*

Sandwich	Calories	Fat
Taco Bell Ranchero Chicken Soft Taco (two tacos)	540	30
KFC Oven Roasted Chicken Tender Wrap	550	23
Subway Buffalo Chicken Sub	550	30
Burger King Chicken Whopper	570	25
Au Bon Pain Arizona Chicken	580	19
Boston Market Rotisserie Chicken Carver	640	29
Chipolte Soft Tacos with Chicken (three tacos)	660	29
Cosi Grilled Chicken with Tomato, Basil, and Mozzarella	720	30
Atlanta Bread Company Chargrilled Chicken Pesto Panini	740	30
Corner Bakery Café Chicken Pomodori Panini	910	40
Panera Bread Tuscan Chicken on Rosemary & Onion Focaccia	950	56

Source: Extracted from “Fast food: Adding Health to the Menu,” Consumer Reports, September 2004, pp. 28–31.

- Construct a scatter plot with calories on the  $X$  axis and total fat on the  $Y$  axis.
- What conclusions can you reach about the relationship between the calories and total fat in chicken sandwiches?

**2.40** College basketball is big business, with coaches’ salaries, revenues, and expenses in millions of dollars. The data file [colleges-basketball.xls](#) contains the coaches’ salary and revenue for college basketball at selected schools in a recent year (extracted from R. Adams, “Pay for Playoffs,” *The Wall Street Journal*, March 11–12, 2006, pp. P1, P8).

- Construct a scatter plot with coaches’ salaries on the  $X$  axis and revenue on the  $Y$  axis.
- What conclusions can you reach about the relationship between the coach’s salary and the revenue?
- You would expect a school with a higher revenue to have a higher coach’s salary? Is this borne out by the data?

**2.41** College football players trying out for the NFL are given the Wonderlic standardized intelligence test. The data file [wonderlic.xls](#) contains the average Wonderlic scores of football players trying out for the NFL and the graduation rate for football players at selected schools (extracted from S. Walker, “The NFL’s Smartest Team,” *The Wall Street Journal*, September 30, 2005, pp. W1, W10).

- Construct a scatter plot with average Wonderlic score on the  $X$  axis and graduation rate on the  $Y$  axis.
- What conclusions can you reach about the relationship between the average Wonderlic score and graduation rate?

**2.42** The U.S. Bureau of Labor Statistics compiles data on a wide variety of workforce issues. The following table (contained in the file [unemploy.xls](#)) gives the monthly seasonally adjusted civilian unemployment rate for the United States from 2000 to 2005:

Month	2000	2001	2002	2003	2004	2005
January	4.0	4.2	5.7	5.8	5.7	5.2
February	4.1	4.2	5.7	5.9	5.6	5.4
March	4.0	4.3	5.7	5.9	5.7	5.1
April	3.8	4.4	5.9	6.0	5.5	5.1
May	4.0	4.3	5.8	6.1	5.6	5.1
June	4.0	4.5	5.8	6.3	5.6	5.0
July	4.0	4.6	5.8	6.2	5.5	5.0
August	4.1	4.9	5.7	6.1	5.4	4.9
September	3.9	5.0	5.7	6.1	5.4	5.1
October	3.9	5.3	5.7	6.0	5.4	4.9
November	3.9	5.5	5.9	5.9	5.4	5.0
December	3.9	5.7	6.0	5.7	5.4	4.9

Source: U.S. Bureau of Labor Statistics, [www.bls.gov](#), Sept. 19, 2006.


- Construct a time-series plot of the U.S. unemployment rate.
- Does there appear to be any pattern?

**2.43** In 2005, five million people in the United States subscribed to online dating services. The subscribers spent, on average, \$99 for the services provided, thus generating nearly \$500 million for the online dating companies. The following table (contained in the file [dating.xls](#)) gives the number of subscribers (in millions) to U.S. dating Web sites from 2000 to 2005:

Year	Number of Subscribers (millions)
2000	0.7
2001	1.6
2002	2.8
2003	4.3
2004	4.7
2005	5.0

Source: Extracted from “Making an E-Match,” National Geographic, February, 2006, p. 128.

- Construct a time-series plot.
- Do you think the number of subscribers is increasing or decreasing? Do you think the *rate* of growth is increasing or decreasing? Explain.

 **2.44** The following table contained in the file [prescriptions.xls](#) (extracted from “Price of Sickness

Rising,” *USA Today*, April 7, 2006, p. A1), shows the average price of prescription drugs from 2000 to 2004:

Year	Price (\$)
2000	46
2001	50
2002	55
2003	60
2004	64

- Construct a time-series plot for the average price of prescription drugs from 2000 to 2004.
- What pattern, if any, is present in the data?
- If you had to make a prediction of the average price of prescription drugs in 2005, what would you predict?

**2.45** The following data, contained in the file [deals.xls](#), provide the number of mergers and acquisitions made during January 1 through January 11 of each year from 1995 to 2006 (extracted from “Back of the Envelope,” *The New York Times*, January 13, 2006, p. C7):

Year	Deals
1995	715
1996	865
1997	708
1998	861
1999	931
2000	939
2001	1,031
2002	893
2003	735
2004	759
2005	1,013
2006	622

- Construct a time-series plot.
- What pattern, if any, is present in the data?
- If you had to make a prediction of the mergers and acquisitions made during January 1 through January 11, 2007, what would you predict?

## 2.6 MISUSING GRAPHS AND ETHICAL ISSUES

Good graphical displays reveal what the data convey. Unfortunately, many graphs presented in newspapers and magazines as well as graphs that can be developed using the Chart Wizard of Microsoft Excel either are incorrect, misleading, or so unnecessarily complicated that they should never be used. To illustrate the misuse of graphs, the graph presented in Figure 2.17 is similar to one that was printed in *Time* magazine as part of an article on increasing exports of wine from Australia to the United States.

**FIGURE 2.17**

“Improper” display of Australian wine exports to the United States, in millions of gallons

Source: Adapted from S. Watterson, “Liquid Gold—Australians Are Changing the World of Wine. Even the French Seem Grateful,” *Time*, November 22, 1999, p. 68.



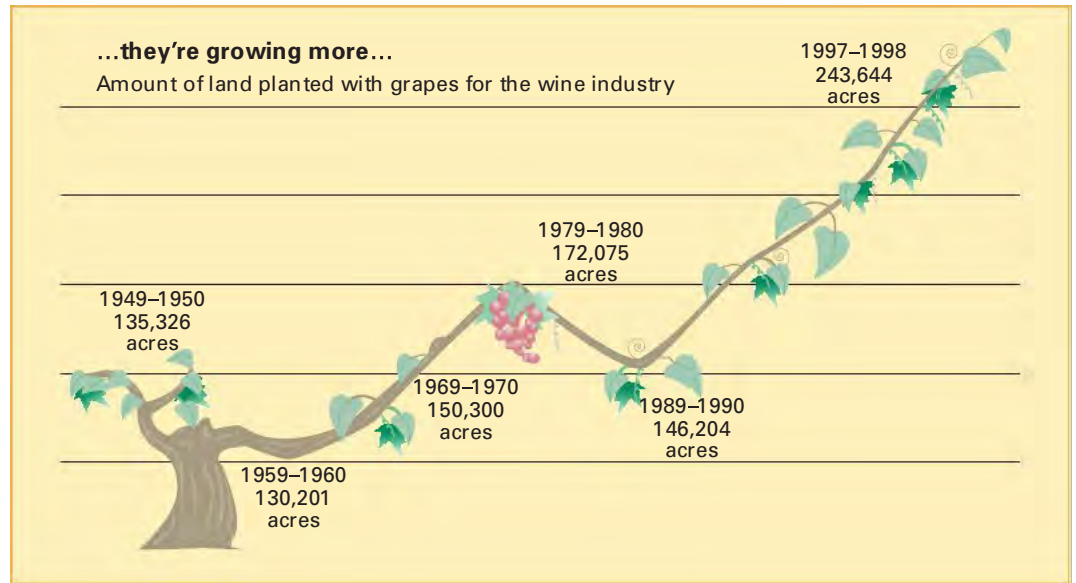
In Figure 2.17, the wineglass icon representing the 6.77 million gallons for 1997 does not appear to be almost twice the size of the wineglass icon representing the 3.67 million gallons for 1995, nor does the wineglass icon representing the 2.25 million gallons for 1992 appear to be twice the size of the wineglass icon representing the 1.04 million gallons for 1989. Part of the reason for this is that the three-dimensional wineglass icon is used to represent the two dimensions of exports and time. Although the wineglass presentation may catch the eye, the data should instead be presented in a summary table or a time-series plot.

In addition to the type of distortion created by the wineglass icons in the *Time* magazine graph displayed in Figure 2.17, improper use of the vertical and horizontal axes leads to distortions. Figure 2.18 presents another graph used in the same *Time* magazine article.

**FIGURE 2.18**

“Improper” display of amount of land planted with grapes for the wine industry

Source: Adapted from S. Watterson, “Liquid Gold—Australians Are Changing the World of Wine. Even the French Seem Grateful,” *Time*, November 22, 1999, pp. 68–69.



There are several problems in this graph. First, there is no zero point on the vertical axis. Second, the acreage of 135,326 for 1949 to 1950 is plotted above the acreage of 150,300 for 1969 to 1970. Third, it is not obvious that the difference between 1979 to 1980 and 1997 to 1998 (71,569 acres) is approximately 3.5 times the difference between 1979–1980 and 1969–1970 (21,775 acres). Fourth, there are no scale values on the horizontal axis. Years are plotted next to the acreage totals, not on the horizontal axis. Fifth, the values for the time dimension are not properly spaced along the horizontal axis. The value for 1979–1980 is much closer to 1989–1990 than it is to 1969–1970.

Other types of eye-catching displays that you typically see in magazines and newspapers often include information that is not necessary and just adds excessive clutter. Figure 2.19 represents one such display. The graph in Figure 2.19 shows those products with the largest market share for soft drinks in 1999. The graph suffers from too much clutter, although it is designed to show the differences in market share among the soft drinks. The display of the fizz for each soft drink takes up too much of the graph relative to the data. The same information could have been conveyed with a bar chart or pie chart.

**FIGURE 2.19**

Plot of market share of soft drinks in 1999

Source: Adapted from Anne B. Carey and Sam Ward, “Coke Still Has Most Fizz,” *USA Today*, May 10, 2000, p. 1B.



Some guidelines for developing good graphs are as follows:

- The graph should not distort the data.
- The graph should not contain unnecessary adornments (sometimes referred to as **chartjunk**).

- Any two-dimensional graph should contain a scale for each axis.
- The scale on the vertical axis should begin at zero.
- All axes should be properly labeled.
- The graph should contain a title.
- The simplest possible graph should be used for a given set of data.

Often these guidelines are unknowingly violated by individuals unaware of how to construct appropriate graphs. However, ethical issues arise when these guidelines are purposely violated in an effort to mislead the reader.

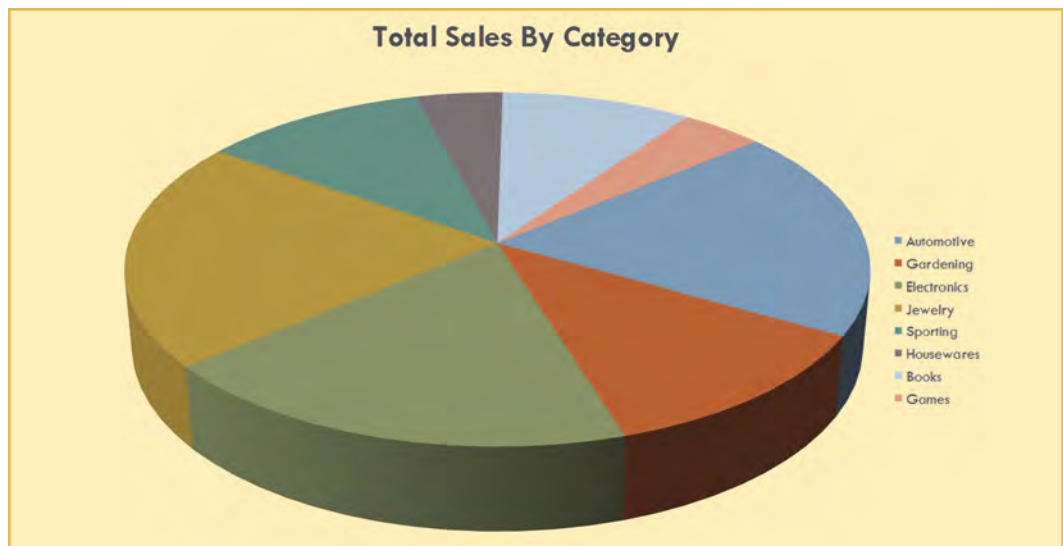
### Microsoft Excel Graphs

Unfortunately, to some extent, Microsoft Excel encourages you to violate these guidelines when you use the Chart feature to create graphs. Microsoft Excel offers you many types of unusual charts, such as doughnut, radar, surface, bubble, cone, and pyramid charts, that obscure the data for most analyses. Even for the simple graphs discussed in this chapter, Microsoft Excel creates graphs with unnecessary adornments or other examples of poor style. Fortunately, for the types of graphs discussed in this text, you will find instructions in Excel Companion sections that correct the style errors that Excel makes.

Oddly, many of the sample graphs used in marketing or educational materials by Microsoft for Microsoft Excel 2007, feature “three-dimensional” charts, such as the 3-D pie chart shown in Figure 2.20. Although you may see many examples of such “3-D” charts in Microsoft-supplied materials, you should avoid creating this type of chart as the perspective of the faked third dimension serves only to distort the relative proportions of each category shown on the chart.

**FIGURE 2.20**

“Poor-style” 3-D pie chart



## PROBLEMS FOR SECTION 2.6

### Applying the Concepts

**2.46 (Student Project)** Bring to class a chart from a newspaper or magazine that you believe to be a poorly drawn representation of a numerical variable. Be prepared to submit the chart to the instructor with comments as to why you believe it is inappropriate. Do you believe that the intent of the chart is to purposely mislead the reader? Also, be prepared to present and comment on this in class.

**2.47 (Student Project)** Bring to class a chart from a newspaper or magazine that you believe to be a poorly

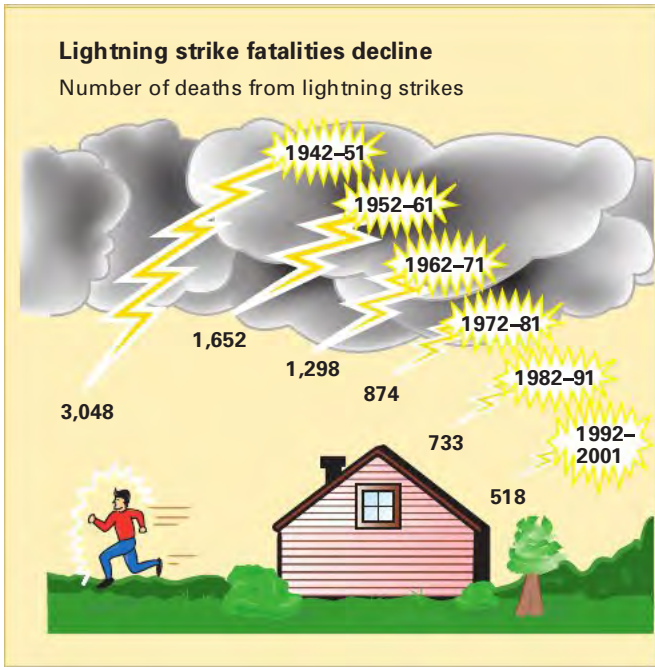
drawn representation of a categorical variable. Be prepared to submit the chart to the instructor with comments as to why you consider it inappropriate. Do you believe that the intent of the chart is to purposely mislead the reader? Also, be prepared to present and comment on this in class.

**2.48 (Student Project)** Bring to class a chart from a newspaper or magazine that you believe to contain too many unnecessary adornments (i.e., chartjunk) that may cloud the message given by the data. Be prepared to submit the chart to the instructor with comments about why you



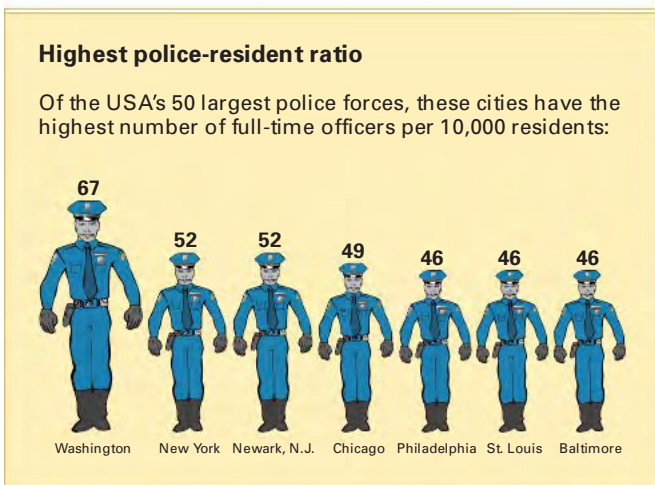
think it is inappropriate. Also, be prepared to present and comment on this in class.

**2.49** The following visual display contains an overembellished chart similar to one that appeared in *USA Today*, dealing with the number of deaths from lightning strikes in the United States:



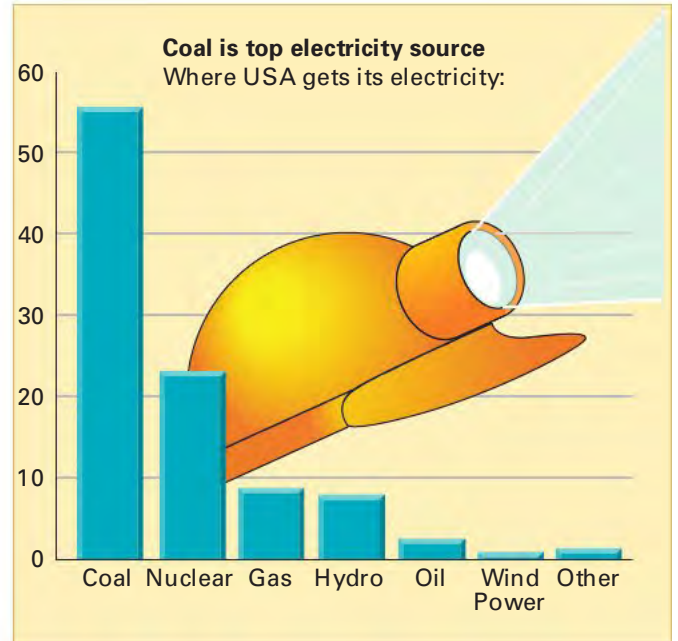
- a. Describe at least one good feature of this visual display.
- b. Describe at least one bad feature of this visual display.
- c. Redraw the graph, using the guidelines given on pages 63 and 64.

**2.50** The following visual display is similar to one that appeared in *USA Today* concerning the relative size of police departments in major U.S. cities:



- a. Indicate a feature of this chart that violates the principles of good graphs.
- b. Set up an alternative graph for the data provided in this figure.

**2.51** The following visual display concerning where the United States gets its electricity is similar to one that appeared in *USA Today*:



- a. Describe at least one good feature of this visual display.
- b. Describe at least one bad feature of this visual display.
- c. Redraw the graph, using the guidelines given on pages 63 and 64.

**2.52** An article in *The New York Times* (Donna Rosato, "Worried About the Numbers? How About the Charts?" *The New York Times*, September 15, 2002, p. B7) reported on research done on annual reports of corporations by Professor Deanna Oxender Burgess of Florida Gulf Coast University. Professor Burgess found that even slight distortions in a chart changed readers' perception of the information. The article displayed sales information from the annual report of Zale Corporation and showed how results were exaggerated.

Go to the World Wide Web or the library and study the most recent annual report of a selected corporation. Find at least one chart in the report that you think needs improvement and develop an improved version of the chart. Explain why you believe the improved chart is better than the one included in the annual report.

**2.53** Figures 2.1, 2.3, and 2.6 consist of a bar chart, a pie chart, and a Pareto diagram for how respondents would spend \$1,000.

- a. Use the Chart Wizard of Microsoft Excel to construct a doughnut chart, a cone chart, and a pyramid chart for how respondents spend \$1,000.
- b. Which graphs do you prefer—the bar chart, pie chart, and Pareto diagram or the doughnut chart, cone chart, and pyramid chart? Explain.
- a. Use the Chart Wizard of Microsoft Excel to develop a doughnut chart, a cone chart, and a pyramid chart for the risk level of the mutual funds.
- b. Which graphs do you prefer—the bar chart and pie chart or the doughnut chart, cone chart, and pyramid chart? Explain.

**2.54** Figures 2.2 and 2.4 consist of a bar chart and a pie chart for the risk level for the mutual fund data.

## SUMMARY

As you can see in Table 2.20, this chapter discusses data presentation. You have used various tables and charts to draw conclusions about spending an extra \$1,000, about the cost of restaurant meals in a city and its suburbs, and about the set of mutual funds that were first introduced in the

Using Statistics scenario at the beginning of the chapter. Now that you have studied tables and charts, in Chapter 3 you will learn about a variety of numerical descriptive measures useful for data analysis and interpretation.

**TABLE 2.20**

Roadmap for Selecting Tables and Charts

Type of Analysis	Type of Data	
	Numerical	Categorical
Tabulating, organizing, and graphically presenting the values of a variable	Ordered array, stem-and-leaf display, frequency distribution, relative frequency distribution, percentage distribution, cumulative percentage distribution, histogram, polygon, cumulative percentage polygon (Sections 2.2 and 2.3)	Summary table, bar chart, pie chart, Pareto diagram (Section 2.1)
Graphically presenting the relationship between two variables	Scatter plot, time-series plot (Section 2.5)	Contingency table, side-by-side bar chart (Section 2.4)

## KEY TERMS

bar chart 33	cumulative percentage polygon (ogive) 51	percentage polygon 50
cells 55	frequency distribution 44	pie chart 34
chartjunk 63	histogram 48	range 44
class boundaries 44	ogive (cumulative percentage polygon) 51	relative frequency distribution 46
class grouping 44	ordered array 41	scatter plot 58
class midpoint 44	Pareto diagram 35	side-by-side bar chart 56
contingency table 55	Pareto principle 36	stem-and-leaf display 41
cross tabulations 54	percentage distribution 46	summary table 33
cumulative percentage distribution 47		time-series plot 59
		width of class interval 44

# CHAPTER REVIEW PROBLEMS

## Checking Your Understanding

**PH Grade ASSIST** 2.55 How do histograms and polygons differ in terms of construction and use?

**PH Grade ASSIST** 2.56 Why would you construct a summary table?

**PH Grade ASSIST** 2.57 What are the advantages and/or disadvantages of using a bar chart, a pie chart, or a Pareto diagram?

**PH Grade ASSIST** 2.58 Compare and contrast the bar chart for categorical data with the histogram for numerical data.

2.59 What is the difference between a time-series plot and a scatter plot?

**PH Grade ASSIST** 2.60 Why is it said that the main feature of the Pareto diagram is its ability to separate the “vital few” from the “trivial many”? Discuss.

**PH Grade ASSIST** 2.61 What are the three different ways to break down the percentages in a contingency table?

## Applying the Concepts

2.62 The following data represent the breakdown of the price of a new college textbook:

Revenue Categories	Percentage
Publisher	64.8
Manufacturing costs	32.3
Marketing and promotion	15.4
Administrative costs and taxes	10.0
After-tax profit	7.1
Bookstore	22.4
Employee salaries and benefits	11.3
Operations	6.6
Pretax profit	4.5
Author	11.6
Freight	1.2

Source: Extracted from T. Lewin, “When Books Break the Bank,” The New York Times, September 16, 2003, pp. B1, B4.

- Using the four categories publisher, bookstore, author, and freight, construct a bar chart, a pie chart, and a Pareto diagram.
- Using the four subcategories of publisher and three subcategories of bookstore along with the author and freight categories, construct a Pareto diagram.
- Based on the results of (a) and (b), what conclusions can you reach concerning who gets the revenue from the sales of new college textbooks? Do any of these results surprise you? Explain.

2.63 The following data represent the global market share for plasma TVs and LCD TVs in the fourth quarter of 2005:

Company	Plasma TVs (%)	Company	LCD TVs (%)
Hitachi	7.3	LGE	6.5
LGE	15.4	Phillips/Magnavox	14.2
Panasonic	26.0	Samsung	11.6
Phillips	13.3	Sharp	13.6
Samsung	14.4	Sony	14.6
Others	23.6	Others	39.5

Source: Extracted from Y. Kkageyama, “Flat-Panel TVs Proving Savior of Japanese Electronics Makers,” The Palm Beach Post, April 23, 2006, p. F3.

- For plasma TVs and LCD TVs, separately construct a bar chart, a pie chart, and a Pareto diagram.
- Based on the results of (a), explain how the market share of plasma TVs differs from the market share of LCD TVs.

2.64 The following data represent energy consumption and renewable energy consumption in the United States in 2005:

Energy Source	%*	Renewable Energy Source	%*
Coal	23.0	Ethanol	10.1
Hydroelectric power	2.7	Geothermal	10.8
Natural gas	22.2	Solar	1.9
Nuclear electric power	8.1	Waste	17.1
Petroleum	40.5	Wind	4.5
Renewable fuels	3.3	Wood	55.5

\*Percentages do not add to 100% due to rounding.

Source: Energy Information Administration, 2006.

\*Percentages do not add to 100% due to rounding.

Source: Energy Information Administration, 2006.

- For energy consumption and renewable energy consumption, separately construct a bar chart, a pie chart, and a Pareto diagram.
- Based on the results of (a), what conclusions can you reach about energy consumption and renewable energy consumption in the United States in 2005?

2.65 The data on page 68 represent proven conventional oil reserves, in billions of barrels, subdivided by region and country:

Region and Country	Proven Conventional Reserves (Billions of Barrels)
<i>North America</i>	54.8
Mexico	28.3
U.S.	21.8
Canada	4.7
<i>Central and South America</i>	95.2
Venezuela	76.9
Brazil	8.1
Other Central and South America	10.2
<i>Western Europe</i>	17.2
Norway	9.5
Britain	5.0
Other Western Europe	2.7
<i>Africa</i>	74.9
Libya	29.5
Nigeria	22.5
Algeria	9.2
Angola	5.4
Other Africa	8.3
<i>Middle East</i>	683.6
Saudi Arabia	259.2
Iraq	112.5
United Arab Emirates	97.8
Kuwait	94.0
Iran	89.7
Qatar	13.2
Oman	5.5
Other Middle East	11.7
<i>Far East and Oceania</i>	44.0
China	24.0
Indonesia	5.0
India	4.7
Other Far East and Oceania	10.3
<i>Eastern Europe and Former USSR</i>	59.0
Russia	48.6
Kazakhstan	5.4
Other Eastern Europe and Former USSR	5.0

Source: U.S. Department of Energy.

- Using the set of countries, construct a bar chart, a pie chart, and a Pareto diagram.
- Using the set of regions, construct a bar chart, a pie chart, and a Pareto diagram.
- Which graphical method do you think is best to portray these data?
- Based on the results of (a) and (b), what conclusions can you make concerning the proven conventional oil reserves for the different countries and regions?

**2.66** In the aftermath of the attacks of September 11, 2001, statisticians at the National Center for Health Statistics became more concerned with their ability to track and classify victims of terrorism (E. Weinstein, "Tracking Terror's Rising Toll," *The Wall Street Journal*, January 25, 2002, p. A13). The following data represents deaths due to terrorism on U.S. soil from 1990 to 2001 and also the deaths in the United States in 2000 due to various causes:

Year	Deaths Due to Terrorism in the United States
1990	0
1991	0
1992	0
1993	6
1994	1
1995	169
1996	2
1997	0
1998	1
1999	3
2000	0
2001	2,717

Cause	Deaths, in Thousands
Smoke and fire	3.3
Accidental drowning	3.3
Alcohol-induced deaths	18.5
Alzheimer's disease	49.0
Assault by firearms	10.4
Assault by non-firearms	5.7
Asthma	4.4
Cancer	551.8
Strokes and related diseases	166.0
Emphysema	16.9
Diabetes	68.7
Heart disease	710.0
Falls	12.0
HIV	14.4
Influenza and pneumonia	67.0
Injuries at work	5.3
Motor vehicle accidents	41.8
Suicide	28.3
Drug-related deaths	15.9

Source: Federal Bureau of Criminal Justice Statistics, National Center for Health Statistics, National Highway Transportation Safety Administration, Department of Defense.

- Construct a time-series plot of deaths due to terrorism on U.S. soil. Is there any pattern to the deaths due to terrorism on U.S. soil between 1990 and 2001?

- b. For the deaths, in thousands, due to different causes, construct a bar chart, a pie chart, and a Pareto diagram.
- c. Which graphical method do you think is best to portray these data?
- d. Based on the results of (c), what conclusions can you make concerning the deaths in the United States in 2000 due to various causes?

**2.67** The owner of a restaurant serving Continental-style entrées is interested in studying patterns of patron demand for the Friday-to-Sunday weekend time period. Records are maintained that indicate the type of entrée ordered. The data are as follows:

Type of Entrée	Number Served
Beef	187
Chicken	103
Duck	25
Fish	122
Pasta	63
Shellfish	74
Veal	26

- a. Construct a percentage summary table for the types of entrées ordered.
- b. Construct a bar chart, a pie chart, and a Pareto diagram for the types of entrées ordered.
- c. Do you prefer a Pareto diagram or a pie chart for these data? Why?
- d. What conclusions can the restaurant owner draw concerning demand for different types of entrées?

**2.68** Suppose that the owner of the restaurant in Problem 2.67 is also interested in studying the demand for dessert during the same time period. She decided that two other variables, along with whether a dessert was ordered, are to be studied: the gender of the individual and whether a beef entrée is ordered. The results are as follows:

DESSERT ORDERED	GENDER		Total
	Male	Female	
Yes	96	40	136
No	224	240	464
Total	320	280	600

DESSERT ORDERED	BEEF ENTRÉE		Total
	Yes	No	
Yes	71	65	136
No	116	348	464
Total	187	413	600

- a. For each of the two contingency tables, construct a contingency table of row percentages, column percentages, and total percentages.
- b. Which type of percentage (row, column, or total) do you think is most informative for each gender? for beef entrée? Explain.
- c. What conclusions concerning the pattern of dessert ordering can the owner of the restaurant reach?

**2.69** An article in *The New York Times* (William McNulty and Hugh K. Truslow, “How It Looked Inside the Booth,” *The New York Times*, November 6, 2002) provided the following data on the method for recording votes in 1980, 2000, and 2002, broken down by percentage of counties in the United States using each method and the percentage of registered voters using each method. The results are as follows:

Percentage of Counties Using Method	1980	2000	2002
Punch cards	18.5	18.5	15.5
Lever machines	36.7	14.4	10.6
Paper ballots	40.7	11.9	10.5
Optical scan	0.8	41.5	43.0
Electronic	0.2	9.3	16.3
Mixed	3.1	4.4	4.1

Percentage of Registered Voters Using Method	1980	2000	2002
Punch cards	31.7	31.4	22.6
Lever machines	42.9	17.4	15.5
Paper ballots	10.5	1.5	1.3
Optical scan	2.1	30.8	31.8
Electronic	0.7	12.2	19.6
Mixed	12.0	6.7	9.3

- a. Construct separate pie charts for each year for the percentage of counties and the percentage of registered voters using the various methods.
- b. Construct side-by-side bar charts, by year, for the percentage of counties and the percentage of registered voters using the various methods.
- c. Which type of graphical display is more helpful in depicting the data? Explain.
- d. What differences are there in the results for the counties and the registered voters?

**2.70** In summer 2000, a growing number of warranty claims on Firestone tires sold on Ford SUVs prompted Firestone and Ford to issue a major recall. An analysis of

warranty-claims data helped identify which models to recall. A breakdown of 2,504 warranty claims based on tire size is given in the following table:

Tire Size	Warranty Claims
23575R15	2,030
311050R15	137
30950R15	82
23570R16	81
331250R15	58
25570R16	54
Others	62

Source: Extracted from Robert L. Simison, "Ford Steps Up Recall Without Firestone," The Wall Street Journal, August 14, 2000, p. A3.

The 2,030 warranty claims for the 23575R15 tires can be categorized into ATX models and Wilderness models. The type of incident leading to a warranty claim, by model type, is summarized in the following table:

Incident	ATX Model Warranty Claims	Wilderness Warranty Claims
Tread separation	1,365	59
Blowout	77	41
Other/unknown	422	66
Total	1,864	166

Source: Extracted from Robert L. Simison, "Ford Steps Up Recall Without Firestone," The Wall Street Journal, August 14, 2000, p. A3.

- Construct a Pareto diagram for the number of warranty claims by tire size. What tire size accounts for most of the claims?
- Construct a pie chart to display the percentage of the total number of warranty claims for the 23575R15 tires that come from the ATX model and Wilderness model. Interpret the chart.
- Construct a Pareto diagram for the type of incident causing the warranty claim for the ATX model. Does a certain type of incident account for most of the claims?
- Construct a Pareto diagram for the type of incident causing the warranty claim for the Wilderness model. Does a certain type of incident account for most of the claims?

**2.71** One of the major measures of the quality of service provided by any organization is the speed with which the organization responds to customer complaints. A large family-held department store selling furniture and flooring, including carpet, had undergone a major expansion in the past several years. In particular, the flooring department had expanded from 2 installation crews to an installation supervisor, a measurer, and 15 installation crews. During a

recent year, the company got 50 complaints concerning carpet installation. The following data (contained in the file [furniture.xls](#)) represent the number of days between the receipt of the complaint and the resolution of the complaint:

54	5	35	137	31	27	152	2	123	81	74	27
11	19	126	110	110	29	61	35	94	31	26	5
12	4	165	32	29	28	29	26	25	1	14	13
13	10	5	27	4	52	30	22	36	26	20	23
33	68										

- Construct a frequency distribution and a percentage distribution.
- Construct a histogram and a percentage polygon.
- Form a cumulative percentage distribution and plot a cumulative percentage polygon (ogive).
- On the basis of the results of (a) through (c), if you had to tell the president of the company how long a customer should expect to wait to have a complaint resolved, what would you say? Explain.

**2.72** Data concerning 58 of the best-selling domestic beers in the United States are located in the file [domesticbeer.xls](#). The values for three variables are included: percentage alcohol, number of calories per 12 ounces, and number of carbohydrates (in grams) per 12 ounces.

Source: Extracted from [www.Beer100.com](#), March 31, 2006.

- Construct a histogram for each of the three variables.
- Construct three scatter plots: percentage alcohol versus calories, percentage alcohol versus carbohydrates, and calories versus carbohydrates.
- Discuss the information you learned from studying the graphs in (a) and (b).

**2.73** The data in the file [spending.xls](#) are the per-capita spending, in thousands of dollars, for each state in 2004.

- Develop an ordered array.
- Construct a frequency distribution and a percentage distribution.
- Plot a percentage histogram.
- What conclusions can you reach about the differences in federal per-capita spending between the states?

**2.74** The data in the file [savings.xls](#) are the yields for a money market account, a one-year certificate of deposit (CD), and a five-year CD for 40 banks in south Florida, as of December 20, 2005 (extracted from Bankrate.com, December 20, 2005).

- Construct a histogram for each of the three variables.
- Construct three scatter plots: money market account versus one-year CD, money market account versus five-year CD, and one-year CD versus five-year CD.

- c. Discuss the information you learned from studying the graphs in (a) and (b).

**2.75** The data in the file [ceo.xls](#) represent the total compensation (in \$millions) of CEOs of the 100 largest companies, by revenue (extracted from “Special Report: Executive Compensation,” *USA Today*, April 10, 2006, pp. 3B, 4B).

- Construct a frequency distribution and a percentage distribution.
- Construct a histogram and a percentage polygon.
- Construct a cumulative percentage distribution and plot a cumulative percentage polygon (ogive).
- Based on (a) through (c), what conclusions can you reach concerning CEO compensation in 2005?

**2.76** Studies conducted by a manufacturer of “Boston” and “Vermont” asphalt shingles have shown product weight to be a major factor in customers’ perception of quality. Moreover, the weight represents the amount of raw materials being used and is therefore very important to the company from a cost standpoint. The last stage of the assembly line packages the shingles before the packages are placed on wooden pallets. When a pallet is full (a pallet for most brands holds 16 squares of shingles), it is weighed, and the measurement is recorded. The company expects pallets of its “Boston” brand-name shingles to weigh at least 3,050 pounds but less than 3,260 pounds. For the company’s “Vermont” brand-name shingles, pallets should weigh at least 3,600 pounds but less than 3,800. The data file [pallet.xls](#) contains the weights (in pounds) from a sample of 368 pallets of “Boston” shingles and 330 pallets of “Vermont” shingles.

- For the “Boston” shingles, construct a frequency distribution and a percentage distribution having eight class intervals, using 3,015, 3,050, 3,085, 3,120, 3,155, 3,190, 3,225, 3,260, and 3,295 as the class boundaries.
- For the “Vermont” shingles, construct a frequency distribution and a percentage distribution having seven class intervals, using 3,550, 3,600, 3,650, 3,700, 3,750, 3,800, 3,850, and 3,900 as the class boundaries.
- Construct histograms for the “Boston” shingles and for the “Vermont” shingles.
- Comment on the distribution of pallet weights for the “Boston” and “Vermont” shingles. Be sure to identify the percentage of pallets that are underweight and overweight.

**2.77** Do marketing promotions, such as bobble-head giveaways, increase attendance at Major League Baseball games? An article in *Sport Marketing Quarterly* reported on the effectiveness of marketing promotions (extracted from T. C. Boyd and T. C. Krehbiel, “An Analysis of the Effects of Specific Promotion Types on Attendance at Major League Baseball Games,” *Mid-American Journal*

*of Business*, Vol. 21, 2006, pp. 21–32). The data file [royals.xls](#) includes the following variables for the Kansas City Royals during the 2002 baseball season:

GAME = Home games in the order they were played  
 ATTENDANCE = Paid attendance for the game  
 PROMOTION 1 = If a promotion was held; 0 = if no promotion was held

- Construct a percentage histogram for the attendance variable. Interpret the histogram.
- Construct a percentage polygon for the attendance variable. Interpret the polygon.
- Which graphical display do you prefer, the one in (a) or (b)? Explain.
- Construct a graphical display containing two percentage polygons for attendance—one for the 43 games with promotions and the second for the 37 games without promotions. Compare the two attendance distributions.

**2.78** The data in the file [protein.xls](#) indicate fat and cholesterol information concerning popular protein foods (fresh red meats, poultry, and fish).

*Source: U.S. Department of Agriculture.*

For the data relating to the number of calories and the amount of cholesterol for the popular protein foods:

- Construct a frequency distribution and a percentage distribution.
- Construct a histogram and a percentage polygon.
- Form a cumulative percentage distribution and plot a cumulative percentage polygon.
- What conclusions can you draw from these analyses?

**2.79** The data in the file [states.xls](#) represent the results of the American Community Survey, a sampling of households taken in all states during the 2000 U.S. Census. For each of the variables average travel-to-work time in minutes, percentage of homes with eight or more rooms, median household income, and percentage of mortgage-paying homeowners whose housing costs exceed 30% of income:

- Construct a frequency distribution and a percentage distribution.
- Construct a histogram and a percentage polygon.
- Construct a cumulative percentage distribution and plot a cumulative percentage polygon.
- What conclusions about these four variables can you make based on the results of (a) through (c)?

**2.80** The economics of baseball has caused a great deal of controversy, with owners arguing that they are losing money, players arguing that owners are making money, and fans complaining about how expensive it is to attend a game and watch games on cable television. In addition to data related to team statistics for the 2001 season, the file [bb2001.xls](#) contains team-by-team statistics on ticket prices;

the fan cost index; regular-season gate receipts; local television, radio, and cable receipts; all other operating revenue; player compensation and benefits; national and other local expenses; and income from baseball operations. For each of these variables,

- Construct a frequency distribution and a percentage distribution.
- Construct a histogram and a percentage polygon.
- Construct a cumulative percentage distribution and plot a cumulative percentage polygon.
- Construct a scatter plot to predict the number of wins on the  $Y$  axis from the player compensation and benefits on the  $X$  axis. What conclusions can you reach based on this scatter plot?
- What conclusions about these variables can you reach based on the results of (a) through (d)?

**2.81** In Section 2.5 on page 59, a scatter plot of the relationship between the cost of a fast-food hamburger meal and the cost of movie tickets in 10 different cities was constructed. The data file **Cost of Living.xls** also includes the overall cost index, the monthly rent for a two-bedroom apartment, the cost of a cup of coffee with service, the cost of dry cleaning a men's blazer, and the cost of toothpaste.

- Construct six separate scatter plots. For each, use the overall cost index as the  $Y$  axis. Use the monthly rent for a two-bedroom apartment, the costs of a cup of coffee with service, a fast-food hamburger meal, dry cleaning a men's blazer, toothpaste, and movie tickets as the  $X$  axis.
- What conclusions can you reach about the relationship of the overall cost index to these six variables?

**2.82** In Problem 2.39 on page 60, using the data set **chicken.xls**, you constructed a scatter plot of calories with the total fat content of chicken sandwiches.

- Construct a scatter plot of calories on the  $Y$  axis and carbohydrates on the  $X$  axis.
- Construct a scatter plot of calories on the  $Y$  axis and sodium on the  $X$  axis.
- Which variable (total fat, carbohydrates, or sodium) seems to be most closely related to calories? Explain.

**2.83** The data file **gas.xls** contains the weekly average price of gasoline in the United States from March 1, 2004, to March 6, 2006. Prices are in dollars per gallon.

Source: U.S. Department of Energy, [www.eia.doe.gov](http://www.eia.doe.gov).

- Construct a time-series plot.
- What pattern, if any, is present in the data?

**2.84** The data contained in the file **drink.xls** represent the amount of soft drink filled in a sample of 50 consecutive 2-liter bottles. The results are listed horizontally in the order of being filled:

2.109 2.086 2.066 2.075 2.065 2.057 2.052 2.044 2.036 2.038  
2.031 2.029 2.025 2.029 2.023 2.020 2.015 2.014 2.013 2.014  
2.012 2.012 2.012 2.010 2.005 2.003 1.999 1.996 1.997 1.992  
1.994 1.986 1.984 1.981 1.973 1.975 1.971 1.969 1.966 1.967  
1.963 1.957 1.951 1.951 1.947 1.941 1.941 1.938 1.908 1.894

- Construct a time-series plot for the amount of soft drink on the  $Y$  axis and the bottle number (going consecutively from 1 to 50) on the  $X$  axis.
- What pattern, if any, is present in these data?
- If you had to make a prediction of the amount of soft drink filled in the next bottle, what would you predict?
- Based on the results of (a) through (c), explain why it is important to construct a time-series plot and not just a histogram, as was done in Problem 2.27 on page 54.

**2.85** The S&P 500 Index tracks the overall movement of the stock market by considering the stock prices of 500 large corporations. The data file **stocks2005.xls** contains weekly data for this index as well as the weekly closing stock price for three companies during 2005. The variables included are:

WEEK —Week ending on date given  
S&P —Weekly closing value for the S&P 500 Index  
SEARS —Weekly closing stock price for Sears  
TARGET —Weekly closing stock price for the Target  
SARA LEE —Weekly closing stock price for the Sara Lee

Source: Extracted from [finance.yahoo.com](http://finance.yahoo.com).

- Construct a time-series plot for the weekly closing values of the S&P 500 Index, Sears, Target, and Sara Lee.
- Explain any patterns present in the plots.
- Write a short summary of your findings.

**2.86 (Class Project)** Let each student in the class respond to the question “Which carbonated soft drink do you most prefer?” so that the teacher can tally the results into a summary table.

- Convert the data to percentages and construct a Pareto diagram.
- Analyze the findings.

**2.87 (Class Project)** Let each student in the class be cross-classified on the basis of gender (male, female) and current employment status (yes, no) so that the teacher can tally the results.

- Construct a table with either row or column percentages, depending on which you think is more informative.
- What would you conclude from this study?
- What other variables would you want to know regarding employment in order to enhance your findings?

## Report Writing Exercises

**2.88** Referring to the results from Problem 2.76 on page 71 concerning the weight of “Boston” and “Vermont” shingles,



write a report that evaluates whether the weight of the pallets of the two types of shingles are what the company expects. Be sure to incorporate tables and charts into the report.

**2.89** Referring to the results from Problem 2.70 on page 69 concerning the warranty claims on Firestone tires, write a report that evaluates warranty claims on Firestone tires sold on Ford SUVs. Be sure to incorporate tables and charts into the report.

### Team Project

The data file [Mutual Funds.xls](#) contains information regarding nine variables from a sample of 838 mutual funds. The variables are:

- Category—Type of stocks comprising the mutual fund (small cap, mid cap, large cap)
- Objective—Objective of stocks comprising the mutual fund (growth or value)
- Assets—In millions of dollars
- Fees—Sales charges (no or yes)
- Expense ratio—Ratio of expenses to net assets in percentage
- 2005 return—Twelve-month return in 2005
- Three-year return—Annualized return, 2003–2005
- Five-year return—Annualized return, 2001–2005
- Risk—Risk-of-loss factor of the mutual fund (low, average, or high)

**2.90** For the expense ratio:

- a. Construct a histogram.
- b. Plot percentage polygons of the expense ratio for mutual funds that have fees and mutual funds that do not have fees on the same graph.
- c. What conclusions about the expense ratio can you reach based on the results of (a) and (b)?

**2.91** For the five-year annualized return from 2001 to 2005:

- a. Construct a histogram.
- b. Plot percentage polygons of the five-year annualized return from 2001 to 2005 for growth mutual funds and value mutual funds on the same graph.

- c. What conclusions about the five-year annualized return from 2001 to 2005 can you reach based on the results of (a) and (b)?

**2.92** For the return in 2005:

- a. Construct a histogram.
- b. Plot percentage polygons of the return in 2005 for growth mutual funds and value mutual funds on the same graph.
- c. What conclusions about the return in 2005 can you reach based on the results of (a) and (b)?

### Student Survey Database

**2.93** Problem 1.27 on page 15 describes a survey of 50 undergraduate students (see the file [undergradsurvey.xls](#)). For these data, construct all the appropriate tables and charts and write a report summarizing your conclusions.

**2.94** Problem 1.27 on page 15 describes a survey of 50 undergraduate students (see the file [undergradsurvey.xls](#)).

- a. Select a sample of 50 undergraduate students at your school and conduct a similar survey for those students.
- b. For the data collected in (a), construct all the appropriate tables and charts and write a report summarizing your conclusions.
- c. Compare the results of (b) to those of Problem 2.93.

**2.95** Problem 1.28 on page 15 describes a survey of 50 MBA students (see the file [gradsurvey.xls](#)). For these data, construct all appropriate tables and charts and write a report summarizing your conclusions.

**2.96** Problem 1.28 on page 15 describes a survey of 50 MBA students (see the file [gradsurvey.xls](#)).

- a. Select a sample of 50 MBA students in your MBA program and conduct a similar survey for those students.
- b. For the data collected in (a), construct all the appropriate tables and charts and write a report summarizing your conclusions.
- c. Compare the results of (b) to those of Problem 2.93.

## Managing the *Springville Herald*

Advertising fees are an important source of revenue for any newspaper. In an attempt to boost these revenues and to minimize costly errors, the management of the *Herald* has established a task force charged with improving customer service in the advertising department. Open a Web browser and link to [www.prenhall.com/HeraldCase/Ad\\_Errors.htm](http://www.prenhall.com/HeraldCase/Ad_Errors.htm) (or open the [Ad\\_Errors.htm](#) file in the Student CD-ROM's **HeraldCase** folder) to review the task force's data collec-

tion. Identify the data that are important in describing the customer service problems. For each set of data you identify, construct the graphical presentation you think is most appropriate for the data and explain your choice. Also, suggest what other information concerning the different types of errors would be useful to examine. Offer possible courses of action for either the task force or management to take that would support the goal of improving customer service.

## Web Case

In the *Using Statistics* scenario, you were asked to gather information to help make wise investment choices. Sources for such information include brokerage firms and investment counselors. Apply your knowledge about the proper use of tables and charts in this Web Case about the claims of foresight and excellence by a Springville financial services firm.

Visit the EndRun Financial Services Web site at [www.prenhall.com/Springville/EndRun.htm](http://www.prenhall.com/Springville/EndRun.htm) (or open the **EndRun.htm** file in the Student CD-ROM's **WebCase** folder). Review the company's investment claims and supporting data and then answer the following.

1. How does the presentation of the general information about EndRun on its home page affect your perception of the business?
2. Is EndRun's claim about having more winners than losers a fair and accurate reflection of the quality of its investment service? If you do not think that the claim is a fair and accurate one, provide an alternate presentation that you think is fair and accurate.
3. EndRun's "Big Eight" mutual funds are part of the sample found in the **Mutual Funds.xls** workbook. Is there any other relevant data from that file that could have been included in the Big Eight table? How would that new data alter your perception of EndRun's claims?
4. EndRun is proud that all Big Eight funds have gained in value over the past five years. Do you agree that EndRun should be proud of its selections? Why or why not?

## REFERENCES

1. Huff, D., *How to Lie with Statistics* (New York: Norton, 1954).
2. *Microsoft Excel 2007* (Redmond, WA: Microsoft Corporation, 2007).
3. Tufte, E. R., *Envisioning Information* (Cheshire, CT: Graphics Press, 1990).
4. Tufte, E. R., *The Visual Display of Quantitative Information*, 2nd ed. (Cheshire, CT: Graphics Press, 2002).
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6. Wainer, H., *Visual Revelations: Graphical Tales of Fate and Deception from Napoleon Bonaparte to Ross Perot* (New York: Copernicus/Springer-Verlag, 1997).

# Excel Companion

## to Chapter 2

### Introduction

From this point forward, the Excel Companions discuss Excel techniques that support specific statistical methods. For most statistical methods, you will be able to choose between two ways of using Excel—Basic Excel, which uses Excel without any outside enhancements, and PHStat2, which uses the free PHStat2 add-in that is included on this Student CD-ROM. If you haven't already reviewed the From the Authors' Desktop: *Using and Learning Microsoft Excel* feature on page 5, you should do so now in order to decide whether the Basic Excel or PHStat2 approach better suits your learning goals.

Many of the Excel techniques discussed are illustrated with examples based on worksheets you see in the chapter. For most examples, you will have your choice of step-by-step Basic Excel *or* PHStat2 instructions for creating a solution. An added complication in this Excel Companion is that many of the Excel features you would use to present data in tables and charts have been greatly altered in Microsoft Excel 2007. These changes, in turn, necessitate having a special set of Basic Excel instructions for Excel 2007. When this occurs, you will see two labels, (97–2003) and (2007), to distinguish material appropriate for either Excel 97, 2000, 2002, or 2003 (97–2003), from material appropriate only for Excel 2007. (PHStat2 instructions are the same for all versions of Microsoft Excel, other than having to click the **Add-Ins** tab in Excel 2007 in order to see the PHStat2 menu.)

Because the Excel 2007 changes do not affect *most* of the statistical methods discussed in later chapters, you will not see the (97–2003) and (2007) labels as frequently in later Companions. However, when you do see these labels, you should read only the passages that apply to the Excel version you use.

## E2.1 CREATING SUMMARY TABLES

You create a summary table from unsummarized data by creating PivotTables. (If your data are already summarized in table form, you can simply type that table into a blank worksheet and skip creating a PivotTable.) PivotTables are worksheet areas that act as if you had entered formulas to summarize data. PivotTables have many uses beyond those discussed in this book, including giving Excel users the capability to drill down, or

look at the unsummarized, detailed data from which the summarized data are derived.

You create a PivotTable by dragging variable names into a form or template. Because the exact process is different for Excel 2007, this section includes passages specific to Excel 97–2003 and Excel 2007.

*Using the PHStat2 One-Way Tables & Charts procedure, you can create a PivotTable, and finish the summary table, and optionally add the entries needed for a bar chart, pie chart, or Pareto diagram, using the instructions discussed later in this section.*

### Starting a PivotTable (97–2003)

To start a PivotTable in Excel 97–2003, open to the worksheet that contains your unsummarized data and select **Data → PivotTable Report** (Excel 97) or select **Data → PivotTable and PivotChart Report** (Excel 2000–2003). Selecting these commands begins the PivotTable Wizard, a sequence of three dialog boxes (four in Excel 97) that step you through the process of creating a PivotTable. Figure E2.1 shows the dialog boxes for Excel 2003. (Other versions have similar dialog boxes; Excel 97 uses four dialog boxes, breaking the third box into two.)

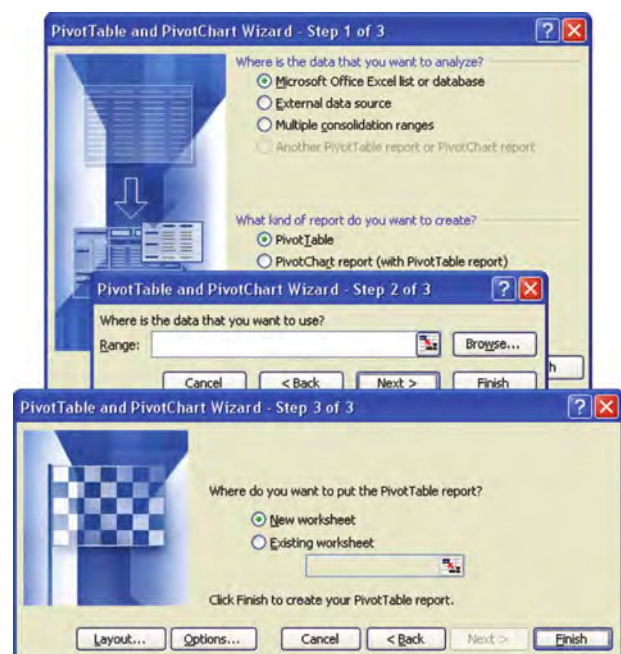


FIGURE E2.1 PivotTable Wizard steps (Excel 2003)

You step through these dialog boxes by clicking **Next** to advance to the next dialog box or **Back** to move back to a previous one. At any point, you can click **Cancel** to stop creating a PivotTable or click **Finish** to end the wizard and create a PivotTable. To create a summary table, you make the following entries in the dialog boxes:

**Step 1** Click **Microsoft Excel list or database** as the source and **PivotTable** as the report type. (You do not select a report type in Microsoft Excel 97; PivotTable is assumed.)

**Step 2** Enter the cell range of the data to be summarized in the PivotTable. This cell range must contain variable labels (column headings) in the first row of the range because the wizard, in Step 3, will use the cells in the first row as the names for your variables.

**Step 3** First click the **New worksheet** option as the location for your PivotTable. Then click **Layout** to display the Layout dialog box (see Figure E2.2).<sup>1</sup> Design your table using the instructions in the next paragraph and then click **OK** to return to the Step 3 box. Then click **Options** to display the PivotTable Options dialog box (see Figure E2.2). Enter **0** as the **For empty cells, show** value and click **OK** to return to the Step 3 dialog box. Then click **Finish** to create the PivotTable.

<sup>1</sup>If you use Excel 97, the Layout dialog box appears as the Step 3 dialog box. When you click **Next** in this box, you then see a Step 4 dialog box that contains location options. There is no **Options** button in either the Step 3 or Step 4 dialog boxes in Excel 97.

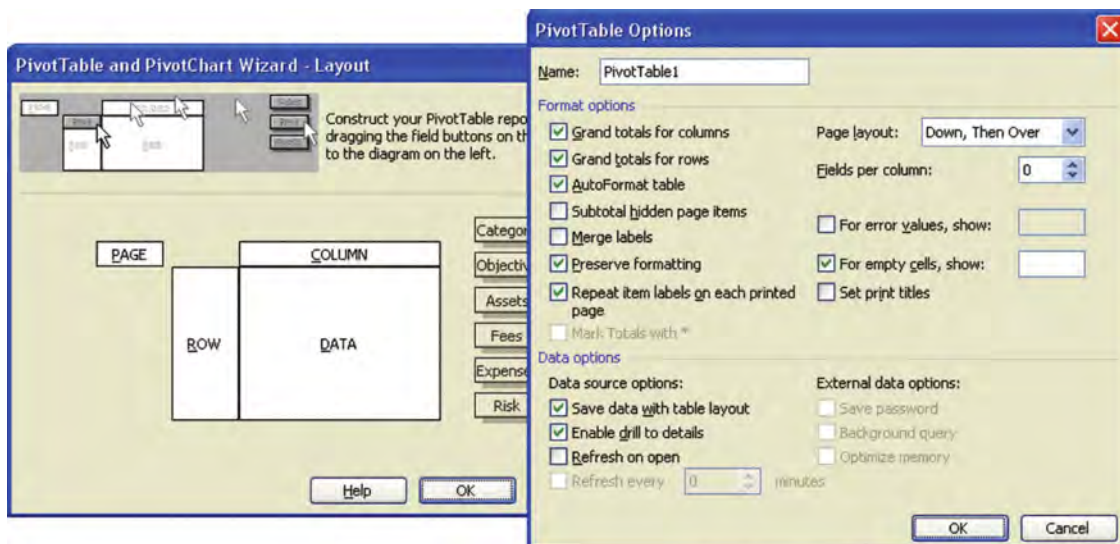
In the Layout dialog box of Step 3, drag the label of the variable to be summarized and drop it in the ROW area. Drag a second copy of this same label and drop it in the DATA area. (This second label changes to **Count of variable name** to indicate that the PivotTable will be counting, or tallying, the occurrences of each category of the variable.)

## Starting a PivotTable (2007)

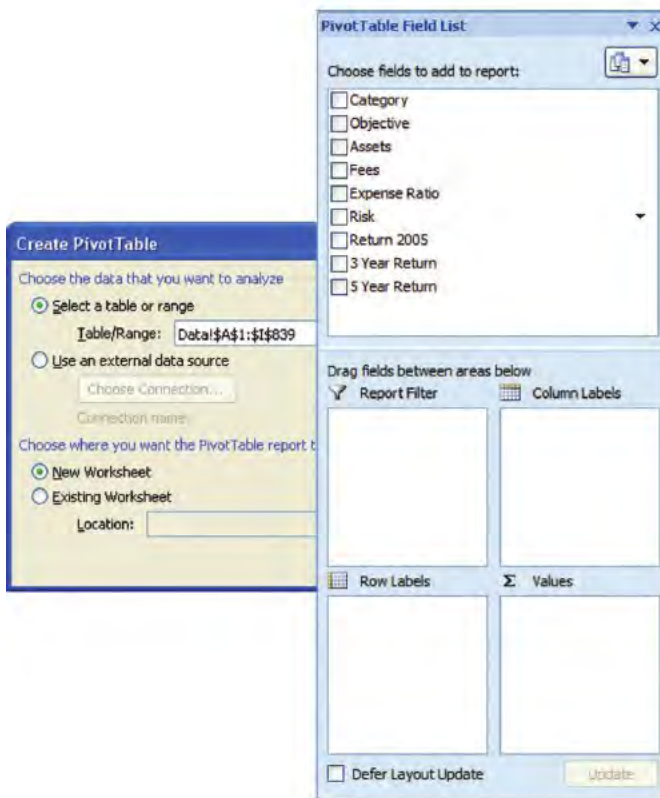
To start a PivotTable in Excel 2007, open to the worksheet that contains your unsummarized data and select **Insert → PivotTable**. In the Create PivotTable dialog box (see Figure E2.3), leave selected the **Select a table or range** option and verify and change, if necessary, the **Table/Range** cell range. (In Figure E2.3, the cell range of the mutual funds data of the **Data** worksheet of **Mutual Funds.xls** is shown.) Select the **New Worksheet** option and click **OK**.

In the PivotTable Field List task pane, drag the label of the variable to be summarized and drop it in the **Row Labels** area. Drag a second copy of this same label and drop it in the **Σ Values** area. (This second label changes to **Count of variable name** to indicate that the PivotTable will be counting, or tallying, the occurrences of each category of the variable.)

Right-click the PivotTable and click **Table Options** in the shortcut menu that appears. In the PivotTable Options dialog box (a reorganized version of the older one shown in Figure E2.2), click the **Layout & Format** tab, check **For empty cells show**, enter **0** as its value, and click **OK**.



**FIGURE E2.2** PivotTable Layout and Options dialog boxes



**FIGURE E2.3** Create PivotTable dialog box and PivotTable Field List task pane (Excel 2007)

### EXAMPLE Table 2.2 Summary Table

To start a PivotTable version of the Table 2.2 summary table on page 33, similar to the PivotTable shown below, open the **Mutual Funds.xls** workbook to the **Data** worksheet and use the appropriate set of instructions.

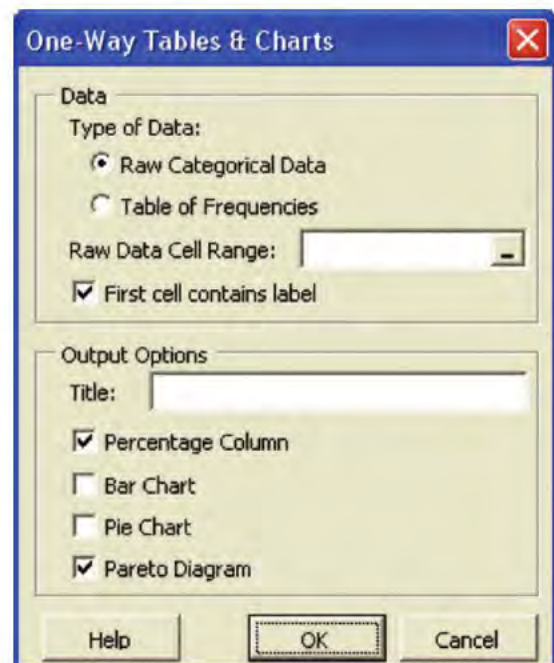
Count of Risk		
Risk	Total	Percentage
Low	247	29.47%
Average	245	29.24%
High	346	41.29%
Grand Total	838	

**Basic Excel 97–2003** Begin the PivotTable Wizard and follow the instructions for “Starting a PivotTable 97–2003” in Section E2.1. In Step 2, enter **F1:F839** as the **Range**. In the Layout dialog box of Step 3, drag the **Risk** variable label twice, once to the ROW area and a second time to the DATA area. (This second label changes to **Count of Risk** when you drop it in the DATA area.) After you click **Finish** to create the PivotTable, continue with “Finish the

PivotTable” instructions that follow this example to complete a PivotTable version of Table 2.2.

**Basic Excel 2007** Select **Insert** → **PivotTable**. In the PivotTable Field List task pane, drag the **Risk** variable label twice, once to the **Row Labels** area and a second time to the **Σ Values** area. (This second label changes to **Count of Risk** when you drop it in the **Σ Values** area.) Continue with “Finish the PivotTable” instructions that follow this example to complete a PivotTable version of Table 2.2.

**PHStat2** Select **PHStat** → **Descriptive Statistics** → **One-Way Tables & Charts**. In the dialog box for this procedure (shown below), click the **Raw Categorical Data** option and enter **F1:F839** as the **Raw Data Cell Range**. Click **First cell contains label** and enter a **Title**. Click **Percentage Column** and **Pareto Diagram** and click **OK**. The worksheet PHStat2 creates columns for percentage frequency and cumulative percentage. (You can safely delete the chart sheet that contains the Pareto diagram if you do not want this chart.) If you want to create charts from the summary table that PHStat2 creates, you can also click **Bar Chart** and/or **Pie Chart** before you click **OK**.



### Finish the PivotTable

After Excel creates the PivotTable, close any floating PivotTable toolbar windows (97–2003) or the PivotTable Field List task pane (2007). Enter a title in cell A1 of the

new worksheet that contains your PivotTable. Also, rename the new worksheet, using a descriptive name. Then in cell C4, enter **Percentage** and in cell C5, enter the formula in the form  $=B5/B\$n$ , in which  $n$  is the row that contains the Grand Total. For example, if the Grand Total row is row 8, enter  $=B5/B\$8$ . (If you do not know the significance of the \$ (dollar sign) symbol, review Section E1.5.) Copy this formula down through all the category rows of the summary table. Format the cell range that contains these formulas for percentage display. Adjust the number of decimals displayed and the width of column C, as necessary, and add borders around the percentage column cells. (If necessary, review the instructions for doing these things presented in the Excel Companion to Chapter 1.)

*If you use PHStat2, these finishing touches are done for you if you select the Percentage Column output option.*

### Cell Entries for a Pareto Diagram

If you plan to create a Pareto diagram, enter the heading **Cumulative Pctage** in cell D4 and enter the formula  $=C5$  in cell D5. Enter the formula  $=C6+D5$  in cell D6 and copy this formula down through all the category rows of the summary table. Format the column D cell range that contains formulas for percentage display. Adjust the number of decimals displayed and the width of column D as necessary.

If you use Excel 2007, click cell B5 (the first frequency) and select **Home** → **Sort & Filter** (in the Editing group) → **Sort Largest to Smallest**. If you are not using Excel 2007, right-click cell A4 and click **Field Settings** in the shortcut menu (**Field**, if using Microsoft Excel 97). In the PivotTable Field dialog box that appears (see Figure E2.4), click **Advanced**. In the PivotTable Field Advanced

Options dialog box that appears (also see Figure E2.4), select the **Descending** option and **Count of variable** from the **Using field** drop-down list. Click **OK** to return to the PivotTable Field dialog box and then click **OK** in that dialog box to return to the worksheet. (These last steps reorder the category rows in the table in descending order.)

*If you use PHStat2, these additional entries are done for you if you select the Pareto Diagram output option.*

**Forum** Click the ALTERNATE TECHNIQUES link to learn how the COUNTIF function can also be used to selectively count occurrences of individual categories.

## E2.2 CREATING CHARTS

You create charts by associating worksheet data with a chart type, which you then customize. To create a chart, you must specify the cell range of the worksheet data, choose the chart type and its subtype, set formatting details such as titles and axis legends, and choose whether to put the chart on its own chart sheet or to embed the chart on a worksheet. Because the process is different for Excel 2007, this section includes passages specific to Excel 97–2003 and Excel 2007.

*Many PHStat2 procedures, including One-Way Tables & Charts, include output options that create charts for you.*

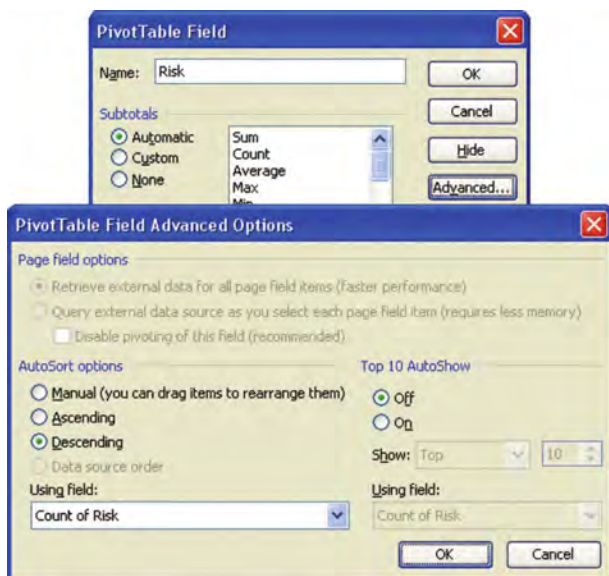
### Creating Charts (97–2003)

To create a chart in Excel versions other than 2007, open to the worksheet containing your unsummarized data and select **Insert** → **Chart**. This begins the Chart Wizard, a sequence of four dialog boxes that step you through the process of creating a chart. Figure E2.5 shows these dialog boxes for Excel 2003. (Other versions have similar dialog boxes.) To create a chart, make the following entries and choices in the Step dialog boxes.

**Step 1** Choose the chart type from either the **Standard Types** or **Custom Types** tab. Most of the charts you create in this text are chart types found in the Standard Types tab.

**Step 2** Enter the cell range of the data to be graphed in the **Data Range** tab.<sup>2</sup> For some types of charts, you also enter the cell range or ranges that contain chart labeling information in the **Series** tab. Cell ranges in the Series tab must always be entered with their worksheet names as a formula, in the form  $=SheetName!CellRange$ .

<sup>2</sup>In some rare situations, with older versions of Excel, if you type a simple cell range, you get an error. In those cases, either retype the range using the SheetName/CellRange form discussed in Section E1.3 or point to the cell range.



**FIGURE E2.4** PivotTable Field and PivotTable Field Advanced Options dialog boxes (Excel 97–2003)

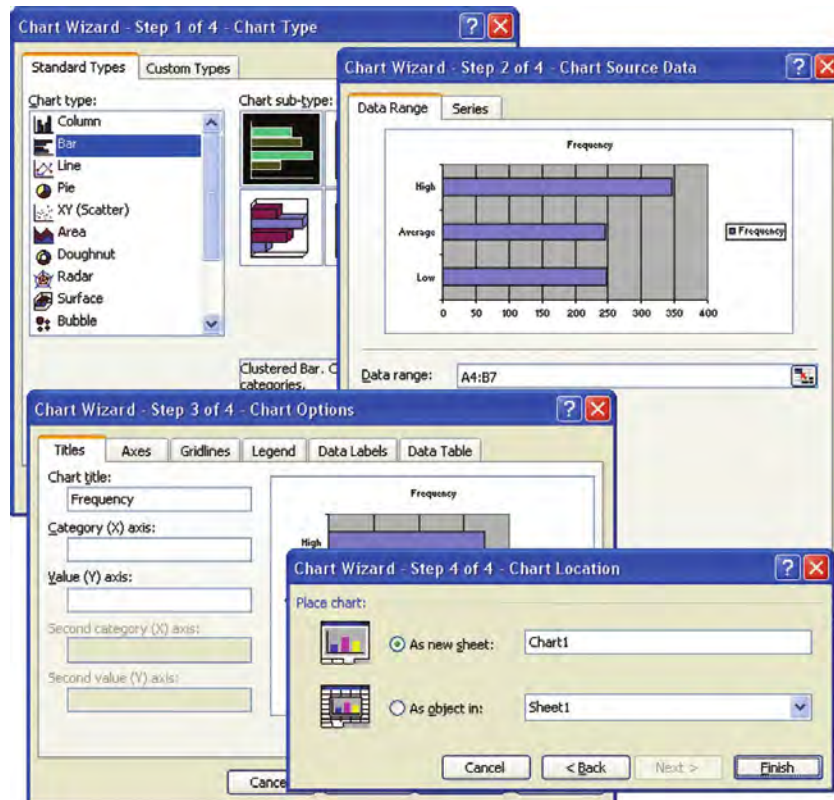


FIGURE E2.5 Chart Wizard dialog boxes (Excel 2003)

**Step 3** Enter titles and select customizing options for your chart. Unless overridden specifically in later instructions, use the following settings (for a given chart type, some of these tabs may not be available).

In the **Titles** tab, enter a title and enter axis labels, if appropriate.

In the **Axes** tab, check both the (X) axis and (Y) axis check boxes and select the **Automatic** option under the (X) axis check box.

In the **Gridlines** tab, clear all the check boxes.

In the **Legend** tab, clear the **Show legend** check box.

In the **Data Labels** tab, click the **None** option under the **Data labels** heading.

In the **Data Table** tab, clear the **Show data table** check box.

**Step 4** Click the **As new sheet** option to place the chart on its own chart sheet. (This choice creates a better-scaled and better-looking chart that can be more easily printed separately.) Then click **Finish** to create the chart.<sup>3</sup>

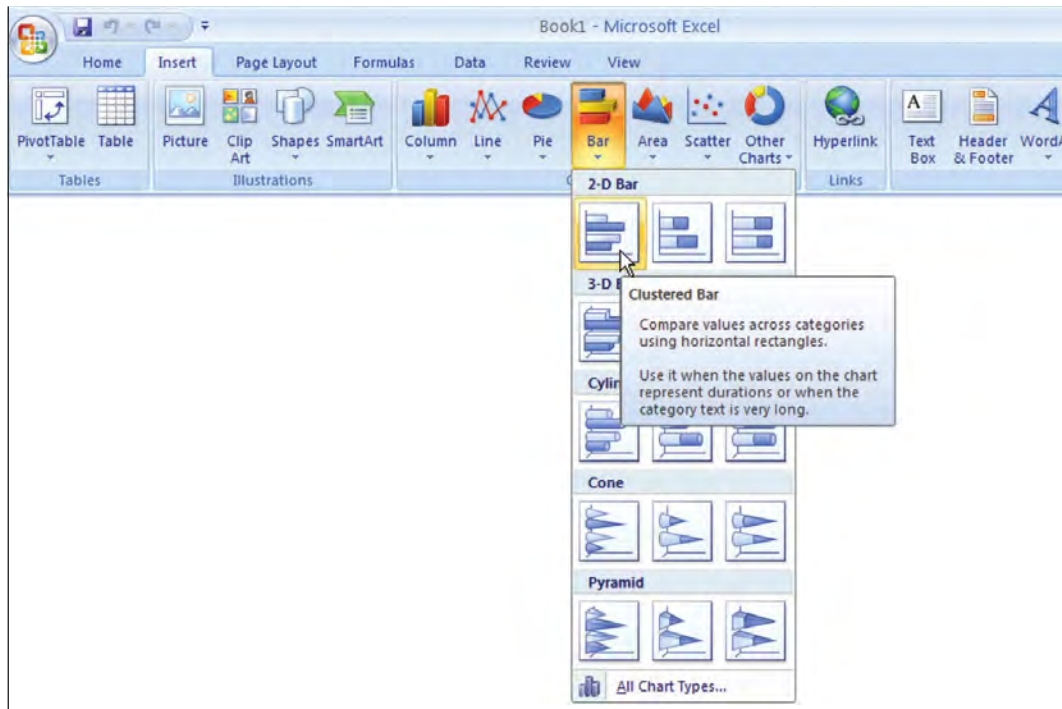
<sup>3</sup>Because the Step 4 dialog box instructions never vary, they are not explicitly listed in the later charting instructions. You should, of course, always click **As New Sheet** and **Finish** in the Step 4 dialog box for every chart you create in Excel 97–2003.

If you discover a mistake in your chart after your chart is created, right-click the chart and select **Chart Type**, **Source Data**, **Chart Options**, or **Location** to return to versions of the Step 1, Step 2, Step 3, or Step 4 dialog boxes, respectively.

## Creating Charts (2007)

To create a chart in Excel 2007, open to the worksheet that contains your unsummarized data. Select the cell range of the data to be charted.<sup>4</sup> If your cell range contains two non-adjacent areas, hold down the Ctrl key as you drag and select each area. Select **Insert** and in the Charts group, click the chart type. From the drop-down gallery that appears, click the chart sub-type you want. To help distinguish sub-types, move the mouse pointer over a sub-type and wait for the sub-type description to appear. In Figure E2.6, the description for the bar chart “clustered bar” sub-type has appeared.

<sup>4</sup>If you fail to select a cell range, Excel may try to construct a chart using all the data in your worksheet. For a worksheet such as **Data in Mutual Funds.xls**, not selecting a range would lead to a very complex—and worthless—chart that could slow down or even disrupt the operation of your computer.



**FIGURE E2.6** Selecting a chart sub-type (Excel 2007)

Customize your chart by clicking the chart and selecting the Layout tab of the Chart Tools ribbon group (or PivotChart Tools, if the chart is based on a PivotTable). Review the settings for the members of the Labels and Axes groups in this tab. Unless overridden specifically in later instructions, use the following settings for these items (for a given chart type, some of these items may be disabled and not available):

**Chart Title** Choose either **Centered Overlay Title** or **Above Chart**.

**Axis Titles** For the **Primary Horizontal Axis Title**, select **Title Below Axis**. For the **Primary Vertical Axis Title**, select **Rotated Title**. Some charts have secondary axes; for such charts, select **Title Below Axis** as the **Secondary Horizontal Axis Title** and **None** as the **Secondary Vertical Axis Title**.

**Data Labels** Select **None**.

**Data Table** Select **None**.

**Axes** For the **Primary Horizontal Axis Title**, select **Show Left to Right Axis**. For the **Primary Vertical Axis Title**, select **Show Default Axis**. Some charts have secondary axes; for such charts, select **None** as the **Secondary Horizontal Axis Title** and **Show Default Axis** as the **Secondary Vertical Axis Title**.

**Gridlines** Select **None** for both the **Primary Horizontal Gridlines** and **Primary Vertical Gridlines**. Also select **None** for secondary gridlines, if the chart contains those as well.

Excel 2007 creates charts on worksheets. To move a newly created chart to its own chart sheet (recommended), right-click the chart frame and click **Move Chart** in the shortcut menu that appears. In the Move Chart dialog box that appears, select the **New sheet** option and click **OK**.

### Additional Chart Tips

Many charts, especially those created in Excel 97-2003, contain colored backgrounds (typically gray) that you may want to remove if you plan either to print charts or display them through a video projector or on an LCD-type device. To remove the background in an Excel 97-2003 chart, right-click the background and click **Format Plot Area** in the shortcut menu. In the dialog box that appears, click the **None** option of the **Area** group and click **OK**. (This was done for all the charts shown in this book.) To remove the background in Excel 2007, right-click the chart background and select **Layout** → **Plot Area** and select **None** in the Plot Area gallery.

Occasionally, you may open to a chart sheet and see only part of a (too-large) chart or see a (too-small) chart surrounded by a too-large frame mat. To display an optimally sized chart in Excel 97-2003, open to the chart sheet and press **Esc**. Select **View** → **Zoom** and then, in the Zoom dialog box, select the **Fit selection** option and click **OK**. To display an optimally sized chart in Excel 2007, use the Zoom slider on the lower right of the Excel window frame or click the chart and then select **Format** and use the items in the Size group.



Finally, if the symbols, captions, and/or legends and titles prove too big or too small for you, you can usually change these elements by right-clicking over them and clicking the shortcut menu choice that contains the word **Format**. For labeling elements, you can use a simpler procedure: Click the element and use the formatting features described in Section E1.4 to change such things as the type size and style. (Most of the charts shown in this text have such elements set to 12 point, boldfaced type.)

## E2.3 CREATING BAR AND PIE CHARTS FROM SUMMARY TABLES

You create bar and pie charts from summary tables (not unsummarized data), using Excel charting features. Because this process is different for Excel 2007, this section includes passages specific to Excel 97–2003 and Excel 2007, as well as passages that apply to all versions.

*The PHStat2 One-WayTables & Charts procedure can create bar and pie charts for you and can use either a summary table or unsummarized data.*

### Creating Bar and Pie Charts (97–2003)

Open to the worksheet that contains your summary table. If your summary table is a PivotTable (like the ones created in Section E2.2), click a cell that is outside your PivotTable.<sup>5</sup> Begin the Chart Wizard and make these entries:

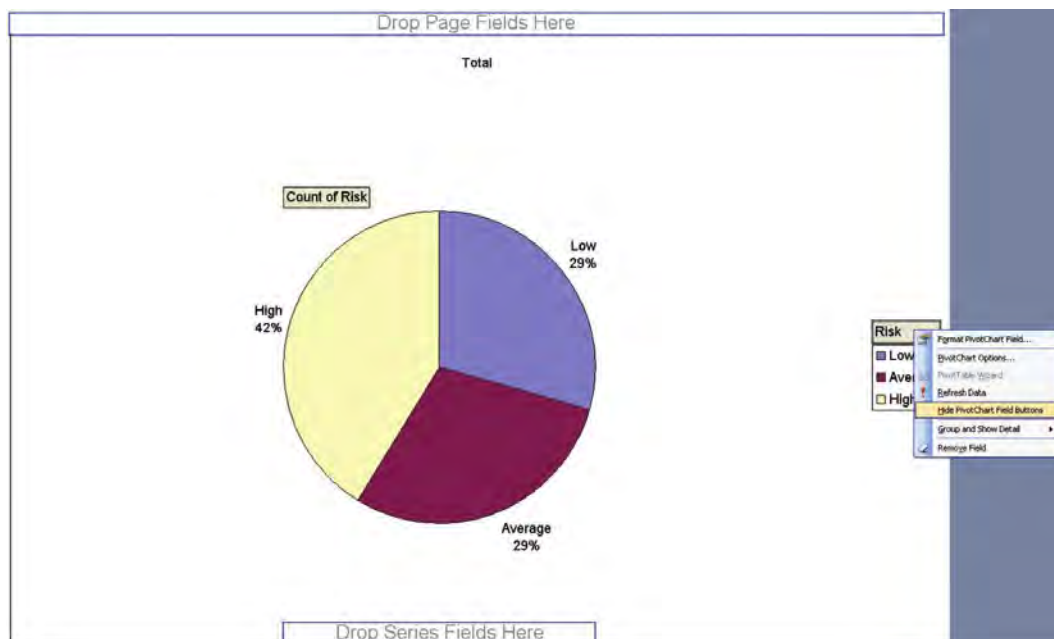
<sup>5</sup>If you forget to click outside the PivotTable, the Chart Wizard will instantly create a (wrong type of) chart. If this occurs, you can press **Ctrl+Z** to undo the chart.

**Step 1** Click the **Standard Types** tab. For a bar chart, click **Bar** as the **Chart type** and then click the first **Chart sub-type** choice, labeled **Clustered Bar** when selected. For a pie chart, click **Pie** as the **Chart type** and then click the first **Chart sub-type** choice, labeled **Pie** when selected.

**Step 2** Click the **Data Range** tab and enter the cell range of the category labels and the frequency counts as the **Data range**. If you used the instructions in Section E2.2 to create your summary table, this range will always start with cell A4 and end with a cell in column B. (You do not include the column C cells that contain the percentages.) Click the **Columns** option if it is visible. In some versions of Excel, the model chart shown in the dialog box contains additional boxed labels that you can ignore for now.

**Step 3** Click the **Titles** tab. Enter a title as the **Chart title** and, if you are creating a bar chart, enter appropriate values for the **Category (X) axis** and **Value (Y) axis** titles. For a bar chart, click, in turn, the **Axes**, **Gridlines**, **Legend**, **Data Labels**, and **Data Table** tabs and adjust the settings, as discussed in the “Creating Charts (97–2003)” part of Section E2.2 on page 78. For a pie chart, click the **Legend** tab and clear **Show legend** and then click the **Data Labels** tab. If you are using Excel 97 or Excel 2000, click the **Show label and percent** option; otherwise, click **Category name and Percentage**.

If the chart created contains the additional boxed labels, such as “Drop Page Fields Here,” that you ignored in the Step 2 dialog box, right-click the category drop-down list on the chart sheet (**Risk** in Figure E2.7) and click **Hide PivotChart Field Buttons**. This eliminates the clutter and makes your chart look more like the one shown in Figure 2.4 on page 35.



**FIGURE E2.7** Selecting Hide PivotChart Field buttons (Excel 97–2003)

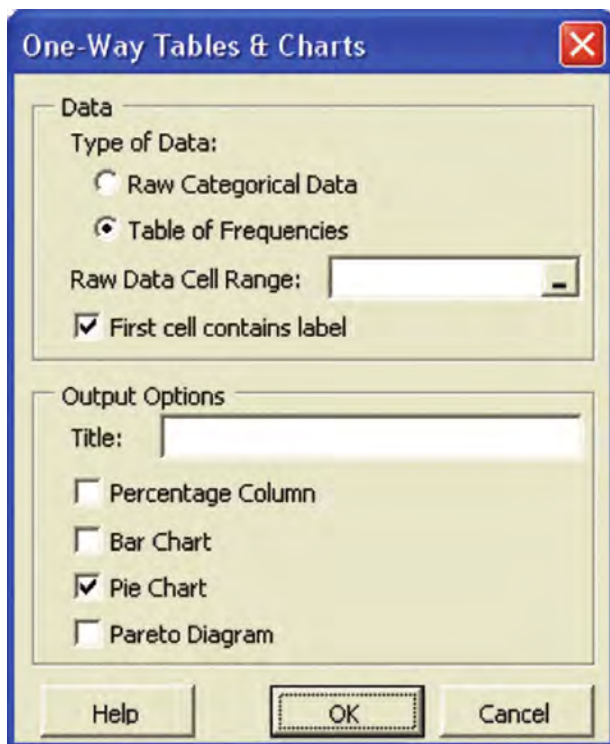
**EXAMPLE Figure 2.4 Pie Chart for Levels of Risk**

To create your own Figure 2.4 pie chart for levels of risk (see page 35), open the **Chapter 2.xls** workbook to the **Table 2.2** worksheet, click cell D1 (a cell outside the PivotTable), and use the appropriate set of instructions.

**Basic Excel 97–2003** Follow the “Creating Bar & Pie Charts (97–2003)” instructions. In **Step 1**, click **Pie** as the **Chart Type** and then click the first chart sub-type choice, labeled as **Pie** when selected. In **Step 2**, enter the cell range **A4:B7** as the **Data Range** in the Step 2 dialog box.

**Basic Excel 2007** Follow the “Creating Bar & Pie Charts 2007” instructions, clicking cell A5 before you select **Insert**.

**PHStat2** Select **PHStat** → **Descriptive Statistics** → **One-Way Tables & Charts**. Click the **Table of Frequencies** option, enter **A4:B7** as the **Freq. Table Cell Range**, and click **First row of table contains labels**. Enter a **Title** and click **Pie Chart**. (You can also click **Bar Chart** and **Pareto Diagram** to create those charts at the same time.)

**Creating Bar & Pie Charts (2007)**

Open to the worksheet that contains your summary table. Click a cell inside your table and then select **Insert**. For a bar chart, click **Column** in the Charts group, and then click **Clustered Column** in the chart gallery. For a pie chart, click **Pie** in the Charts group, and then click **Pie** in the chart

gallery. Adjust chart settings as discussed in the “Creating Charts (2007)” part of Section E2.2 on page 79.

**E2.4 CREATING PARETO DIAGRAMS FROM SUMMARY TABLES**

You create a Pareto diagram from a summary table by using Excel charting features. To begin, open to the worksheet that contains your summary table. Enter, if necessary, the entries discussed in the “Cell Entries for a Pareto Diagram” part of Section E2.1 (see page 78). Because creating charts differs for Excel 2007, this section includes passages specific to Excel 97–2003 and Excel 2007.

*The PHStat2 One-WayTables & Charts procedure can create a Pareto diagram for you when you select the One-WayTables & Charts procedure.*

**Creating a Pareto Diagram (97–2003)**

If your worksheet contains a PivotTable summary table (like the ones created in Section E2.2), click a cell that is outside your PivotTable.<sup>6</sup> Begin the Chart Wizard (by selecting **Insert** → **Chart**) and make these entries:

**Step 1** Click the **Custom Types** tab. Click the **Built-in** option and then select **Line - Column on 2 Axes** as the **Chart type**.

**Step 2** Click the **Data Range** tab. Enter the cell range of the percentage and cumulative percentage frequencies, without their column headings, in the **Data range** edit box. This range will always start with cell C4 and end with a cell in column D. Click the **Columns** option in the **Series in** group and then click the **Series** tab. Enter as a **formula** the column A cell range that contains the category labels as the **Category (X) axis labels**. Leave the **Second category (X) axis labels** box blank, if it appears.

**Step 3** Click the **Titles** tab. Enter a title as the **Chart title**, the name of the variable in the **Category (X) axis** edit box, and **Percentage** in the **Value (Y) axis** edit box. Leave the other two boxes blank. Click, in turn, the **Gridlines**, **Legend**, **Data Labels**, and **Data Table** tabs and use the formatting settings given in the “Creating Charts (97–2003)” part of Section E2.2 on page 78.

The wizard creates a Pareto diagram that contains a secondary (right) y-axis scale that improperly extends past 100%. To correct this error, right-click that axis (you will see the ToolTip<sup>7</sup> **Secondary Value Axis** when your mouse is properly positioned) and click **Format Axis** in the short-

<sup>6</sup>If you forget to click outside the PivotTable, the Chart Wizard instantly creates a chart of the wrong type. If this occurs, you can press **Ctrl+Z** to undo the chart.

<sup>7</sup>ToolTips are pop-up balloons that display informational messages about onscreen objects.

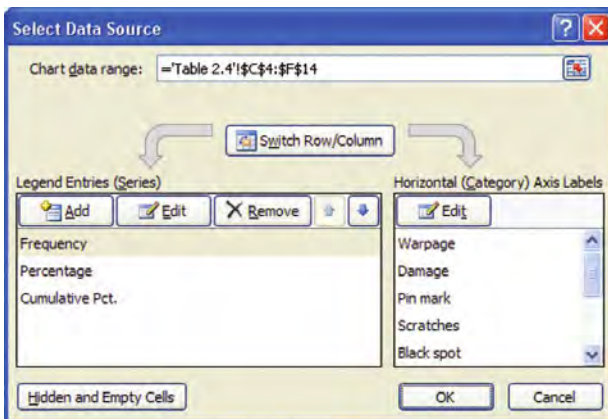
cut menu. In the **Scale** tab of the Format Axis dialog box, change the **Maximum** value to **1** and click **OK**.

*The Pareto Diagram output option of the PHStat2 One-Way Tables & Charts procedure makes this correction for you.*

## Creating a Pareto Diagram (2007)

Select the cell range of the data to be charted. (This range will begin with cell C4, if you used the “Cell Entries for a Pareto Diagram” instructions in Section E2.1.) Select **Insert** → **Column** (in the Charts group) and select the first sub-type, identified as **Clustered Column** when you move the mouse pointer over that sub-type and pause. Select **Format** and select the cumulative percentage series from the drop-down in the Current Selection group. Then select **Format Selection** (from the same group) and in the Format Data Series dialog box select the **Secondary Axis** in the Series Options panel and click **Close**. With the cumulative percentage series still selected in the Current Selection group, select **Design** → **Change Chart Type**, and in the Change Chart Type gallery, select the line chart identified as **Line with Markers** and click **OK**.

If your chart contains extraneous plots—for example, a plot of percentage frequencies—delete this plot one series at a time by doing the following: Select **Format** and then select an extraneous series from the drop-down in the Current Selection group. Select **Design** → **Select Data**. In the Select Data Source dialog box (see Figure E2.8), select the extraneous **Legend Entries** series (**Percentage** in Figure E2.8) and click **Remove** and then **OK**.



**FIGURE E2.8** The Select Data Source dialog box (Excel 2007)

Relocate your chart to a chart sheet and customize your chart, using the instructions in “Creating a Chart (Excel 2007)” in Section E.2.2. If your chart has a secondary (right) y-axis scale that improperly extends past 100%, right-click the axis and click **Format Axis** in the shortcut menu. In the **Scale** tab of the Format Axis dialog box, change the value in the **Maximum** edit box to **1** and click **Close**.

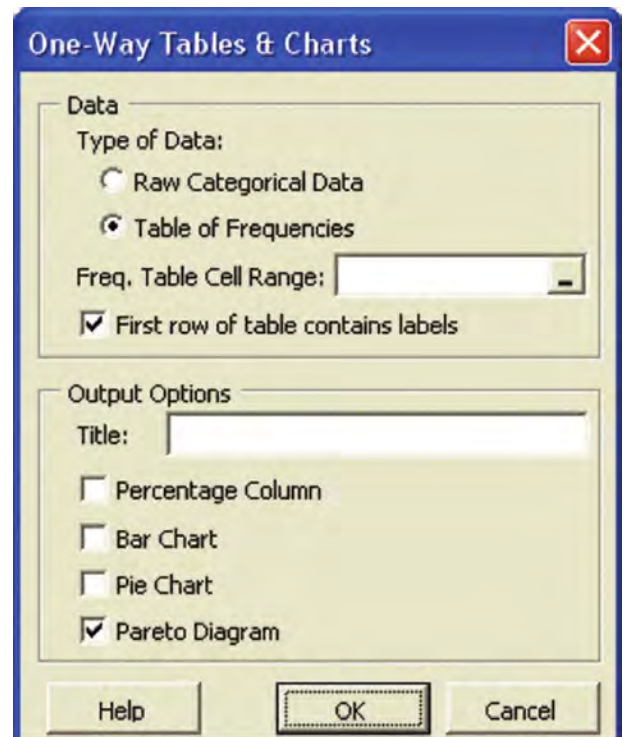
## EXAMPLE Figure 2.5 Pareto Diagram

To create your own Figure 2.5 Pareto diagram (see page 37) from the Table 2.4 keyboard defects summary table on page 36, first open the **Chapter 2.xls** workbook to the **Table 2.4** worksheet, a PivotTable version of Table 2.4. Click cell D2 (a cell outside the PivotTable) and use the appropriate set of instructions.

**Basic Excel 97–2003** Use the instructions of the preceding “Creating a Pareto Diagram (97–2003)” section. In Step 2 of the Chart Wizard, enter **C4:D14** as the **Data Range** in the Data Range tab and enter the formula **=ParetoData!A5:A14** in the **Category (X) axis labels** box in the Series tab.

**Basic Excel 2007** Select the cell range **C4:F14**. (Although this range includes columns you do not need, this helps Excel make a good guess about the type of chart you want.) Follow the instructions of the preceding “Creating a Pareto Diagram (2007)” section. Because your chart range includes the percentage frequency column, you will have to delete this series using the process discussed in those instructions.

**PHStat2** Select **PHStat** → **Descriptive Statistics** → **One-Way Tables & Charts**. Click the **Table of Frequencies** option, enter **A4:B14** as the **Freq. Table Cell Range**, and click **First row of table contains labels**. Enter a **Title** and click **Pareto Diagram**. (You can also click **Bar Chart** and **Pie Chart** to create those charts at the same time.)



## E2.5 CREATING AN ORDERED ARRAY

You create an ordered array by using Excel sorting features. Because this process differs for Excel 2007, this section includes passages specific to Excel 97–2003 and Excel 2007.

### Creating an Ordered Array (97–2003)

Select the data to be sorted and then select **Data** → **Sort**. (In the Sort dialog box (shown below, similar for all versions, but enhanced in Excel 2007), select the column by which to sort from the **Sort by** drop-down list and click either the first **Ascending** or **Descending** option button. (In the dialog box, a column named Price has been selected as an example.) You then click the **Header row** option and **OK** button to complete the sort.



### Creating an Ordered Array (2007)

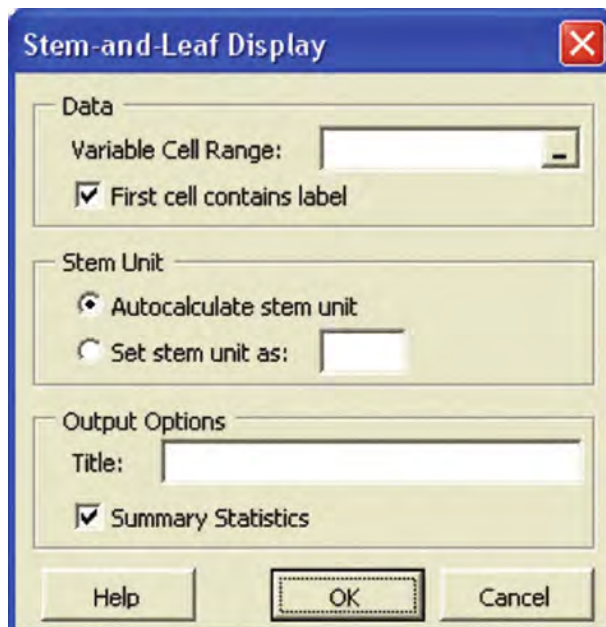
Select the data to be sorted and then select **Home** → **Sort & Filter** (in the Editing group) → **Sort Smallest to Largest**.

## E2.6 CREATING STEM-AND-LEAF DISPLAYS

You create stem-and-leaf displays by using the PHStat2 Stem-and-Leaf Display procedure or by manually creating the stems and leaves in worksheet cells. If you choose to manually create a stem-and-leaf display, you will find that sorting your data to create an ordered array (see previous section) will simplify your task.

## Using PHStat2 Stem-and-Leaf Display

Open to the worksheet containing the data to be plotted. Select **PHStat** → **Descriptive Statistics** → **Stem-and-Leaf Display**. Enter the cell range of the data to be plotted as the **Variable Cell Range**. Click **Select the First cell contains label**. *Always* select the **Autocalculate stem unit** option, enter a **Title**, and click **OK**. (**Summary Statistics** is automatically checked for you.)



### Manually Creating Stems and Leaves

Apply the discussion of Section 2.2 (see page 41) that describes the manual process of creating stems and leaves. Use a new worksheet and use two columns, one for the stems and one for the leaves. You need to enter each leaf as a label value. To do this, begin your cell entry with an apostrophe (') and follow that apostrophe with the digits that form the leaf. As you enter each digit, you may also want to add a space after each digit for greater readability. For example, to enter a leaf that contains 2, 2, 3, and 7, enter '2 2 3 7.

## E2.7 CREATING FREQUENCY DISTRIBUTIONS AND HISTOGRAMS

You create frequency distributions and histograms from unsummarized data by using the ToolPak Histogram procedure or the PHStat2 Histogram & Polygons procedure. Both procedures require that you first translate your class groupings into what Excel calls bins (explained later in this section).

Unlike the frequency distribution discussion in Chapter 2, you typically include frequency, percentage fre-

quency, and cumulative percentage frequencies as columns of one table and not as separate tables, as shown in Tables 2.8, 2.10, and 2.13 on pages 45, 47, and 48. Also, unlike what is done in Chapter 2, in Excel, you create frequency distributions for individual categories, one at a time (growth funds *or* value funds, for example) and not frequency distributions that contain two categories (for example, growth *and* value funds). To create multiple category tables, such as Tables 2.8, 2.10, and 2.13, you must combine columns from the individual tables using copy and paste operations.

If you use the ToolPak procedure, you face a final complication of having to correct several errors that the procedure makes in the frequency distribution and histogram. If you have data already summarized in a frequency distribution, you should skip ahead to Section E2.8, which explains how to create a histogram from that frequency distribution, using Excel charting features.

### Translating Class Grouping into Bins

When summarizing numerical data in Microsoft Excel, you use bins which have implied ranges of values, and not class groupings, which have well-defined boundary values. A **bin** is the set of values that are less than or equal to the current bin value and that are greater than the previous bin value, which must be a lesser value. To create a frequency distribution, you must translate your class groupings into a set of bin values and enter those bin values on the same worksheet as your unsummarized data.

In Chapter 2, Table 2.7 (and Tables 2.8 through 2.11) uses class groupings in a form that enables you to make the translation from class to bin. When you express class groupings in the Table 2.7 form *valueA but less than valueB*, you can create nearly equivalent bins, as Table E2.1 illustrates.

**TABLE E2.1** Class Grouping and Bins

Table 2.7 Classes	The Nearly Equivalent Bins
0 but less than 5	4.99
5 but less than 10	9.99
10 but less than 15	14.99
15 but less than 20	19.99
20 but less than 25	24.99
25 but less than 30	29.99
30 but less than 35	34.99
35 but less than 40	39.99
40 but less than 45	44.99

The first bin value always represents a grouping with no explicit lower boundary (other than negative infinity). Thus, the first bin can never have a midpoint. (You may have noticed in this chapter that in the charts in which mid-

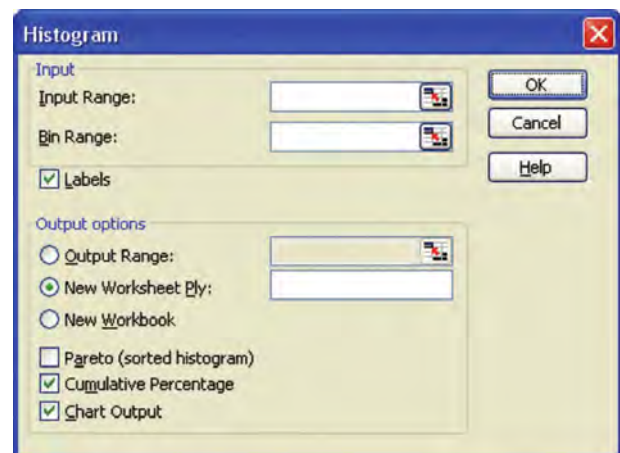
points are used as labels, the first bin is labeled with — and not with a midpoint.)

Sometimes you work with class groupings in the form “all values from *valueA* to *valueB*,” as in this set of classes: 0.0 through 4.9, 5.0 through 9.9, 10.0 through 14.9, and 15.0 through 19.9. In such cases, you can approximate each class grouping by choosing a bin value just above the *valueB* value for the class groupings. For example, the numbers 4.99, 9.99, 14.99, and 19.99 would create bins that approximate the four class groupings, assuming that all data values were measured in units no smaller than tenths. (While the last digit of the bin values could be any digit other than zero, using 9 as the last digit is a standard practice.)

When you have defined your set of bin values, enter them into a new column of the worksheet that contains your unsummarized data. Make sure the first cell in that column contains the column heading “Bins.”

### Using the ToolPak Histogram Procedure

To create a frequency distribution and histogram, open to the worksheet that contains the unsummarized data and verify that bin values have been placed in their own column. Begin the Analysis ToolPak add-in and select **Histogram** from the **Analysis Tools** list and then click **OK**. In the Histogram dialog box (see Figure E2.9), enter the cell range of the data to be summarized as the **Input Range**. Enter the cell range of the bin values (including the cell containing the heading “Bins”) as the **Bin Range**.<sup>8</sup> Click **Labels** to indicate that the first cells of the Input Range and Bin Range contain a label. Select the **New Worksheet Ply** option, click **Cumulative Percentage**, and click **Chart Output** to create a histogram and click **OK**.



**FIGURE E2.9** The Data Analysis Histogram dialog box

<sup>8</sup>If you leave the **Bin Range** blank, the procedure creates a set of bin values for you. However, the better practice is to supply your set of bin values as these instructions require you to do.

The procedure creates a frequency distribution and histogram together on a new worksheet.

## Editing and Enhancing the Frequency Distribution

The frequency distribution you create will improperly contain an open-ended bin labeled “More” and contain some errors in column heading formatting. To eliminate the “More” class, first manually add the More frequency count to the count of the preceding class and set the cumulative percentage of the preceding class to 100%. Then, select the entire row containing the “More” row, select **Edit** → **Delete**, and in the Delete dialog box, select the **Shift cells up** option and click **OK**.

Next, enhance your frequency distribution. Select a cell in row 1 and then select **Insert** → **Rows**. In the “new” row 1, enter a title for the frequency distribution. Then add a column for the percentage frequency by selecting column C and then selecting **Insert** → **Columns**. In the new column C, enter the heading **Percentage** in cell C2. Enter the formula  $=B3/SUM(B:B)$  in cell C3 and copy the formula down the column through the rest of the frequency distribution. Format column C for percentage display to complete the column. Finally, add a column for midpoints. Select cell E2 and enter the heading **Midpoints**. Then use the cells in column E, starting with cell E3, to enter your midpoints. If your cell E3 entry is --- (as it typically will be), enter the value '--- (with a leading apostrophe) to avoid an error.

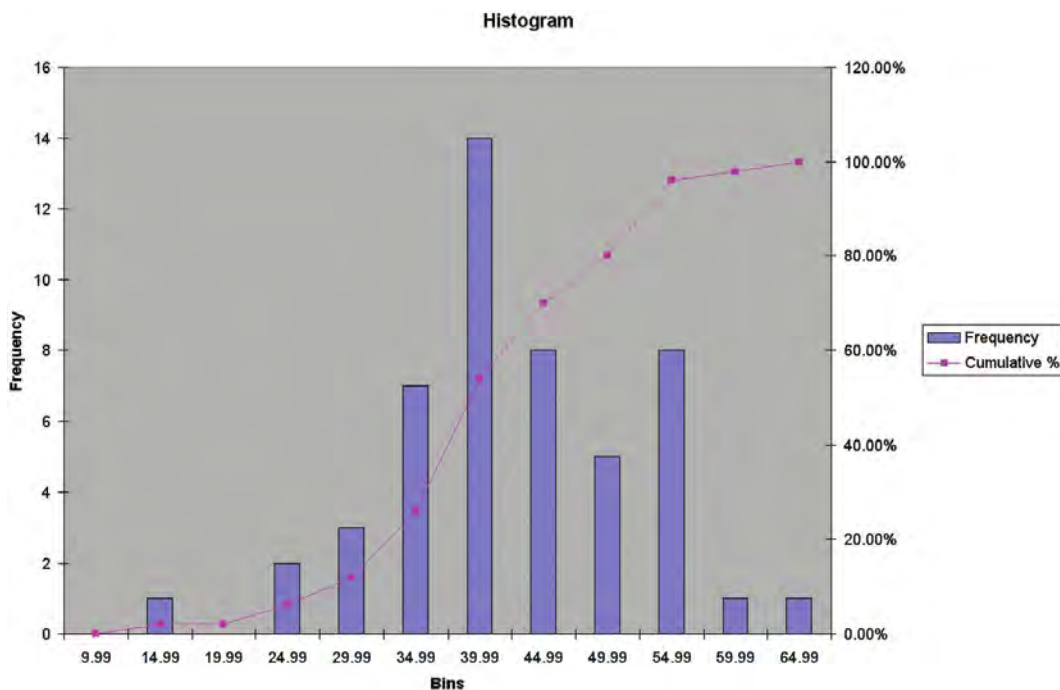
*If you use the PHStat2 Histogram & Polygons procedure, these edits and enhancements are done for you if you click the appropriate output options.*

## Correcting the Histogram

As you correct your frequency distribution, your histogram also changes, and the bar representing the in-error “More” group disappears. At this point, the histogram still contains the errors shown in Figure E2.10: Gaps between the bars correspond to the class intervals in the histogram, the bins are labeled with their maximum bin values and not with their midpoint values, and the secondary y-axis scale exceeds 100%.

To correct these errors, you need to first relocate your histogram to its own chart sheet in order to more easily work with the histogram. Right-click the background area of the histogram and click **Location (97–2003)** or **Move Chart (2007)** in the shortcut menu. In the dialog box that appears, select the **As new sheet** option (97–2003) or **New sheet (2007)** and click **OK**. The histogram now appears (much larger) in its own chart sheet, and you can continue with the following instructions to correct the histogram.

**To eliminate the gaps between bars**, right-click inside one of the histogram bars. (You will see a ToolTip that begins with **Series ‘Frequency’** when your mouse is properly positioned.) Click **Format Data Series** in the shortcut menu to display the Format Data Series dialog box. If you use Excel 97–2003, click the **Options** tab, change the value of **Gap width** to **0**, and click **OK**. If you



**FIGURE E2.10** Histogram before applying corrections (Figure 2.8 on page 49 shows the corrected histogram.)

use Excel 2007, move the **Gap Width** slider to **No Gap** in the **Series Options** panel of this dialog box.

**To change the labels for the bins**, enter midpoint values in column E. Right-click the tinted background of the chart. (You will see the ToolTip **Plot Area** when your mouse is properly positioned.) Click **Source Data** (Excel 97–2003) or **Select Data** (Excel 2007) in the shortcut menu. If you use Excel 97–2003, select the **Series** tab in the Source Data dialog box and enter the cell range of the midpoints as a formula, in the form `=SheetName!CellRange`, in the **Category (X) axis labels** box. Then delete the entry for the **Second category(X) axis labels** and click **OK**. If you use Excel 2007, click the **Edit** button under the **Horizontal (Categories) Axis Labels** heading in the Select Data Source dialog box. In the Axis Labels dialog box, enter the cell range of the midpoints as a formula, in the form `=SheetName!CellRange`, and click **OK**. (This range should start with the first midpoint value and not with the midpoint column heading). Click **OK** a second time (in the original dialog box) to complete the task.

**To rescale the secondary Y-axis**, right-click on the secondary (right) Y-axis. (You will see a ToolTip that includes the words **Secondary** and **Axis** when your mouse is properly positioned.) Click **Format Axis** in the shortcut menu. If you use Excel 97–2003, change the **Maximum** in the **Scale** tab of the Format Axis dialog box to **1** and click **OK**. If you use Excel 2007, select the **Fixed** option for **Maximum** and enter **1** as the maximum value in the **Axis Options** panel of the Format Axis dialog box, and then click **OK**.

*If you use the PHStat2 Histogram & Polygons procedure, these corrections are done for you, although PHStat2 keeps the histogram on the same worksheet as the frequency distribution.*

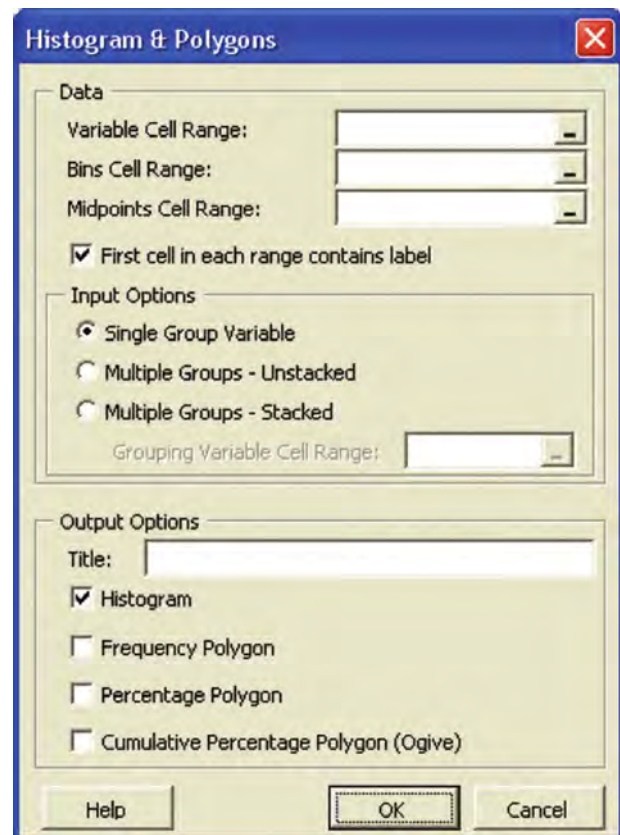
### EXAMPLE Three-Year Annualized Return for Growth Funds

To create a frequency distribution that combines the best of Tables 2.8, 2.10, and 2.13, and your own Figure 2.9 Panel A histogram (see page 49) for the three-year annualized return for growth funds, first open the **Mutual Funds.xls** workbook to the **Growth** worksheet. Column J of this worksheet contains bin values that translate the class groupings used throughout Chapter 2, thereby eliminating the need for you to do this translation yourself. (Column K contains midpoint values, used by the PHStat2 procedure.) Continue with the appropriate set of instructions.

**Basic Excel** Use the ToolPak Histogram procedure. Enter **H1:H839** as the **Input Range** and **J1:J10** as the **Bin Range** in the Histogram dialog box. Click **Labels**, select

the **New Worksheet Ply** option, click **Cumulative Percentage**, and click **Chart Output**. Click **OK** to create a frequency distribution and histogram together on a new worksheet. Apply the changes discussed in “Editing and Enhancing the Frequency Distribution” and “Correcting the Histogram,” starting on page 86. (If you successfully make these changes, the last row of the frequency distribution will be row 11.)

**PHStat2** Select **PHStat** → **Descriptive Statistics** → **Histogram & Polygons**. Enter **H1:H839** as the **Variable Cell Range**, **J1:J10** as the **Bins Cell Range** and **K1:K9** as the **Midpoints Cell Range**. Click **First cell in each range contains label** and select the **Single Group Variable** option. Enter a **Title** and click **OK**. (**Histogram** is already checked for you.)



### Enhancing the Histogram

You can enhance the histogram created by the ToolPak and PHStat2 procedures in several ways to improve the presentation quality of the histogram. You can change the generic title and axis labels and/or delete the cumulative percentage plot that is superimposed over the histogram. To change the generic title, click the title to make a halo appear around the title. In the formula bar, enter a better title and press the **Enter** key. Then follow a similar process to change each of

the axis legends. To delete the cumulative percentage plot, click the cumulative percentage plot to highlight the plot and then press the **Delete** key. Then click the (now unnecessary) chart legend box to the right of the histogram (a halo appears around the box) and press **Delete** to eliminate this, too. Also consider removing the chart background and making other changes, if appropriate, as described in the “Additional Chart Tips” part of Section E2.2 on page 80.

**Forum** Click the **ALTERNATIVE METHODS** link to learn the Excel technique you can use to create a frequency distribution that updates itself automatically when the underlying data are changed.

## E2.8 CREATING A HISTOGRAM FROM SUMMARIZED DATA

Sometimes you work with data that are already summarized in a frequency distribution. In such cases, you cannot use the ToolPak Histogram or PHStat2 Histogram & Polygons procedures because those procedures require unsummarized data. Instead, you first enter the frequency distribution into a worksheet and then use the Chart Wizard to create a histogram.

### Entering the Frequency Distribution

Open to a new worksheet and enter a title in cell A1 and column headings for your classes and frequencies in cells A2 and B2. Enter your classes and frequency counts under these headings. Add columns for percentage frequency, cumulative percentage frequency, and midpoints, using the following instructions.

Enter the heading **Percentage** in cell C2. Enter the formula  $=B3/SUM(B:B)$  in cell C3 and copy the formula down the column through the rest of the frequency distribution. Format column C for percentage display to complete the column. Enter the heading **Cumulative Percentage** in cell D2. Enter the formula  $=C3$  in cell D3. Enter the formula  $=D3+C4$  in cell D4 and copy the formula down the column through the rest of the frequency distribution. Enter the heading **Midpoints** in cell E2. Then use the cells in column E, starting with cell E3, to enter your midpoints. If your cell E3 entry is --- (as it typically will be), enter that value '--- (with a leading apostrophe) to avoid an error.

### Creating the Histogram (97–2003)

Open to the worksheet that contains your frequency distribution and use the Chart Wizard. Make these entries in the Chart Wizard dialog boxes:

**Step 1** Click the **Standard Types** tab. Click **Column** as the **Chart type** and then click the first **Chart sub-type** icon, captioned **Clustered Column** when selected.

**Step 2** Click the **Data Range** tab. Enter the cell range of the frequencies as the **Data range** and click the **Columns** option in the **Series in** group. Click the **Series** tab. Enter the cell range of the midpoints (as a formula in the form  $=SheetName!CellRange$ ) in the **Category (X) axis labels** box.

**Step 3** Click the **Titles** tab. Enter a title as the **Chart title**, **Midpoints** as the **Category (X) axis title**, and **Frequency** as the **Value (Y) axis title**. Use the formatting settings for **Axes**, **Gridlines**, **Legend**, **Data Labels**, and **Data Table** tabs that are given in the “Creating Charts (97–2003)” part of Section E2.2 on page 78.

To eliminate the gaps between the bars of the histogram the wizard created, right-click one of the histogram bars. (You will see a ToolTip that begins with **Series ‘Frequency’** when your mouse is properly positioned.) Click **Format Data Series** in the shortcut menu. In the Format Data Series dialog box, click the **Options** tab, change the value of **Gap width** to **0**, and click **OK**.

### Creating the Histogram (2007)

Select the cell range that is the column that contains your frequencies. Then select **Insert** → **Column** and click the **Clustered Column** gallery choice. Right-click the chart that appears and click **Select Data** in the shortcut menu. In the Select Data Source dialog box, click the **Edit** button under the **Horizontal (Categories) Axis Labels** heading, enter the cell range of the midpoints as a formula in the form  $=SheetName!CellRange$ , and click **OK**. Click **OK** in the Select Data Source dialog box to complete this task. (This range should start with the first midpoint value and not with the midpoint column heading). Relocate your chart to a chart sheet and customize your chart, using the instructions in “Creating Charts (2007)” in Section E.2.2 on page 79.

## E2.9 CREATING POLYGONS

You create polygons by modifying a frequency distribution previously created using the instructions from Section E2.7 or Section E2.8, and then using Excel charting features. (Because charting is different in Excel 2007, this section includes passages specific to Excel 97–2003 and Excel 2007.)

*The PHStat2 Histogram & Polygons procedure can create polygons for you from either an unmodified frequency distribution or unsummarized data.*



## Modifying the Frequency Distribution

To modify a frequency distribution for creating a polygon, open to the worksheet containing the frequency distribution. Right-click **row 3** (that is, right-click the 3 row number legend at the left of the worksheet) and click **Insert** in the shortcut menu. Enter **0** in the new cells **B3**, **C3**, and **D3**. Then enter **0** in the column B and C cells that are in the first empty row under your frequency distribution.

### EXAMPLE Figure 2.12 Cumulative Percentage Polygons (Part I)

To start the process of creating your own Figure 2.12 cumulative percentage polygons for the cost of restaurant meals at city and suburban restaurants (page 52), open the **Restaurants.xls** workbook and use the appropriate set of instructions.

**Basic Excel** Open to the **CityFrequencies** worksheet. Right-click **row 3** (that is, right-click the 3 row number legend at the left of the worksheet) and click **Insert** in the shortcut menu. Enter **0** in the new cells **B3**, **C3**, and **D3** as well as in cells **B16** and **C16**. Open to the **SuburbanFrequencies** worksheet and make the same modifications.

**PHStat2** You do not need to make any modifications to your frequency distribution if you are using the **PHStat2 Histogram & Polygons** procedure. (The procedure makes these modifications for you.)

## Creating Polygons (97–2003)

With your workbook opened to the modified frequency distribution, begin the Chart Wizard and make these entries in the Chart Wizard dialog boxes:

**Step 1** Click the **Standard Types** tab and then click **Line** as the **Chart type**. Click the first choice in the second row of **Chart sub-type** choices, labeled **Line with markers displayed at each data value** when selected.

**Step 2** Click the **Data Range** tab. As the **Data range**, enter a cell range that contains both the leading and trailing zero entries added to the worksheet earlier. (This cell range will begin in row 3.) For a frequency polygon, enter the column B cell range that contains the zeros. For a percentage polygon, enter the column C cell range that contains the zeros. For a cumulative percentage polygon, enter the column D cell range that contains the initial zero. Click the **Columns** option in the **Series in** group. Click the **Series** tab. Enter a column E cell range that begins with cell E3 (as a formula in the form  $=\text{SheetName!CellRange}$ ) in the

**Category (X) axis labels** box. For either a frequency or percentage polygon, enter the column E cell range that ends with the row that contains the trailing zero (the ending cell will be blank in column E). For a cumulative percentage polygon, enter the column E cell range that ends with the last entry in column E. Then enter a self-descriptive legend value to replace the default **Series 1** in the **Name** box.

**Step 3** Click the **Titles** tab. Enter a title as the **Chart title** and appropriate values for the **Category (X) axis** and **Value (Y) axis** titles. Use the formatting settings for **Axes**, **Gridlines**, **Data Labels**, and **Data Table** tabs that are given in the “Creating Charts (97–2003)” part of Section E2.2 (see page 78). Click the **Legend** tab and click **Show legend**.

Additional data series can be added to the polygon to produce several polygons on one chart, similar to Figures 2.10 through Figures 2.13 on pages 50–52. With your workbook opened to your chart, select **Chart** → **Add Data**. In the Add Data dialog box, enter the cell range of the new data series as a formula in the form  $=\text{Sheetname!CellRange}$  and click **OK**. If the Paste Special dialog box appears, select the **New series** and **Columns** options and click **OK**.

## Creating Polygons (2007)

With your workbook opened to the modified frequency distribution, select **Insert** → **Line** and click the **Line with markers** gallery choice. Right-click the chart that appears and click **Select Data** in the shortcut menu. In the Select Data Source dialog box, click the **Edit** button under the **Horizontal (Categories) Axis Labels** heading. In the Axis Labels dialog box, enter the cell range of the midpoints (not including the midpoints column heading) as a formula in the form  $=\text{SheetName!CellRange}$  and click **OK**. Click **OK** a second time (in the original dialog box) to complete the task.

Relocate your chart to a chart sheet and customize your chart using the instructions in “Creating Charts (2007)” in Section E.2.2 on page 79. If you plan to have more than one data series, select **Show Legend at Right** and not **None** (as Section E2.2 states) for **Legend**.

Additional data series can be added to the polygon to produce several polygons on one chart, similar to Figures 2.10 through Figures 2.13. With your workbook opened to your chart, right-click the chart and click **Select Data** in the shortcut menu. In the Edit Data Source dialog box, click **Add** and enter the cell range of the new data series as a formula in the form  $=\text{Sheetname!CellRange}$  and click **OK**. (This cell range should exclude the column heading cell.) Click **OK** a second time in the original dialog box to complete the task.

**EXAMPLE Figure 2.12 Cumulative Percentage Polygons (Part II)**

Having completed the appropriate Part I instructions (see page 89), continue with the appropriate set of instructions.

**Basic Excel (97–2003)** Begin the Chart Wizard. Enter **D3:D15** as the **Data Range** and click the **Columns** option in the **Series in** group in the **Data Range** tab of the Step 2 dialog box. In the **Series** tab of the same dialog box, enter **=CityFrequencies!E3:E15** as in the **Category (X) axis labels**.

After you complete the wizard, select **Chart** → **Add Data**. In the Add Data dialog box, enter **=SuburbanFrequencies!E3:E15** as the **Range** and click **OK**. (If the Paste Special dialog box appears, select the **New series** and **Columns** options and click **OK**.)

**Basic Excel 2007** Select cell range **D3:D15** and use the preceding “Creating Polygons (2007)” instructions. Enter **=CityFrequencies!E3:E15** as the cell range for the midpoints.

**PHStat2** Open to the **Data** worksheet. Because PHStat2 creates frequency distributions, histograms, and polygons in one step, you will be using a procedure similar to the one

in the example for Section E2.7. Select **PHStat** → **Descriptive Statistics** → **Histogram & Polygons**. Enter **G1:G101** as the **Variable Cell Range**, **H1:H13** as the **Bins Cell Range**, and **I1:I12** as the **Midpoints Cell Range**. Click **First cell in each range contains label**, click **Multiple Groups - Stacked**, and enter **A1:A101** as the **Grouping Variable Cell Range**. Enter a **Title** and click **Percentage Polygon** and **Cumulative Percentage Polygon (Ogive)**. (**Histogram** is already checked for you.) Click **OK** to create the worksheets and chart sheets.

## E2.10 CREATING CONTINGENCY TABLES

You use Excel PivotTable features to create a contingency table from the unsummarized data of two categorical variables. You create a contingency table by using a process almost identical to the process of creating a PivotTable summary table discussed in Section E2.1 on page 76. Because this process differs for Excel 2007, this section includes passages specific to Excel 97–2003 and Excel 2007.

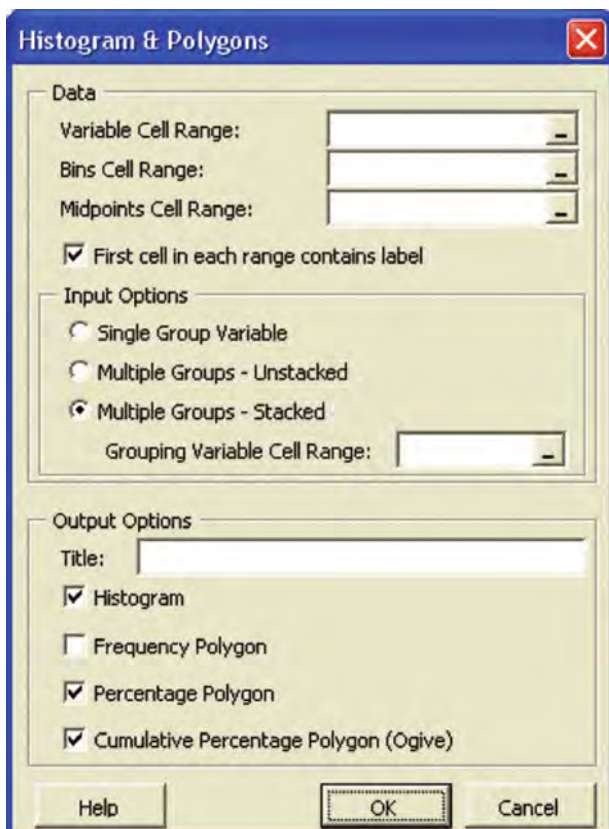
*The PHStat2 Two-WayTables & Charts procedure can create a contingency table for you, and, optionally, a side-by-side chart (see Section E2.11).*

### Creating a Contingency Table (97–2003)

Begin with the instructions in “Starting a PivotTable (97–2003)” in Section E2.1. When you get to the Step 3 instructions, modify them as follows. In the Layout dialog box in Step 3, first drag the label of the first variable to be summarized and drop it in the ROW area. Drag a second copy of this same label and drop it in the DATA area. (The label changes to **Count of variable name**.) Then drag the label of the second variable and drop it in the COLUMN area. In the PivotTable Options dialog box, also in Step 3, verify that both **Grand total for columns** and **Grand totals for rows** are checked (they should be) and that you have entered **0** as the **For empty cells, show** value. (Excel 97 does not contain this options dialog box.)

### Creating a Contingency Table (2007)

Begin with the instructions in “Starting a PivotTable (97–2003)” in Section E2.1. When you get to the instructions for using the PivotTable Field List task pane, modify them as follows. In the PivotTable Field List task pane, drag the label of the variable to be summarized and drop it in the **Row Labels** area. Drag a second copy of this same



label and drop it in the  $\Sigma$  **Values** area. (This second label changes to **Count of variable name**.) Then drag the label of the second variable and drop it in the **Column Labels** area. When you later right-click the PivotTable, click **Table Options** in the shortcut menu. In the Total & Filters tab of the PivotTable Options dialog box, verify that both **Show grand total for columns** and **Show grand totals for rows** are checked. (Remember to also click the **Layout & Format** tab, check **For empty cells show**, and enter 0 as its value, as stated in the Section E2.1 instructions.)

### EXAMPLE Table 2.14 Contingency Table

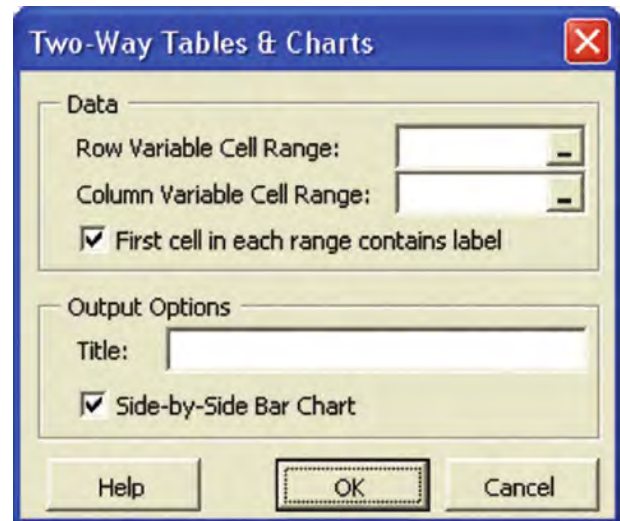
To create a PivotTable version of Table 2.14 on page 55 for fund objective and risk from your PivotTable, open the **Mutual Funds.xls** workbook to the **Data** worksheet and use the appropriate set of instructions.

**Basic Excel 97–2003** Follow the instructions in “Creating a Contingency Table (97–2003).” In the Chart Wizard Step 2 dialog box, enter **A1:F839** as the **Range**. In the Layout dialog box of Step 3, drag the **Objective** variable label and drop it in the ROW area. Drag a second copy of this same label and drop it in the DATA area. (The second label changes to **Count of Objective** when you drop it in the DATA area.) Drag the **Risk** variable label and drop it in the COLUMN area.

When you complete the PivotTable Wizard, open to the worksheet that contains the PivotTable and click a cell outside the PivotTable. Begin the Chart Wizard and use the “Creating a Side-by-Side Chart (97–2003)” instructions of Section E2.11.

**Basic Excel 2007** Follow the instructions in “Creating a Contingency Table (2007).” In the PivotTable Field List task pane, drag the **Objective** variable label and drop it in the **Row Labels** area. Drag a second copy of this same label and drop it in the  $\Sigma$  **Values** area. (This second label changes to **Count of variable name**.) Then drag the **Risk** variable label and drop it in the **Column Labels** area. When you complete your PivotTable, use the “Creating a Side-by-Side Chart (2007)” instructions of Section E2.11.

**PHStat2** Select **PHStat** → **Descriptive Statistics** → **Two-Way Tables & Charts**. In the Two-Way Tables & Charts dialog box (see the top of the next column), enter B1:B839 as the **Row Variable Cell Range** and enter F1:F839 as the **Column Variable Cell Range**. Click **First cell in each range contains label**, enter a **Title**, and optionally click **Side-by-Side Bar Chart**. Click **OK** to produce the PivotTable and chart.



## E2.11 CREATING SIDE-BY-SIDE CHARTS

You use Excel charting features to create a side-by-side chart from a contingency table of two categorical variables. Because Excel charting is different in Excel 2007, this section includes passages specific to Excel 97–2003 and Excel 2007

*PHStat2 Two-Way Tables & Charts procedure creates a side-by-side chart for you when you click the Side-by-Side Chart output option.*

### Creating a Side-by-Side Chart (97–2003)

To create a side-by-side chart, open to the worksheet containing the contingency table. If you created a contingency table that is a PivotTable, first click a cell that is outside the PivotTable.<sup>9</sup> Begin the Chart Wizard and make these entries:

**Step 1** Click **Bar** from the **Standard Types Chart type** box and leave the first **Chart sub-type** selected.

**Step 2** If you are using a PivotTable, click the PivotTable when the **Data range** box is selected, to have Excel fill in the proper cell range. If you are using a manually entered contingency table, enter the rectangular range that excludes the total row and the total column but that includes the row and column headings.

**Step 3** Click the **Titles** tab. Enter a title as the **Chart title** and appropriate values for the **Category (X) axis** and **Value (Y) axis** titles. (Unlike other charts, Excel considers the horizontal axis to be the *Value (Y) axis* and the vertical

<sup>9</sup>If you forget to click outside the PivotTable, the Chart Wizard instantly creates a (wrong) chart. If this occurs, you can press **Ctrl+Z** to undo the chart.

axis of this chart to be the *Category (X) axis*, which is different from what you might otherwise expect.) Click the **Legend** tab and click **Show legend**. Use the formatting settings for **Axes**, **Gridlines**, **Data Labels**, and **Data Table** tabs that are given in the “Creating Charts (97–2003)” part of Section E2.2 on page 78.

If field buttons appear on the chart, right-click any button and click **Hide PivotChart Field Buttons** in the shortcut menu.

### Creating a Side-by-Side Chart (2007)

To create a side-by-side chart, open to the worksheet that contains the contingency table. If you created a contingency table that is a PivotTable, click a cell inside the PivotTable; otherwise, select the cell range of the contingency table, including row and column headings, but excluding the total row and total column.

Select **Insert** → **Bar** and click the **Clustered Bar** gallery choice. If your chart contains reversed row and column variables and is not based on a PivotTable, right-click the chart and click **Select Data** in the shortcut menu. In the **Select Data Source** dialog box, click **Switch Row/Column** and then click **OK**. If your chart has reversed variables and is based on a PivotTable, you have to reorder your variables in the PivotTable in order to get the correct chart. Finish by relocating your chart to a chart sheet and customizing your chart, using the instructions in “Creating Charts (Excel 2007)” in Section E.2.2 on page 79, with this exception: After you click **Legend**, select **Show Legend at Right**.

---

#### EXAMPLE Figure 2.14 Side-by-Side Chart

If you are using PHStat2 to create your own Figure 2.14 side-by-side bar chart for fund objective and risk (page 56), first open to the **Data** worksheet of the **Mutual Funds.xls** workbook. Otherwise, first create a PivotTable version of Table 2.14, as is done in the example in Section E2.10, and open your workbook to that PivotTable. Continue by using the appropriate set of instructions.

**Basic Excel 97–2003** Follow the “Creating a Side-by-Side Chart (97–2003)” instructions from this section.

**Basic Excel 2007** Follow the “Creating a Side-by-Side Chart (2007)” instructions from this section.

**PHStat2** Use the PHStat2 instructions for the example in Section 2.10. Click the **Side-by-Side Chart** output option before clicking **OK**.

---

## E2.12 CREATING SCATTER PLOTS

You use Excel charting features to create scatter plots. To create a scatter plot, open to the worksheet that contains the columns for the two numerical variables you wish to analyze. Your variables should be arranged *X* variable first, then *Y* variable, reading left-to-right. (If your data are arranged first *Y* and then *X*, cut and paste the *Y* variable column so it appears to the right of the *X* variable column.) Because charting has been changed in Excel 2007, continue with the appropriate set of instructions.

*You can use the Scatter Diagram output option of the PHStat2 Simple Linear Regression procedure (see the Excel Companion to Chapter 13) if you want PHStat2 to create a scatter plot for you.*

### Creating a Scatter Plot (97–2003)

Continue by beginning the Chart Wizard and make these entries:

**Step 1** Click **XY (Scatter)** from the **Standard Types Chart type** box and leave the first **Chart sub-type** selected.

**Step 2** Enter the cell range of the two variables in the **Data range** box and select the **Columns** option. If the two variables are in non-adjacent columns, first type or point to the cell range of the first variable, then type a comma, then type or point to the cell range of the second variable, and then press **Enter**. Do not make any entries in the **Series** tab of this dialog box.

**Step 3** Click the **Titles** tab. Enter a title as the **Chart title** and enter appropriate values for the **Value (X) axis** and **Value (Y) axis** titles. Click, in turn, the **Axes**, **Gridlines**, **Legend**, and **Data Labels** tabs and use the formatting settings given in the “Creating Charts (97–2003)” part of Section E2.2 on page 78.

### Creating a Scatter Plot (2007)

Continue by selecting the cell range of the *X* and *Y* variables. Then select **Insert** → **Scatter** and click the **Scatter with only Markers** gallery choice. Finish by relocating your chart to a chart sheet and customizing your chart using the instructions in “Creating Charts (2007)” in Section E.2.2.

---

#### EXAMPLE Figure 2.15 Scatter Plot

To create your own Figure 2.15 scatter plot of the cost of a fast-food hamburger meal and the cost of two movie tickets (page 59), first open the **Cost of Living.xls** work-

book to the **Data** worksheet and then use the following instructions.

**Basic Excel 97–2003** Use the preceding “Creating a Scatter Plot (97–2003)” instructions. In the Chart Wizard Step 2 dialog box, enter **E1:E11,H1:H11** as the **Data Range** and click the **Columns** option in the **Series in** group in the **Data Range** tab.

**Basic Excel 2007** Copy the cell ranges **E1:E11** and **H1:H11** to consecutive columns of a blank worksheet. (If you need to insert a new worksheet, right-click the **Data** sheet tab and select **Insert**. In the gallery that appears, click **Worksheet**.) Select the copied columns and then use the preceding “Creating a Scatter Plot (2007)” instructions.

**PHStat2** Select **PHStat** → **Regression** → **Simple Linear Regression**. Enter **H1:H11** as the **Y Variable Cell Range** and **E1:E11** as the **X Variable Cell Range**. Enter a title as the **Title**, click **Scatter Diagram**, and then click **OK**. (For more information about this procedure, see the Excel Companion to Chapter 13).

## E2.13 CREATING TIME-SERIES PLOTS

You use Excel charting features to create time-series plots. To create a time-series plot, open to the worksheet that contains the time variable and the numerical variable to plot. The column for your time variable should appear first, reading left-to-right. (If the column that contains the time variable appears after the column that contains the numerical variable, cut and paste the numerical variable column so it appears to the right of the time variable column.) Because charting is different in Excel 2007, continue with the appropriate set of instructions.

### Creating a Time Series Plot (97–2003)

Begin the Chart Wizard and make these entries:

**Step 1** Click **Line** from the **Standard Types Chart type** box and select the first **Chart sub-type** in the second row, identified as **Line with markers displayed at each data value**.

**Step 2** Enter the cell range of the two variables in the **Data range** box and select the **Columns** option. If the variables are in non-adjacent columns, first type or point to the cell range of the first variable, then type a comma, then

type or point to the cell range of the second variable, and then press **Enter**. Do not make any entries in the **Series** tab of this dialog box.

**Step 3** Click the **Titles** tab. Enter a title as the **Chart title** and enter appropriate values for the **Value (X) axis** and **Value (Y) axis** titles. Click, in turn, the **Axes**, **Gridlines**, **Legend**, **Data Labels**, and **Data Table** tabs and adjust the formatting settings given in the “Creating Charts (97–2003)” part of Section E2.2 on page 78.

### Creating a Time Series Plot (2007)

Continue by selecting the cell range of the numerical variable. Then select **Insert** → **Line** and click the **Line with Markers** gallery choice. Right-click the new chart and click **Select Data**. Click the **Edit** button under the **Horizontal (Categories) Axis Labels** heading in the Select Data Source dialog box. In the Axis Labels dialog box, enter the cell range of the time variable as a formula in the form `=SheetName!CellRange` and click **OK**. (This range should start with the first time value and not with the column heading for the time variable). Click **OK** a second time (in the original dialog box) to complete the task.

Finish by relocating your chart to a chart sheet and customizing your chart using the instructions in “Creating Charts (2007)” in Section E.2.2 on page 79.

*PHStat2 does not contain a time-series plot command. PHStat2 users should use the instructions given above to create time-series plots.*

### EXAMPLE Figure 2.16 Time-Series Plot

To create your own Figure 2.16 time-series plot of the movie attendance (in billions) from 1999 to 2005 (page 60), first open the **Movies.xls** workbook to the **Data** worksheet and then use the appropriate set of instructions.

**Basic Excel 97–2003** Use the preceding “Creating a Time-Series Plot (97–2003)” instructions. In the Chart Wizard Step 2 dialog box, enter **A1:B8** as the **Data Range** and click the **Columns** option in the **Series in** group in the **Data Range** tab.

**Basic Excel 2007** Select **B1:B11** and then use the preceding “Creating a Time-Series Plot (2007)” instructions. Use `=Data!A2:A11` as the formula to enter in the Axis Labels dialog box.



# CHAPTER 3

## Numerical Descriptive Measures

USING STATISTICS @ Choice Is Yours, Part II

### 3.1 MEASURES OF CENTRAL TENDENCY

The Mean  
The Median  
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### EXCEL COMPANION TO CHAPTER 3

E3.1 Computing Measures of Central Tendency, Variation, and Shape  
E3.2 Creating Dot Scale Diagrams  
E3.3 Computing Measures for a Population  
E3.4 Creating Box-and-Whisker Plots  
E3.5 Computing the Covariance  
E3.6 Computing the Correlation Coefficient

## LEARNING OBJECTIVES

In this chapter, you learn:

- To describe the properties of central tendency, variation, and shape in numerical data
- To calculate descriptive summary measures for a population
- To construct and interpret a box-and-whisker plot
- To describe the covariance and the coefficient of correlation

## USING STATISTICS @ Choice Is Yours, Part II



The tables and charts you prepared for the sample of 838 mutual funds has proved useful to the customers of the Choice Is Yours service. However, customers have become frustrated trying to evaluate mutual fund performance. While they know how the 838 three-year rates of return are distributed, they have no idea what a typical three-year rate of return would be for a particular category of mutual funds, such as low-risk funds, nor do they know how that typical value compares to the typical values of other categories. They also have no idea of the extent of the variability in the three-year rate of return. Are all the values relatively similar, or do they include very small and very large values? Are there a lot of small values and a few large ones, or vice versa, or are there a similar number of small and large values?

How could you help the customers get answers to these questions so that they could better evaluate the mutual funds?

The customers in the Using Statistics scenario are asking questions about numerical variables. When summarizing and describing numerical variables, you need to do more than just prepare the tables and charts discussed in Chapter 2. You need to consider the central tendency, variation, and shape of each numerical variable.

### CENTRAL TENDENCY

The **central tendency** is the extent to which all the data values group around a typical or central value.

### VARIATION

The **variation** is the amount of dispersion, or scattering, of values away from a central value.

### SHAPE

The **shape** is the pattern of the distribution of values from the lowest value to the highest value.

This chapter discusses ways you can measure the central tendency, variation, and shape of a variable. You will also learn about the covariance and the coefficient of correlation, which help measure the strength of the association between two numerical variables. Using these measures would give the customers of the Choice Is Yours service the answers they seek.

## 3.1 MEASURES OF CENTRAL TENDENCY

Most sets of data show a distinct tendency to group around a central point. When people talk about an “average value” or the “middle value” or the “most frequent value,” they are talking informally about the mean, median, and mode—three measures of central tendency.



## The Mean

The **arithmetic mean** (typically referred to as the **mean**) is the most common measure of central tendency. The mean is the only common measure in which all the values play an equal role. The mean serves as a “balance point” in a set of data (like the fulcrum on a seesaw). You calculate the mean by adding together all the values in a data set and then dividing that sum by the *number* of values in the data set.

The symbol  $\bar{X}$ , called *X-bar*, is used to represent the mean of a sample. For a sample containing  $n$  values, the equation for the mean of a sample is written as

$$\bar{X} = \frac{\text{Sum of the values}}{\text{Number of values}}$$

Using the series  $X_1, X_2, \dots, X_n$  to represent the set of  $n$  values and  $n$  to represent the number of values, the equation becomes:

$$\bar{X} = \frac{X_1 + X_2 + \dots + X_n}{n}$$

By using summation notation (discussed fully in Appendix B), you replace the numerator  $X_1 + X_2 + \dots + X_n$  by the term  $\sum_{i=1}^n X_i$ , which means sum all the  $X_i$  values from the first  $X$  value,  $X_1$ , to the last  $X$  value,  $X_n$ , to form Equation (3.1), a formal definition of the sample mean.

### SAMPLE MEAN

The **sample mean** is the sum of the values divided by the number of values.

$$\bar{X} = \frac{\sum_{i=1}^n X_i}{n} \quad (3.1)$$

where

$\bar{X}$  = sample mean

$n$  = number of values or sample size

$X_i$  =  $i$ th value of the variable  $X$

$\sum_{i=1}^n X_i$  = summation of all  $X_i$  values in the sample

Because all the values play an equal role, a mean is greatly affected by any value that is greatly different from the others in the data set. When you have such extreme values, you should avoid using the mean.

The mean can suggest a typical or central value for a data set. For example, if you knew the typical time it takes you to get ready in the morning, you might be able to better plan your morning and minimize any excessive lateness (or earliness) going to your destination. Suppose you define the time to get ready as the time (rounded to the nearest minute) from when you get out of bed to when you leave your home. You collect the times shown below for 10 consecutive work days (stored in the data file [times.xls](#)):

Day:	1	2	3	4	5	6	7	8	9	10
Time (minutes):	39	29	43	52	39	44	40	31	44	35

The mean time is 39.6 minutes, computed as follows:

$$\begin{aligned}\bar{X} &= \frac{\text{Sum of the values}}{\text{Number of values}} \\ \bar{X} &= \frac{\sum_{i=1}^n X_i}{n} \\ \bar{X} &= \frac{39 + 29 + 43 + 52 + 39 + 44 + 40 + 31 + 44 + 35}{10} \\ \bar{X} &= \frac{396}{10} = 39.6\end{aligned}$$

Even though no one day in the sample actually had the value 39.6 minutes, allotting about 40 minutes to get ready would be a good rule for planning your mornings. The mean is a good measure of central tendency in this case because the data set does not contain any exceptionally small or large values.

Consider a case in which the value on Day 4 is 102 minutes instead of 52 minutes. This extreme value causes the mean to rise to 44.6 minutes, as follows:

$$\begin{aligned}\bar{X} &= \frac{\text{Sum of the values}}{\text{Number of values}} \\ \bar{X} &= \frac{\sum_{i=1}^n X_i}{n} \\ \bar{X} &= \frac{446}{10} = 44.6\end{aligned}$$

The one extreme value has increased the mean by more than 10%, from 39.6 to 44.6 minutes. In contrast to the original mean that was in the “middle” (that is, greater than 5 of the getting-ready times and less than the 5 other times), the new mean is greater than 9 of the 10 getting-ready times. Because of the extreme value, the mean is now a poor measure of central tendency.

### EXAMPLE 3.1

#### THE MEAN THREE-YEAR ANNUALIZED RETURN FOR SMALL-CAP GROWTH MUTUAL FUNDS WITH LOW RISK

The 838 mutual funds ([Mutual Funds.xls](#)) that are part of the Using Statistics scenario (see page 96) are classified according to the category (small cap, mid cap, and large cap), the type (growth or value), and the risk level of the mutual funds (low, average, and high). You are particularly interested in small companies with a lot of growth potential. Moreover, you want to investigate only those funds with low risk. Thus, you sort the mutual fund data and locate those funds that are classified as specializing in small-cap companies, have a growth objective, and are perceived to be low risk. The following seven companies meet all these criteria:

Fund	Category	Objective	Risk	Three-Year Return
Baron Growth	Small Cap	Growth	Low	20.8
Columbia Acorn Z	Small Cap	Growth	Low	26.0
FBR Small Cap	Small Cap	Growth	Low	24.9
Perritt Micro Cap Opportunities	Small Cap	Growth	Low	29.9
Schroder Capital US Opportunities Inv	Small Cap	Growth	Low	22.3
Value Line Emerging Opportunities	Small Cap	Growth	Low	19.0
Wells Fargo Advtg Small Cap Opp Adm	Small Cap	Growth	Low	22.4

Compute the mean three-year annualized return for the small-cap growth funds with low risk.

**SOLUTION** The mean three-year annualized return for the small-cap growth funds with low risk is 23.61, calculated as follows:

$$\begin{aligned}\bar{X} &= \frac{\text{Sum of the values}}{\text{Number of values}} \\ &= \frac{\sum_{i=1}^n X_i}{n} \\ &= \frac{165.3}{7} = 23.6143\end{aligned}$$

The ordered array for the seven small-cap growth funds with low risk is:

19.0 20.8 22.3 22.4 24.9 26.0 29.9

Four of these returns are below the mean of 23.61, and three of them are above the mean.

## The Median

The **median** is the middle value in a set of data that has been ranked from smallest to largest. Half the values are smaller than or equal to the median, and half the values are larger than or equal to the median. The median is not affected by extreme values, so you can use the median when extreme values are present.

To calculate the median for a set of data, you first rank the values from smallest to largest and then use Equation (3.2) to compute the rank of the value that is the median.

### MEDIAN

$$\text{Median} = \frac{n+1}{2} \text{ranked value} \quad (3.2)$$

You compute the median value by following one of two rules:

- **Rule 1** If there are an *odd* number of values in the data set, the median is the middle-ranked value.
- **Rule 2** If there are an *even* number of values in the data set, then the median is the *average* of the two middle ranked values.

To compute the median for the sample of 10 times to get ready in the morning, you rank the daily times as follows:

Ranked values:

29    31    35    39    39    40    43    44    44    52

Ranks:

1    2    3    4    5    6    7    8    9    10

↑

Median = 39.5

Because the result of dividing  $n + 1$  by 2 is  $(10 + 1)/2 = 5.5$  for this sample of 10, you must use Rule 2 and average the fifth and sixth ranked values, 39 and 40. Therefore, the median is 39.5. The median of 39.5 means that for half the days, the time to get ready is less than or equal to 39.5 minutes, and for half the days, the time to get ready is greater than or equal to 39.5 minutes. In this case, the median time to get ready of 39.5 minutes is very close to the mean time to get ready of 39.6 minutes.

### EXAMPLE 3.2

#### COMPUTING THE MEDIAN FROM AN ODD-SIZED SAMPLE

The 838 mutual funds ([Mutual Funds.xls](#)) that are part of the Using Statistics scenario (see page 96) are classified according to the category (small cap, mid cap, and large cap), the type (growth or value), and the risk level of the mutual funds (low, average, and high). Compute the median three-year annualized return for the small-cap growth funds with low risk.

**SOLUTION** Because the result of dividing  $n + 1$  by 2 is  $(7 + 1)/2 = 4$  for this sample of seven, using Rule 1, the median is the fourth ranked value. The three-year annualized returns for the seven small-cap growth funds with low risk (see page 99) are ranked from the smallest to the largest:

Ranked values:

19.0    20.8    22.3    22.4    24.9    26.0    29.9

Ranks:

1    2    3    4    5    6    7

↑

Median

The median three-year annualized return is 22.4. Half the three-year annualized returns are equal to or below 22.4, and half the returns are equal to or above 22.4.

### The Mode

The **mode** is the value in a set of data that appears most frequently. Like the median and unlike the mean, extreme values do not affect the mode. Often, there is no mode or there are several modes in a set of data. For example, consider the time-to-get-ready data shown below:

29    31    35    39    39    40    43    44    44    52

There are two modes, 39 minutes and 44 minutes, because each of these values occurs twice.

**EXAMPLE 3.3** COMPUTING THE MODE

A systems manager in charge of a company's network keeps track of the number of server failures that occur in a day. Compute the mode for the following data, which represents the number of server failures in a day for the past two weeks:

1 3 0 3 26 2 7 4 0 2 3 3 6 3

**SOLUTION** The ordered array for these data is

0 0 1 2 2 3 3 3 3 3 4 6 7 26

Because 3 appears five times, more times than any other value, the mode is 3. Thus, the systems manager can say that the most common occurrence is having three server failures in a day. For this data set, the median is also equal to 3, and the mean is equal to 4.5. The extreme value 26 is an outlier. For these data, the median and the mode better measure central tendency than the mean.

A set of data has no mode if none of the values is “most typical.” Example 3.4 presents a data set with no mode.

**EXAMPLE 3.4** DATA WITH NO MODE

Compute the mode for the three-year annualized return for the small-cap growth funds ([Mutual Funds.xls](#)) with low risk (see page 99).

**SOLUTION** The ordered array for these data is

19.0 20.8 22.3 22.4 24.9 26.0 29.9

These data have no mode. None of the values is most typical because each value appears once.

**Quartiles**

<sup>1</sup>The  $Q_1$ , median, and  $Q_3$  are also the 25th, 50th, and 75th percentiles, respectively. Equations (3.2), (3.3), and (3.4) can be expressed generally in terms of finding percentiles:  $(p \times 100)$ th percentile =  $p \times (n + 1)$  ranked value.

**Quartiles** split a set of data into four equal parts—the **first quartile,  $Q_1$** , divides the smallest 25.0% of the values from the other 75.0% that are larger. The **second quartile,  $Q_2$** , is the median—50.0% of the values are smaller than the median and 50.0% are larger. The **third quartile,  $Q_3$** , divides the smallest 75.0% of the values from the largest 25.0%. Equations (3.3) and (3.4) define the first and third quartiles.<sup>1</sup>

**FIRST QUARTILE,  $Q_1$** 

25.0% of the values are smaller than or equal to  $Q_1$ , the first quartile, and 75.0% are larger than or equal to the first quartile,  $Q_1$ .

$$Q_1 = \frac{n+1}{4} \text{ ranked value} \quad (3.3)$$

**THIRD QUARTILE,  $Q_3$** 

75.0% of the values are smaller than or equal to the third quartile,  $Q_3$ , and 25.0% are larger than or equal to the third quartile,  $Q_3$ .

$$Q_3 = \frac{3(n+1)}{4} \text{ ranked value} \quad (3.4)$$

Use the following rules to calculate the quartiles:

- **Rule 1** If the result is a whole number, then the quartile is equal to that ranked value. For example, if the sample size  $n = 7$ , the first quartile,  $Q_1$ , is equal to the  $(7 + 1)/4 = 2$  second ranked value.
- **Rule 2** If the result is a fractional half (2.5, 4.5, etc.), then the quartile is equal to the average of the corresponding ranked values. For example, if the sample size  $n = 9$ , the first quartile,  $Q_1$ , is equal to the  $(9 + 1)/4 = 2.5$  ranked value, halfway between the second ranked value and the third ranked value.
- **Rule 3** If the result is neither a whole number nor a fractional half, you round the result to the nearest integer and select that ranked value. For example, if the sample size  $n = 10$ , the first quartile,  $Q_1$ , is equal to the  $(10 + 1)/4 = 2.75$  ranked value. Round 2.75 to 3 and use the third ranked value.

To illustrate the computation of the quartiles for the time-to-get-ready data, rank the following data from smallest to largest:

Ranked values:

29    31    35    39    39    40    43    44    44    52

Ranks:

1    2    3    4    5    6    7    8    9    10

The first quartile is the  $(n + 1)/4 = (10 + 1)/4 = 2.75$  ranked value. Using Rule 3, you round up to the third ranked value. The third ranked value for the time-to-get-ready data is 35 minutes. You interpret the first quartile of 35 to mean that on 25% of the days, the time to get ready is less than or equal to 35 minutes, and on 75% of the days, the time to get ready is greater than or equal to 35 minutes.

The third quartile is the  $3(n + 1)/4 = 3(10 + 1)/4 = 8.25$  ranked value. Using Rule 3 for quartiles, you round this down to the eighth ranked value. The eighth ranked value is 44 minutes. Thus, on 75% of the days, the time to get ready is less than or equal to 44 minutes, and on 25% of the days, the time to get ready is greater than or equal to 44 minutes.

### EXAMPLE 3.5

#### COMPUTING THE QUARTILES

The 838 mutual funds ([Mutual Funds.xls](#)) that are part of the Using Statistics scenario (see page 96) are classified according to the category (small cap, mid cap, and large cap), the type (growth or value), and the risk level of the mutual funds (low, average, and high). Compute the first quartile ( $Q_1$ ) and third quartile ( $Q_3$ ) three-year annualized return for the small-cap growth funds with low risk.

**SOLUTION** Ranked from smallest to largest, the three-year annualized returns for the seven small-cap growth funds with low risk (see page 99) are:

Ranked value:

19.0    20.8    22.3    22.4    24.9    26.0    29.9

Ranks:

1    2    3    4    5    6    7

For these data

$$\begin{aligned} Q_1 &= \frac{(n+1)}{4} \text{ ranked value} \\ &= \frac{7+1}{4} \text{ ranked value} = 2\text{nd ranked value} \end{aligned}$$

Therefore, using Rule 1,  $Q_1$  is the second ranked value. Because the second ranked value is 20.8, the first quartile,  $Q_1$ , is 20.8.

To find the third quartile,  $Q_3$ :

$$\begin{aligned} Q_3 &= \frac{3(n+1)}{4} \text{ ranked value} \\ &= \frac{3(7+1)}{4} \text{ ranked value} = 6\text{th ranked value} \end{aligned}$$

Therefore, using Rule 1,  $Q_3$  is the sixth ranked value. Because the sixth ranked value is 26.0,  $Q_3$  is 26.

The first quartile of 20.8 indicates that 25% of the returns are below or equal to 20.8 and 75% are greater than or equal to 20.8. The third quartile of 26.0 indicates that 75% of the returns are below or equal to 26.0 and 25% are greater than or equal to 26.0.

## The Geometric Mean

The **geometric mean** measures the rate of change of a variable over time. Equation (3.5) defines the geometric mean.

### GEOMETRIC MEAN

The geometric mean is the  $n$ th root of the product of  $n$  values.

$$\bar{X}_G = (X_1 \times X_2 \times \cdots \times X_n)^{1/n} \quad (3.5)$$

The **geometric mean rate of return** measures the average percentage return of an investment over time. Equation (3.6) defines the geometric mean rate of return.

### GEOMETRIC MEAN RATE OF RETURN

$$\bar{R}_G = [(1 + R_1) \times (1 + R_2) \times \cdots \times (1 + R_n)]^{1/n} - 1 \quad (3.6)$$

where

$R_i$  is the rate of return in time period  $i$

To illustrate these measures, consider an investment of \$100,000 that declined to a value of \$50,000 at the end of Year 1 and then rebounded back to its original \$100,000 value at the end

of Year 2. The rate of return for this investment for the two-year period is 0 because the starting and ending value of the investment is unchanged. However, the arithmetic mean of the yearly rates of return of this investment is

$$\bar{X} = \frac{(-0.50) + (1.00)}{2} = 0.25, \text{ or } 25\%$$

because the rate of return for Year 1 is

$$R_1 = \left( \frac{50,000 - 100,000}{100,000} \right) = -0.50, \text{ or } -50\%$$

and the rate of return for Year 2 is

$$R_2 = \left( \frac{100,000 - 50,000}{50,000} \right) = 1.00, \text{ or } 100\%$$

Using Equation (3.6), the geometric mean rate of return for the two years is

$$\begin{aligned} \bar{R}_G &= [(1 + R_1) \times (1 + R_2)]^{1/n} - 1 \\ &= [(1 + (-0.50)) \times (1 + (1.0))]^{1/2} - 1 \\ &= [(0.50) \times (2.0)]^{1/2} - 1 \\ &= [1.0]^{1/2} - 1 \\ &= 1 - 1 = 0 \end{aligned}$$

Thus, the geometric mean rate of return more accurately reflects the (zero) change in the value of the investment for the two-year period than does the arithmetic mean.

### EXAMPLE 3.6

#### COMPUTING THE GEOMETRIC MEAN RATE OF RETURN

The percentage change in the Russell 2000 Index of the stock prices of 2,000 small companies was +18.33% in 2004 and +4.55% in 2005. Compute the geometric rate of return.

**SOLUTION** Using Equation (3.6), the geometric mean rate of return in the Russell 2000 Index for the two years is

$$\begin{aligned} \bar{R}_G &= [(1 + R_1) \times (1 + R_2)]^{1/n} - 1 \\ &= [(1 + (0.1833)) \times (1 + (0.0455))]^{1/2} - 1 \\ &= [(1.1833) \times (1.0455)]^{1/2} - 1 \\ &= [1.23714]^{1/2} - 1 \\ &= 1.1123 - 1 = 0.1123 \end{aligned}$$

The geometric mean rate of return in the Russell 2000 Index for the two years is 11.23%.



## 3.2 VARIATION AND SHAPE

In addition to central tendency, every data set can be characterized by its variation and shape. Variation measures the **spread**, or **dispersion**, of values in a data set. One simple measure of variation is the range, the difference between the largest and smallest values. More commonly used in statistics are the standard deviation and variance, two measures explained later in this section. The shape of a data set represents a pattern of all the values, from the lowest to highest value. As you will learn later in this section, many data sets have a pattern that looks approximately like a bell, with a peak of values somewhere in the middle.

### The Range

The **range** is the simplest numerical descriptive measure of variation in a set of data.

#### RANGE

The range is equal to the largest value minus the smallest value.

$$\text{Range} = X_{\text{largest}} - X_{\text{smallest}} \quad (3.7)$$

To determine the range of the times to get ready in the morning, you rank the data from smallest to largest:

29 31 35 39 39 40 43 44 44 52

Using Equation (3.7), the range is  $52 - 29 = 23$  minutes. The range of 23 minutes indicates that the largest difference between any two days in the time to get ready in the morning is 23 minutes.

#### EXAMPLE 3.7

#### COMPUTING THE RANGE IN THE THREE-YEAR ANNUALIZED RETURNS FOR SMALL-CAP GROWTH MUTUAL FUNDS WITH LOW RISK

The 838 mutual funds ([Mutual Funds.xls](#)) that are part of the Using Statistics scenario (see page 96) are classified according to the category (small cap, mid cap, and large cap), the type (growth or value), and the risk level of the mutual funds (low, average, and high). Compute the range of the three-year annualized returns for the small-cap growth funds with low risk (see page 99).

**SOLUTION** Ranked from smallest to largest, the three-year annualized returns for the seven small-cap growth funds with low risk are

19.0 20.8 22.3 22.4 24.9 26.0 29.9

Therefore, using Equation (3.7), the range =  $29.9 - 19.0 = 10.9$ .

The largest difference between any two returns is 10.9.

The range measures the *total spread* in the set of data. Although the range is a simple measure of the total variation in the data, it does not take into account *how* the data are distributed between the smallest and largest values. In other words, the range does not indicate whether the values are evenly distributed throughout the data set, clustered near the middle, or clustered near one or both extremes. Thus, using the range as a measure of variation when at least one value is an extreme value is misleading.

## The Interquartile Range

The **interquartile range** (also called **midsread**) is the difference between the *third* and *first* quartiles in a set of data.

### INTERQUARTILE RANGE

The interquartile range is the difference between the third quartile and the first quartile.

$$\text{Interquartile range} = Q_3 - Q_1 \quad (3.8)$$

The interquartile range measures the spread in the middle 50% of the data. Therefore, it is not influenced by extreme values. To determine the interquartile range of the times to get ready

29 31 35 39 39 40 43 44 44 52

you use Equation (3.8) and the earlier results on page 102,  $Q_1 = 35$  and  $Q_3 = 44$ :

$$\text{Interquartile range} = 44 - 35 = 9 \text{ minutes}$$

Therefore, the interquartile range in the time to get ready is 9 minutes. The interval 35 to 44 is often referred to as the *middle ffy*.

### EXAMPLE 3.8

#### COMPUTING THE INTERQUARTILE RANGE FOR THE THREE-YEAR ANNUALIZED RETURNS FOR SMALL-CAP GROWTH MUTUAL FUNDS WITH LOW RISK

The 838 mutual funds ([Mutual Funds.xls](#)) that are part of the Using Statistics scenario (see page 96) are classified according to the category (small cap, mid cap, and large cap), the type (growth or value), and the risk level of the mutual funds (low, average, and high). Compute the interquartile range of the three-year annualized returns for the small-cap growth funds with low risk (see page 99).

**SOLUTION** Ranked from smallest to largest, the three-year annualized returns for the seven small-cap growth funds with low risk are

19.0 20.8 22.3 22.4 24.9 26.0 29.9

Using Equation (3.8) and the earlier results on page 103,  $Q_1 = 20.8$  and  $Q_3 = 26.0$ :

$$\text{Interquartile range} = 26.0 - 20.8 = 5.2$$

Therefore, the interquartile range in the three-year annualized return is 5.2.

Because the interquartile range does not consider any value smaller than  $Q_1$  or larger than  $Q_3$ , it cannot be affected by extreme values. Summary measures such as the median,  $Q_1$ ,  $Q_3$ , and the interquartile range, which cannot be influenced by extreme values, are called **resistant measures**.

## The Variance and the Standard Deviation

Although the range and the interquartile range are measures of variation, they do not take into consideration *how* the values distribute or cluster between the extremes. Two commonly used measures of variation that take into account how all the values in the data are distributed are the **variance** and the **standard deviation**. These statistics measure the “average” scatter around the mean—how larger values fluctuate above it and how smaller values distribute below it.

A simple measure of variation around the mean might take the difference between each value and the mean and then sum these differences. However, if you did that, you would find that because the mean is the balance point in a set of data, for *every* set of data, these differences would sum to zero. One measure of variation that differs from data set to data set *squares* the difference between each value and the mean and then sums these squared differences. In statistics, this quantity is called a **sum of squares** (or **SS**). This sum is then divided by the number of values minus 1 (for sample data) to get the sample variance ( $S^2$ ). The square root of the sample variance is the sample standard deviation ( $S$ ).

Because the sum of squares is a sum of squared differences that by the rules of arithmetic will always be nonnegative, *neither the variance nor the standard deviation can ever be negative*. For virtually all sets of data, the variance and standard deviation will be a positive value, although both of these statistics will be zero if there is no variation at all in a set of data and each value in the sample is the same.

For a sample containing  $n$  values,  $X_1, X_2, X_3, \dots, X_n$ , the sample variance (given by the symbol  $S^2$ ) is

$$S^2 = \frac{(X_1 - \bar{X})^2 + (X_2 - \bar{X})^2 + \dots + (X_n - \bar{X})^2}{n - 1}$$

Equation (3.9) expresses the sample variance using summation notation, and Equation (3.10) expresses the sample standard deviation.

### SAMPLE VARIANCE

The **sample variance** is the sum of the squared differences around the mean divided by the sample size minus one.

$$S^2 = \frac{\sum_{i=1}^n (X_i - \bar{X})^2}{n - 1} \quad (3.9)$$

where

$\bar{X}$  = mean

$n$  = sample size

$X_i$  =  $i$ th value of the variable  $X$

$\sum_{i=1}^n (X_i - \bar{X})^2$  = summation of all the squared differences between the  $X_i$  values and  $\bar{X}$

### SAMPLE STANDARD DEVIATION

The **sample standard deviation** is the square root of the sum of the squared differences around the mean divided by the sample size minus one.

$$S = \sqrt{S^2} = \sqrt{\frac{\sum_{i=1}^n (X_i - \bar{X})^2}{n - 1}} \quad (3.10)$$

If the denominator were  $n$  instead of  $n - 1$ , Equation (3.9) [and the inner term in Equation (3.10)] would calculate the average of the squared differences around the mean. However,  $n - 1$  is used because of certain desirable mathematical properties possessed by the statistic  $S^2$  that

make it appropriate for statistical inference (which is discussed in Chapter 7). As the sample size increases, the difference between dividing by  $n$  and by  $n - 1$  becomes smaller and smaller.

You will most likely use the sample standard deviation as your measure of variation [defined in Equation (3.10)]. Unlike the sample variance, which is a squared quantity, the standard deviation is always a number that is in the same units as the original sample data. The standard deviation helps you to know how a set of data clusters or distributes around its mean. For almost all sets of data, the majority of the observed values lie within an interval of plus and minus one standard deviation above and below the mean. Therefore, knowledge of the mean and the standard deviation usually helps define where at least the majority of the data values are clustering.

To hand-calculate the sample variance,  $S^2$ , and the sample standard deviation,  $S$ :

**Step 1.** Compute the difference between each value and the mean.

**Step 2.** Square each difference.

**Step 3.** Add the squared differences.

**Step 4.** Divide this total by  $n - 1$  to get the sample variance.

**Step 5.** Take the square root of the sample variance to get the sample standard deviation.

Table 3.1 shows the first four steps for calculating the variance and standard deviation for the getting-ready-times data with a mean ( $\bar{X}$ ) equal to 39.6. (See page 98 for the calculation of the mean.) The second column of Table 3.1 shows step 1. The third column of Table 3.1 shows step 2. The sum of the squared differences (step 3) is shown at the bottom of Table 3.1. This total is then divided by  $10 - 1 = 9$  to compute the variance (step 4).

**TABLE 3.1**

Computing the Variance of the Getting-Ready Times

$\bar{X} = 39.6$		
Time ( $X$ )	Step 1: $(X_i - \bar{X})$	Step 2: $(X_i - \bar{X})^2$
39	-0.60	0.36
29	-10.60	112.36
43	3.40	11.56
52	12.40	153.76
39	-0.60	0.36
44	4.40	19.36
40	0.40	0.16
31	-8.60	73.96
44	4.40	19.36
35	-4.60	21.16
	<b>Step 3: Sum:</b>	<b>Step 4: Divide by <math>(n - 1)</math>:</b>
	412.40	45.82

You can also calculate the variance by substituting values for the terms in Equation (3.9):

$$\begin{aligned}
 S^2 &= \frac{\sum_{i=1}^n (X_i - \bar{X})^2}{n - 1} \\
 &= \frac{(39 - 39.6)^2 + (29 - 39.6)^2 + \cdots + (35 - 39.6)^2}{10 - 1} \\
 &= \frac{412.4}{9} \\
 &= 45.82
 \end{aligned}$$

Because the variance is in squared units (in squared minutes, for these data), to compute the standard deviation, you take the square root of the variance. Using Equation (3.10) on page 107, the sample standard deviation,  $S$ , is

$$S = \sqrt{S^2} = \sqrt{\frac{\sum_{i=1}^n (X_i - \bar{X})^2}{n-1}} = \sqrt{45.82} = 6.77$$

This indicates that the getting-ready times in this sample are clustering within 6.77 minutes around the mean of 39.6 minutes (i.e., clustering between  $\bar{X} - 1S = 32.83$  and  $\bar{X} + 1S = 46.37$ ). In fact, 7 out of 10 getting-ready times lie within this interval.

Using the second column of Table 3.1, you can also calculate the sum of the differences between each value and the mean to be zero. For any set of data, this sum will always be zero:

$$\sum_{i=1}^n (X_i - \bar{X}) = 0 \text{ for all sets of data}$$

This property is one of the reasons that the mean is used as the most common measure of central tendency.

### EXAMPLE 3.9

#### COMPUTING THE VARIANCE AND STANDARD DEVIATION OF THE THREE-YEAR ANNUALIZED RETURNS FOR SMALL-CAP GROWTH MUTUAL FUNDS WITH LOW RISK

The 838 mutual funds ([Mutual Funds.xls](#)) that are part of the Using Statistics scenario (see page 96) are classified according to the category (small cap, mid cap, and large cap), the type (growth or value), and the risk level of the mutual funds (low, average, and high). Compute the variance and standard deviation of the three-year annualized returns for the small-cap growth funds with low risk (see page 99).

**SOLUTION** Table 3.2 illustrates the computation of the variance and standard deviation for the three-year annualized returns for the small-cap growth funds with low risk.

**TABLE 3.2**

Computing the Variance of the Three-Year Annualized Returns for the Small-Cap Growth Mutual Funds with Low Risk

$\bar{X} = 23.6143$		
Three-Year Annualized Return	Step 1: $(X_i - \bar{X})$	Step 2: $(X_i - \bar{X})^2$
20.8	-2.8143	7.9202
26.0	2.3857	5.6916
24.9	1.2857	1.6531
29.9	6.2857	39.5102
22.3	-1.3143	1.7273
19.0	-4.6143	21.2916
22.4	-1.2143	1.4745
	Step 3: Sum:	Step 4: Divide by $(n - 1)$ :
	79.2686	13.2114

Using Equation (3.9) on page 107:

$$\begin{aligned}
 S^2 &= \frac{\sum_{i=1}^n (X_i - \bar{X})^2}{n-1} \\
 &= \frac{(20.8 - 23.6143)^2 + (26.0 - 23.6143)^2 + \cdots + (22.4 - 23.6143)^2}{7-1} \\
 &= \frac{79.2686}{6} \\
 &= 13.2114
 \end{aligned}$$

Using Equation (3.10) on page 107, the sample standard deviation,  $S$ , is

$$S = \sqrt{S^2} = \sqrt{\frac{\sum_{i=1}^n (X_i - \bar{X})^2}{n-1}} = \sqrt{13.2114} = 3.635$$

The standard deviation of 3.635 indicates that the returns are clustering within 3.635 around the mean of 23.61 (i.e., clustering between  $\bar{X} - 1S = 19.975$  and  $\bar{X} + 1S = 27.245$ ). In fact, 71.4% (5 out of 7) of the three-year annualized returns lie within this interval.

The following summarizes the characteristics of the range, interquartile range, variance, and standard deviation:

- The more the data are spread out or dispersed, the larger the range, interquartile range, variance, and standard deviation.
- The more the data are concentrated or homogeneous, the smaller the range, interquartile range, variance, and standard deviation.
- If the values are all the same (so that there is no variation in the data), the range, interquartile range, variance, and standard deviation will all equal zero.
- None of the measures of variation (the range, interquartile range, standard deviation, and variance) can *ever* be negative.

## The Coefficient of Variation

Unlike the previous measures of variation presented, the **coefficient of variation** is a *relative measure* of variation that is always expressed as a percentage rather than in terms of the units of the particular data. The coefficient of variation, denoted by the symbol  $CV$ , measures the scatter in the data relative to the mean.

### COEFFICIENT OF VARIATION

The coefficient of variation is equal to the standard deviation divided by the mean, multiplied by 100%.

$$CV = \left( \frac{S}{\bar{X}} \right) 100\% \quad (3.11)$$

where

$S$  = sample standard deviation

$\bar{X}$  = sample mean

For the sample of 10 getting-ready times, because  $\bar{X} = 39.6$  and  $S = 6.77$ , the coefficient of variation is

$$CV = \left( \frac{S}{\bar{X}} \right) 100\% = \left( \frac{6.77}{39.6} \right) 100\% = 17.10\%$$

For the getting-ready times, the standard deviation is 17.1% of the size of the mean.

The coefficient of variation is very useful when comparing two or more sets of data that are measured in different units, as Example 3.10 illustrates.

### EXAMPLE 3.10

#### COMPARING TWO COEFFICIENTS OF VARIATION WHEN TWO VARIABLES HAVE DIFFERENT UNITS OF MEASUREMENT

The operations manager of a package delivery service is deciding whether to purchase a new fleet of trucks. When packages are stored in the trucks in preparation for delivery, you need to consider two major constraints—the weight (in pounds) and the volume (in cubic feet) for each item.

The operations manager samples 200 packages and finds that the mean weight is 26.0 pounds, with a standard deviation of 3.9 pounds, and the mean volume is 8.8 cubic feet, with a standard deviation of 2.2 cubic feet. How can the operations manager compare the variation of the weight and the volume?

**SOLUTION** Because the measurement units differ for the weight and volume constraints, the operations manager should compare the relative variability in the two types of measurements.

For weight, the coefficient of variation is

$$CV_W = \left( \frac{3.9}{26.0} \right) 100\% = 15.0\%$$

For volume, the coefficient of variation is

$$CV_V = \left( \frac{2.2}{8.8} \right) 100\% = 25.0\%$$

Thus, relative to the mean, the package volume is much more variable than the package weight.

### Z Scores

An **extreme value** or **outlier** is a value located far away from the mean. *Z* scores are useful in identifying outliers. The larger the *Z* score, the greater the distance from the value to the mean. The **Z score** is the difference between the value and the mean, divided by the standard deviation.

#### Z SCORES

$$Z = \frac{X - \bar{X}}{S} \quad (3.12)$$

For the time-to-get-ready data, the mean is 39.6 minutes, and the standard deviation is 6.77 minutes. The time to get ready on the first day is 39.0 minutes. You compute the *Z* score for Day 1 by using Equation (3.12):

$$\begin{aligned} Z &= \frac{X - \bar{X}}{S} \\ &= \frac{39.0 - 39.6}{6.77} \\ &= -0.09 \end{aligned}$$

Table 3.3 shows the  $Z$  scores for all 10 days. The largest  $Z$  score is 1.83 for Day 4, on which the time to get ready was 52 minutes. The lowest  $Z$  score was  $-1.57$  for Day 2, on which the time to get ready was 29 minutes. As a general rule, a  $Z$  score is considered an outlier if it is less than  $-3.0$  or greater than  $+3.0$ . None of the times met that criterion to be considered outliers.

**TABLE 3.3**

$Z$  Scores for the  
10 Getting-Ready  
Times

	Time ( $X$ )	$Z$ Score
	39	$-0.09$
	29	$-1.57$
	43	0.50
	52	1.83
	39	$-0.09$
	44	0.65
	40	0.06
	31	$-1.27$
	44	0.65
	35	$-0.68$
<b>Mean</b>	39.6	
<b>Standard deviation</b>	6.77	

**EXAMPLE 3.11**

### COMPUTING THE $Z$ SCORES OF THE THREE-YEAR ANNUALIZED RETURNS FOR SMALL-CAP GROWTH MUTUAL FUNDS WITH LOW RISK

The 838 mutual funds ([Mutual Funds.xls](#)) that are part of the Using Statistics scenario (see page 96) are classified according to the category (small cap, mid cap, and large cap), the type (growth or value), and the risk level of the mutual funds (low, average, and high). Compute the  $Z$  scores of the three-year annualized returns for the small-cap growth funds with low risk (see page 99).

**SOLUTION** Table 3.4 illustrates the  $Z$  scores of the three-year annualized returns for the small-cap growth funds with low risk. The largest  $Z$  score is 1.73, for an annualized return of 29.9. The lowest  $Z$  score is  $-1.27$ , for an annualized return of 19.0. There are no apparent outliers in these data because none of the  $Z$  scores are less than  $-3$  or greater than  $+3$ .

**TABLE 3.4**

$Z$  Scores of the  
Three-Year Annualized  
Returns for the Small-  
Cap Growth Mutual  
Funds with Low Risk

	Three-Year Return	$Z$ Scores
	20.8	$-0.77$
	26.0	0.66
	24.9	0.35
	29.9	1.73
	22.3	$-0.36$
	19.0	$-1.27$
	22.4	$-0.33$
<b>Mean</b>	23.61	
<b>Standard Deviation</b>	3.63	

## Shape

Shape is the pattern of the distribution of data values throughout the entire range of all the values. A distribution is either symmetrical or skewed. In a **symmetrical** distribution, the values below the mean are distributed exactly as the values above the mean. In this case, the low and high values balance each other out. In a **skewed** distribution, the values are not symmetrical around the mean. This skewness results in an imbalance of low values or high values.



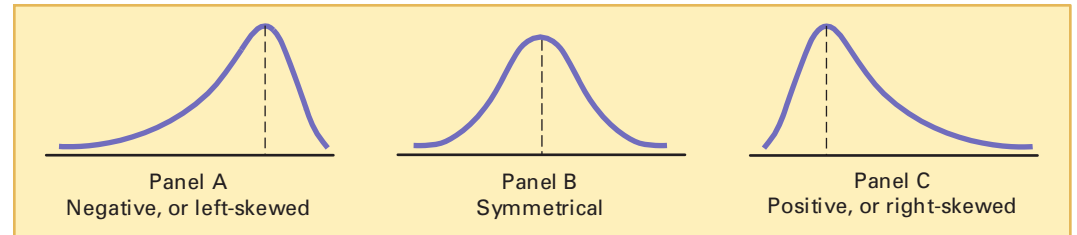
Shape influences the relationship of the mean to the median in the following ways:

- Mean < median: negative, or left-skewed
- Mean = median: symmetric, or zero skewness
- Mean > median: positive, or right-skewed

Figure 3.1 depicts three data sets, each with a different shape.

**FIGURE 3.1**

A comparison of three data sets differing in shape



The data in Panel A are negative, or **left-skewed**. In this panel, most of the values are in the upper portion of the distribution. A long tail and distortion to the left is caused by some extremely small values. These extremely small values pull the mean downward so that the mean is less than the median.

The data in Panel B are symmetrical. Each half of the curve is a mirror image of the other half of the curve. The low and high values on the scale balance, and the mean equals the median.

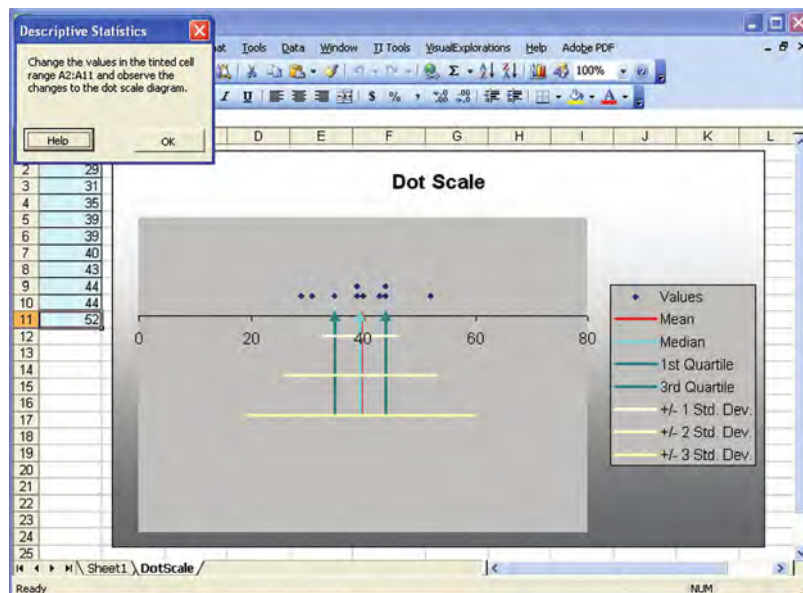
The data in Panel C are positive, or **right-skewed**. In this panel, most of the values are in the lower portion of the distribution. A long tail on the right is caused by some extremely large values. These extremely large values pull the mean upward so that the mean is greater than the median.

## VISUAL EXPLORATIONS Exploring Descriptive Statistics

You can use the Visual Explorations Descriptive Statistics procedure to see the effect of changing data values on measures of central tendency, variation, and shape. Open the [visual explorations.xls](#) add-in workbook (see Appendix D) and select **VisualExplorations → Descriptive Statistics** (Excel 97–2003) or **Add-ins → Visual Explorations → Descriptive Statistics** (Excel 2007) from the Microsoft Excel menu bar. Read the instructions in the pop-up box (see illustration below) and click **OK** to examine a dot-scale

diagram for the sample of 10 getting-ready times used throughout this chapter.

Experiment by entering an extreme value such as 10 minutes into one of the tinted cells of column A. Which measures are affected by this change? Which ones are not? You can flip between the “before” and “after” diagrams by repeatedly pressing **Ctrl + Z** (undo) followed by **Ctrl + Y** (redo) to help see the changes the extreme value caused in the diagram.



## Microsoft Excel Descriptive Statistics Results

The Descriptive Statistics procedure of the ToolPak add-in (see Section E3.1) computes the mean, median, mode, standard deviation, variance, range, minimum, maximum, and count (sample size) and displays these statistics on a new worksheet. In addition, the procedure calculates and displays the standard error, the kurtosis, and skewness, three statistics not discussed previously in this section. The *standard error*, discussed in Chapter 7, is the standard deviation divided by the square root of the sample size. *Skewness* measures the lack of symmetry in the data. A skewness value of zero indicates a symmetrical distribution. A positive value indicates right skewness while a negative value indicates left skewness. *Kurtosis* measures the relative concentration of values in the center of the distribution, as compared with the tails. A kurtosis value of zero indicates a bell-shaped distribution. A negative value indicates a distribution that is flatter than a bell-shaped distribution. A positive value indicates a distribution with a sharper peak than a bell-shaped distribution.

Figure 3.2 shows the results of using the Descriptive Statistics procedure to calculate separate results for low-risk, average-risk, and high-risk mutual funds. (The procedure was used three times, and the results that appeared on three separate worksheets were consolidated on one sheet.)

**FIGURE 3.2**

Microsoft Excel descriptive statistics of the three-year annualized returns based on risk level



See Section E3.1 to create this.

	A	B	C	D
1	<b>Descriptive Statistics for 3-Year Return</b>			
2		<b>Low</b>	<b>Average</b>	<b>High Risk</b>
3	<b>Mean</b>	18.4101	16.9661	17.9549
4	<b>Standard Error</b>	0.3158	0.3161	0.2621
5	<b>Median</b>	17.5	16.1	17.6
6	<b>Mode</b>	14.1	15.5	16.4
7	<b>Standard Deviation</b>	4.9636	4.9478	4.8762
8	<b>Sample Variance</b>	24.6376	24.4804	23.7774
9	<b>Kurtosis</b>	0.2838	1.9893	1.1686
10	<b>Skewness</b>	0.5069	0.8118	0.7069
11	<b>Range</b>	28	35.6	33.2
12	<b>Minimum</b>	6.7	6.7	7.3
13	<b>Maximum</b>	34.7	42.3	40.5
14	<b>Sum</b>	4547.3	4156.7	6212.4
15	<b>Count</b>	247	245	346
16	<b>Largest(1)</b>	34.7	42.3	40.5
17	<b>Smallest(1)</b>	6.7	6.7	7.3

In examining the results, there appear to be slight differences in the three-year annualized return for the three risk levels. Low-risk and high-risk funds had a slightly higher mean and median than did average-risk funds. There was very little difference in the standard deviations of the three groups. Each of the risk levels showed some evidence of positive skewness.

## PROBLEMS FOR SECTIONS 3.1 AND 3.2

### Learning the Basics

**PH Grade ASSIST** **3.1** The following is a set of data from a sample of  $n = 5$ :

7 4 9 8 2

- Compute the mean, median, and mode.
- Compute the range, interquartile range, variance, standard deviation, and coefficient of variation.
- Compute the  $Z$  scores. Are there any outliers?
- Describe the shape of the data set.

**PH Grade ASSIST** **3.2** The following is a set of data from a sample of  $n = 6$ :

7 4 9 7 3 12

- Compute the mean, median, and mode.
- Compute the range, interquartile range, variance, standard deviation, and coefficient of variation.
- Compute the  $Z$  scores. Are there any outliers?
- Describe the shape of the data set.

**PH Grade ASSIST** **3.3** The following set of data is from a sample of  $n = 7$ :

12 7 4 9 0 7 3

- Compute the mean, median, and mode.
- Compute the range, interquartile range, variance, standard deviation, and coefficient of variation.
- Compute the  $Z$  scores. Are there any outliers?
- Describe the shape of the data set.

**PH Grade ASSIST** **3.4** The following is a set of data from a sample of  $n = 5$ :

7 -5 -8 7 9

- Compute the mean, median, and mode.
- Compute the range, interquartile range, variance, standard deviation, and coefficient of variation.
- Compute the  $Z$  scores. Are there any outliers?
- Describe the shape of the data set.

**PH Grade ASSIST** **3.5** Suppose the rate of return for a particular stock during the past two years was 10% and 30%. Compute the geometric mean rate of return. (Note: A rate of return of 10% is recorded as 0.10, and a rate of return of 30% is recorded as 0.30.)

### Applying the Concepts

**PH Grade ASSIST** **3.6** The operations manager of a plant that manufactures tires wants to compare the actual inner diameters of two grades of tires, each of which is expected to be 575 millimeters. A sample of five

tires of each grade was selected, and the results representing the inner diameters of the tires, ranked from smallest to largest, are as follows:

Grade X					Grade Y				
568	570	575	578	584	573	574	575	577	578

- For each of the two grades of tires, compute the mean, median, and standard deviation.
- Which grade of tire is providing better quality? Explain.
- What would be the effect on your answers in (a) and (b) if the last value for grade Y were 588 instead of 578? Explain.

**3.7** The data in the file [movieprices.xls](#) contain the price for two tickets with online service charges, large popcorn, and two medium soft drinks at a sample of six theatre chains:

\$36.15 \$31.00 \$35.05 \$40.25 \$33.75 \$43.00

Source: Extracted from K. Kelly, "The Multiplex Under Siege," The Wall Street Journal, December 24–25, 2005, pp. P1, P5.

- Compute the mean, median, first quartile, and third quartile.
- Compute the variance, standard deviation, range, interquartile range, and coefficient of variation.
- Are the data skewed? If so, how?
- Based on the results of (a) through (c), what conclusions can you reach concerning the cost of going to the movies?

**3.8** A total of 92,000 new single-family homes were sold in the United States during February 2006. The median price of the homes was \$230,400, a decrease of 2.9% from February 2005 (U.S. Census Bureau, [www.census.gov](#)). Why do you think the Census Bureau refers to the median price instead of the mean price?

**3.9** The data in the file [bankcost1.xls](#) contain the bounced check fees, in dollars, for a sample of 23 banks for direct-deposit customers who maintain a \$100 balance:

26 28 20 20 21 22 25 25 18 25 15 20  
18 20 25 25 22 30 30 30 15 20 29

Source: Extracted from "The New Face of Banking," June 2000. Copyright © 2000 by Consumers Union of U.S., Inc., Yonkers, NY 10703-1057.

- Compute the mean, median, first quartile, and third quartile.
- Compute the variance, standard deviation, range, interquartile range, coefficient of variation, and  $Z$  scores.
- Are the data skewed? If so, how?
- Based on the results of (a) through (c), what conclusions can you reach concerning the bounced check fees?



**3.10** The data in the file `coffeedrink.xls` represent the calories and fat (in grams) of 16-ounce iced coffee drinks at Dunkin' Donuts and Starbucks:

Product	Calories	Fat
Dunkin' Donuts Iced Mocha Swirl latte (whole milk)	240	8.0
Starbucks Coffee Frappuccino blended coffee	260	3.5
Dunkin' Donuts Coffee Coolatta (cream)	350	22.0
Starbucks Iced Coffee Mocha Espresso (whole milk and whipped cream)	350	20.0
Starbucks Mocha Frappuccino blended coffee (whipped cream)	420	16.0
Starbucks Chocolate Brownie Frappuccino blended coffee (whipped cream)	510	22.0
Starbucks Chocolate Frappuccino Blended Crème (whipped cream)	530	19.0

Source: Extracted from "Coffee as Candy at Dunkin' Donuts and Starbucks," Consumer Reports, June 2004, p. 9.

- For each variable (calories and fat), compute the mean, median, first quartile, and third quartile.
- For each variable (calories and fat), compute the variance, standard deviation, range, interquartile range, coefficient of variation, and Z scores. Are there any outliers? Explain.
- Are the data skewed? If so, how?
- Based on the results of (a) through (c), what conclusions can you reach concerning the calories and fat in iced coffee drinks at Dunkin' Donuts and Starbucks?

**3.11** The data in the file `chicken.xls` contain the total fat, in grams per serving, for a sample of 20 chicken sandwiches from fast-food chains. The data is as follows:

7 8 4 5 16 20 20 24 19 30  
23 30 25 19 29 29 30 30 40 56

Source: Extracted from "Fast Food: Adding Health to the Menu," Consumer Reports, September 2004, pp. 28–31.

- Compute the mean, median, first quartile, and third quartile.
- Compute the variance, standard deviation, range, interquartile range, coefficient of variation, and Z scores. Are there any outliers? Explain.
- Are the data skewed? If so, how?
- Based on the results of (a) through (c), what conclusions can you reach concerning the total fat of chicken sandwiches?

**3.12** The data in the file `batterylife.xls` represents the battery life (in shots) for three pixel digital cameras:

300 180 85 170 380 460  
260 35 380 120 110 240

Source: Extracted from "Cameras: More Features in the Mix," Consumer Reports, July 2005, pp. 14–18.

- Compute the mean, median, first quartile, and third quartile.
- Compute the variance, standard deviation, range, interquartile range, coefficient of variation, and Z scores. Are there any outliers? Explain.
- Are the data skewed? If so, how?
- Based on the results of (a) through (c), what conclusions can you reach concerning the battery life (in shots) for three pixel digital cameras?

**3.13** Is there a difference in the variation of the yields of different types of investments between banks? The data in the file `bankyield.xls` represents the nationwide highest yields for money market accounts and one-year CDs as of January 24, 2006:

Money Market Accounts					One-Year CD				
4.55	4.50	4.40	4.38	4.38	4.94	4.90	4.85	4.85	4.85

Source: Extracted from Bankrate.com, January 24, 2006.

- For money market accounts and one-year CDs, separately compute the variance, standard deviation, range, interquartile range, and coefficient of variation.
- Based on the results of (a), do money market accounts or one-year CDs have more variation in the highest yields offered? Explain.

**3.14** The data in the file `themeparks.xls` contain the starting admission price (in \$) for one-day tickets to 10 theme parks in the United States:

58 63 41 42 29 50 62 43 40 40

Source: Extracted from C. Jackson and E. Gamerman, "Rethinking the Thrill Factor," The Wall Street Journal, April 15–16, 2006, pp. P1, P4.

- Compute the mean, median, first quartile, and third quartile.
- Compute the range, variance, and standard deviation.
- Based on the results of (a) and (b), what conclusions can you reach concerning the starting admission price for one-day tickets.
- Suppose that the first value was 98 instead of 58. Repeat (a) through (c), using this value. Comment on the difference in the results.

**3.15** A bank branch located in a commercial district of a city has developed an improved process for serving customers during the noon-to-1:00 p.m. lunch period. The waiting time, in minutes (defined as the time the customer enters the line to when he or she reaches the teller window), of a sample of 15 customers during this hour is recorded over a period of one week. The results are contained in the data file `bank1.xls` and are listed below:

4.21 5.55 3.02 5.13 4.77 2.34 3.54 3.20  
4.50 6.10 0.38 5.12 6.46 6.19 3.79

- a. Compute the mean, median, first quartile, and third quartile.
- b. Compute the variance, standard deviation, range, interquartile range, coefficient of variation, and  $Z$  scores. Are there any outliers? Explain.
- c. Are the data skewed? If so, how?
- d. As a customer walks into the branch office during the lunch hour, she asks the branch manager how long she can expect to wait. The branch manager replies, "Almost certainly less than five minutes." On the basis of the results of (a) through (c), evaluate the accuracy of this statement.

**3.16** Suppose that another branch, located in a residential area, is also concerned with the noon-to-1 p.m. lunch hour. The waiting time, in minutes (defined as the time the customer enters the line to when he or she reaches the teller window), of a sample of 15 customers during this hour is recorded over a period of one week. The results are contained in the data file [bank2.xls](#) and are listed below:

9.66 5.90 8.02 5.79 8.73 3.82 8.01 8.35  
 10.49 6.68 5.64 4.08 6.17 9.91 5.47

- a. Compute the mean, median, first quartile, and third quartile.
- b. Compute the variance, standard deviation, range, interquartile range, and coefficient of variation. Are there any outliers? Explain.
- c. Are the data skewed? If so, how?
- d. As a customer walks into the branch office during the lunch hour, he asks the branch manager how long he can expect to wait. The branch manager replies, "Almost certainly less than five minutes." On the basis of the results of (a) through (c), evaluate the accuracy of this statement.

**3.17** General Electric (GE) is one of the world's largest companies; it develops, manufactures, and markets a wide range of products, including medical diagnostic imaging devices, jet engines, lighting products, and chemicals. Through its affiliate, NBC Universal, GE produces and delivers network television and motion pictures. In 2004, GE's stock price rose 20.6%, but in 2005, the price dropped 1.4% (Source: Extracted from [finance.yahoo.com](#), April 17, 2006).

- a. Compute the geometric mean rate of increase for the two-year period 2004–2005. (Hint: Denote an increase of 20.6% as  $R_1 = 0.206$ .)
- b. If you purchased \$1,000 of GE stock at the start of 2004, what was its value at the end of 2005?
- c. Compare the result of (b) to that of Problem 3.18 (b).

**3.18** TASER International, Inc., develops, manufactures, and sells nonlethal self-defense devices known as tasers. Marketing primarily to law enforcement, correc-

tions institutions, and the military, TASER's popularity has enjoyed a roller-coaster ride. The stock price in 2004 increased 361.4%, but in 2005, it decreased 78.0%

(Source: Extracted from [finance.yahoo.com](#), April 17, 2006).

- a. Compute the geometric mean rate of increase for the two-year period 2004–2005. (Hint: Denote an increase of 361.4% as  $R_1 = 3.614$ .)
- b. If you purchased \$1,000 of TASER stock at the start of 2004, what was its value at the end of 2005?
- c. Compare the result of (b) to that of Problem 3.17 (b).

**3.19** In 2002, all the major stock market indexes decreased dramatically as the attacks on 9/11 drove stock prices spiraling downward. Stocks soon rebounded, but what type of mean return did investors experience over the four-year period from 2002 to 2005? The data in the following table (contained in the data file [indexes.xls](#)) represent the total rate of return (in percentage) for the Dow Jones Industrial Average (DJIA), the Standard & Poor's 500 (S&P 500), and the technology-heavy NASDAQ Composite (Nasdaq).

Year	DJIA	S&P 500	NASDAQ
2005	-0.6	2.9	1.4
2004	3.4	9.1	8.6
2003	30.0	26.4	50.0
2002	-16.8	-24.2	-31.5

Source: Extracted from [finance.yahoo.com](#), April 14, 2006.

- a. Calculate the geometric mean rate of return for the DJIA, S&P 500, and Nasdaq.
- b. What conclusions can you reach concerning the geometric rates of return of the three market indexes?
- c. Compare the results of (b) to those of Problem 3.20 (b).

**3.20** In 2002–2005 precious metals changed rapidly in value. The data in the following table (contained in the data file [metals.xls](#)) represent the total rate of return (in percentage) for platinum, gold, and silver:

Year	Platinum	Gold	Silver
2005	12.3	17.8	29.5
2004	5.7	4.6	14.2
2003	36.0	19.9	27.8
2002	24.6	25.6	3.3

Source: Extracted from [www.kitco.com](#), April 14, 2006.

- a. Calculate the geometric mean rate of return for platinum, gold, and silver.
- b. What conclusions can you reach concerning the geometric rates of return of the three precious metals?
- c. Compare the results of (b) to those of Problem 3.19 (b).

### 3.3 NUMERICAL DESCRIPTIVE MEASURES FOR A POPULATION

Sections 3.1 and 3.2 present various *statistics* that described the properties of central tendency and variation for a *sample*. If your data set represents numerical measurements for an entire *population*, you need to calculate and interpret *parameters*, summary measures for a population. In this section, you will learn about three descriptive population parameters: the population mean, population variance, and population standard deviation.

To help illustrate these parameters, first review Table 3.5, which contains the one-year return for the five largest bond funds (in terms of total assets) as of January 31, 2006. (The data are contained in the file [largest bonds.xls](#).)

**TABLE 3.5**

One-Year Return for the Population Consisting of the Five Largest Bond Funds

Bond Fund	One-Year Return
Pimco:Total Rtn;Inst	2.74
Vanguard Tot Bd;Inv	1.62
American Funds Bond;A	2.25
Vanguard GNMA;Inv	2.88
Franklin CA TF Inc;A	3.66

Source: Extracted from The Wall Street Journal, February 6, 2006, p. R6.

#### The Population Mean

The **population mean** is represented by the symbol  $\mu$ , the Greek lowercase letter *mu*. Equation (3.13) defines the population mean.

##### POPULATION MEAN

The population mean is the sum of the values in the population divided by the population size  $N$ .

$$\mu = \frac{\sum_{i=1}^N X_i}{N} \quad (3.13)$$

where

$\mu$  = population mean

$X_i$  =  $i$ th value of the variable  $X$

$\sum_{i=1}^N X_i$  = summation of all  $X_i$  values in the population

To compute the mean one-year return for the population of bond funds given in Table 3.5, use Equation (3.13):

$$\mu = \frac{\sum_{i=1}^N X_i}{N} = \frac{2.74 + 1.62 + 2.25 + 2.88 + 3.66}{5} = \frac{13.15}{5} = 2.63$$

Thus, the mean percentage return for these bond funds is 2.63.

## The Population Variance and Standard Deviation

The **population variance** and the **population standard deviation** measure variation in a population. Like the related sample statistics, the population standard deviation is the square root of the population variance. The symbol  $\sigma^2$ , the Greek lowercase letter *sigma* squared, represents the population variance, and the symbol  $\sigma$ , the Greek lowercase letter *sigma*, represents the population standard deviation. Equations (3.14) and (3.15) define these parameters. The denominators for the right-side terms in these equations use  $N$  and not the  $(n - 1)$  term that is used in the equations for the sample variance and standard deviation [see Equations (3.9) and (3.10) on page 107].

### POPULATION VARIANCE

The population variance is the sum of the squared differences around the population mean divided by the population size  $N$ .

$$\sigma^2 = \frac{\sum_{i=1}^N (X_i - \mu)^2}{N} \quad (3.14)$$

where

$\mu$  = population mean

$X_i$  =  $i$ th value of the variable  $X$

$\sum_{i=1}^N (X_i - \mu)^2$  = summation of all the squared differences between the  $X_i$  values and  $\mu$

### POPULATION STANDARD DEVIATION

$$\sigma = \sqrt{\frac{\sum_{i=1}^N (X_i - \mu)^2}{N}} \quad (3.15)$$

To compute the population variance for the data of Table 3.5, you use Equation (3.14):

$$\begin{aligned} \sigma^2 &= \frac{\sum_{i=1}^N (X_i - \mu)^2}{N} \\ &= \frac{(2.74 - 2.63)^2 + (1.62 - 2.63)^2 + (2.25 - 2.63)^2 + (2.88 - 2.63)^2 + (3.66 - 2.63)^2}{5} \\ &= \frac{0.0121 + 1.0201 + 0.1444 + 0.0625 + 1.0609}{5} \\ &= \frac{2.30}{5} = 0.46 \end{aligned}$$

Thus, the variance of the one-year returns is 0.46 squared percentage return. The squared units make the variance hard to interpret. You should use the standard deviation that is expressed in the original units of the data (percentage return). From Equation (3.15),

$$\sigma = \sqrt{\sigma^2} = \sqrt{\frac{\sum_{i=1}^N (X_i - \mu)^2}{N}} = \sqrt{0.46} = 0.68$$

Therefore, the typical percentage return differs from the mean of 2.63 by approximately 0.68. This small amount of variation suggests that these large bond funds produce results that do not differ greatly.

## The Empirical Rule

In most data sets, a large portion of the values tend to cluster somewhat near the median. In right-skewed data sets, this clustering occurs to the left of the mean—that is, at a value less than the mean. In left-skewed data sets, the values tend to cluster to the right of the mean—that is, at a value greater than the mean. In symmetrical data sets, where the median and mean are the same, the values often tend to cluster around the median and mean, producing a bell-shaped distribution. You can use the **empirical rule** to examine the variability in bell-shaped distributions:

- Approximately 68% of the values are within a distance of  $\pm 1$  standard deviation from the mean.
- Approximately 95% of the values are within a distance of  $\pm 2$  standard deviations from the mean.
- Approximately 99.7% are within a distance of  $\pm 3$  standard deviations from the mean.

The empirical rule helps you measure how the values distribute above and below the mean and can help you identify outliers. The empirical rule implies that for bell-shaped distributions, only about 1 out of 20 values will be beyond two standard deviations from the mean in either direction. As a general rule, you can consider values not found in the interval  $\mu \pm 2\sigma$  as potential outliers. The rule also implies that only about 3 in 1,000 will be beyond three standard deviations from the mean. Therefore, values not found in the interval  $\mu \pm 3\sigma$  are almost always considered outliers. For heavily skewed data sets, or those not appearing bell shaped for any other reason, the Chebyshev rule discussed below should be applied instead of the empirical rule.

### EXAMPLE 3.12

#### USING THE EMPIRICAL RULE

A population of 12-ounce cans of cola is known to have a mean fill-weight of 12.06 ounces and a standard deviation of 0.02. The population is known to be bell shaped. Describe the distribution of fill-weights. Is it very likely that a can will contain less than 12 ounces of cola?

#### SOLUTION

$$\mu \pm \sigma = 12.06 \pm 0.02 = (12.04, 12.08)$$

$$\mu \pm 2\sigma = 12.06 \pm 2(0.02) = (12.02, 12.10)$$

$$\mu \pm 3\sigma = 12.06 \pm 3(0.02) = (12.00, 12.12)$$

Using the empirical rule, approximately 68% of the cans will contain between 12.04 and 12.08 ounces, approximately 95% will contain between 12.02 and 12.10 ounces, and approximately 99.7% will contain between 12.00 and 12.12 ounces. Therefore, it is highly unlikely that a can will contain less than 12 ounces.

## The Chebyshev Rule

The **Chebyshev rule** (reference 1) states that for any data set, regardless of shape, the percentage of values that are found within distances of  $k$  standard deviations from the mean must be at least

$$(1 - 1/k^2) \times 100\%$$



You can use this rule for any value of  $k$  greater than 1. Consider  $k = 2$ . The Chebyshev rule states that at least  $[1 - (1/2)^2] \times 100\% = 75\%$  of the values must be found within  $\pm 2$  standard deviations of the mean.

The Chebyshev rule is very general and applies to any type of distribution. The rule indicates *at least* what percentage of the values fall within a given distance from the mean. However, if the data set is approximately bell shaped, the empirical rule will more accurately reflect the greater concentration of data close to the mean. Table 3.6 compares the Chebyshev and empirical rules.

**TABLE 3.6**

How Data Vary Around the Mean

Interval	% of Values Found in Intervals Around the Mean	
	Chebyshev (any distribution)	Empirical Rule (bell-shaped distribution)
$(\mu - \sigma, \mu + \sigma)$	At least 0%	Approximately 68%
$(\mu - 2\sigma, \mu + 2\sigma)$	At least 75%	Approximately 95%
$(\mu - 3\sigma, \mu + 3\sigma)$	At least 88.89%	Approximately 99.7%

**EXAMPLE 3.13****USING THE CHEBYSHEV RULE**

As in Example 3.12, a population of 12-ounce cans of cola is known to have a mean fill-weight of 12.06 ounces and a standard deviation of 0.02. However, the shape of the population is unknown, and you cannot assume that it is bell shaped. Describe the distribution of fill-weights. Is it very likely that a can will contain less than 12 ounces of cola?

**SOLUTION**

$$\mu \pm \sigma = 12.06 \pm 0.02 = (12.04, 12.08)$$

$$\mu \pm 2\sigma = 12.06 \pm 2(0.02) = (12.02, 12.10)$$

$$\mu \pm 3\sigma = 12.06 \pm 3(0.02) = (12.00, 12.12)$$

Because the distribution may be skewed, you cannot use the empirical rule. Using the Chebyshev rule, you cannot say anything about the percentage of cans containing between 12.04 and 12.08 ounces. You can state that at least 75% of the cans will contain between 12.02 and 12.10 ounces and at least 88.89% will contain between 12.00 and 12.12 ounces. Therefore, between 0 and 11.11% of the cans will contain less than 12 ounces.

You can use these two rules for understanding how data are distributed around the mean when you have sample data. In each case, you use the value you calculated for  $\bar{X}$  in place of  $\mu$  and the value you calculated for  $S$  in place of  $\sigma$ . The results you compute using the sample statistics are *approximations* because you used sample statistics ( $\bar{X}$ ,  $S$ ) and not population parameters ( $\mu$ ,  $\sigma$ ).

**PROBLEMS FOR SECTION 3.3****Learning the Basics**

**PH Grade ASSIST** **3.21** The following is a set of data for a population with  $N = 10$ :

7 5 11 8 3 6 2 1 9 8

- Compute the population mean.
- Compute the population standard deviation.

**PH Grade ASSIST** **3.22** The following is a set of data for a population with  $N = 10$ :

7 5 6 6 6 4 8 6 9 3

- Compute the population mean.
- Compute the population standard deviation.

### Applying the Concepts

**3.23** The data in the file [tax.xls](#) represent the quarterly sales tax receipts (in thousands of dollars) submitted to the comptroller of the Village of Fair Lake for the period ending March 2006 by all 50 business establishments in that locale:

10.3	11.1	9.6	9.0	14.5
13.0	6.7	11.0	8.4	10.3
13.0	11.2	7.3	5.3	12.5
8.0	11.8	8.7	10.6	9.5
11.1	10.2	11.1	9.9	9.8
11.6	15.1	12.5	6.5	7.5
10.0	12.9	9.2	10.0	12.8
12.5	9.3	10.4	12.7	10.5
9.3	11.5	10.7	11.6	7.8
10.5	7.6	10.1	8.9	8.6

- Compute the mean, variance, and standard deviation for this population.
- What proportion of these businesses have quarterly sales tax receipts within  $\pm 1$ ,  $\pm 2$ , or  $\pm 3$  standard deviations of the mean?
- Compare and contrast your findings with what would be expected on the basis of the empirical rule. Are you surprised at the results in (b)?

**PH Grade ASSIST** **3.24** Consider a population of 1,024 mutual funds that primarily invested in large companies. You have determined that  $\mu$ , the mean one-year total percentage return achieved by all the funds, is 8.20 and that  $\sigma$ , the standard deviation, is 2.75. In addition, suppose you have determined that the range in the one-year total returns is from  $-2.0$  to  $17.1$  and that the quartiles are  $5.5$  ( $Q_1$ ) and  $10.5$  ( $Q_3$ ). According to the empirical rule, what percentage of these funds is expected to be

- within  $\pm 1$  standard deviation of the mean?
- within  $\pm 2$  standard deviations of the mean?
- According to the Chebyshev rule, what percentage of these funds are expected to be within  $\pm 1$ ,  $\pm 2$ , or  $\pm 3$  standard deviations of the mean?
- According to the Chebyshev rule, at least 93.75% of these funds are expected to have one-year total returns between what two amounts?

**3.25** The data in the file [stockassets.xls](#) represent the assets, in billions of dollars, of the five largest stock funds.

Fund	Assets
American Funds Gro; A	73.6
Vanguard 500 Index; Inv	69.4
American Funds ICA; A	67.0
American Funds Wsh; A	62.4
Fidelity Contrafund	60.1

- Compute the mean for this population of the five largest stock funds. Interpret this parameter.
- Compute the variance and standard deviation for this population. Interpret these parameters.
- Is there a lot of variability in the assets of these stock funds? Explain.

**3.26** The data in the file [energy.xls](#) contains the per capita energy consumption, in kilowatt hours, for each of the 50 states and the District of Columbia during a recent year.

- Compute the mean, variance, and standard deviation for the population.
- What proportion of these states has average per capita energy consumption within  $\pm 1$  standard deviation of the mean, within  $\pm 2$  standard deviations of the mean, and within  $\pm 3$  standard deviations of the mean?
- Compare and contrast your findings versus what would be expected based on the empirical rule. Are you surprised at the results in (b)?
- Repeat (a) through (c) with the District of Columbia removed. How have the results changed?

**3.27** Thirty companies comprise the DJIA. Just how big are these companies? One common method to measure the size of a company is to use its market capitalization, which is computed by taking the number of stock shares multiplied by the price of a share of stock. On April 4, 2006, the market capitalization of these companies ranged from Hewlett-Packard's \$3.55 billion to Exxon-Mobil's \$376.64 billion. The entire population of market capitalization values is recorded in the file [dowmc.xls](#).

Source: Extracted from [money.cnn.com](#), April 4, 2006.

- Calculate the mean and standard deviation of the market capitalization for this population of 30 companies.
- Interpret the parameters calculated in (a).

## 3.4 EXPLORATORY DATA ANALYSIS

Sections 3.1–3.3 discuss measures of central tendency, variation, and shape. Another way of describing numerical data is through exploratory data analysis that includes the five-number summary and the box-and-whisker plot (references 3 and 4).

## The Five-Number Summary

A **five-number summary** that consists of

$$X_{\text{smallest}} \quad Q_1 \quad \text{Median} \quad Q_3 \quad X_{\text{largest}}$$

provides a way to determine the shape of a distribution. Table 3.7 explains how the relationships among the “five numbers” allows you to recognize the shape of a data set.

**TABLE 3.7** Relationships Among the Five-Number Summary and the Type of Distribution

Comparison	Type of Distribution		
	Left-Skewed	Symmetric	Right-Skewed
The distance from $X_{\text{smallest}}$ to the median versus the distance from the median to $X_{\text{largest}}$	The distance from $X_{\text{smallest}}$ to the median is greater than the distance from the median to $X_{\text{largest}}$ .	Both distances are the same.	The distance from $X_{\text{smallest}}$ to the median is less than the distance from the median to $X_{\text{largest}}$ .
The distance from $X_{\text{smallest}}$ to $Q_1$ versus the distance from $Q_3$ to $X_{\text{largest}}$	The distance from $X_{\text{smallest}}$ to $Q_1$ is greater than the distance from $Q_3$ to $X_{\text{largest}}$ .	Both distances are the same.	The distance from $X_{\text{smallest}}$ to $Q_1$ is less than the distance from $Q_3$ to $X_{\text{largest}}$ .
The distance from $Q_1$ to the median versus the distance from the median to $Q_3$ .	The distance from $Q_1$ to the median is greater than the distance from the median to $Q_3$ .	Both distances are the same.	The distance from $Q_1$ to the median is less than the distance from the median to $Q_3$ .

For the sample of 10 getting-ready times, the smallest value is 29 minutes and the largest value is 52 minutes (see page 100). Calculations done in Section 3.1 show that the median = 39.5,  $Q_1 = 35$ , and  $Q_3 = 44$ . Therefore, the five-number summary is

$$29 \quad 35 \quad 39.5 \quad 44 \quad 52$$

The distance from  $X_{\text{smallest}}$  to the median ( $39.5 - 29 = 10.5$ ) is slightly less than the distance from the median to  $X_{\text{largest}}$  ( $52 - 39.5 = 12.5$ ). The distance from  $X_{\text{smallest}}$  to  $Q_1$  ( $35 - 29 = 6$ ) is slightly less than the distance from  $Q_3$  to  $X_{\text{largest}}$  ( $52 - 44 = 8$ ). Therefore, the getting-ready times are slightly right-skewed.

### EXAMPLE 3.14

#### COMPUTING THE FIVE-NUMBER SUMMARY OF THE THREE-YEAR ANNUALIZED RETURNS FOR SMALL-CAP GROWTH MUTUAL FUNDS WITH LOW RISK

The 838 mutual funds ([Mutual Funds.xls](#)) that are part of the Using Statistics scenario (see page 96) are classified according to the category (small cap, mid cap, and large cap), the type (growth or value), and the risk level of the mutual funds (low, average, and high). Compute the five-number summary of the three-year annualized returns for the small-cap growth funds with low risk (see page 99).

**SOLUTION** From previous computations for the three-year annualized returns for the small-cap growth funds with low risk (see pages 100, 102, and 103), the median = 22.4,  $Q_1 = 20.8$ , and  $Q_3 = 26.0$ . In addition, the smallest value in the data set is 19.0, and the largest value is 29.9. Therefore, the five-number summary is

$$19.0 \quad 20.8 \quad 22.4 \quad 26.0 \quad 29.9$$

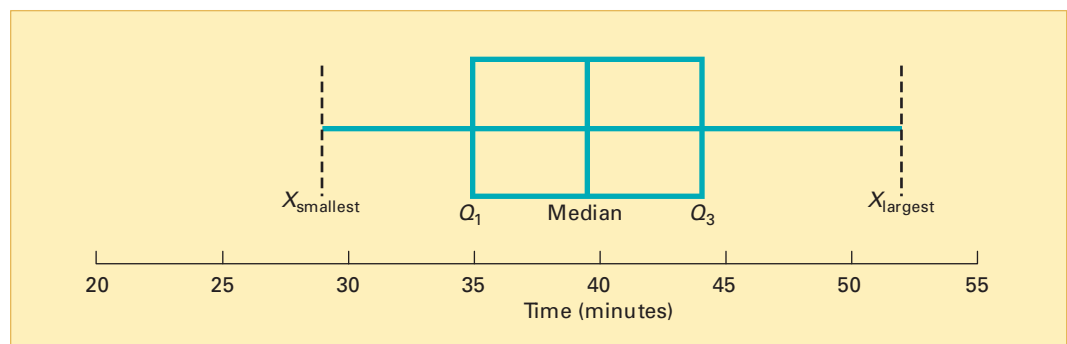
The three comparisons listed in Table 3.7 are used to evaluate skewness. The distance from  $X_{\text{smallest}}$  to the median ( $22.4 - 19.0 = 3.4$ ) is less than the distance ( $29.9 - 22.4 = 7.5$ ) from the median to  $X_{\text{largest}}$ . The distance from  $X_{\text{smallest}}$  to  $Q_1$  ( $20.8 - 19.0 = 1.8$ ) is less than the distance from  $Q_3$  to  $X_{\text{largest}}$  ( $29.9 - 26.0 = 3.9$ ). The distance from  $Q_1$  to the median ( $22.4 - 20.8 = 1.6$ ) is less than the distance from the median to  $Q_3$  ( $26.0 - 22.4 = 3.6$ ). All three comparisons indicate a right-skewed distribution.

### The Box-and-Whisker Plot

A **box-and-whisker plot** provides a graphical representation of the data based on the five-number summary. Figure 3.3 illustrates the box-and-whisker plot for the getting-ready times.

**FIGURE 3.3**

Box-and-whisker plot of the time to get ready



The vertical line drawn within the box represents the median. The vertical line at the left side of the box represents the location of  $Q_1$ , and the vertical line at the right side of the box represents the location of  $Q_3$ . Thus, the box contains the middle 50% of the values. The lower 25% of the data are represented by a line (i.e., a *whisker*) connecting the left side of the box to the location of the smallest value,  $X_{\text{smallest}}$ . Similarly, the upper 25% of the data are represented by a whisker connecting the right side of the box to  $X_{\text{largest}}$ .

The box-and-whisker plot of the getting-ready times in Figure 3.3 indicates very slight right-skewness because the distance between the median and the highest value is slightly greater than the distance between the lowest value and the median. Also, the right whisker is slightly longer than the left whisker.

### EXAMPLE 3.15

#### THE BOX-AND-WHISKER PLOTS OF THE THREE-YEAR ANNUALIZED RETURNS OF LOW-, AVERAGE-, AND HIGH-RISK MUTUAL FUNDS

The 838 mutual funds ([Mutual Funds.xls](#)) that are part of the Using Statistics scenario (see page 96) are classified according to the risk level of the mutual funds (low, average, and high). Construct the box-and-whisker plot of the three-year annualized returns for low-risk, average-risk, and high-risk mutual funds.

**SOLUTION** Figure 3.4 is a Microsoft Excel box-and-whisker plot of the three-year annualized return for low-risk, average-risk, and high-risk mutual funds. The median return and the quartiles are slightly higher for the low-risk and high-risk funds than for the average-risk funds. All three types of funds are right-skewed due to the long upper whisker.

**FIGURE 3.4**

Microsoft Excel box-and-whisker plots of the three-year annualized return for low-risk, average-risk, and high-risk mutual funds



See Section E3.4 to create this.

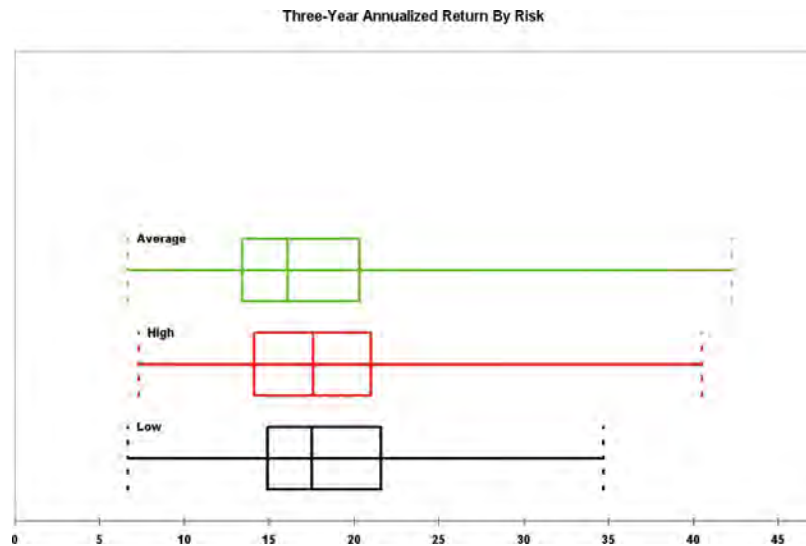
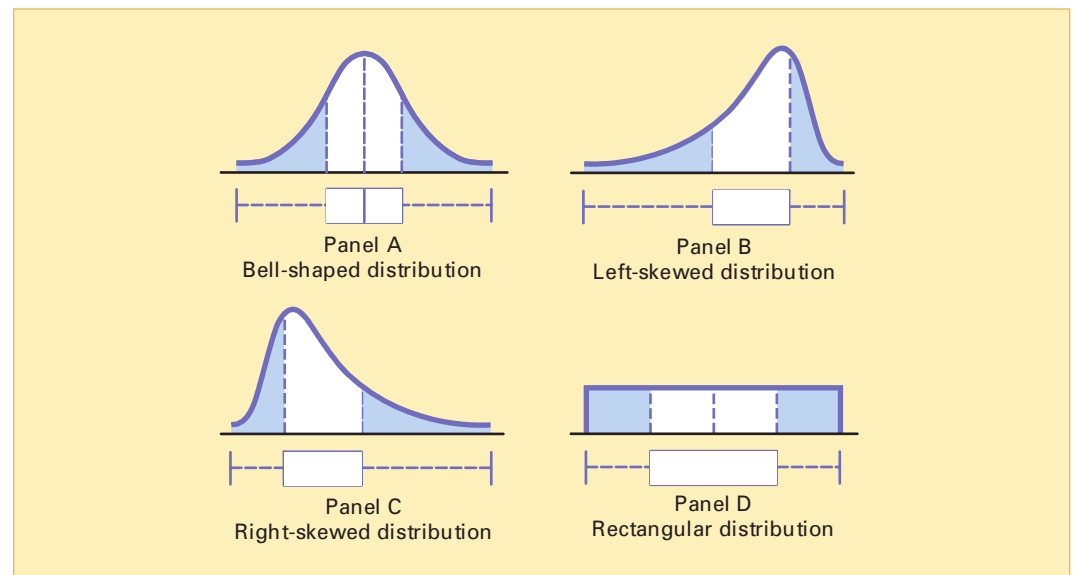


Figure 3.5 demonstrates the relationship between the box-and-whisker plot and the polygon for four different types of distributions. (Note: The area under each polygon is split into quartiles corresponding to the five-number summary for the box-and-whisker plot.)

**FIGURE 3.5**

Box-and-whisker plots and corresponding polygons for four distributions



Panels A and D of Figure 3.5 are symmetrical. In these distributions, the mean and median are equal. In addition, the length of the left whisker is equal to the length of the right whisker, and the median line divides the box in half.

Panel B of Figure 3.5 is left-skewed. The few small values distort the mean toward the left tail. For this left-skewed distribution, the skewness indicates that there is a heavy clustering of values at the high end of the scale (i.e., the right side); 75% of all values are found between the left edge of the box ( $Q_1$ ) and the end of the right whisker ( $X_{\text{largest}}$ ). Therefore, the long left whisker contains the smallest 25% of the values, demonstrating the distortion from symmetry in this data set.

Panel C of Figure 3.5 is right-skewed. The concentration of values is on the low end of the scale (i.e., the left side of the box-and-whisker plot). Here, 75% of all data values are found between the beginning of the left whisker ( $X_{\text{smallest}}$ ) and the right edge of the box ( $Q_3$ ), and the remaining 25% of the values are dispersed along the long right whisker at the upper end of the scale.

## PROBLEMS FOR SECTION 3.4

### Learning the Basics

**PH Grade ASSIST** **3.28** The following is a set of data from a sample of  $n = 6$ :

7 4 9 7 3 12

- List the five-number summary.
- Construct a box-and-whisker plot and describe the shape.
- Compare your answer in (b) with that from Problem 3.2 (d) on page 115. Discuss.

**PH Grade ASSIST** **3.29** The following is a set of data from a sample of  $n = 7$ :

12 7 4 9 0 7 3

- List the five-number summary.
- Construct a box-and-whisker plot and describe the shape.
- Compare your answer in (b) with that from Problem 3.3 (d) on page 115. Discuss.

**3.30** The following is a set of data from a sample of  $n = 5$ :

7 -5 -8 7 9

- List the five-number summary.
- Construct a box-and-whisker plot and describe the shape.
- Compare your answer in (b) with that from Problem 3.4 (d) on page 115. Discuss.

### Applying the Concepts

**3.31** The data file **chicken.xls** contains the total fat, in grams per serving, for a sample of 20 chicken sandwiches from fast-food chains. The data are as follows:

7 8 4 5 16 20 20 24 19 30  
23 30 25 19 29 29 30 30 40 56

Source: Extracted from "Fast Food: Adding Health to the Menu," Consumer Reports, September 2004, pp. 28–31.

- List the five-number summary.
- Construct a box-and-whisker plot and describe the shape.

**3.32** The data in the file **batterylife.xls** represent the battery life (in shots) for three pixel digital cameras:

300 180 85 170 380 460  
260 35 380 120 110 240

Source: Extracted from "Cameras: More Features in the Mix," Consumer Reports, July 2005, pp. 14–18.

- List the five-number summary.
- Construct a box-and-whisker plot and describe the shape.

**3.33** The data file **themeparks.xls** contains data on the starting admission price (in \$) for one-day tickets to 10 theme parks in the United States:

58 63 41 42 29 50 62 43 40 40

Source: Extracted from C. Jackson and E. Gamerman, "Rethinking the Thrill Factor," The Wall Street Journal, April 15–16, 2006, pp. P1, P4.

- List the five-number summary.
- Construct a box-and-whisker plot and describe the shape of the data.

**SELF Test** **3.34** The data in the file **coffeedrink.xls** represent the calories and fat (in grams) of 16-ounce iced coffee drinks at Dunkin' Donuts and Starbucks:

Product	Calories	Fat
Dunkin' Donuts Iced Mocha Swirl latte (whole milk)	240	8.0
Starbucks Coffee Frappuccino blended coffee	260	3.5
Dunkin' Donuts Coffee Coolatta (cream)	350	22.0
Starbucks Iced Coffee Mocha Espresso (whole milk and whipped cream)	350	20.0
Starbucks Mocha Frappuccino blended coffee (whipped cream)	420	16.0
Starbucks Chocolate Brownie Frappuccino blended coffee (whipped cream)	510	22.0
Starbucks Chocolate Frappuccino Blended Crème (whipped cream)	530	19.0

Source: Extracted from "Coffee as Candy at Dunkin' Donuts and Starbucks," Consumer Reports, June 2004, p. 9.

- For each variable (calories and fat), list the five-number summary.
- Construct a box-and-whisker plot for calories and for fat.
- What similarities and differences are there in the distributions for calories and for fat?

**3.35** The data in the file **savings.xls** are the yields for a money market account, a one-year certificate of deposit (CD), and a five-year CD for 40 banks in South Florida as of December 20, 2005 (extracted from Bankrate.com, December 20, 2005).

- List the five-number summary for the yield of the money market account, one-year CD, and a five-year CD.
- Construct a box-and-whisker plot for the yield of the money market account, one-year CD, and a five-year CD.
- What similarities and differences are there in the distributions for the yield of the money market account, one-year CD, and a five-year CD?

**3.36** A bank branch located in a commercial district of a city has developed an improved process for serving customers during the noon-to-1:00 p.m. lunch period. The waiting time, in minutes (defined as the time the customer enters the line to when he or she reaches the teller window), of a sample of 15 customers during this hour is recorded over a period of one week. The results are contained in the data file [bank1.xls](#) and are listed below:

4.21 5.55 3.02 5.13 4.77 2.34 3.54 3.20  
4.50 6.10 0.38 5.12 6.46 6.19 3.79

Another branch, located in a residential area, is also concerned with the noon-to-1 p.m. lunch hour. The waiting

time, in minutes (defined as the time the customer enters the line to when he or she reaches the teller window), of a sample of 15 customers during this hour is recorded over a period of one week. The results are contained in the data file [bank2.xls](#) and are listed below:

9.66 5.90 8.02 5.79 8.73 3.82 8.01 8.35  
10.49 6.68 5.64 4.08 6.17 9.91 5.47

- List the five-number summaries of the waiting times at the two bank branches.
- Construct box-and-whisker plots and describe the shape of the distribution of each for the two bank branches.
- What similarities and differences are there in the distributions of the waiting time at the two bank branches?

### 3.5 THE COVARIANCE AND THE COEFFICIENT OF CORRELATION

In Section 2.5, you used scatter plots to *visually* examine the relationship between two numerical variables. This section presents two *numerical* measures that examine the relationship between two numerical variables: the covariance and the coefficient of correlation.

#### The Covariance

The **covariance** measures the strength of the linear relationship between two numerical variables ( $X$  and  $Y$ ). Equation (3.16) defines the **sample covariance**, and Example 3.16 illustrates its use.

#### THE SAMPLE COVARIANCE

$$\text{cov}(X, Y) = \frac{\sum_{i=1}^n (X_i - \bar{X})(Y_i - \bar{Y})}{n - 1} \tag{3.16}$$

#### EXAMPLE 3.16

#### COMPUTING THE SAMPLE COVARIANCE

In Section 2.5 on page 58, you examined the relationship between the cost of a fast-food hamburger meal and the cost of two movie tickets in 10 cities around the world (extracted from K. Spors, “Keeping Up with . . . Yourself,” *The Wall Street Journal*, April 11, 2005, p. R4). The data file [cost of living.xls](#) contains the complete data set. Compute the sample covariance.

**SOLUTION** Table 3.8 provides the cost of a fast-food hamburger meal and the cost of two movie tickets in 10 cities around the world.

**TABLE 3.8**

Cost of a Fast-Food Hamburger Meal and Cost of Two Movie Tickets in 10 Cities

City	Hamburger	Movie Tickets
Tokyo	5.99	32.66
London	7.62	28.41
New York	5.75	20.00
Sydney	4.45	20.71
Chicago	4.99	18.00
San Francisco	5.29	19.50
Boston	4.39	18.00
Atlanta	3.70	16.00
Toronto	4.62	18.05
Rio de Janeiro	2.99	9.90

**FIGURE 3.6**

Microsoft Excel worksheet for the covariance between cost of a fast-food hamburger meal and cost of two movie tickets in 10 cities

	A	B	C	
1	<b>Covariance Analysis</b>			
2				
3	<b>Hamburger Meal</b>	<b>Movie Ticket</b>	<b>(X-XBar)(Y-YBar)</b>	
4	5.99	32.66	12.67491	=(A4 - \$C\$16) * (B4 - \$C\$17)
5	7.62	28.41	21.88597	=(A5 - \$C\$16) * (B5 - \$C\$17)
6	5.75	20	-0.09483	=(A6 - \$C\$16) * (B6 - \$C\$17)
7	4.45	20.71	-0.31052	=(A7 - \$C\$16) * (B7 - \$C\$17)
8	4.99	18	-0.02335	=(A8 - \$C\$16) * (B8 - \$C\$17)
9	5.29	19.5	-0.19375	=(A9 - \$C\$16) * (B9 - \$C\$17)
10	4.39	18	1.25045	=(A10 - \$C\$16) * (B10 - \$C\$17)
11	3.7	16	5.27332	=(A11 - \$C\$16) * (B11 - \$C\$17)
12	4.62	18.05	0.74421	=(A12 - \$C\$16) * (B12 - \$C\$17)
13	2.99	9.9	20.33355	=(A13 - \$C\$16) * (B13 - \$C\$17)
14				
15		<b>Calculations</b>		
16	XBar		4.979	=AVERAGE(A4:A13)
17	YBar		20.123	=AVERAGE(B4:B13)
18	n-1		9	=COUNT(A4:A13) - 1
19	Sum		61.53993	=SUM(C4:C13)
20	<b>Covariance</b>		<b>6.83777</b>	=C19 / C18

Figure 3.6 contains a Microsoft Excel worksheet that calculates the covariance for these data. The Calculations area of Figure 3.6 breaks down Equation (3.16) into a set of smaller calculations. From cell C20, or by using Equation (3.16) directly, you find that the covariance is 6.83777:

$$\begin{aligned} \text{cov}(X, Y) &= \frac{61.53993}{10 - 1} \\ &= 6.83777 \end{aligned}$$

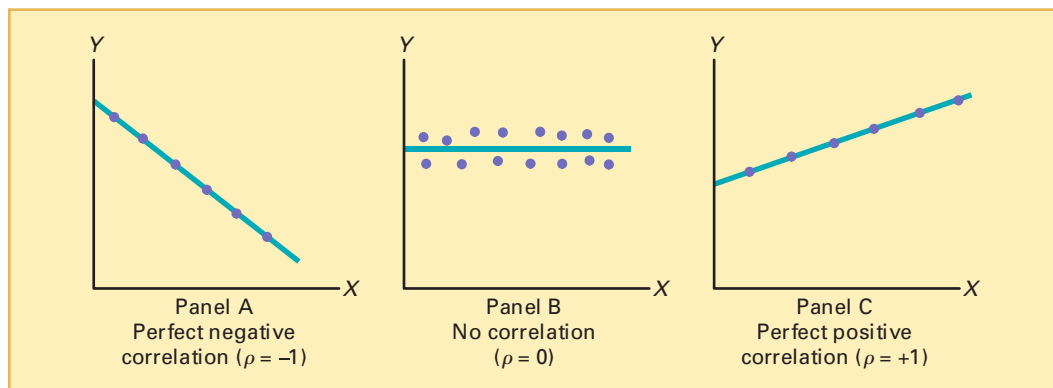
The covariance has a major flaw as a measure of the linear relationship between two numerical variables. Because the covariance can have any value, you are unable to determine the relative strength of the relationship. In other words, you cannot tell whether the value 6.83777 is an indication of a strong relationship or a weak relationship. To better determine the relative strength of the relationship, you need to compute the coefficient of correlation.

### The Coefficient of Correlation

The **coefficient of correlation** measures the relative strength of a linear relationship between two numerical variables. The values of the coefficient of correlation range from  $-1$  for a perfect negative correlation to  $+1$  for a perfect positive correlation. *Perfect* means that if the points were plotted in a scatter plot, all the points could be connected with a straight line. When dealing with population data for two numerical variables, the Greek letter  $\rho$  is used as the symbol for the coefficient of correlation. Figure 3.7 illustrates three different types of association between two variables.

**FIGURE 3.7**

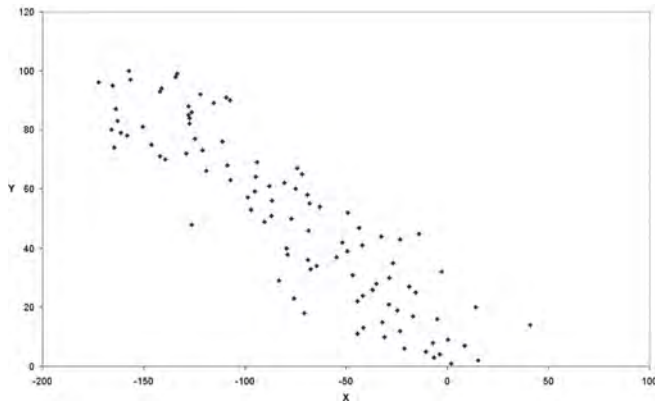
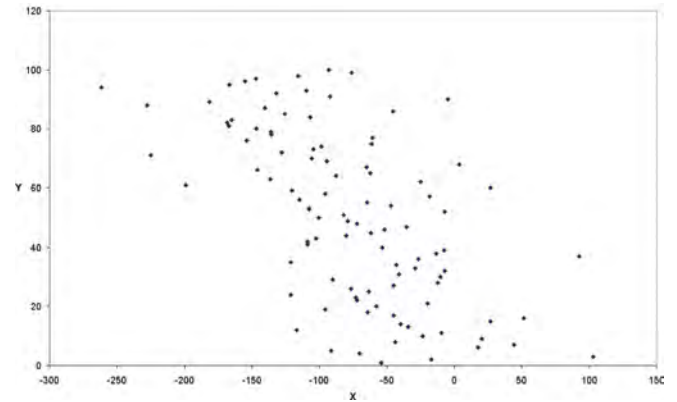
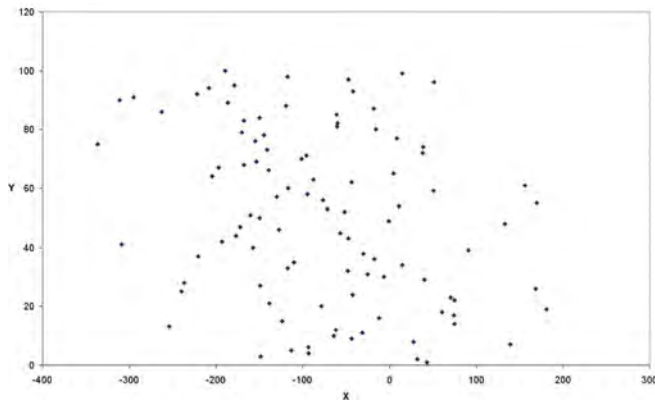
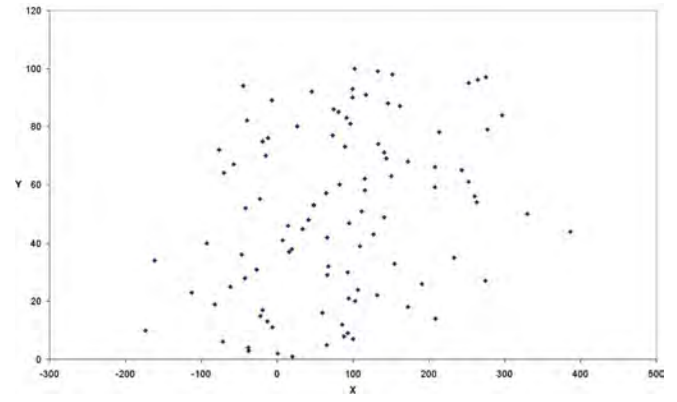
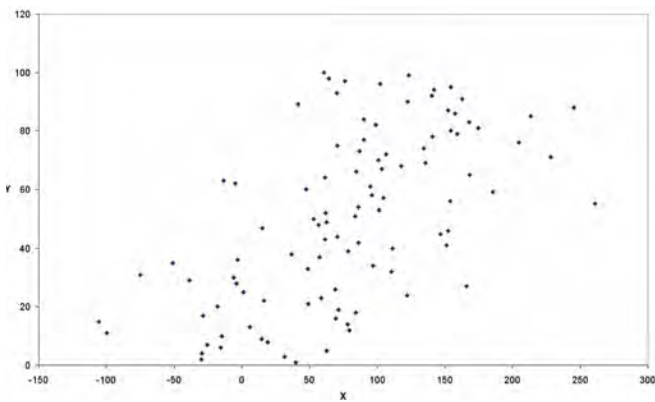
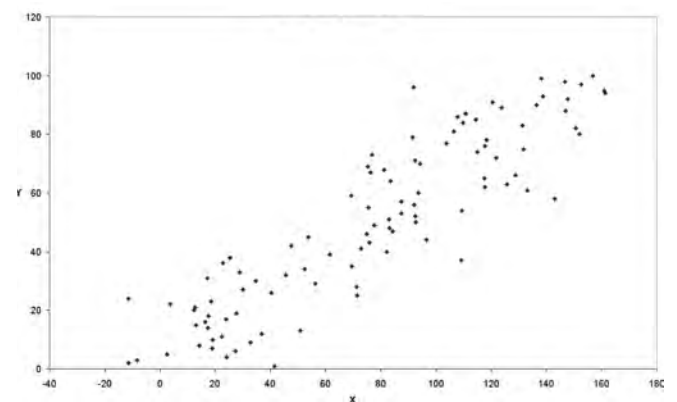
Types of association between variables





In Panel A of Figure 3.7, there is a perfect negative linear relationship between  $X$  and  $Y$ . Thus, the coefficient of correlation,  $\rho$ , equals  $-1$ , and when  $X$  increases,  $Y$  decreases in a perfectly predictable manner. Panel B shows a situation in which there is no relationship between  $X$  and  $Y$ . In this case, the coefficient of correlation,  $\rho$ , equals  $0$ , and as  $X$  increases, there is no tendency for  $Y$  to increase or decrease. Panel C illustrates a perfect positive relationship where  $\rho$  equals  $+1$ . In this case,  $Y$  increases in a perfectly predictable manner when  $X$  increases.

When you have sample data, the sample coefficient of correlation,  $r$ , is calculated. When using sample data, you are unlikely to have a sample coefficient of exactly  $+1$ ,  $0$ , or  $-1$ . Figure 3.8 presents scatter plots along with their respective sample coefficients of correlation,  $r$ , for six data sets, each of which contains 100 values of  $X$  and  $Y$ .

Panel A ( $r = -0.9$ )Panel B ( $r = -0.6$ )Panel C ( $r = -0.3$ )Panel D ( $r = 0.3$ )Panel E ( $r = 0.6$ )Panel F ( $r = 0.9$ )**FIGURE 3.8**

Six scatter plots created from Microsoft Excel and their sample coefficients of correlation,  $r$

In Panel A, the coefficient of correlation,  $r$ , is  $-0.9$ . You can see that for small values of  $X$ , there is a very strong tendency for  $Y$  to be large. Likewise, the large values of  $X$  tend to be paired with small values of  $Y$ . The data do not all fall on a straight line, so the association between  $X$  and  $Y$  cannot be described as *perfect*. The data in Panel B have a coefficient of correlation equal to  $-0.6$ , and the small values of  $X$  tend to be paired with large values of  $Y$ . The linear relationship between  $X$  and  $Y$  in Panel B is not as strong as that in Panel A. Thus, the coefficient of correlation in Panel B is not as negative as that in Panel A. In Panel C, the linear relationship between  $X$  and  $Y$  is very weak,  $r = -0.3$ , and there is only a slight tendency for the small values of  $X$  to be paired with the larger values of  $Y$ . Panels D through F depict data sets that have positive coefficients of correlation because small values of  $X$  tend to be paired with small values of  $Y$ , and the large values of  $X$  tend to be associated with large values of  $Y$ .

In the discussion of Figure 3.8, the relationships were deliberately described as *tendencies* and not as *causes and effects*. This wording was used on purpose. Correlation alone cannot prove that there is a causation effect—that is, that the change in the value of one variable *caused* the change in the other variable. A strong correlation can be produced simply by chance, by the effect of a third variable not considered in the calculation of the correlation, or by a cause-and-effect relationship. You would need to perform additional analysis to determine which of these three situations actually produced the correlation. Therefore, you can say that causation implies correlation, but correlation alone does not imply causation.

Equation (3.17) defines the **sample coefficient of correlation**,  $r$ , and Example 3.17 illustrates its use.

### THE SAMPLE COEFFICIENT OF CORRELATION

$$r = \frac{\text{cov}(X, Y)}{S_X S_Y} \quad (3.17)$$

where

$$\text{cov}(X, Y) = \frac{\sum_{i=1}^n (X_i - \bar{X})(Y_i - \bar{Y})}{n - 1}$$

$$S_X = \sqrt{\frac{\sum_{i=1}^n (X_i - \bar{X})^2}{n - 1}}$$

$$S_Y = \sqrt{\frac{\sum_{i=1}^n (Y_i - \bar{Y})^2}{n - 1}}$$

**EXAMPLE 3.17****COMPUTING THE SAMPLE COEFFICIENT OF CORRELATION**

Consider the cost of a fast-food hamburger meal and the cost of two movie tickets in 10 cities around the world (see Table 3.8 on page 127). From Figure 3.9 and Equation (3.17), compute the sample coefficient of correlation.

**SOLUTION**

$$\begin{aligned}
 r &= \frac{\text{cov}(X, Y)}{S_X S_Y} \\
 &= \frac{6.83777}{(1.2925)(6.337)} \\
 &= 0.8348
 \end{aligned}$$

**FIGURE 3.9**

Microsoft Excel worksheet for the sample coefficient of correlation,  $r$ , between the cost of a fast-food hamburger meal and two movie tickets



See Sections E3.5 and E3.6 to create this.

	A	B	C	D	E
1	Hamburger Meal	Movie Ticket	(X-XBar) <sup>2</sup>	(Y-YBar) <sup>2</sup>	(X-XBar)(Y-YBar)
2	5.99	32.66	1.0221	157.1764	12.6749
3	7.62	28.41	6.9749	68.6744	21.8860
4	5.75	20	0.5944	0.0151	-0.0948
5	4.45	20.71	0.2798	0.3446	-0.3105
6	4.99	18	0.0001	4.5071	-0.0234
7	5.29	19.5	0.0967	0.3881	-0.1938
8	4.39	18	0.3469	4.5071	1.2504
9	3.7	16	1.6358	16.9991	5.2733
10	4.62	18.05	0.1289	4.2973	0.7442
11	2.99	9.9	3.9561	104.5097	20.3335
12	Sums:		15.03589	361.41901	61.5399
13					
14	Calculations				
15			XBar	4.979	=AVERAGE(A2:A11)
16			YBar	20.123	=AVERAGE(B2:B11)
17			n-1	9	=COUNT(A2:A11)-1
18			Covariance	6.83777	=E12 / E17
19			S <sub>X</sub>	1.2925	=SQRT(C12 / E17)
20			S <sub>Y</sub>	6.3370	=SQRT(D12 / E17)
21			r	0.8348	=CORREL(A2:A11, B2:B11)

The cost of a fast-food hamburger meal and the cost of two movie tickets are positively correlated. Those cities with the lowest cost of a fast-food hamburger meal tend to be associated with the lowest cost of two movie tickets. Those cities with the highest cost of a fast-food hamburger meal tend to be associated with the highest cost of two movie tickets. This relationship is fairly strong, as indicated by a coefficient of correlation,  $r = 0.8348$ .

You cannot assume that having a low cost of a fast-food hamburger meal caused the low cost of two movie tickets. You can only say that this is what tended to happen in the sample.

In summary, the coefficient of correlation indicates the linear relationship, or association, between two numerical variables. When the coefficient of correlation gets closer to +1 or -1, the linear relationship between the two variables is stronger. When the coefficient of correlation is near 0, little or no linear relationship exists. The sign of the coefficient of correlation indicates whether the data are positively correlated (i.e., the larger values of  $X$  are typically paired with the larger values of  $Y$ ) or negatively correlated (i.e., the larger values of  $X$  are typically paired with the smaller values of  $Y$ ). The existence of a strong correlation does not imply a causation effect. It only indicates the tendencies present in the data.

## PROBLEMS FOR SECTION 3.5

### Learning the Basics

**3.37** The following is a set of data from a sample of  $n = 11$  items:

$X$	7	5	8	3	6	10	12	4	9	15	18
$Y$	21	15	24	9	18	30	36	12	27	45	54

- Compute the covariance.
- Compute the coefficient of correlation.
- How strong is the relationship between  $X$  and  $Y$ ? Explain.

### Applying the Concepts

**3.38** An article (J. Clements, “Why Investors Should Put up to 30% of Their Stock Portfolio in Foreign Funds,” *The Wall Street Journal*, November 26, 2003, p. D1) that discussed investment in foreign stocks stated that the coefficient of correlation between the return on investment of U.S. stocks and international large cap stocks was 0.80, U.S. stocks and international small cap stocks was 0.53, U.S. stocks and international bonds was 0.03, U.S. stocks and emerging markets stocks was 0.71, and U.S. stocks and emerging markets debt was 0.58.

- What conclusions can you make about the strength of the relationship between the return on investment of U.S. stocks and each of these five other types of investments?
- Compare the results of (a) to those of Problem 3.39 (a).

**3.39** An article (J. Clements, “Why Investors Should Put up to 30% of Their Stock Portfolio in Foreign Funds,” *The Wall Street Journal*, November 26, 2003, p. D1) that discussed investment in foreign bonds stated that the coefficient of correlation between the return on investment of U.S. bonds and international large cap stocks was  $-0.13$ , U.S. bonds and international small cap stocks was  $-0.18$ , U.S. bonds and international bonds was 0.48, U.S. bonds and emerging markets stocks was  $-0.20$ , and U.S. bonds and emerging markets debt was 0.10.

- What conclusions can you make about the strength of the relationship between the return on investment of U.S. bonds and each of these five other types of investments?
- Compare the results of (a) to those of Problem 3.38 (a).



**3.40** The data in the file [coffeedrink.xls](#) represent the calories and fat, in grams, of 16-ounce iced coffee drinks at Dunkin’ Donuts and Starbucks:

Product	Calories	Fat
Dunkin’ Donuts Iced Mocha Swirl latte (whole milk)	240	8.0
Starbucks Coffee Frappuccino blended coffee	260	3.5
Dunkin’ Donuts Coffee Coolatta (cream)	350	22.0
Starbucks Iced Coffee Mocha Espresso (whole milk and whipped cream)	350	20.0
Starbucks Mocha Frappuccino blended coffee (whipped cream)	420	16.0
Starbucks Chocolate Brownie Frappuccino blended coffee (whipped cream)	510	22.0
Starbucks Chocolate Frappuccino Blended Crème (whipped cream)	530	19.0

Source: Extracted from “Coffee as Candy at Dunkin’ Donuts and Starbucks,” *Consumer Reports*, June 2004, p. 9.

- Compute the covariance.
- Compute the coefficient of correlation.
- Which do you think is more valuable in expressing the relationship between calories and fat—the covariance or the coefficient of correlation? Explain.
- What conclusions can you reach about the relationship between calories and fat?

**3.41** There are several methods for calculating fuel economy. The following table (contained in the file [mileage.xls](#)) indicates the mileage, as calculated by owners and by current government standards:

Car	Owner	Government
2005 Ford F-150	14.3	16.8
2005 Chevrolet Silverado	15.0	17.8
2002 Honda Accord LX	27.8	26.2
2002 Honda Civic	27.9	34.2
2004 Honda Civic Hybrid	48.8	47.6
2002 Ford Explorer	16.8	18.3
2005 Toyota Camry	23.7	28.5
2003 Toyota Corolla	32.8	33.1
2005 Toyota Prius	37.3	56.0

Source: Extracted from J. Healey, “Fuel Economy Calculations to be Altered,” *USA Today*, January 11, 2006, p. 1B.

- Compute the covariance.
- Compute the coefficient of correlation.
- Which do you think is more valuable in expressing the relationship between owner and current government standards mileage—the covariance or the coefficient of correlation? Explain.
- What conclusions can you reach about the relationship between owner and current government standards mileage?

**3.42** College basketball is big business, with coaches' salaries, revenues, and expenses in millions of dollars. The data file [colleges-basketball.xls](#) contains the coaches' salaries and revenue for college basketball at selected schools in a recent year (extracted from R. Adams, "Pay for Playoffs," *The Wall Street Journal*, March 11–12, 2006, pp. P1, P8).

- Compute the covariance.
- Compute the coefficient of correlation.
- What conclusions can you reach about the relationship between a coach's salary and revenue?

**3.43** College football players trying out for the NFL are given the Wonderlic standardized intelligence test. The data in the file [wonderlic.xls](#) contains the average Wonderlic score of football players trying out for the NFL and the graduation rate for football players at selected schools (extracted from S. Walker, "The NFL's Smartest Team," *The Wall Street Journal*, September 30, 2005, pp. W1, W10).

- Compute the covariance.
- Compute the coefficient of correlation.
- What conclusions can you reach about the relationship between the average Wonderlic score and graduation rate?

### 3.6 PITFALLS IN NUMERICAL DESCRIPTIVE MEASURES AND ETHICAL ISSUES

In this chapter, you have studied how a set of numerical data can be characterized by various statistics that measure the properties of central tendency, variation, and shape. Your next step is analysis and interpretation of the calculated statistics. Your analysis is *objective*; your interpretation is *subjective*. You must avoid errors that may arise either in the objectivity of your analysis or in the subjectivity of your interpretation.

The analysis of the mutual funds is *objective* and reveals several impartial findings. Objectivity in data analysis means reporting the most appropriate numerical descriptive measures for a given data set. Now that you have read the chapter and have become familiar with various numerical descriptive measures and their strengths and weaknesses, how should you proceed with the objective analysis? Because the data distribute in a slightly asymmetrical manner, shouldn't you report the median in addition to the mean? Doesn't the standard deviation provide more information about the property of variation than the range? Should you describe the data set as right-skewed?

On the other hand, data interpretation is *subjective*. Different people form different conclusions when interpreting the analytical findings. Everyone sees the world from different perspectives. Thus, because data interpretation is subjective, you must do it in a fair, neutral, and clear manner.

#### Ethical Issues

Ethical issues are vitally important to all data analysis. As a daily consumer of information, you need to question what you read in newspapers and magazines, what you hear on the radio or television, and what you see while surfing the Internet. Over time, much skepticism has been expressed about the purpose, the focus, and the objectivity of published studies. Perhaps no comment on this topic is more telling than a quip often attributed to the famous nineteenth-century British statesman Benjamin Disraeli: "There are three kinds of lies: lies, damned lies, and statistics."

Ethical considerations arise when you are deciding what results to include in a report. You should document both good and bad results. In addition, when making oral presentations and presenting written reports, you need to give results in a fair, objective, and neutral manner. Unethical behavior occurs when you willfully choose an inappropriate summary measure (for example, the mean for a very skewed set of data) to distort the facts in order to support a particular position. In addition, unethical behavior occurs when you selectively fail to report pertinent findings because it would be detrimental to the support of a particular position.

## SUMMARY

In this and the previous chapter, you studied descriptive statistics—how data are presented in tables and charts, and then summarized, described, analyzed, and interpreted. When dealing with the mutual fund data, you were able to present useful information through the use of pie charts, histograms, and other graphical methods. You explored characteristics of past performance, such as central tendency, variability, and shape, using numerical descriptive

measures, such as the mean, median, quartiles, range, standard deviation, and coefficient of correlation. Table 3.9 provides a list of the numerical descriptive measures covered in this chapter.

In the next chapter, the basic principles of probability are presented in order to bridge the gap between the subject of descriptive statistics and the subject of inferential statistics.

**TABLE 3.9**

Summary of Numerical Descriptive Measures

Type of Analysis	Numerical Data
Describing central tendency, variation, and shape of a numerical variable	Mean, median, mode, quartiles, geometric mean, range, interquartile range, standard deviation, variance, coefficient of variation, Z scores, box-and-whisker plot ( <b>Sections 3.1–3.4</b> )
Describing the relationship between two numerical variables	Covariance, coefficient of correlation ( <b>Section 3.5</b> )

## KEY EQUATIONS

### Sample Mean

$$\bar{X} = \frac{\sum_{i=1}^n X_i}{n} \quad (3.1)$$

### Median

$$\text{Median} = \frac{n+1}{2} \text{ ranked value} \quad (3.2)$$

### First Quartile, $Q_1$

$$Q_1 = \frac{n+1}{4} \text{ ranked value} \quad (3.3)$$

### Third Quartile, $Q_3$

$$Q_3 = \frac{3(n+1)}{4} \text{ ranked value} \quad (3.4)$$

### Geometric Mean

$$\bar{X}_G = (X_1 \times X_2 \times \cdots \times X_n)^{1/n} \quad (3.5)$$

### Geometric Mean Rate of Return

$$\bar{R}_G = [(1 + R_1) \times (1 + R_2) \times \cdots \times (1 + R_n)]^{1/n} - 1 \quad (3.6)$$

### Range

$$\text{Range} = X_{\text{largest}} - X_{\text{smallest}} \quad (3.7)$$

### Interquartile Range

$$\text{Interquartile range} = Q_3 - Q_1 \quad (3.8)$$

### Sample Variance

$$S^2 = \frac{\sum_{i=1}^n (X_i - \bar{X})^2}{n-1} \quad (3.9)$$

### Sample Standard Deviation

$$S = \sqrt{S^2} = \sqrt{\frac{\sum_{i=1}^n (X_i - \bar{X})^2}{n-1}} \quad (3.10)$$

**Coefficient of Variation**

$$CV = \left( \frac{S}{\bar{X}} \right) 100\% \quad (3.11)$$

**Z Scores**

$$Z = \frac{X - \bar{X}}{S} \quad (3.12)$$

**Population Mean**

$$\mu = \frac{\sum_{i=1}^N X_i}{N} \quad (3.13)$$

**Population Variance**

$$\sigma^2 = \frac{\sum_{i=1}^N (X_i - \mu)^2}{N} \quad (3.14)$$

**Population Standard Deviation**

$$\sigma = \sqrt{\frac{\sum_{i=1}^N (X_i - \mu)^2}{N}} \quad (3.15)$$

**Sample Covariance**

$$\text{cov}(X, Y) = \frac{\sum_{i=1}^n (X_i - \bar{X})(Y_i - \bar{Y})}{n - 1} \quad (3.16)$$

**Sample Coefficient of Correlation**

$$r = \frac{\text{cov}(X, Y)}{S_X S_Y} \quad (3.17)$$

**KEY TERMS**

arithmetic mean 97	mean 97	sample coefficient of correlation 130
box-and-whisker plot 124	median 99	sample covariance 127
central tendency 96	midsread 106	sample mean 97
Chebyshev rule 120	mode 100	sample standard deviation 107
coefficient of correlation 128	outlier 111	sample variance 107
coefficient of variation 110	population mean 118	shape 96
covariance 127	population standard deviation 119	skewed 112
dispersion 105	population variance 119	spread 105
empirical rule 120	$Q_1$ : first quartile 101	standard deviation 106
extreme value 111	$Q_2$ : second quartile 101	sum of squares (SS) 107
five-number summary 123	$Q_3$ : third quartile 101	symmetrical 112
geometric mean 103	quartiles 101	variance 106
geometric mean rate of return 103	range 105	variation 96
interquartile range 106	resistant measure 106	Z score 111
left-skewed 113	right-skewed 113	

**CHAPTER REVIEW PROBLEMS****Checking Your Understanding**

**3.44** What are the properties of a set of numerical data?

**PH Grade ASSIST** **3.45** What is meant by the property of central tendency?

**PH Grade ASSIST** **3.46** What are the differences among the mean, median, and mode, and what are the advantages and disadvantages of each?

**3.47** How do you interpret the first quartile, median, and third quartile?

**PH Grade ASSIST** **3.48** What is meant by the property of variation?

**3.49** What does the Z score measure?

**PH Grade ASSIST** **3.50** What are the differences among the various measures of variation, such as the range, interquartile range, variance, standard deviation,

and coefficient of variation, and what are the advantages and disadvantages of each?

**PH Grade ASSIST** **3.51** How does the empirical rule help explain the ways in which the values in a set of numerical data cluster and distribute?

**3.52** How do the empirical rule and the Chebychev rule differ?

**3.53** What is meant by the property of shape?

**3.54** How do the covariance and the coefficient of correlation differ?

### Applying the Concepts

**3.55** The American Society for Quality (ASQ) conducted a salary survey of all its members. ASQ members work in all areas of manufacturing and service-related institutions, with a common theme of an interest in quality. For the U.S. survey, emails were sent to 38,599 members, and 5,642 valid responses were received. The two most common job titles were manager and quality engineer. Descriptive statistics concerning salaries for these two titles are given below (Extracted from Debbie Phillips-Donaldson, “Good Year for Traditional Quality Professionals,” *Quality Progress*, December, 2004, pp. 24–48). Compare the salaries of managers and quality engineers.

Title	Sample Size	Minimum	Maximum	Standard Deviation	Mean	Median
Manager	1,553	11,000	142,000	21,421	71,130	70,000
Quality Engineer	899	9,200	125,000	16,571	61,739	60,000

**3.56** In New York State, savings banks are permitted to sell a form of life insurance called savings bank life insurance (SBLI). The approval process consists of underwriting, which includes a review of the application, a medical information bureau check, possible requests for additional medical information and medical exams, and a policy compilation stage during which the policy pages are generated and sent to the bank for delivery. The ability to deliver approved policies to customers in a timely manner is critical to the profitability of this service to the bank. During a period of one month, a random sample of 27 approved policies was selected, and the following total processing times in days, were recorded; the data are contained in the file [insurance.xls](#):

73 19 16 64 28 28 31 90 60 56 31 56 22 18  
45 48 17 17 17 91 92 63 50 51 69 16 17

- Compute the mean, median, first quartile, and third quartile.
- Compute the range, interquartile range, variance, standard deviation, and coefficient of variation.

- Construct a box-and-whisker plot. Are the data skewed? If so, how?
- What would you tell a customer who enters the bank to purchase this type of insurance policy and asks how long the approval process takes?

**3.57** One of the major measures of the quality of service provided by any organization is the speed with which it responds to customer complaints. A large family-held department store selling furniture and flooring, including carpet, had undergone a major expansion in the past several years. In particular, the flooring department had expanded from 2 installation crews to an installation supervisor, a measurer, and 15 installation crews. A sample of 50 complaints concerning carpet installation was selected during a recent year. The data in the file [furniture.xls](#) represent the number of days between the receipt of a complaint and the resolution of the complaint:

54 5 35 137 31 27 152 2 123 81 74 27  
11 19 126 110 110 29 61 35 94 31 26 5  
12 4 165 32 29 28 29 26 25 1 14 13  
13 10 5 27 4 52 30 22 36 26 20 23  
33 68

- Compute the mean, median, first quartile, and third quartile.
- Compute the range, interquartile range, variance, standard deviation, and coefficient of variation.
- Construct a box-and-whisker plot. Are the data skewed? If so, how?
- On the basis of the results of (a) through (c), if you had to tell the president of the company how long a customer should expect to wait to have a complaint resolved, what would you say? Explain.

**3.58** A manufacturing company produces steel housings for electrical equipment. The main component part of the housing is a steel trough that is made out of a 14-gauge steel coil. It is produced using a 250-ton progressive punch press with a wipe-down operation, putting two 90-degree forms in the flat steel to make the trough. The distance from one side of the form to the other is critical because of weatherproofing in outdoor applications. The company requires that the width of the trough be between 8.31 inches and 8.61 inches. The data file [trough.xls](#) contains the widths of the troughs, in inches, for a sample of  $n = 49$ :

8.312 8.343 8.317 8.383 8.348 8.410 8.351 8.373 8.481 8.422  
8.476 8.382 8.484 8.403 8.414 8.419 8.385 8.465 8.498 8.447  
8.436 8.413 8.489 8.414 8.481 8.415 8.479 8.429 8.458 8.462  
8.460 8.444 8.429 8.460 8.412 8.420 8.410 8.405 8.323 8.420  
8.396 8.447 8.405 8.439 8.411 8.427 8.420 8.498 8.409



- Calculate the mean, median, range, and standard deviation for the width. Interpret these measures of central tendency and variability.
- List the five-number summary.
- Construct a box-and-whisker plot and describe its shape.
- What can you conclude about the number of troughs that will meet the company's requirement of troughs being between 8.31 and 8.61 inches wide?

**3.59** The manufacturing company in Problem 3.58 also produces electric insulators. If the insulators break when in use, a short circuit is likely to occur. To test the strength of the insulators, destructive testing is carried out to determine how much force is required to break the insulators. Force is measured by observing how many pounds must be applied to the insulator before it breaks. The data from 30 insulators from this experiment are contained in the file **force.xls**:

1,870 1,728 1,656 1,610 1,634 1,784 1,522 1,696 1,592 1,662  
1,866 1,764 1,734 1,662 1,734 1,774 1,550 1,756 1,762 1,866  
1,820 1,744 1,788 1,688 1,810 1,752 1,680 1,810 1,652 1,736

- Calculate the mean, median, range, and standard deviation for the force variable.
- Interpret the measures of central tendency and variability in (a).
- Construct a box-and-whisker plot and describe its shape.
- What can you conclude about the strength of the insulators if the company requires a force measurement of at least 1,500 pounds before breakage?

**3.60** Problems with a telephone line that prevent a customer from receiving or making calls are disconcerting to both the customer and the telephone company. The data contained in the file **phone.x** represent samples of 20 problems reported to two different offices of a telephone company and the time to clear these problems, in minutes, from the customers' lines:

*Central Office I Time to Clear Problems (minutes)*

1.48 1.75 0.78 2.85 0.52 1.60 4.15 3.97 1.48 3.10  
1.02 0.53 0.93 1.60 0.80 1.05 6.32 3.93 5.45 0.97

*Central Office II Time to Clear Problems (minutes)*

7.55 3.75 0.10 1.10 0.60 0.52 3.30 2.10 0.58 4.02  
3.75 0.65 1.92 0.60 1.53 4.23 0.08 1.48 1.65 0.72

For each of the two central office locations:

- Compute the mean, median, first quartile, and third quartile.
- Compute the range, interquartile range, variance, standard deviation, and coefficient of variation.

- Construct side-by-side box-and-whisker plots. Are the data skewed? If so, how?
- On the basis of the results of (a) through (c), are there any differences between the two central offices? Explain.

**3.61** In many manufacturing processes, the term *work-in-process* (often abbreviated WIP) is used. In a book manufacturing plant, the WIP represents the time it takes for sheets from a press to be folded, gathered, sewn, tipped on end sheets, and bound. The data contained in the file **wip.xls** represent samples of 20 books at each of two production plants and the processing time (operationally defined as the time, in days, from when the books came off the press to when they were packed in cartons) for these jobs:

*Plant A*

5.62 5.29 16.25 10.92 11.46 21.62 8.45 8.58 5.41 11.42  
11.62 7.29 7.50 7.96 4.42 10.50 7.58 9.29 7.54 8.92

*Plant B*

9.54 11.46 16.62 12.62 25.75 15.41 14.29 13.13 13.71 10.04  
5.75 12.46 9.17 13.21 6.00 2.33 14.25 5.37 6.25 9.71

For each of the two plants:

- Compute the mean, median, first quartile, and third quartile.
- Compute the range, interquartile range, variance, standard deviation, and coefficient of variation.
- Construct side-by-side box-and-whisker plots. Are the data skewed? If so, how?
- On the basis of the results of (a) through (c), are there any differences between the two plants? Explain.

**3.62** The data contained in the file **tuition2006.xls** consist of the in-state tuition and fees and the out-of-state tuition and fees for four-year colleges with the highest percentage of students graduating within six years.

*Source: U.S. Department of Education, 2006.*

For each variable:

- Compute the mean, median, first quartile, and third quartile.
- Compute the range, interquartile range, variance, standard deviation, and coefficient of variation.
- Construct a box-and-whisker plot. Are the data skewed? If so, how?
- Compute the coefficient of correlation between the in-state tuition and fees and the out-of-state tuition and fees.
- What conclusions can you reach concerning the in-state tuition and fees and the out-of-state tuition and fees?

**3.63** A quality characteristic of interest for a tea-bag-filling process is the weight of the tea in the individual bags. If the bags are underfilled, two problems arise. First, customers may not be able to brew the tea to be as strong as they wish. Second, the company may be in violation of the truth-in-labeling laws. For this product, the label weight on the package indicates that, on average, there are 5.5 grams of tea in a bag. If the mean amount of tea in a bag exceeds the label weight, the company is giving away product. Getting an exact amount of tea in a bag is problematic because of variation in the temperature and humidity inside the factory, differences in the density of the tea, and the extremely fast filling operation of the machine (approximately 170 bags per minute). The data in the file [teabags.xls](#) shown below provide the weight, in grams, of a sample of 50 tea bags produced in one hour by a single machine:

5.65 5.44 5.42 5.40 5.53 5.34 5.54 5.45 5.52 5.41  
 5.57 5.40 5.53 5.54 5.55 5.62 5.56 5.46 5.44 5.51  
 5.47 5.40 5.47 5.61 5.53 5.32 5.67 5.29 5.49 5.55  
 5.77 5.57 5.42 5.58 5.58 5.50 5.32 5.50 5.53 5.58  
 5.61 5.45 5.44 5.25 5.56 5.63 5.50 5.57 5.67 5.36

- Compute the mean, median, first quartile, and third quartile.
- Compute the range, interquartile range, variance, standard deviation, and coefficient of variation.
- Interpret the measures of central tendency and variation within the context of this problem. Why should the company producing the tea bags be concerned about the central tendency and variation?
- Construct a box-and-whisker plot. Are the data skewed? If so, how?
- Is the company meeting the requirement set forth on the label that, on average, there are 5.5 grams of tea in a bag? If you were in charge of this process, what changes, if any, would you try to make concerning the distribution of weights in the individual bags?

**3.64** Do marketing promotions, such as bobble-head giveaways, increase attendance at Major League Baseball games? An article in the *Mid-American Journal of Business* reported on the effectiveness of marketing promotions (T. C. Boyd and T. C. Krehbiel, “An Analysis of the Effects of Specific Promotion Types on Attendance at Major League Baseball Games,” *Mid-American Journal of Business*, 2006, 21, pp. 21–32). The data file [royals.xls](#) includes the following variables for the Kansas City Royals during the 2002 baseball season:

GAME —Home games in the order in which they were played

ATTENDANCE —Paid attendance for the game

PROMOTION—Y = a promotion was held; N = no promotion was held

- Calculate the mean and standard deviation of attendance for the 43 games where promotions were held and for the 37 games without promotions.
- Construct a five-number summary for the 43 games where promotions were held and for the 37 games without promotions.
- Construct a graphical display containing two box-and-whisker plots—one for the 43 games where promotions were held and one for the 37 games without promotions.
- Discuss the results of (a) through (c) and comment on the effectiveness of promotions at Royals’ games during the 2002 season.

**3.65** A study conducted by Zagat Survey concluded that many first-rate restaurants are located in hotels across the United States. Travelers can find quality food, service, and décor without leaving their hotels. The top-rated hotel restaurant is The French Room, located in The Adolphus Hotel in Dallas, Texas. The estimated price for dinner, including one drink and tip at The French Room, is \$80. The highest price reported is \$179 at Alain Ducasse, located in the Jumeirah Essex House in New York City (Extracted from Gary Stoller, “Top Restaurants Check into Luxury Hotels,” *USA Today*, April 11, 2006, p. 5B). The file [bestrest.xls](#) contains the top 100 hotel restaurants in the United States and the variables state, city, restaurant, hotel, cost (estimated price of dinner including one drink and tip), and rating (1 to 100, with 1 the top-rated restaurant).

- Construct the five-number summary of dinner price.
- Construct a box-and-whisker plot of dinner price and interpret the distribution of dinner prices.
- Calculate and interpret the correlation coefficient of the rating and dinner price.

**3.66** The manufacturer of Boston and Vermont asphalt shingles provides its customers with a 20-year warranty on most of its products. To determine whether a shingle will last as long as the warranty period, accelerated-life testing is conducted at the manufacturing plant. Accelerated-life testing exposes the shingle to the stresses it would be subject to in a lifetime of normal use in a laboratory setting via an experiment that takes only a few minutes to conduct. In this test, a shingle is repeatedly scraped with a brush for a short period of time, and the shingle granules removed by the brushing are weighed (in grams). Shingles that experience low amounts of granule loss are expected to last longer in normal use than shingles that experience high amounts of granule loss. In this situation, a shingle should experience no more than 0.8 gram of granule loss if it is expected to last the length of the warranty period. The data file [granule.xls](#) contains a

sample of 170 measurements made on the company's Boston shingles and 140 measurements made on Vermont shingles.

- List the five-number summary for the Boston shingles and for the Vermont shingles.
- Construct side-by-side box-and-whisker plots for the two brands of shingles and describe the shapes of the distributions.
- Comment on the shingles' ability to achieve a granule loss of 0.8 gram or less.

**3.67** The data in the file `states.xls` represent the results of the American Community Survey, a sampling of 700,000 households taken in each state during the 2000 U.S. Census. For each of the variables average travel-to-work time in minutes, percentage of homes with eight or more rooms, median household income, and percentage of mortgage-paying homeowners whose housing costs exceed 30% of income:

- Compute the mean, median, first quartile, and third quartile.
- Compute the range, interquartile range, variance, standard deviation, and coefficient of variation.
- Construct a box-and-whisker plot. Are the data skewed? If so, how?
- What conclusions can you reach concerning the mean travel-to-work time in minutes, percentage of homes with eight or more rooms, median household income, and percentage of mortgage-paying homeowners whose housing costs exceed 30% of income?

**3.68** The economics of baseball has caused a great deal of controversy, with owners arguing that they are losing money, players arguing that owners are making money, and fans complaining about how expensive it is to attend a game and watch games on cable television. In addition to data related to team statistics for the 2001 season, the file `bb2001.xls` contains team-by-team statistics on ticket prices; the fan cost index; regular season gate receipts; local television, radio, and cable receipts; all other operating revenue; player compensation and benefits; national and other local expenses; and income from baseball operations. For each of these variables,

- Compute the mean, median, first quartile, and third quartile.
- Compute the range, interquartile range, variance, standard deviation, and coefficient of variation.
- Construct a box-and-whisker plot. Are the data skewed? If so, how?
- Compute the correlation between the number of wins and player compensation and benefits. How strong is the relationship between these two variables?
- What conclusions can you reach concerning the regular season gate receipts; local television, radio, and cable receipts; all other operating revenue; player compensa-

tion and benefits; national and other local expenses; and income from baseball operations?

**3.69** In Section 3.5 on page 131, the correlation coefficient between the cost of a fast-food hamburger meal and the cost of movie tickets in 10 different cities was computed. The data file `cost of living.xls` also includes the overall cost index, the monthly rent for a two bedroom apartment, and the costs of a cup of coffee with service, dry cleaning for a men's blazer, and toothpaste.

- Compute the correlation coefficient between the overall cost index and the monthly rent for a two-bedroom apartment, the cost of a cup of coffee with service, the cost of a fast food hamburger meal, the cost of dry cleaning a men's blazer, the cost of toothpaste, and the cost of movie tickets. (There will be six separate correlation coefficients.)
- What conclusions can you reach about the relationship of the overall cost index to each of these six variables?

**3.70** The data in the file `chicken.xls` contains the characteristics for a sample of 20 chicken sandwiches from fast-food chains.

- Compute the correlation coefficient between calories and carbohydrates.
- Compute the correlation coefficient between calories and sodium.
- Compute the correlation coefficient between calories and total fat.
- Which variable (total fat, carbohydrates, or sodium) seems to be most closely related to calories? Explain.

**3.71** The data in the file `ceo.xls` represent the total compensation (in \$millions) of CEOs of the 100 largest companies, by revenue (extracted from "Special Report: Executive Compensation," *USA Today*, April 10, 2006, pp. 3B, 4B).

- Compute the mean, median, first quartile, and third quartile.
- Compute the range, interquartile range, variance, standard deviation, and coefficient of variation.
- Construct a box-and-whisker plot. Are the data skewed? If so, how?
- What conclusions can you draw concerning the total compensation (in \$millions) of CEOs?

**3.72** The data in the file `spending.xls` is the per capita spending, in thousands of dollars, for each state in 2004.

- Compute the mean, median, first quartile, and third quartile.
- Compute the range, interquartile range, variance, standard deviation, and coefficient of variation.
- Construct side-by-side box-and-whisker plots. Are the data skewed? If so, how?
- What conclusions can you reach concerning per capita spending, in thousands of dollars, for each state in 2004?

**3.73** As an illustration of the misuse of statistics, an article by Glenn Kramon (“Coaxing the Stanford Elephant to Dance,” *The New York Times* Sunday Business Section, November 11, 1990) implied that costs at Stanford Medical Center had been driven up higher than at competing institutions because the former was more likely than other organizations to treat indigent, Medicare, Medicaid, sicker, and more complex patients. The chart below was provided to compare the average 1989 to 1990 hospital charges for three medical procedures (coronary bypass, simple birth, and hip replacement) at three competing institutions (El Camino, Sequoia, and Stanford).

Suppose you were working in a medical center. Your CEO knows you are currently taking a course in statistics and calls you in to discuss this. She tells you that the article was presented in a discussion group setting as part of a meeting of regional area medical center CEOs last night and that one of them mentioned that this chart was totally meaningless and asked her opinion. She now requests that you prepare her response. You smile, take a deep breath, and reply . . .

**3.74** You are planning to study for your statistics examination with a group of classmates, one of whom you particularly want to impress. This individual has volunteered to use Microsoft Excel to get the needed summary information, tables, and charts for a data set containing several numerical and categorical variables assigned by the

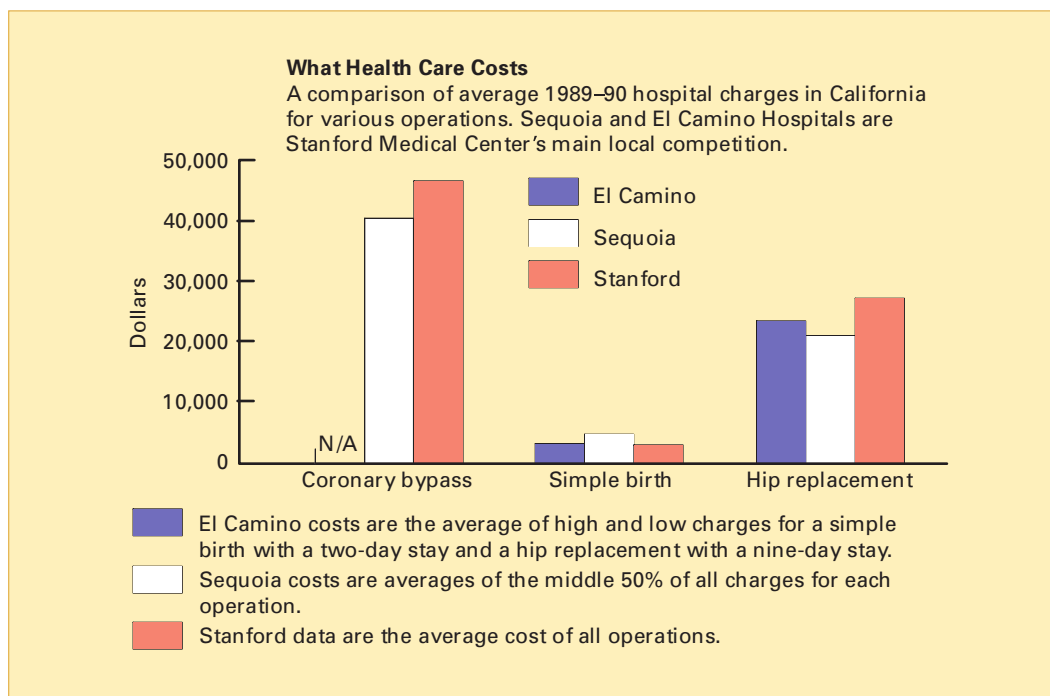
instructor for study purposes. This person comes over to you with the printout and exclaims, “I’ve got it all—the means, the medians, the standard deviations, the box-and-whisker plots, the pie charts—for all our variables. The problem is, some of the output looks weird—like the box-and-whisker plots for gender and for major and the pie charts for grade point index and for height. Also, I can’t understand why Professor Krehbiel said we can’t get the descriptive stats for some of the variables—I got them for everything! See, the mean for height is 68.23, the mean for grade point index is 2.76, the mean for gender is 1.50, the mean for major is 4.33.” What is your reply?

### Report Writing Exercises

**3.75** Data concerning 58 of the best-selling domestic beers in the U.S. are located in the file [domesticbeer.xls](#). The values for three variables are included: percentage alcohol, number of calories per 12 ounces, and number of carbohydrates (in grams) per 12 ounces.

Source: Extracted from [www.Beer100.com](#), March 31, 2006.

Your task is to write a report based on a complete descriptive evaluation of each of the numerical variables—percentage alcohol, number of calories per 12 ounces, and number of carbohydrates (in grams) per 12 ounces. Appended to your report should be all appropriate tables, charts, and numerical descriptive measures.



Source: Adapted from Stanford Medical Center, Sequoia Hospital, and El Camino Hospital.

## Team Projects

The data file [Mutual Funds.xls](#) contains information regarding nine variables from a sample of 838 mutual funds:

Category—Type of stocks comprising the mutual fund (small cap, mid cap, large cap)

Objective—Objective of stocks comprising the mutual fund (growth or value)

Assets—In millions of dollars

Fees—Sales charges (no or yes)

Expense ratio—ratio of expenses to net assets in percentage

Risk—Risk-of-loss factor of the mutual fund (low, average, high)

2005 return—Twelve-month return in 2005

Three-year return—Annualized return, 2003–2005

Five-year return—Annualized return, 2001–2005

**3.76** For expense ratio in percentage, 2005 return, three-year return, and five-year return,

- Compute the mean, median, first quartile, and third quartile.
- Compute the range, interquartile range, variance, standard deviation, and coefficient of variation.
- Construct a box-and-whisker plot. Are the data skewed? If so, how?
- What conclusions can you reach concerning these variables?

**3.77** You wish to compare mutual funds that have fees to those that do not have fees. For each of these two groups, for the variables expense ratio in percentage, 2005 return, three-year return, and five-year return,

- Compute the mean, median, first quartile, and third quartile.
- Compute the range, interquartile range, variance, standard deviation, and coefficient of variation.
- Construct a box-and-whisker plot. Are the data skewed? If so, how?
- What conclusions can you reach about differences between mutual funds that have fees and those that do not have fees?

**3.78** You wish to compare mutual funds that have a growth objective to those that have a value objective. For each of these two groups, for the variables expense ratio in percentage, 2005 return, three-year return, and five-year return,

- Compute the mean, median, first quartile, and third quartile.
- Compute the range, interquartile range, variance, standard deviation, and coefficient of variation.
- Construct a box-and-whisker plot. Are the data skewed? If so, how?

**d.** What conclusions can you reach about differences between mutual funds that have a growth objective and those that have a value objective?

**3.79** You wish to compare small cap, mid cap, and large cap mutual funds. For each of these three groups, for the variables expense ratio in percentage, 2005 return, three-year return, and five-year return,

- Compute the mean, median, first quartile, and third quartile.
- Compute the range, interquartile range, variance, standard deviation, and coefficient of variation.
- Construct a box-and-whisker plot. Are the data skewed? If so, how?
- What conclusions can you reach about differences between small cap, mid cap, and large cap mutual funds?

## Student Survey Data Base

**3.80** Problem 1.27 on page 15 describes a survey of 50 undergraduate students (see the file [undergradsurvey.xls](#)). For these data, for each numerical variable

- Compute the mean, median, first quartile, and third quartile.
- Compute the range, interquartile range, variance, standard deviation, and coefficient of variation.
- Construct a box-and-whisker plot. Are the data skewed? If so, how?
- Write a report summarizing your conclusions.

**3.81** Problem 1.27 on page 15 describes a survey of 50 undergraduate students (see the file [undergradsurvey.xls](#)).

- Select a sample of 50 undergraduate students at your school and conduct a similar survey for those students.
- For the data collected in (a), repeat (a) through (d) of Problem 3.80.
- Compare the results of (b) to those of Problem 3.80.

**3.82** Problem 1.28 on page 15 describes a survey of 50 MBA students (see the file [gradsurvey.xls](#)). For these data, for each numerical variable,

- Compute the mean, median, first quartile, and third quartile.
- Compute the range, interquartile range, variance, standard deviation, and coefficient of variation.
- Construct a box-and-whisker plot. Are the data skewed? If so, how?
- Write a report summarizing your conclusions.

**3.83** Problem 1.28 on page 15 describes a survey of 50 MBA students (see the file [gradsurvey.xls](#)).

- Select a sample of 50 graduate students from your MBA program and conduct a similar survey for those students.
- For the data collected in (a), repeat (a) through (d) of Problem 3.82.
- Compare the results of (b) to those of Problem 3.82.

## Managing the *Springville Herald*

For what variable in the Chapter 2 “Managing the *Springville Herald*” case (see page 73) are numerical descriptive measures needed? For the variable you identify:

1. Compute the appropriate numerical descriptive measures, and generate a box-and-whisker plot.
2. Identify another graphical display that might be useful and construct it. What conclusions can you form

from that plot that cannot be made from the box-and-whisker plot?

3. Summarize your findings in a report that can be included with the task force’s study.

## Web Case

Apply your knowledge about the proper use of numerical descriptive measures in this continuing Web Case from Chapter 2.

Visit EndRun Investing Services, at [www.prenhall.com/Springville/EndRun.htm](http://www.prenhall.com/Springville/EndRun.htm) (or open the **EndRun.htm** file in the Student CD-ROM Web Case folder) a second time and reexamine their supporting data and then answer the following:

1. Can descriptive measures be computed for any variables? How would such summary statistics support EndRun’s

claims? How would those summary statistics affect your perception of EndRun’s record?

2. Evaluate the methods EndRun used to summarize the results of its customer survey (see [www.prenhall.com/Springville/ER\\_Survey.htm](http://www.prenhall.com/Springville/ER_Survey.htm) or the **ER\_Survey.htm** file on the Student CD-ROM Web Case folder. Is there anything you would do differently to summarize these results?
3. Note that the last question of the survey has fewer responses than the other questions. What factors may have limited the number of responses to that question?

## REFERENCES

1. Kendall, M. G., A. Stuart, and J. K. Ord, *Kendall’s Advanced Theory of Statistics, Volume 1: Distribution Theory*, 6th ed. (New York: Oxford University Press, 1994).
2. *Microsoft Excel 2007* (Redmond, WA: Microsoft Corporation, 2007).
3. Tukey, J., *Exploratory Data Analysis* (Reading, MA: Addison-Wesley, 1977).
4. Velleman, P. F., and D. C. Hoaglin, *Applications, Basics, and Computing of Exploratory Data Analysis* (Boston: Duxbury Press, 1981).

# Excel Companion

## to Chapter 3

This companion discusses how to compute the descriptive statistics discussed in Chapter 3. (Instructions presented in this companion apply to all Excel versions.)

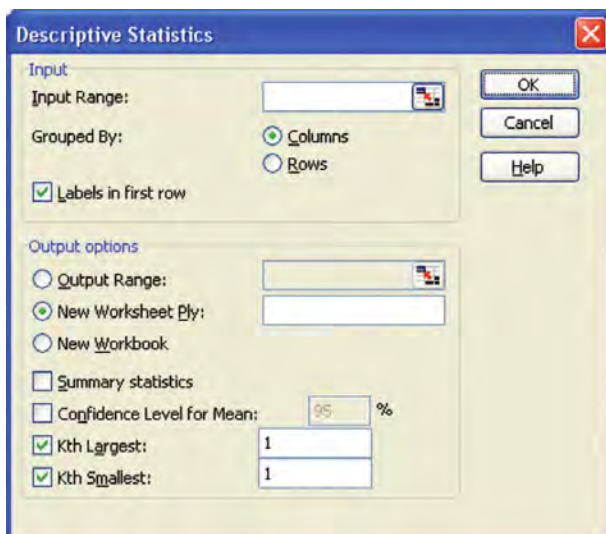
### E3.1 COMPUTING MEASURES OF CENTRAL TENDENCY, VARIATION, AND SHAPE

You compute measures of central tendency, variation, and shape by either using the ToolPak Descriptive Statistics procedure or by using worksheet functions.

*PHStat2 does not contain a descriptive statistics procedure, although some of the descriptive statistics are computed as output options to other procedures. PHStat2 users should use one of the ways discussed below to compute descriptive statistics.*

#### Using ToolPak Descriptive Statistics

Begin the Analysis ToolPak add-in and select **Descriptive Statistics** from the **Analysis Tools** list and then click **OK**. In the Descriptive Statistics dialog box (shown below), enter the cell range of the data as the **Input Range**. Click the **Columns** option and **Labels in first row** (for data organized according to the conventions in “Designing Effective Worksheets” in Section 1.6). Finish by clicking **New**



**Worksheet Ply, Summary statistics, Kth Largest, and Kth Smallest**, and then **OK**. A table of results that does not contain any formulas appears on a new worksheet.

To compute descriptive statistics for a variable by subgroups, as was done in Figure 3.2 on page 114 for the low-risk, average risk, and high-risk subgroups, first sort your data by the categorical variable to which the subgroups belong. Next, repeat the procedure for each subgroup, using the cell range of the subgroup and the **New Worksheet Ply** option each time. Then copy and paste the results found in the second and subsequent worksheets to this first sheet to create a multiple-column table similar to the one shown in Figure 3.2.

When you use this procedure, consider formatting the results to reduce the number of significant digits, many of which may not be accurate. For example, the procedure reported the mean for the low-risk group as 18.4101214574899, but that number has been reformatted to a more manageable 18.4101 in Figure 3.2.

#### Using Worksheet Functions

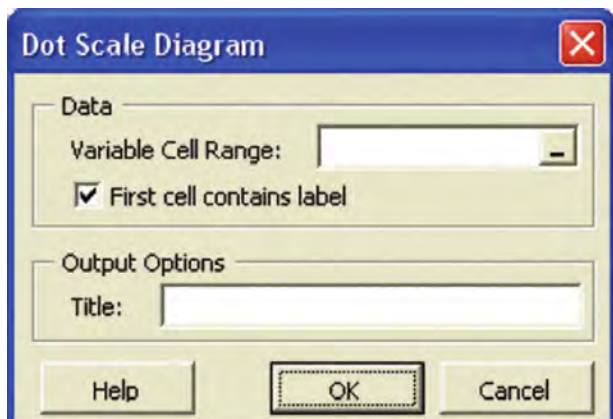
You use the worksheet functions **SUM**, **COUNT**, **AVERAGE** (for mean), **MEDIAN**, **MODE**, **QUARTILE**, or **GEOMEAN** to compute measures of central tendency and **STDEV**, **VAR**, **MIN**, **MAX**, **LARGE**, or **SMALL** to compute measures of shape and variation. For all but **QUARTILE**, you enter a formula in the form **=WorksheetFunction(cell range of data to be summarized)**. For **QUARTILE**, you enter the formula in the form **=QUARTILE(cell range of data to be summarized, quartile number)**. Use 1 as the *quartile number* to compute the first quartile, 2 to compute the second quartile (the median), or 3 to compute the third quartile. In many Excel versions you may encounter some minor errors in results when using the **QUARTILE** function.

### E3.2 CREATING DOT SCALE DIAGRAMS

You create dot scale diagrams by using the PHStat2 Dot Scale Diagram procedure (recommended) or by manually adjusting the **Dot Scale** worksheet of the **Dot Scale.xls** workbook.

## Using PHStat2 Dot Scale Diagram

Open to the worksheet that contains the data to be plotted. Select **PHStat** → **Descriptive Statistics** → **Dot Scale Diagram**. In the Dot Scale Diagram dialog box (shown below) enter the cell range of the data to be plotted as the **Variable Cell Range** and click **First cell contains label**. Enter a title as the **Title** and click **OK**.



## Adjusting the Dot Scale Worksheet

Open to the **DotScale** worksheet. Enter up to 20 values in the tinted cell range **A2:A21**, starting with cell A2, and not skipping any cells. As you enter each value, the dot scale diagram changes to reflect each new value.

**Forum** Click the **ALTERNATE TECHNIQUES** link to learn how to modify this worksheet for sets of data that contain more than 20 values.

## E3.3 COMPUTING MEASURES FOR A POPULATION

You use the worksheet functions **VARP** and **STDEVP** to compute the population variance and standard deviation. To use either function, you enter a formula in the form **=WorksheetFunction(cell range of data to be summarized)**, as shown below.

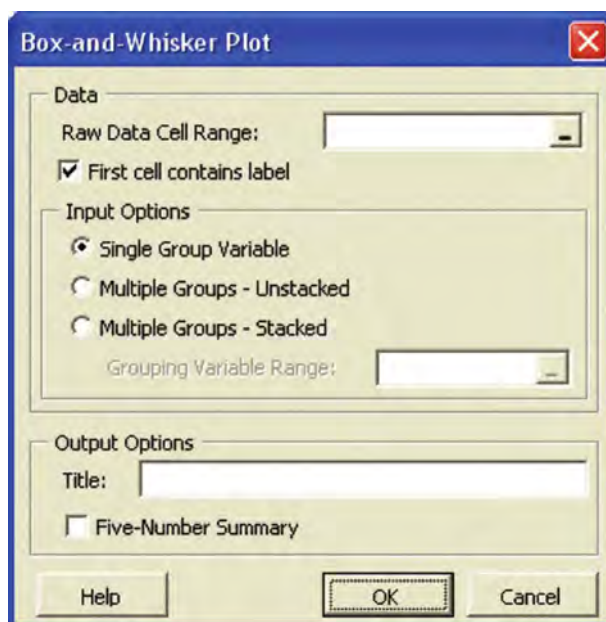
	A	B	
1	<b>Bond Fund</b>	<b>One-Year Return</b>	
2	Pimco: Total Rtn; Inst	2.74	
3	Vanguard Tot Bd; Inv	1.82	
4	American Funds Bond; A	2.25	
5	Vanguard GNMA; Inv	2.88	
6	Franklin CA TF Inc; A	3.66	
7			
8	<b>Population Parameters</b>		
9	<b>Population Variance</b>	<b>0.46</b>	<b>=VARP(B2:B6)</b>
10	<b>Population Std. Deviation</b>	<b>0.68</b>	<b>=STDEVP(B2:B6)</b>

## E3.4 CREATING BOX-AND-WHISKER PLOTS

You create box-and-whisker plots by using the PHStat2 **Box-and-Whisker Plot** procedure (recommended) or by entering a five-number summary into the Plot worksheet of the **Box-and-whisker.xls** workbook.

## Using PHStat2 Box-and-Whisker Plot

Open to the worksheet that contains the data to be plotted. Select **PHStat** → **Descriptive Statistics** → **Box-and-Whisker Plot**. In the Box-and-Whisker Plot dialog box (shown below), enter the cell range of the data to be plotted as the **Raw Data Cell Range**. Click the appropriate option from Input Options and then enter a title as the **Title** and click **OK**.



For the input option, click **Single Group Variable** if your data represents a single group. Click **Multiple Groups-Unstacked** if your cell range contains two or more columns, with each column representing a group. Click **Multiple Groups-Stacked** if you have stacked group data. (You need to enter the cell range of the grouping variable if you choose this option.)

## Using the Plot Worksheet

First, compute the five-number summary (see page 123) for your data. Then open to the **Plot** worksheet of the **Box-and-whisker.xls** workbook and enter this table into the tinted cell range B2:B6. The Plot worksheet can display only one box-and-whisker plot at a time.



**EXAMPLE Figure 3.4 Box-and-Whisker Plot**

To create your own Figure 3.4 box-and-whisker plot of the three-year annualized return for low-risk, average-risk, and high-risk mutual funds, use the appropriate set of instructions.

**Basic Excel** You can create three separate plots that are equivalent to Figure 3.4. Open the **Box-and-whisker.xls** workbook and create three copies of the **Plot** worksheet. Compute the five-number summary for the low-risk, average-risk, and high-risk mutual funds (see the Data worksheet of the **Mutual Funds.xls** workbook). Enter each five-number summary into a different copy of the Plot worksheet.

**PHStat2** Open to the Data sheet of the **Mutual Funds.xls** workbook. Select **PHStat** → **Descriptive Statistics** → **Box-and-Whisker Plot**. Enter **H1:H839** as the **Raw Data Cell Range** and click the **Multiple Groups–Stacked** option. This option activates the **Grouping Variable Range** box. In that box enter **F1:F839**. Enter a title as the **Title** and click **OK**.

**E3.5 COMPUTING THE COVARIANCE**

Open to the **Covariance** worksheet of the **Covariance.xls** workbook, shown in Figure 3.6 on page 128. If you want to use this worksheet with other pairs of variables, follow the instructions in the worksheet for modifying the table area. The worksheet gains its flexibility by the cell C18 formula that uses the COUNT function to determine the sample size  $n$ . This allows the worksheet to always use the proper value of  $n - 1$  for the covariance calculation when you change the size of the table area.

**E3.6 COMPUTING THE CORRELATION COEFFICIENT**

Open the **Correlation** worksheet of the **Correlation.xls** workbook, shown in Figure 3.9 on page 131. If you want to use this worksheet with other pairs of variables, follow the instructions in the worksheet for modifying the table area. This worksheet shares some of the design of the covariance worksheet discussed in the previous section.

This worksheet uses the CORREL function in the formula =CORREL(A2:A11, B2:B11) in cell E21 to compute the correlation coefficient. Because the covariance,  $S_x$ , and  $S_y$  are computed elsewhere in this worksheet, the formula =E18/(E19 \* E20) could also be used to compute the correlation coefficient statistic.



# CHAPTER 4

## Basic Probability

### USING STATISTICS @ The Consumer Electronics Company

#### 4.1 BASIC PROBABILITY CONCEPTS

Events and Sample Spaces  
Contingency Tables  
Simple (Marginal) Probability  
Joint Probability  
General Addition Rule

#### 4.2 CONDITIONAL PROBABILITY

Computing Conditional Probabilities  
Decision Trees  
Statistical Independence

Multiplication Rules  
Marginal Probability Using the General  
Multiplication Rule

#### 4.3 BAYES' THEOREM

#### 4.4 ETHICAL ISSUES AND PROBABILITY

#### 4.5 (CD-ROM TOPIC) COUNTING RULES

#### EXCEL COMPANION TO CHAPTER 4

E4.1 Computing Basic Probabilities  
E4.2 Using Bayes' Theorem

### LEARNING OBJECTIVES

In this chapter, you learn:

- Basic probability concepts
- Conditional probability
- To use Bayes' theorem to revise probabilities

## USING STATISTICS @ The Consumer Electronics Company



As the marketing manager for the Consumer Electronics Company, you are analyzing the survey results of an intent-to-purchase study. This study asked the heads of 1,000 households about their intentions to purchase a big-screen television (defined as 36 inches or larger) sometime during the next 12 months. Investigations of this type are known as *intent-to-purchase studies*. As a follow-up, you plan to survey the same people 12 months later to see whether such a television was purchased. In addition, for households purchasing big-screen televisions, you would like to know whether the television they purchased was a plasma screen, whether they also purchased a digital video recorder (DVR) in the past 12 months, and whether they were satisfied with their purchase of the big-screen television.

You are expected to use the results of this survey to plan a new marketing strategy that will enhance sales and better target those households likely to purchase multiple or more expensive products. What questions can you ask in this survey? How can you express the relationships among the various intent-to-purchase responses of individual households?

In previous chapters, you learned descriptive methods to summarize categorical and numerical variables. In this chapter, you will learn about probability to answer questions such as the following:

- What is the probability that a household is planning to purchase a big-screen television in the next year?
- What is the probability that a household will actually purchase a big-screen television?
- What is the probability that a household is planning to purchase a big-screen television and actually purchases the television?
- Given that the household is planning to purchase a big-screen television, what is the probability that the purchase is made?
- Does knowledge of whether a household *plans* to purchase the television change the likelihood of predicting whether the household *will* purchase the television?
- What is the probability that a household that purchases a big-screen television will purchase a plasma-screen television?
- What is the probability that a household that purchases a big-screen television will also purchase a DVR?
- What is the probability that a household that purchases a big-screen television will be satisfied with the purchase?

With answers to questions such as these, you can begin to make decisions about your marketing strategy. Should your strategy for selling more big-screen televisions target those households that have indicated an intent to purchase? Should you concentrate on selling plasma screens? Is it likely that households that purchase televisions with plasma screens can be easily persuaded to also purchase DVRs?

**T**he principles of probability help bridge the worlds of descriptive statistics and inferential statistics. Reading this chapter will help you learn about different types of probabilities, how to compute probability values, and then revise those values in light of new information. Probability principles are the foundation for the probability distribution, the concept of mathematical expectation, and the binomial, hypergeometric, and Poisson distributions, topics that are discussed in Chapter 5.

## 4.1 BASIC PROBABILITY CONCEPTS

What is meant by the word *probability*? A **probability** is the numeric value representing the chance, likelihood, or possibility a particular event will occur, such as the price of a stock increasing, a rainy day, a nonconforming unit of production, or the outcome five in a single toss of a die. In all these instances, the probability involved is a proportion or fraction whose value ranges between 0 and 1, inclusive. An event that has no chance of occurring (i.e., the **impossible event**) has a probability of 0. An event that is sure to occur (i.e., the **certain event**) has a probability of 1. There are three approaches to the subject of probability:

- *A priori* classical probability
- Empirical classical probability
- Subjective probability

In *a priori* classical probability, the probability of success is based on prior knowledge of the process involved. In the simplest case, where each outcome is equally likely, the chance of occurrence of the event is defined in Equation (4.1).

### PROBABILITY OF OCCURRENCE

$$\text{Probability of occurrence} = \frac{X}{T} \quad (4.1)$$

where

$X$  = number of ways in which the event occurs

$T$  = total number of possible outcomes

Consider a standard deck of cards that has 26 red cards and 26 black cards. The probability of selecting a black card is  $26/52 = 0.50$  because there are  $X = 26$  black cards and  $T = 52$  total cards. What does this probability mean? If each card is replaced after it is drawn, does it mean that 1 out of the next 2 cards selected will be black? No, because you cannot say for certain what will happen on the next several selections. However, you can say that in the long run, if this selection process is continually repeated, the proportion of black cards selected will approach 0.50.

### EXAMPLE 4.1

#### FINDING A PRIORI PROBABILITIES

A standard six-sided die has six faces. Each face of the die contains either one, two, three, four, five, or six dots. If you roll a die, what is the probability that you will get a face with five dots?

**SOLUTION** Each face is equally likely to occur. Because there are six faces, the probability of getting a face with five dots is  $\frac{1}{6}$ .

The preceding examples use the *a priori* classical probability approach because the number of ways the event occurs and the total number of possible outcomes are known from the composition of the deck of cards or the faces of the die.

In the **empirical classical probability** approach, the outcomes are based on observed data, not on prior knowledge of a process. Examples of this type of probability are the proportion of individuals in the Using Statistics scenario who actually purchase a big-screen television, the proportion of registered voters who prefer a certain political candidate, and the proportion of students who have part-time jobs. For example, if you take a survey of students, and 60% state that they have part-time jobs, then there is a 0.60 probability that an individual student has a part-time job.

The third approach to probability, **subjective probability**, differs from the other two approaches because subjective probability differs from person to person. For example, the development team for a new product may assign a probability of 0.6 to the chance of success for the product, while the president of the company may be less optimistic and assign a probability of 0.3. The assignment of subjective probabilities to various outcomes is usually based on a combination of an individual's past experience, personal opinion, and analysis of a particular situation. Subjective probability is especially useful in making decisions in situations in which you cannot use *a priori* classical probability or empirical classical probability.

## Events and Sample Spaces

The basic elements of probability theory are the individual outcomes of a variable under study. You need the following definitions to understand probabilities.

### EVENT

Each possible outcome of a variable is referred to as an **event**. A **simple event** is described by a single characteristic.

For example, when you toss a coin, the two possible outcomes are heads and tails. Each of these represents a simple event. When you roll a standard six-sided die in which the six faces of the die contain either one, two, three, four, five, or six dots, there are six possible simple events. An event can be any one of these simple events, a set of them, or a subset of all of them. For example, the event of an *even number of dots* consists of three simple events (i.e., two, four, or six dots).

### JOINT EVENT

A **joint event** is an event that has two or more characteristics.

Getting two heads on the toss of two coins is an example of a joint event because it consists of heads on the toss of the first coin and heads on the toss of the second coin.

### COMPLEMENT

The **complement** of event  $A$  (represented by the symbol  $A'$ ) includes all events that are not part of  $A$ .

The complement of a head is a tail because that is the only event that is not a head. The complement of face five is not getting face five. Not getting face five consists of getting face one, two, three, four, or six.

### SAMPLE SPACE

The collection of all the possible events is called the **sample space**.

The sample space for tossing a coin consists of heads and tails. The sample space when rolling a die consists of one, two, three, four, five, and six dots.

Example 4.2 demonstrates events and sample spaces.

**EXAMPLE 4.2**      **EVENTS AND SAMPLE SPACES**

The Using Statistics scenario on page 148 concerns the Consumer Electronics Company. Table 4.1 presents the results of the sample of 1,000 households in terms of purchase behavior for big-screen televisions.

**TABLE 4.1**  
Purchase Behavior for  
Big-Screen Televisions

PLANNED TO PURCHASE	ACTUALLY PURCHASED		
	Yes	No	Total
Yes	200	50	250
No	100	650	750
Total	300	700	1,000

What is the sample space? Give examples of simple events and joint events.

**SOLUTION** The sample space consists of the 1,000 respondents. Simple events are “planned to purchase,” “did not plan to purchase,” “purchased,” and “did not purchase.” The complement of the event “planned to purchase” is “did not plan to purchase.” The event “planned to purchase and actually purchased” is a joint event because the respondent must plan to purchase the television *and* actually purchase it.

### Contingency Tables

There are several ways to present a sample space. The method used in this text is called a **contingency table** (see Section 2.4). You get the values in the cells of the table by subdividing the sample space of 1,000 households according to whether someone planned to purchase and actually purchased the big-screen television. For example, 200 of the respondents planned to purchase a big-screen television and subsequently did purchase the big-screen television.

### Simple (Marginal) Probability

Now you can answer some of the questions posed in the Using Statistics scenario. Because the results are based on data collected in a survey (refer to Table 4.1 above), you can use the empirical classical probability approach.

As stated previously, the most fundamental rule for probabilities is that they range in value from 0 to 1. An impossible event has a probability of 0, and an event that is certain to occur has a probability of 1.

**Simple probability** refers to the probability of occurrence of a simple event,  $P(A)$ . A simple probability in the Using Statistics scenario is the probability of planning to purchase a big-screen television. How can you determine the probability of selecting a household that planned to purchase a big-screen television? Using Equation (4.1) on page 149:

$$\begin{aligned} \text{Probability of occurrence} &= \frac{X}{T} \\ P(\text{Planned to purchase}) &= \frac{\text{Number who planned to purchase}}{\text{Total number of households}} \\ &= \frac{250}{1,000} = 0.25 \end{aligned}$$

Thus, there is a 0.25 (or 25%) chance that a household planned to purchase a big-screen television.

Simple probability is also called **marginal probability** because it enables you to compute the total number of successes (those who planned to purchase) from the appropriate margin of the contingency table (see Table 4.1 on page 151). Example 4.3 illustrates another application of simple probability.

**EXAMPLE 4.3****COMPUTING THE PROBABILITY THAT THE BIG-SCREEN TELEVISION PURCHASED IS A PLASMA SCREEN**

In the Using Statistics follow-up survey, additional questions were asked of the 300 households that actually purchased big-screen televisions. Table 4.2 indicates the consumers' responses to whether the television purchased was a plasma screen and whether they also purchased a DVR in the past 12 months.

**TABLE 4.2**

Purchase Behavior Regarding Plasma-Screen Televisions and DVR

PURCHASED PLASMA SCREEN	PURCHASED DVR		
	Yes	No	Total
Plasma screen	38	42	80
Not plasma screen	70	150	220
<b>Total</b>	108	192	300

Find the probability that if a household that purchased a big-screen television is randomly selected, the television purchased is a plasma screen.

**SOLUTION** Using the following definitions:

$A$  = purchased a plasma screen

$B$  = purchased a DVR

$A'$  = did not purchase a plasma screen

$B'$  = did not purchase a DVR

$$\begin{aligned}
 P(\text{Plasma screen}) &= \frac{\text{Number of plasma screen televisions}}{\text{Total number of televisions}} \\
 &= \frac{80}{300} = 0.267
 \end{aligned}$$

There is a 26.7% chance that a randomly selected big-screen television purchase is a purchase of a plasma-screen television.

## Joint Probability

Whereas marginal probability refers to the probability of occurrence of simple events, **joint probability** refers to the probability of an occurrence involving two or more events. An example of joint probability is the probability that you will get heads on the first toss of a coin and heads on the second toss of a coin.

Referring to Table 4.1 on page 151, those individuals who planned to purchase and actually purchased a big-screen television consist only of the outcomes in the single cell “yes—planned to purchase *and* yes—actually purchased.” Because this group consists of 200 households, the



probability of picking a household that planned to purchase *and* actually purchased a big-screen television is

$$\begin{aligned} P(\text{Planned to purchase} \\ \text{and actually purchased}) &= \frac{\text{Planned to purchase and actually purchased}}{\text{Total number of respondents}} \\ &= \frac{200}{1,000} = 0.20 \end{aligned}$$

Example 4.4 also demonstrates how to determine joint probability.

#### EXAMPLE 4.4

#### DETERMINING THE JOINT PROBABILITY THAT A BIG-SCREEN TELEVISION CUSTOMER PURCHASED A PLASMA-SCREEN TELEVISION AND A DVR

In Table 4.2 on page 152, the purchases are cross-classified as plasma screen or not plasma screen and whether or not the household purchased a DVR. Find the probability that a randomly selected household that purchased a big-screen television also purchased a plasma-screen television and a DVR.

**SOLUTION** Using Equation (4.1) on page 149,

$$\begin{aligned} P(\text{Plasma screen and DVR}) &= \frac{\text{Number that purchased a plasma screen and a DVR}}{\text{Total number of big-screen television purchasers}} \\ &= \frac{38}{300} = 0.127 \end{aligned}$$

Therefore, there is a 12.7% chance that a randomly selected household that purchased a big-screen television purchased a plasma-screen television and a DVR.

You can view the marginal probability of a particular event by using the concept of joint probability just discussed. The marginal probability of an event consists of a set of joint probabilities. For example, if  $B$  consists of two events,  $B_1$  and  $B_2$ , then  $P(A)$ , the probability of event  $A$ , consists of the joint probability of event  $A$  occurring with event  $B_1$  and the joint probability of event  $A$  occurring with event  $B_2$ . Use Equation (4.2) to compute marginal probabilities.

#### MARGINAL PROBABILITY

$$P(A) = P(A \text{ and } B_1) + P(A \text{ and } B_2) + \dots + P(A \text{ and } B_k) \quad (4.2)$$

where  $B_1, B_2, \dots, B_k$  are  $k$  mutually exclusive and collectively exhaustive events.

Mutually exclusive events and collectively exhaustive events are defined as follows.

#### MUTUALLY EXCLUSIVE

Two events are **mutually exclusive** if both the events cannot occur simultaneously.

Heads and tails in a coin toss are mutually exclusive events. The result of a coin toss cannot simultaneously be a head and a tail.

#### COLLECTIVELY EXHAUSTIVE

A set of events is **collectively exhaustive** if one of the events must occur.

Heads and tails in a coin toss are collectively exhaustive events. One of them must occur. If heads does not occur, tails must occur. If tails does not occur, heads must occur.

Being male and being female are mutually exclusive and collectively exhaustive events. No one is both (the two are mutually exclusive), and everyone is one or the other (the two are collectively exhaustive).

You can use Equation (4.2) to compute the marginal probability of “planned to purchase” a big-screen television:

$$\begin{aligned} P(\text{Planned to purchase}) &= P(\text{Planned to purchase and purchased}) \\ &\quad + P(\text{Planned to purchase and did not purchase}) \\ &= \frac{200}{1,000} + \frac{50}{1,000} \\ &= \frac{250}{1,000} = 0.25 \end{aligned}$$

You get the same result if you add the number of outcomes that make up the simple event “planned to purchase.”

### General Addition Rule

The general addition rule allows you to find the probability of event “*A or B*.” This rule considers the occurrence of either event *A* or event *B* or both *A* and *B*. How can you determine the probability that a household planned to purchase *or* actually purchased a big-screen television? The event “planned to purchase *or* actually purchased” includes all households that planned to purchase and all households that actually purchased the big-screen television. You examine each cell of the contingency table (Table 4.1 on page 151) to determine whether it is part of this event. From Table 4.1, the cell “planned to purchase *and* did not actually purchase” is part of the event because it includes respondents who planned to purchase. The cell “did not plan to purchase *and* actually purchased” is included because it contains respondents who actually purchased. Finally, the cell “planned to purchase *and* actually purchased” has both characteristics of interest. Therefore, the probability of “planned to purchase *or* actually purchased” is

$$\begin{aligned} P(\text{Planned to purchase or actually purchased}) &= P(\text{Planned to purchase and did not actually} \\ &\quad \text{purchase}) + P(\text{Did not plan to purchase and} \\ &\quad \text{actually purchased}) + P(\text{Planned to purchase} \\ &\quad \text{and actually purchased}) \\ &= \frac{50}{1,000} + \frac{100}{1,000} + \frac{200}{1,000} = \frac{350}{1,000} = 0.35 \end{aligned}$$

Often, it is easier to determine  $P(A \text{ or } B)$ , the probability of the event *A or B*, by using the **general addition rule**, defined in Equation (4.3).

#### GENERAL ADDITION RULE

The probability of *A or B* is equal to the probability of *A* plus the probability of *B* minus the probability of *A and B*.

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) \quad (4.3)$$

Applying Equation (4.3) to the previous example produces the following result:

$$\begin{aligned}
 P(\text{Planned to purchase } \textit{or} \text{ actually purchased}) &= P(\text{Planned to purchase}) + P(\text{Actually purchased}) \\
 &\quad - P(\text{Planned to purchase } \textit{and} \text{ actually purchased}) \\
 &= \frac{250}{1,000} + \frac{300}{1,000} - \frac{200}{1,000} \\
 &= \frac{350}{1,000} = 0.35
 \end{aligned}$$

The general addition rule consists of taking the probability of  $A$  and adding it to the probability of  $B$  and then subtracting the probability of the joint event  $A$  and  $B$  from this total because the joint event has already been included in computing both the probability of  $A$  and the probability of  $B$ . Referring to Table 4.1 on page 151, if the outcomes of the event “planned to purchase” are added to those of the event “actually purchased,” the joint event “planned to purchase *and* actually purchased” has been included in each of these simple events. Therefore, because this joint event has been double-counted, you must subtract it to provide the correct result. Example 4.5 illustrates another application of the general addition rule.

#### EXAMPLE 4.5

#### USING THE GENERAL ADDITION RULE FOR THE HOUSEHOLDS THAT PURCHASED BIG-SCREEN TELEVISIONS

In Example 4.3 on page 152, the purchases were cross-classified as a plasma screen or not a plasma screen and whether or not the household purchased a DVR. Find the probability that among households that purchased a big-screen television, they purchased a plasma-screen television or a DVR.

**SOLUTION** Using Equation (4.3),

$$\begin{aligned}
 P(\text{Plasma screen } \textit{or} \text{ DVR}) &= P(\text{Plasma screen}) + P(\text{DVR}) - P(\text{Plasma screen } \textit{and} \text{ DVR}) \\
 &= \frac{80}{300} + \frac{108}{300} - \frac{38}{300} \\
 &= \frac{150}{300} = 0.50
 \end{aligned}$$

Therefore, there is a 50.0% chance that a randomly selected household that purchased a big-screen television purchased a plasma-screen television or a DVR.

## PROBLEMS FOR SECTION 4.1

### Learning the Basics

PH Grade  
ASSIST

**4.1** Two coins are tossed.

- Give an example of a simple event.
- Give an example of a joint event.
- What is the complement of a head on the first toss?

**4.2** An urn contains 12 red balls and 8 white balls. One ball is to be selected from the urn.

- Give an example of a simple event.
- What is the complement of a red ball?

**PH Grade ASSIST** 4.3 Given the following contingency table:

	$B$	$B'$
$A$	10	20
$A'$	20	40

What is the probability of

- event  $A$ ?
- event  $A'$ ?
- event  $A$  and  $B$ ?
- event  $A$  or  $B$ ?

**PH Grade ASSIST** 4.4 Given the following contingency table:

	$B$	$B'$
$A$	10	30
$A'$	25	35

What is the probability of

- event  $A'$ ?
- event  $A$  and  $B$ ?
- event  $A'$  and  $B'$ ?
- event  $A'$  or  $B'$ ?

### Applying the Concepts

**PH Grade ASSIST** 4.5 For each of the following, indicate whether the type of probability involved is an example of *a priori* classical probability, empirical classical probability, or subjective probability.

- The next toss of a fair coin will land on heads.
- Italy will win soccer's World Cup the next time the competition is held.
- The sum of the faces of two dice will be seven.
- The train taking a commuter to work will be more than 10 minutes late.

4.6 For each of the following, state whether the events created are mutually exclusive and collectively exhaustive. If they are not mutually exclusive and collectively exhaustive, either reword the categories to make them mutually exclusive and collectively exhaustive or explain why that would not be useful.

- Registered voters in the United States were asked whether they registered as Republicans or Democrats.
- Each respondent was classified by the type of car he or she drives: American, European, Japanese, or none.
- People were asked, "Do you currently live in (i) an apartment or (ii) a house?"
- A product was classified as defective or not defective.

4.7 The probability of each of the following events is zero. For each, state why.

- A voter in the United States who is registered as a Republican and a Democrat

- A product that is defective and not defective
- An automobile that is a Ford and a Toyota

4.8 According to an Ipsos poll, the perception of unfairness in the U.S. tax code is spread fairly evenly across income groups, age groups, and education levels. In an April 2006 survey of 1,005 adults, Ipsos reported that almost 60% of all people said the code is unfair, while slightly more than 60% of those making more than \$50,000 viewed the code as unfair ("People Cry Unfairness," *The Cincinnati Enquirer*, April 16, 2006, p. A8). Suppose that the following contingency table represents the specific breakdown of responses:

U.S. TAX CODE	INCOME LEVEL		Total
	Less Than \$50,000	More Than \$50,000	
Fair	225	180	405
Unfair	280	320	600
Total	505	500	1,005

- Give an example of a simple event.
- Give an example of a joint event.
- What is the complement of "tax code is fair"?
- Why is "tax code is fair and makes less than \$50,000" a joint event?

4.9 Referring to the contingency table in Problem 4.8, if a respondent is selected at random, what is the probability that he or she

- thinks the tax code is unfair?
- thinks the tax code is unfair and makes less than \$50,000?
- thinks the tax code is unfair or makes less than \$50,000?
- Explain the difference in the results in (b) and (c).



4.10 A yield improvement study at a semiconductor manufacturing facility provided defect data for a sample of 450 wafers. The following table presents a summary of the responses to two questions: "Was a particle found on the die that produced the wafer?" and "Is the wafer good or bad?"

QUALITY OF WAFER	CONDITION OF DIE		
	No Particles	Particles	Totals
Good	320	14	334
Bad	80	36	116
Totals	400	50	450

Source: Extracted from S. W. Hall, "Analysis of Defectivity of Semiconductor Wafers by Contingency Table," Proceedings Institute of Environmental Sciences, Vol. 1, 1994, pp. 177-183.

- Give an example of a simple event.
- Give an example of a joint event.

- c. What is the complement of a good wafer?
- d. Why is a “good wafer” and a die “with particles” a joint event?

- 4.11** Referring to the contingency table in Problem 4.10, if a wafer is selected at random, what is the probability that
- a. it was produced from a die with no particles?
  - b. it is a bad wafer and was produced from a die with no particles?
  - c. it is a bad wafer or was produced from a die with particles?
  - d. Explain the difference in the results in (b) and (c).

**4.12** An experiment was conducted to study the choices made in mutual fund selection. Undergraduate and MBA students were presented with different S&P 500 Index funds that were identical except for fees. Suppose 100 undergraduate students and 100 MBA students were selected. Partial results are shown in the following table:

FUND	STUDENT GROUP	
	Undergraduate	MBA
Highest-cost fund	27	18
Not Highest-cost fund	73	82

Source: Extracted from J. J. Choi, D. Laibson, and B. C. Madrian, Why Does the Law of One Price Fail? [www.som.yale.edu/faculty/jjk83/kes.pdf](http://www.som.yale.edu/faculty/jjk83/kes.pdf)

If a student is selected at random, what is the probability that he or she

- a. selected the highest-cost fund?
- b. selected the highest-cost fund *and* is an undergraduate?
- c. selected the highest-cost fund *or* is an undergraduate?
- d. Explain the difference in the results in (b) and (c).

**4.13** Where people turn to for news is different for various age groups. Suppose that a study conducted on this issue (extracted from P. Johnson, “Young People Turn to the Web for News,” *USA Today*, March 23, 2006, p. 9D) was based on 200 respondents who were between the ages of 36 and 50 and 200 respondents who were over age 50. Of the 200 respondents who were between the ages of 36 and 50, 82 got their news primarily from newspapers. Of

the 200 respondents who were over age 50, 104 got their news primarily from newspapers. Construct a contingency table to evaluate the probabilities. If a respondent is selected at random, what is the probability that he or she

- a. got news primarily from newspapers?
- b. got news primarily from newspapers and is over 50 years old?
- c. got news primarily from newspapers or is over 50 years old?
- d. Explain the difference in the results in (b) and (c).

**4.14** A sample of 500 respondents was selected in a large metropolitan area to study consumer behavior. Among the questions asked was “Do you enjoy shopping for clothing?” Of 240 males, 136 answered yes. Of 260 females, 224 answered yes. Construct a contingency table to evaluate the probabilities. What is the probability that a respondent chosen at random

- a. enjoys shopping for clothing?
- b. is a female and enjoys shopping for clothing?
- c. is a female or enjoys shopping for clothing?
- d. is a male or a female?

**4.15** Each year, ratings are compiled concerning the performance of new cars during the first 90 days of use. Suppose that the cars have been categorized according to whether the car needs warranty-related repair (yes or no) and the country in which the company manufacturing the car is based (United States or not United States). Based on the data collected, the probability that the new car needs warranty repair is 0.04, the probability that the car was manufactured by a U.S.-based company is 0.60, and the probability that the new car needs a warranty repair *and* was manufactured by a U.S.-based company is 0.025. Construct a contingency table to evaluate the probabilities of a warranty-related repair. What is the probability that a new car selected at random

- a. needs a warranty repair?
- b. needs a warranty repair and was manufactured by a U.S.-based company?
- c. needs a warranty repair or was manufactured by a U.S.-based company?
- d. needs a warranty repair or was not manufactured by a U.S.-based company?

## 4.2 CONDITIONAL PROBABILITY

Each example in Section 4.1 involves finding the probability of an event when sampling from the entire sample space. How do you determine the probability of an event if certain information about the events involved is already known?

### Computing Conditional Probabilities

**Conditional probability** refers to the probability of event  $A$ , given information about the occurrence of another event  $B$ .

**CONDITIONAL PROBABILITY**

The probability of  $A$  given  $B$  is equal to the probability of  $A$  and  $B$  divided by the probability of  $B$ .

$$P(A | B) = \frac{P(A \text{ and } B)}{P(B)} \quad (4.4a)$$

The probability of  $B$  given  $A$  is equal to the probability of  $A$  and  $B$  divided by the probability of  $A$ .

$$P(B | A) = \frac{P(A \text{ and } B)}{P(A)} \quad (4.4b)$$

where

$P(A \text{ and } B)$  = joint probability of  $A$  and  $B$

$P(A)$  = marginal probability of  $A$

$P(B)$  = marginal probability of  $B$

Referring to the Using Statistics scenario involving the purchase of big-screen televisions, suppose you were told that a household planned to purchase a big-screen television. Now, what is the probability that the household actually purchased the television? In this example, the objective is to find  $P(\text{Actual purchase} | \text{Planned to purchase})$ . Here you are given the information that the household planned to purchase the big-screen television. Therefore, the sample space does not consist of all 1,000 households in the survey. It consists of only those households that planned to purchase the big-screen television. Of 250 such households, 200 actually purchased the big-screen television. Therefore, based on Table 4.1 on page 151, the probability that a household actually purchased the big-screen television given that he or she planned to purchase is

$$\begin{aligned} P(\text{Actually purchased} | \text{Planned to purchase}) &= \frac{\text{Planned to purchase and actually purchased}}{\text{Planned to purchase}} \\ &= \frac{200}{250} = 0.80 \end{aligned}$$

You can also use Equation (4.4b) to compute this result:

$$P(B | A) = \frac{P(A \text{ and } B)}{P(A)}$$

where

$A$  = planned to purchase

$B$  = actually purchased

then

$$\begin{aligned} P(\text{Actually purchased} | \text{Planned to purchase}) &= \frac{200/1,000}{250/1,000} \\ &= \frac{200}{250} = 0.80 \end{aligned}$$

Example 4.6 further illustrates conditional probability.

**EXAMPLE 4.6****FINDING A CONDITIONAL PROBABILITY OF PURCHASING A DVR**

Table 4.2 on page 152 is a contingency table for whether the household purchased a plasma-screen television and whether the household purchased a DVR. Of the households that purchased plasma-screen televisions, what is the probability that they also purchased DVRs?

**SOLUTION** Because you know that the household purchased a plasma-screen television, the sample space is reduced to 80 households. Of these 80 households, 38 also purchased a DVR. Therefore, the probability that a household purchased a DVR, given that the household purchased a plasma-screen television, is:

$$\begin{aligned} P(\text{Purchased DVR} \mid \text{Purchased plasma screen}) &= \frac{\text{Number purchasing plasma screen and DVR}}{\text{Number purchasing plasma screen}} \\ &= \frac{38}{80} = 0.475 \end{aligned}$$

If you use Equation (4.4a) on page 158:

$$A = \text{Purchased DVR} \quad B = \text{Purchased plasma-screen television}$$

then

$$P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)} = \frac{38/300}{80/300} = 0.475$$

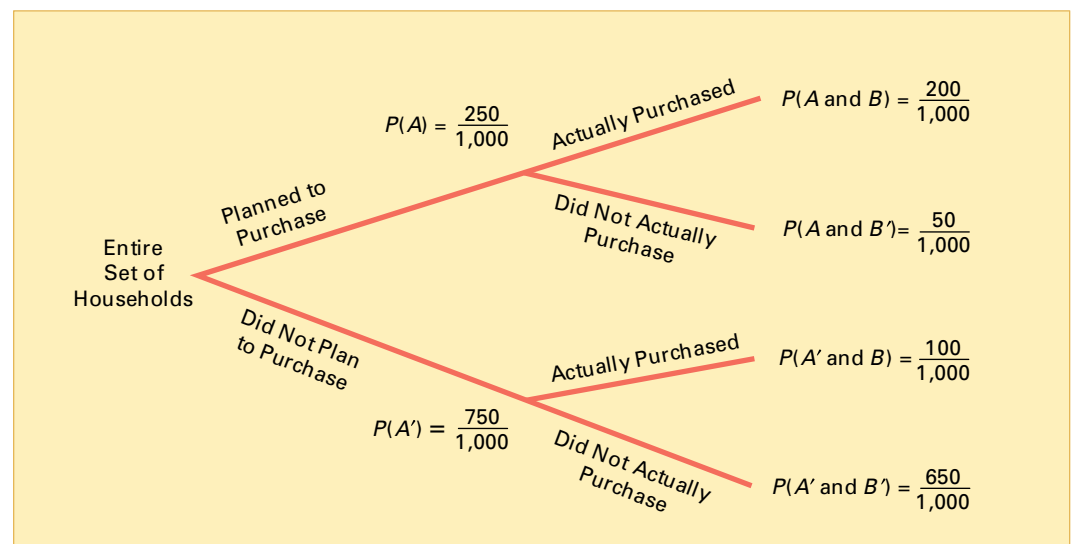
Therefore, given that the household purchased a plasma-screen television, there is a 47.5% chance that the household also purchased a DVR. You can compare this conditional probability to the marginal probability of purchasing a DVR, which is  $108/300 = 0.36$ , or 36%. These results tell you that households that purchased plasma-screen televisions are more likely to purchase DVRs than are households that purchased big-screen televisions that are not plasma-screen televisions.

**Decision Trees**

In Table 4.1 on page 151, households are classified according to whether they planned to purchase and whether they actually purchased big-screen televisions. A **decision tree** is an alternative to the contingency table. Figure 4.1 represents the decision tree for this example.

**FIGURE 4.1**

Decision tree for the Consumer Electronics Company example



In Figure 4.1, beginning at the left with the entire set of households, there are two “branches” for whether or not the household planned to purchase a big-screen television. Each of these branches has two subbranches, corresponding to whether the household actually purchased or did not actually purchase the big-screen television. The probabilities at the end of the initial branches represent the marginal probabilities of  $A$  and  $A'$ . The probabilities at the end of each of the four subbranches represent the joint probability for each combination of events  $A$  and  $B$ . You compute the conditional probability by dividing the joint probability by the appropriate marginal probability.

For example, to compute the probability that the household actually purchased, given that the household planned to purchase the big-screen television, take  $P(\text{Planned to purchase and actually purchased})$  and divide by  $P(\text{Planned to purchase})$ . From Figure 4.1:

$$\begin{aligned} P(\text{Actually purchased} \mid \text{Planned to purchase}) &= \frac{200/1,000}{250/1,000} \\ &= \frac{200}{250} = 0.80 \end{aligned}$$

Example 4.7 illustrates how to construct a decision tree.

#### EXAMPLE 4.7

#### FORMING THE DECISION TREE FOR THE HOUSEHOLDS THAT PURCHASED BIG-SCREEN TELEVISIONS

Using the cross-classified data in Table 4.2 on page 152, construct the decision tree. Use the decision tree to find the probability that a household purchased a DVR, given that the household purchased a plasma-screen television.

**SOLUTION** The decision tree for purchased a DVR and a plasma-screen television is displayed in Figure 4.2. Using Equation (4.4b) on page 158 and the following definitions,

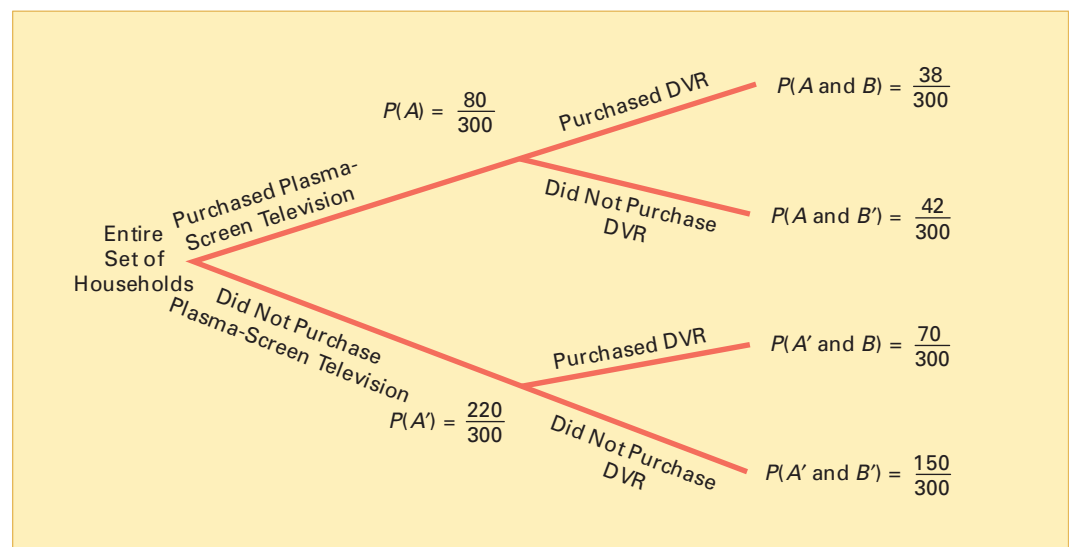
$A$  = Purchased plasma-screen television

$B$  = Purchased DVR

$$P(B \mid A) = \frac{P(A \text{ and } B)}{P(A)} = \frac{38/300}{80/300} = 0.475$$

**FIGURE 4.2**

Decision tree for purchased a DVR and a plasma-screen television





## Statistical Independence

In the example concerning the purchase of big-screen televisions, the conditional probability is  $200/250 = 0.80$  that the selected household actually purchased the big-screen television, given that the household planned to purchase. The simple probability of selecting a household that actually purchased is  $300/1,000 = 0.30$ . This result shows that the prior knowledge that the household planned to purchase affected the probability that the household actually purchased the television. In other words, the outcome of one event is *dependent* on the outcome of a second event.

When the outcome of one event does *not* affect the probability of occurrence of another event, the events are said to be statistically independent. **Statistical independence** can be determined by using Equation (4.5).

### STATISTICAL INDEPENDENCE

Two events,  $A$  and  $B$ , are statistically independent if and only if

$$P(A | B) = P(A) \quad (4.5)$$

where

$P(A | B)$  = conditional probability of  $A$  given  $B$

$P(A)$  = marginal probability of  $A$

Example 4.8 demonstrates the use of Equation (4.5).

### EXAMPLE 4.8

### DETERMINING STATISTICAL INDEPENDENCE

In the follow-up survey of the 300 households that actually purchased big-screen televisions, the households were asked if they were satisfied with their purchases. Table 4.3 cross-classifies the responses to the satisfaction question with the responses to whether the television was a plasma-screen television.

**TABLE 4.3**

Satisfaction with Purchase of Big-Screen Televisions

TYPE OF TELEVISION	SATISFIED WITH PURCHASE?		
	Yes	No	Total
Plasma screen	64	16	80
Not plasma screen	176	44	220
<b>Total</b>	240	60	300

Determine whether being satisfied with the purchase and type of television purchased are statistically independent.

**SOLUTION** For these data,

$$P(\text{Satisfied} | \text{Plasma screen}) = \frac{64/300}{80/300} = \frac{64}{80} = 0.80$$

which is equal to

$$P(\text{Satisfied}) = \frac{240}{300} = 0.80$$

Thus, being satisfied with the purchase and type of television purchased are statistically independent. Knowledge of one event does not affect the probability of the other event.

## Multiplication Rules

By manipulating the formula for conditional probability, you can determine the joint probability  $P(A \text{ and } B)$  from the conditional probability of an event. The **general multiplication rule** is derived using Equation (4.4a) on page 158:

$$P(A | B) = \frac{P(A \text{ and } B)}{P(B)}$$

and solving for the joint probability  $P(A \text{ and } B)$ .

### GENERAL MULTIPLICATION RULE

The probability of  $A$  and  $B$  is equal to the probability of  $A$  given  $B$  times the probability of  $B$ .

$$P(A \text{ and } B) = P(A | B)P(B) \quad (4.6)$$

Example 4.9 demonstrates the use of the general multiplication rule.

### EXAMPLE 4.9

#### USING THE MULTIPLICATION RULE

Consider the 80 households that purchased plasma-screen televisions. In Table 4.3 on page 161 you see that 64 households are satisfied with their purchase and 16 households are dissatisfied. Suppose two households are randomly selected from the 80 customers. Find the probability that both households are satisfied with their purchase.

**SOLUTION** Here you can use the multiplication rule in the following way. If:

$A$  = second household selected is satisfied

$B$  = first household selected is satisfied

then, using Equation (4.6),

$$P(A \text{ and } B) = P(A | B)P(B)$$

The probability that the first household is satisfied with the purchase is  $64/80$ . However, the probability that the second household is also satisfied with the purchase depends on the result of the first selection. If the first household is not returned to the sample after the satisfaction level is determined (that is, sampling without replacement), the number of households remaining is 79. If the first household is satisfied, the probability that the second is also satisfied is  $63/79$  because 63 satisfied households remain in the sample. Therefore,

$$P(A \text{ and } B) = \left(\frac{63}{79}\right)\left(\frac{64}{80}\right) = 0.6380$$

There is a 63.80% chance that both of the households sampled will be satisfied with their purchase.

The **multiplication rule for independent events** is derived by substituting  $P(A)$  for  $P(A|B)$  in Equation (4.6).

### MULTIPLICATION RULE FOR INDEPENDENT EVENTS

If  $A$  and  $B$  are statistically independent, the probability of  $A$  and  $B$  is equal to the probability of  $A$  times the probability of  $B$ .

$$P(A \text{ and } B) = P(A)P(B) \quad (4.7)$$

If this rule holds for two events,  $A$  and  $B$ , then  $A$  and  $B$  are statistically independent. Therefore, there are two ways to determine statistical independence:

1. Events  $A$  and  $B$  are statistically independent if, and only if,  $P(A|B) = P(A)$ .
2. Events  $A$  and  $B$  are statistically independent if, and only if,  $P(A \text{ and } B) = P(A)P(B)$ .

### Marginal Probability Using the General Multiplication Rule

In Section 4.1, marginal probability was defined using Equation (4.2) on page 153. You can state the formula for marginal probability by using the general multiplication rule. If

$$P(A) = P(A \text{ and } B_1) + P(A \text{ and } B_2) + \dots + P(A \text{ and } B_k)$$

then, using the general multiplication rule, Equation (4.8) defines the marginal probability.

### MARGINAL PROBABILITY USING THE GENERAL MULTIPLICATION RULE

$$P(A) = P(A|B_1)P(B_1) + P(A|B_2)P(B_2) + \dots + P(A|B_k)P(B_k) \quad (4.8)$$

where  $B_1, B_2, \dots, B_k$  are  $k$  mutually exclusive and collectively exhaustive events.

To illustrate this equation, refer to Table 4.1 on page 151. Using Equation (4.8), the probability of planning to purchase is:

$$P(A) = P(A|B_1)P(B_1) + P(A|B_2)P(B_2)$$

where

$P(A)$  = probability of “planned to purchase”

$P(B_1)$  = probability of “actually purchased”

$P(B_2)$  = probability of “did not actually purchase”

$$\begin{aligned} P(A) &= \left(\frac{200}{300}\right)\left(\frac{300}{1,000}\right) + \left(\frac{50}{700}\right)\left(\frac{700}{1,000}\right) \\ &= \frac{200}{1,000} + \frac{50}{1,000} = \frac{250}{1,000} = 0.25 \end{aligned}$$

## PROBLEMS FOR SECTION 4.2

### Learning the Basics

**PH Grade ASSIST** 4.16 Given the following contingency table:

	$B$	$B'$
$A$	10	20
$A'$	20	40

What is the probability of

- $A | B$ ?
- $A | B'$ ?
- $A' | B$ ?
- Are events  $A$  and  $B$  statistically independent?

4.17 Given the following contingency table:

	$B$	$B'$
$A$	10	30
$A'$	25	35

What is the probability of

- $A | B$ ?
- $A' | B'$ ?
- $A | B'$ ?
- Are events  $A$  and  $B$  statistically independent?

**PH Grade ASSIST** 4.18 If  $P(A \text{ and } B) = 0.4$  and  $P(B) = 0.8$ , find  $P(A | B)$ .

**PH Grade ASSIST** 4.19 If  $P(A) = 0.7$ ,  $P(B) = 0.6$ , and  $A$  and  $B$  are statistically independent, find  $P(A \text{ and } B)$ .

**PH Grade ASSIST** 4.20 If  $P(A) = 0.3$ ,  $P(B) = 0.4$ , and if  $P(A \text{ and } B) = 0.2$ , are  $A$  and  $B$  statistically independent?

### Applying the Concepts

4.21 Where people turn to for news is different for various age groups. Suppose that a study conducted on this issue (extracted from P. Johnson, “Young People Turn to the Web for News,” *USA Today*, March 23, 2006, p. 9D) was based on 200 respondents who were between the ages of 36 and 50 and 200 respondents who were over age 50. Of the 200 respondents who were between the ages of 36 and 50, 82 got their news primarily from newspapers. Of the 200 respondents who were over age 50, 104 got their news primarily from newspapers.

- Given that a respondent is over age 50, what then is the probability that he or she gets news primarily from newspapers?
- Given that a respondent gets news primarily from newspapers, what is the probability that he or she is over age 50?
- Explain the difference in the results in (a) and (b).

- Are the two events, whether the respondent is over age 50 and whether he or she gets news primarily from newspapers, statistically independent?



4.22 A yield improvement study at a semiconductor manufacturing facility provided defect data for a sample of 450 wafers. The following table presents a summary of the responses to two questions: “Were particles found on the die that produced the wafer?” and “Is the wafer good or bad?”

QUALITY OF WAFER	CONDITION OF DIE		
	No Particles	Particles	Totals
Good	320	14	334
Bad	<u>80</u>	<u>36</u>	<u>116</u>
Totals	400	50	450

Source: Extracted from S. W. Hall, “Analysis of Defectivity of Semiconductor Wafers by Contingency Table,” Proceedings Institute of Environmental Sciences, Vol. 1, 1994, pp. 177–183.

- Suppose you know that a wafer is bad. What is the probability that it was produced from a die that had particles?
- Suppose you know that a wafer is good. What is the probability that it was produced from a die that had particles?
- Are the two events, a good wafer and a die with no particles, statistically independent? Explain.

4.23 According to an Ipsos poll, the perception of unfairness in the U.S. tax code is spread fairly evenly across income groups, age groups, and education levels. In an April 2006 survey of 1,005 adults, Ipsos reported that almost 60% of all people said the code is unfair, while slightly more than 60% of those making more than \$50,000 viewed the code as unfair (“People Cry Unfairness,” *The Cincinnati Enquirer*, April 16, 2006, p. A8). Suppose that the following contingency table represents the specific breakdown of responses:

	INCOME LEVEL		Total
	Less Than \$50,000	More Than \$50,000	
Fair	225	180	405
Unfair	<u>280</u>	<u>320</u>	<u>600</u>
Total	505	500	1,005

- Given that a respondent earns less than \$50,000, what is the probability that he or she said that the tax code is fair?
- Given that a respondent earns more than \$50,000, what is the probability that he or she said that the tax code is fair?
- Is income level statistically independent of attitude about whether the tax code is fair? Explain.

4.24 An experiment was conducted to study the choices made in mutual fund selection. Undergraduate and MBA

students were presented with different S&P 500 Index funds that were identical except for fees. Suppose 100 undergraduate students and 100 MBA students were selected. Partial results are shown in the following table:

FUND	STUDENT GROUP	
	Undergraduate	MBA
Highest-cost fund	27	18
Not Highest-cost fund	73	82

Source: Extracted from J. J. Choi, D. Laibson, and B. C. Madrian, Why Does the Law of One Price Fail? [www.som.yale.edu/faculty/jj83/pes.pdf](http://www.som.yale.edu/faculty/jj83/pes.pdf)

- Given that a student is an undergraduate, what is the probability that he or she selected the highest-cost fund?
- Given that a student selected the highest-cost fund, what is the probability that he or she is an undergraduate?
- Explain the difference in the results in (a) and (b).
- Are the two events “student group” and “fund selected” statistically independent? Explain.

**4.25** A sample of 500 respondents was selected in a large metropolitan area to study consumer behavior, with the following results:

ENJOYS SHOPPING FOR CLOTHING	GENDER		
	Male	Female	Total
Yes	136	224	360
No	104	36	140
Total	240	260	500

- Suppose the respondent chosen is a female. What is the probability that she does not enjoy shopping for clothing?
- Suppose the respondent chosen enjoys shopping for clothing. What is the probability that the individual is a male?
- Are enjoying shopping for clothing and the gender of the individual statistically independent? Explain.

**4.26** Each year, ratings are compiled concerning the performance of new cars during the first 90 days of use. Suppose that the cars have been categorized according to whether the car needs warranty-related repair (yes or no) and the country in which the company manufacturing the car is based (United States or not United States). Based on the data collected, the probability that the new car needs a warranty repair is 0.04, the probability that the car is manufactured by a U.S.-based company is 0.60, and the probability that the new car needs a warranty repair *and* was manufactured by a U.S.-based company is 0.025.

- Suppose you know that a company based in the United States manufactured a particular car. What is the probability that the car needs warranty repair?
- Suppose you know that a company based in the United States did not manufacture a particular car. What is the probability that the car needs warranty repair?

- Are need for warranty repair and location of the company manufacturing the car statistically independent?

**4.27** In 35 of the 56 years from 1950 through 2005, the S&P 500 finished higher after the first 5 days of trading. In 30 of those 35 years, the S&P 500 finished higher for the year. Is a good first week a good omen for the upcoming year? The following table gives the first-week and annual performance over this 56-year period:

First Week	S&P 500's Annual Performance	
	Higher	Lower
Higher	30	5
Lower	11	10

- If a year is selected at random, what is the probability that the S&P 500 finished higher for the year?
- Given that the S&P 500 finished higher after the first five days of trading, what is the probability that it finished higher for the year?
- Are the two events “first-week performance” and “annual performance” statistically independent? Explain.
- Look up the performance after the first five days of 2006 and the 2006 annual performance of the S&P 500 at [finance.yahoo.com](http://finance.yahoo.com). Comment on the results.

**4.28** A standard deck of cards is being used to play a game. There are four suits (hearts, diamonds, clubs, and spades), each having 13 faces (ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, jack, queen, and king), making a total of 52 cards. This complete deck is thoroughly mixed, and you will receive the first 2 cards from the deck without replacement.

- What is the probability that both cards are queens?
- What is the probability that the first card is a 10 and the second card is a 5 or 6?
- If you were sampling with replacement, what would be the answer in (a)?
- In the game of blackjack, the picture cards (jack, queen, king) count as 10 points, and the ace counts as either 1 or 11 points. All other cards are counted at their face value. Blackjack is achieved if 2 cards total 21 points. What is the probability of getting blackjack in this problem?

**PH Grade ASSIST** **4.29** A box of nine gloves contains two left-handed gloves and seven right-handed gloves.

- If two gloves are randomly selected from the box without replacement, what is the probability that both gloves selected will be right-handed?
- If two gloves are randomly selected from the box without replacement, what is the probability there will be one right-handed glove and one left-handed glove selected?
- If three gloves are selected with replacement, what is the probability that all three will be left-handed?
- If you were sampling with replacement, what would be the answers to (a) and (b)?

### 4.3 BAYES' THEOREM

**Bayes' theorem** is used to revise previously calculated probabilities based on new information. Developed by Thomas Bayes in the eighteenth century (see references 1, 2, and 5), Bayes' theorem is an extension of what you previously learned about conditional probability.

You can apply Bayes' theorem to the situation in which the Consumer Electronics Company is considering marketing a new model of television. In the past, 40% of the televisions introduced by the company have been successful, and 60% have been unsuccessful. Before introducing the television to the marketplace, the marketing research department conducts an extensive study and releases a report, either favorable or unfavorable. In the past, 80% of the successful televisions had received favorable market research reports, and 30% of the unsuccessful televisions had received favorable reports. For the new model of television under consideration, the marketing research department has issued a favorable report. What is the probability that the television will be successful?

Bayes' theorem is developed from the definition of conditional probability. To find the conditional probability of  $B$  given  $A$ , consider Equation (4.4b) [originally presented on page 158 and given below]:

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)} = \frac{P(A|B)P(B)}{P(A)}$$

Bayes' theorem is derived by substituting Equation (4.8) on page 163 for  $P(A)$  in Equation (4.4b).

#### BAYES' THEOREM

$$P(B_i|A) = \frac{P(A|B_i)P(B_i)}{P(A|B_1)P(B_1) + P(A|B_2)P(B_2) + \cdots + P(A|B_k)P(B_k)} \quad (4.9)$$

where  $B_i$  is the  $i$ th event out of  $k$  mutually exclusive and collectively exhaustive events.

To use Equation (4.9) for the television marketing example, let

event $S$ = successful television	event $F$ = favorable report
event $S'$ = unsuccessful television	event $F'$ = unfavorable report

and

$$\begin{aligned} P(S) &= 0.40 & P(F|S) &= 0.80 \\ P(S') &= 0.60 & P(F|S') &= 0.30 \end{aligned}$$

Then, using Equation (4.9),

$$\begin{aligned} P(S|F) &= \frac{P(F|S)P(S)}{P(F|S)P(S) + P(F|S')P(S')} \\ &= \frac{(0.80)(0.40)}{(0.80)(0.40) + (0.30)(0.60)} \\ &= \frac{0.32}{0.32 + 0.18} = \frac{0.32}{0.50} \\ &= 0.64 \end{aligned}$$

The probability of a successful television, given that a favorable report was received, is 0.64. Thus, the probability of an unsuccessful television, given that a favorable report was received, is  $1 - 0.64 = 0.36$ . Table 4.4 summarizes the computation of the probabilities, and Figure 4.3 presents the decision tree.

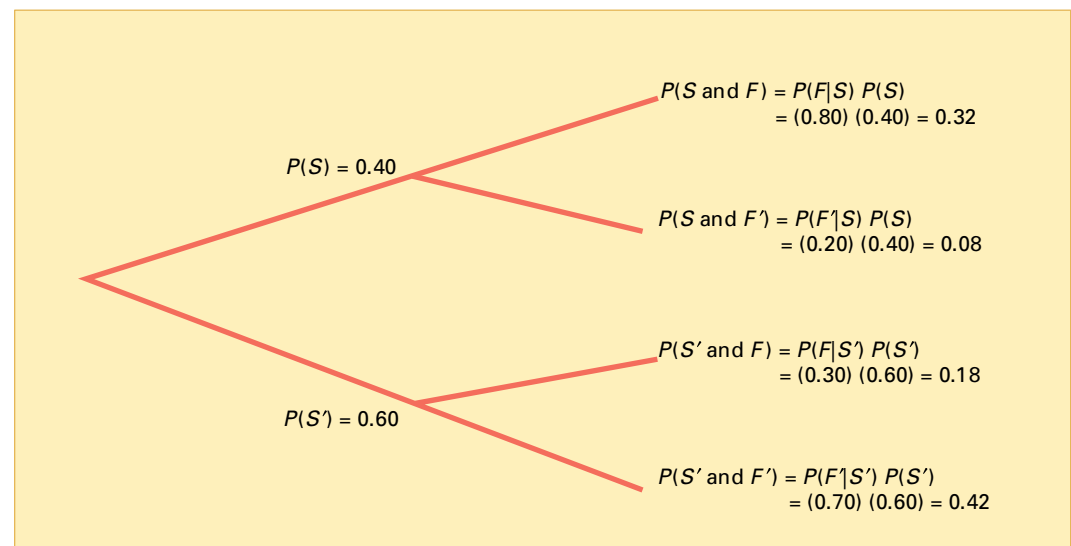
**TABLE 4.4**

Bayes' Theorem  
Calculations for the  
Television Marketing  
Example

Event $S_i$	Prior Probability $P(S_i)$	Conditional Probability $P(F   S_i)$	Joint Probability $P(F   S_i)P(S_i)$	Revised Probability $P(S_i   F)$
$S =$ successful television	0.40	0.80	0.32	$0.32/0.50 = 0.64$ $= P(S   F)$
$S' =$ unsuccessful television	0.60	0.30	$\frac{0.18}{0.50}$	$0.18/0.50 = 0.36$ $= P(S'   F)$

**FIGURE 4.3**

Decision tree for  
marketing a new  
television



Example 4.10 applies Bayes' theorem to a medical diagnosis problem.

**EXAMPLE 4.10****USING BAYES' THEOREM IN A MEDICAL DIAGNOSIS PROBLEM**

The probability that a person has a certain disease is 0.03. Medical diagnostic tests are available to determine whether the person actually has the disease. If the disease is actually present, the probability that the medical diagnostic test will give a positive result (indicating that the disease is present) is 0.90. If the disease is not actually present, the probability of a positive test result (indicating that the disease is present) is 0.02. Suppose that the medical diagnostic test has given a positive result (indicating that the disease is present). What is the probability that the disease is actually present? What is the probability of a positive test result?

**SOLUTION** Let

event  $D$  = has disease

event  $D'$  = does not have disease

event  $T$  = test is positive

event  $T'$  = test is negative

and

$$\begin{aligned}
 P(D) &= 0.03 & P(T|D) &= 0.90 \\
 P(D') &= 0.97 & P(T|D') &= 0.02
 \end{aligned}$$

Using Equation (4.9) on page 166,

$$\begin{aligned}
 P(D|T) &= \frac{P(T|D)P(D)}{P(T|D)P(D) + P(T|D')P(D')} \\
 &= \frac{(0.90)(0.03)}{(0.90)(0.03) + (0.02)(0.97)} \\
 &= \frac{0.0270}{0.0270 + 0.0194} = \frac{0.0270}{0.0464} \\
 &= 0.582
 \end{aligned}$$

The probability that the disease is actually present, given that a positive result has occurred (indicating that the disease is present), is 0.582. Table 4.5 summarizes the computation of the probabilities, and Figure 4.4 presents the decision tree.

The denominator in Bayes' theorem represents  $P(T)$ , the probability of a positive test result, which in this case is 0.0464, or 4.64%.

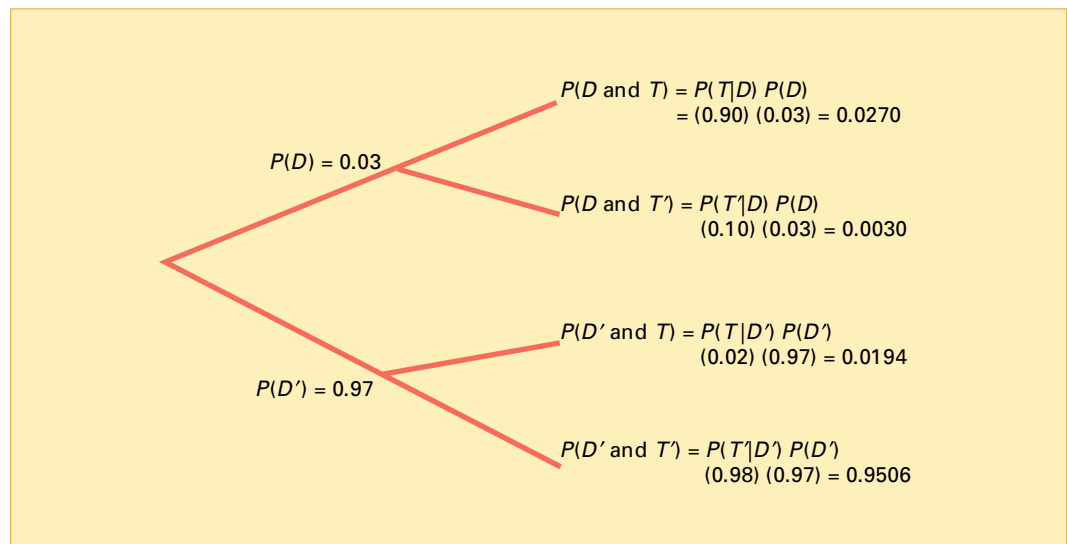
**TABLE 4.5**

Bayes' Theorem Calculations for the Medical Diagnosis Problem

Event $D_i$	Prior Probability $P(D_i)$	Conditional Probability $P(T   D_i)$	Joint Probability $P(T   D_i)P(D_i)$	Revised Probability $P(D_i   T)$
$D = \text{has disease}$	0.03	0.90	0.0270	$0.0270/0.0464 = 0.582$ $= P(D   T)$
$D' = \text{does not have disease}$	0.97	0.02	<u>0.0194</u>	$0.0194/0.0464 = 0.418$ $= P(D'   T)$
			0.0464	

**FIGURE 4.4**

Decision tree for the medical diagnosis problem





# Divine Providence and Spam

**Y**ou wouldn't normally think that the author of *Divine Benevolence: Or, An Attempt to Prove That the Principal End of the Divine Providence and Government Is the Happiness of His Creatures* would have anything to contribute to the discussion of probability, would you? But you might say that the author of *An Essay Towards Solving a Problem in the Doctrine of Chances* would, wouldn't you? As it turns out, both essays were written by Thomas Bayes, who was a minister and theologian by training. If you didn't guess that both essays were written by the same person, don't feel too bad because your wrong guess helps illustrate a modern-day application of Bayesian statistics—spam filters—about which you'll learn more later.

*An Essay Towards Solving . . .* introduced Bayes' theorem, discussed in Section 4.3. After writing the essay, Bayes put it aside, and we only know about the paper because a friend, Richard Price, published it in 1763 after Bayes' death. In Price's view, the essay proved the existence of God by showing that "the frame of the world must be the effect of wisdom and power of an intelligent cause; and thus to confirm . . . the existence of the Deity." Bayes, himself, was more modest about the essay, noting "so far as mathematics do not tend to make men more sober and rational thinkers . . . they are only to be considered as an amusement, which ought not to take us off from serious business." (By the way, Price deleted those words when he published *An Essay Towards Solving . . .*)

In spite of Price's enthusiasm, Bayes' theorem was soon dismissed by the majority of statisticians, who felt, among other reasons, that establishing the prior probabilities needed by the theorem would ultimately be nothing more than an example of setting subjective probabilities. Others, who had a more accepting view, noted that the prior probabilities would need to be constantly updated and therefore were concerned about the impossibility of keeping track of all the results that would be needed in order to constantly update those probabilities (to better reflect current experience).

While always championed by a minority of statisticians, and notably, a number of non-statisticians, Bayes' theorem remained at the fringes of respectable statistics for more than 150 years. The theorem began its long return from statistical exile when *An Essay Towards . . .* was republished in 1940, with commentary by W. Edwards Deming. (Ironically, like

Bayes' theorem, many of Deming's own ideas would also be ignored, only to be widely accepted later on, as Chapter 18 explains.) But it took the data processing power of modern computer systems to bring Bayesian statistics into the mainstream of research.

Today, applications of Bayesian statistics are making computers more "artificially intelligent." Programs that "mine" large databases, seeking to discover hidden patterns, use Bayesian techniques. Your favorite Internet search engine may use some Bayesian techniques, as well, to deliver Internet links most likely to be relevant to your search. But of all the Bayesian applications in use, the one you've most likely experienced involves spam, the unwanted junk mail that clogs your email inbox.

Return to the questions of the first paragraph concerning the titles of two essays authored by Thomas Bayes. Implicit in those questions are expectations that word frequencies vary by subject matter. A text about statistics would very likely contain the word statistics as well as words such as chance, problem, and solving. An eighteenth-century book about theology and religion would be more likely to contain the uppercase forms of Divine and Providence. Likewise, there are words very unlikely to appear in either books, such as the words sedimentary, igneous, and metamorphic (types of rocks), and there are words that are likely to appear in both types of books—common words such as *a*, *an*, and *the*.

The key words in these sentences are likely and unlikely because they suggest that we might be able to use probabilities. Of course, likely and unlikely are fuzzy concepts, and we could never be completely sure if we had classified a book correctly. After all, a book about the underground film director John Waters (of *Hairspray* fame) might talk about him traveling with the star of many of his movies, *Divine*, to see *Providence* (as in Rhode Island). Still, we would probably not mistake such a book for an eighteenth-century book on theology if we considered other words, such as *film* and *movie*, which are unlikely to be found in an eighteenth-century publication.

Classifying books into one of several categories creates complications too difficult to explain here, but what if we looked at the simpler problem of trying to classify a newly arrived email message as either spam or a legitimate message (sometimes called "ham" in this context)? More specifically, what if we

focused on all the emails sent to one address? And what if we added something to that person's email program that would track word frequencies associated with spam and ham messages in order to constantly update the prior probabilities that Bayes' theorem needs? When researchers realized all this was possible, Bayesian spam filters got their start.

These filters ask, "What is the probabilities that an email is spam, given the presence of a certain word?" Using Bayes' theorem, the filters can calculate this probabilities by multiplying the probability of finding the word in a spam email,  $P(A|B)$  in Equation 4.9, by the probability that the email is spam,  $P(B)$ , and dividing by the probability of finding the word in an email, the denominator in Equation 4.9.

Bayesian spam filters quickly proved superior to simple keyword filters in both identifying spam and avoiding labeling a ham message as spam. Most Bayesian spam filters ignore common words found in both ham and spam emails. These filters also take shortcuts by focusing on a small set of words that have a high probability of being found in a spam email and on words that have a low probability of being found in a spam email.

As spammers (people who sent junk email) learned of these new filters, they tried to outfox them. Having learned that Bayesian filters might be assigning a high  $P(A|B)$  value to words like *Viagra*, commonly found in early spam messages, spammers thought they could fool the filter by misspelling the word as *Vi@gr@* or *V1 agra*. What they failed to realize was that the misspelled variants were even more likely to be found in a spam message than the original word. Thus the misspelled variants made the job even easier for the Bayesian filters. Of course, through all the emails, the filters are constantly being retrained—that is, updating probabilities and noting when the user indicates a particular email marked as ham is really spam. So, the initial *Vi@agr@* emails were mismarked as ham, but the filter quickly learns to treat such messages as spam. (Don't worry about filters marking a ham message as spam; most filters are set up with a bias in their probabilities that minimizes such occurrences.)

Other spammers, reading a little bit about filters, decided to add "good" words, words that would have a very low  $P(A|B)$  value, or "rare" words, words not frequently encountered in emails. This, they thought, should change the result as the filter evaluated the text of an email and led the filter to enable their spam messages be viewed as

(Continued)

## Divine Providence and Spam (continued)

ham messages. But these spammers overlooked the fact that the conditional probabilities are constantly updated and that words once considered “good” would be discarded by the filter as their  $P(A|B)$  value increased. They also failed to realize that, as “rare” words started to show up in spam messages (and yet still rarely in ham messages), those words acted like the misspelled variants they had tried earlier.

After these attacks were beaten back by Bayesian filters, spammers apparently started learning more about Bayesian statistics. Some figured that they could “break” Bayesian filters by inserting random words in their messages. Those random words would affect the filter by causing it to see many words whose  $P(A|B)$  value would be low. The Bayesian filter would begin to label many spam messages as ham and end up being of no practical use. Because the Internet still contains some Web pages and posts that triumph this approach,

we will leave it to you to figure out why this method of attack cannot succeed in the long run and why this method is not as initially successful as some would claim.

Today, spammers are still trying to outwit Bayesian filters. Some spammers have decided to eliminate all or most of the words in their messages and replace them with graphics so that Bayesian filters will have very few words with which to work. But this approach will fail too, as Bayes’ filters are rewritten to consider things other than words in a message. After all, Bayes’ theorem is all about events, and “graphics present with no text” is as valid an event as “some word present in email.”

In truth, for a variety of reasons, including expediency, most email filters today use a combination of techniques that includes applying Bayesian statistics. And while the war against Bayesian filters escalates over

time, it’s a war that statistics dooms spammers to lose in the long run.

You can gain a number of insights from knowing the story of Bayes’ theorem and its application to the problem of email spam. You can understand better the nature of intellectual discovery, especially how fringe knowledge can become mainstream knowledge. You can see that statisticians have disagreed over important concepts as they have tried to extend the boundaries of their field, and yes, you can even reflect about the search for a proof that would confirm a benevolent Deity. If nothing else, perhaps you will begin to think how applications of statistics can show up in your life in very unexpected ways and how a knowledge of statistics is an important component of being a well-educated person.

**Forum** > Click the **BAYES’ THEOREM** link to continue this discussion. <<

## PROBLEMS FOR SECTION 4.3

### Learning the Basics

**PH Grade ASSIST** **4.30** If  $P(B) = 0.05$ ,  $P(A|B) = 0.80$ ,  $P(B') = 0.95$ , and  $P(A|B') = 0.40$ , find  $P(B|A)$ .

**PH Grade ASSIST** **4.31** If  $P(B) = 0.30$ ,  $P(A|B) = 0.60$ ,  $P(B') = 0.70$ , and  $P(A|B') = 0.50$ , find  $P(B|A)$ .

### Applying the Concepts

**4.32** In Example 4.10 on page 167, suppose that the probability that a medical diagnostic test will give a positive result if the disease is not present is reduced from 0.02 to 0.01. Given this information,

- if the medical diagnostic test has given a positive result (indicating that the disease is present), what is the probability that the disease is actually present?
- if the medical diagnostic test has given a negative result (indicating that the disease is not present), what is the probability that the disease is not present?

**PH Grade ASSIST** **4.33** An advertising executive is studying television viewing habits of married men and women during prime-time hours. Based on past viewing records, the executive has determined that during prime time, husbands are watching television 60% of the time. When the husband is watching television, 40% of the time the wife is also watching. When the husband is not watch-

ing television, 30% of the time the wife is watching television. Find the probability that

- if the wife is watching television, the husband is also watching television.
- the wife is watching television in prime time.

**PH Grade ASSIST** **4.34** Olive Construction Company is determining whether it should submit a bid for a new shopping center. In the past, Olive’s main competitor, Base Construction Company, has submitted bids 70% of the time. If Base Construction

Company does not bid on a job, the probability that Olive Construction Company will get the job is 0.50. If Base Construction Company bids on a job, the probability that Olive Construction Company will get the job is 0.25.

- If Olive Construction Company gets the job, what is the probability that Base Construction Company did not bid?
- What is the probability that Olive Construction Company will get the job?

**4.35** Laid-off workers who become entrepreneurs because they cannot find meaningful employment with another company are known as *entrepreneurs by necessity*. *The Wall Street Journal* reports that these entrepreneurs by necessity are less likely to grow into large businesses than are *entrepreneurs by choice* (J. Bailey, “Desire—More Than Need—Builds a Business,” *The Wall Street Journal*, May 21, 2001,

p. B4). This article states that 89% of the entrepreneurs in the United States are entrepreneurs by choice and 11% are entrepreneurs by necessity. Only 2% of entrepreneurs by necessity expect their new business to employ 20 or more people within five years, while 14% of entrepreneurs by choice expect to employ at least 20 people within five years.

- a. If an entrepreneur is selected at random, and that individual expects that his or her new business will employ 20 or more people within five years, what is the probability that this individual is an entrepreneur by choice?
- b. Discuss several possible reasons why entrepreneurs by choice are more likely to believe that they will grow their businesses.

**4.36** The editor of a textbook publishing company is trying to decide whether to publish a proposed business statistics textbook. Information on previous textbooks published indicates that 10% are huge successes, 20% are modest successes, 40% break even, and 30% are losers. However, before a publishing decision is made, the book will be reviewed. In the past, 99% of the huge successes received favorable reviews,

70% of the moderate successes received favorable reviews, 40% of the break-even books received favorable reviews, and 20% of the losers received favorable reviews.

- a. If the proposed text receives a favorable review, how should the editor revise the probabilities of the various outcomes to take this information into account?
- b. What proportion of textbooks receives favorable reviews?

**4.37** A municipal bond service has three rating categories (*A*, *B*, and *C*). Suppose that in the past year, of the municipal bonds issued throughout the United States, 70% were rated *A*, 20% were rated *B*, and 10% were rated *C*. Of the municipal bonds rated *A*, 50% were issued by cities, 40% by suburbs, and 10% by rural areas. Of the municipal bonds rated *B*, 60% were issued by cities, 20% by suburbs, and 20% by rural areas. Of the municipal bonds rated *C*, 90% were issued by cities, 5% by suburbs, and 5% by rural areas.

- a. If a new municipal bond is to be issued by a city, what is the probability that it will receive an *A* rating?
- b. What proportion of municipal bonds is issued by cities?
- c. What proportion of municipal bonds is issued by suburbs?

## 4.4 ETHICAL ISSUES AND PROBABILITY

Ethical issues can arise when any statements related to probability are presented to the public, particularly when these statements are part of an advertising campaign for a product or service. Unfortunately, many people are not comfortable with numerical concepts (see reference 4) and tend to misinterpret the meaning of the probability. In some instances, the misinterpretation is not intentional, but in other cases, advertisements may unethically try to mislead potential customers.

One example of a potentially unethical application of probability relates to advertisements for state lotteries. When purchasing a lottery ticket, the customer selects a set of numbers (such as 6) from a larger list of numbers (such as 54). Although virtually all participants know that they are unlikely to win the lottery, they also have very little idea of how unlikely it is for them to select all 6 winning numbers from the list of 54 numbers. They have even less idea of the probability of winning a consolation prize by selecting either 4 or 5 winning numbers.

Given this background, you might consider a recent commercial for a state lottery that stated, “We won’t stop until we have made everyone a millionaire” to be deceptive and possibly unethical. Given the fact that the lottery brings millions of dollars into the state treasury, the state is never going to stop running it, although in our lifetime, no one can be sure of becoming a millionaire by winning the lottery.

Another example of a potentially unethical application of probability relates to an investment newsletter promising a 90% probability of a 20% annual return on investment. To make the claim in the newsletter an ethical one, the investment service needs to (a) explain the basis on which this probability estimate rests, (b) provide the probability statement in another format, such as 9 chances in 10, and (c) explain what happens to the investment in the 10% of the cases in which a 20% return is not achieved (for example, is the entire investment lost?).

## PROBLEMS FOR SECTION 4.4

### Applying the Concepts

**4.38** Write an advertisement for the state lottery that ethically describes the probability of winning.

**4.39** Write an advertisement for the investment newsletter that ethically states the probability of a 20% return.

## 4.5 (CD-ROM Topic) COUNTING RULES

In many instances, there are a large number of possible outcomes, and determining the exact number can be difficult. In these circumstances, rules for counting the number of possible outcomes have been developed. To study this topic, go to the [section 4.5.pdf](#) file located on the CD-ROM that accompanies this book.

## SUMMARY

This chapter develops concepts concerning basic probability, conditional probability, and Bayes' theorem. In the next chapter, important discrete probability distributions such as

the binomial, hypergeometric, and Poisson distributions are developed.

## KEY EQUATIONS

### Probability of Occurrence

$$\text{Probability of occurrence} = \frac{X}{T} \quad (4.1)$$

### Marginal Probability

$$P(A) = P(A \text{ and } B_1) + P(A \text{ and } B_2) + \dots + P(A \text{ and } B_k) \quad (4.2)$$

### General Addition Rule

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) \quad (4.3)$$

### Conditional Probability

$$P(A | B) = \frac{P(A \text{ and } B)}{P(B)} \quad (4.4a)$$

$$P(B | A) = \frac{P(A \text{ and } B)}{P(A)} \quad (4.4b)$$

### Statistical Independence

$$P(A | B) = P(A) \quad (4.5)$$

### General Multiplication Rule

$$P(A \text{ and } B) = P(A | B)P(B) \quad (4.6)$$

### Multiplication Rule for Independent Events

$$P(A \text{ and } B) = P(A)P(B) \quad (4.7)$$

### Marginal Probability Using the General Multiplication Rule

$$P(A) = P(A | B_1)P(B_1) + P(A | B_2)P(B_2) + \dots + P(A | B_k)P(B_k) \quad (4.8)$$

### Bayes' Theorem

$$P(B_i | A) = \frac{P(A | B_i)P(B_i)}{P(A | B_1)P(B_1) + P(A | B_2)P(B_2) + \dots + P(A | B_k)P(B_k)} \quad (4.9)$$

## KEY TERMS

*a priori* classical probability 149  
Bayes' theorem 166  
certain event 149

collectively exhaustive 153  
complement 150  
conditional probability 157

contingency table 151  
decision tree 159  
empirical classical probability 149

event 150  
 general addition rule 154  
 general multiplication rule 162  
 impossible event 149  
 joint event 150  
 joint probability 152

marginal probability 152  
 multiplication rule for independent  
 events 163  
 mutually exclusive 153  
 probability 149  
 sample space 150

simple event 150  
 simple probability 151  
 statistical independence 161  
 subjective probability 150

## CHAPTER REVIEW PROBLEMS

### Checking Your Understanding

**4.40** What are the differences between *a priori* classical probability, empirical classical probability, and subjective probability?

**4.41** What is the difference between a simple event and a joint event?

**4.42** How can you use the addition rule to find the probability of occurrence of event *A* or *B*?

**4.43** What is the difference between mutually exclusive events and collectively exhaustive events?

**4.44** How does conditional probability relate to the concept of statistical independence?

**4.45** How does the multiplication rule differ for events that are and are not independent?

**4.46** How can you use Bayes' theorem to revise probabilities in light of new information?

**4.47** In Bayes' theorem, how does the prior probability differ from the revised probability?

### Applying the Concepts

**4.48** A soft drink bottling company maintains records concerning the number of unacceptable bottles of soft drink from the filling and capping machines. Based on past data, the probability that a bottle came from machine I and was nonconforming is 0.01, and the probability that a bottle came from machine II and was nonconforming is 0.005. Half the bottles are filled on machine I. The other half are filled on machine II. If a filled bottle of soft drink is randomly selected, what is the probability that

- it is a nonconforming bottle?
- it was filled on machine I *and* is a conforming bottle?
- it was filled on machine I *or* is a conforming bottle?
- Suppose you know that the bottle was filled on machine I. What is the probability that it is nonconforming?
- Suppose you know that the bottle is nonconforming. What is the probability that it was filled on machine I?
- Explain the difference in the answers to (d) and (e).

(Hint: Construct a  $2 \times 2$  contingency table to evaluate the probabilities.)

**4.49** A survey of 500 men and 500 women, designed to study financial tensions between couples, asked how likely each was to hide purchases, cash, or investments from his or her partner. The results were as follows:

Type	Likely to Hide	
	Men	Women
Auto-related	66	36
Cash	126	133
Clothes	62	116
Electronics	79	56
Entertainment	96	76
Food	74	94
Investments	76	52
Travel	53	39

Source: Extracted from L. Wei, "Your Money Manager as Financial Therapist," *The Wall Street Journal*, November 5–6, 2005, p. B4.

Construct a contingency table of likely to hide (yes or no) with gender for each type. If a respondent is chosen at random, what is the probability that

- he or she is likely to hide a clothing purchase?
- he or she is likely to hide investments?
- the person is a male and is likely to hide auto-related purchases?
- the person is a female and is likely to hide clothing purchases?
- Given that the person hides investments, what is the probability that the person is a male?
- Are any of the types of things that couples hide statistically independent of the gender of the respondent? Explain.

**4.50** Many companies use Web sites to conduct business transactions, such as taking orders or performing financial exchanges. These sites are referred to as *transactional public Web sites*. An analysis of 490 firms listed in the Fortune 500 identified firms by their level of sales and whether the firm had a transactional public Web site (D. Young and J. Benamati, "A Cross-Industry Analysis of Large Firm Transactional Public Web Sites," *Mid-American Journal of*

*Business*, Vol. 19, 2004, pp. 37–46). The results of this analysis are given in the following table:

#### TRANSACTIONAL PUBLIC WEB SITE

SALES (IN BILLIONS OF DOLLARS)	Yes	No
Greater than \$10 billion	71	88
Up to \$10 billion	99	232

- Give an example of a simple event and a joint event.
- What is the probability that a firm in the Fortune 500 has a transactional public Web site?
- What is the probability that a firm in the Fortune 500 has sales in excess of \$10 billion and a transactional Web site?
- Are the events “sales in excess of \$10 billion” and “has a transactional public Web site” independent? Explain.

**4.51** The owner of a restaurant serving Continental-style entrées was interested in studying ordering patterns of patrons for the Friday-to-Sunday weekend time period. Records were maintained that indicated the demand for dessert during the same time period. The owner decided to study two other variables, along with whether a dessert was ordered: the gender of the individual and whether a beef entrée was ordered. The results are as follows:

DESSERT ORDERED	GENDER		Total
	Male	Female	
Yes	96	40	136
No	224	240	464
Total	320	280	600

DESSERT ORDERED	BEEF ENTRÉE		Total
	Yes	No	
Yes	71	65	136
No	116	348	464
Total	187	413	600

A waiter approaches a table to take an order. What is the probability that the first customer to order at the table

- orders a dessert?
- orders a dessert *or* a beef entrée?
- is a female *and* does not order a dessert?
- is a female *or* does not order a dessert?
- Suppose the first person that the waiter takes the dessert order from is a female. What is the probability that she does not order dessert?
- Are gender and ordering dessert statistically independent?
- Is ordering a beef entrée statistically independent of whether the person orders dessert?

**4.52** Unsolicited commercial email messages containing product advertisements, commonly referred to as spam,

are routinely deleted before being read by more than 80% of all email users. Furthermore, a small percentage of those reading the spam actually follow through and purchase items. Yet many companies use these unsolicited email advertisements because of the extremely low cost involved. Movies Unlimited, a mail-order video and DVD business in Philadelphia, is one of the more successful companies in terms of generating sales through this form of e-marketing. Ed Weiss, general manager of Movies Unlimited, estimates that somewhere from 15% to 20% of the company’s email recipients read the advertisements. Moreover, approximately 15% of those who read the advertisements place orders (S. Forster, “E-Marketers Look to Polish Spam’s Rusty Image,” *The Wall Street Journal*, May 20, 2002, p. D2).

- Using Mr. Weiss’s lower estimate that the probability a recipient will read the advertisement is 0.15, what is the probability that a recipient will read the advertisement and place an order?
- Movies Unlimited uses a 175,000-customer database to send email advertisements. If an email advertisement is sent to everyone in its customer database, how many customers do you expect will read the advertisement and place an order?
- If the probability a recipient will read the advertisement is 0.20, what is the probability that a recipient will read the advertisement and place an order?
- What is your answer to (b) if the probability that a recipient will read the advertisement is 0.20?

**4.53** An experiment was conducted by James Choi, David Laibson, and Brigitte Madrian to study the choices made in fund selection. Suppose 100 undergraduate students and 100 MBA students were selected. When presented with four S&P 500 Index funds that were identical except for their fees, undergraduate and MBA students chose the funds as follows:

FUND	STUDENT GROUP	
	Undergraduate	MBA
Lowest-cost fund	19	19
Second-lowest-cost fund	37	40
Third-lowest-cost fund	17	23
Highest-cost fund	27	18

Source: Extracted from J. J. Choi, D. Laibson, and B. C. Madrian, *Why Does the Law of One Price Fail?* [www.som.yale.edu/faculty/jj83/fes.pdf](http://www.som.yale.edu/faculty/jj83/fes.pdf)

If a student is selected at random, what is the probability that he or she

- selected the lowest or second-lowest cost fund?
- selected the lowest-cost fund *and* is an undergraduate?
- selected the lowest-cost fund *or* is an undergraduate?
- Given that the student is an undergraduate, what is the probability that he or she selected the highest-cost fund?

- e. Do you think undergraduate students and graduate students differ in their fund selection? Explain.

**4.54** Sport utility vehicles (SUVs), vans, and pickups are generally considered to be more prone to roll over than cars. In 1997, 24.0% of all highway fatalities involved rollovers; 15.8% of all fatalities in 1997 involved SUVs, vans, and pickups, given that the fatality involved a rollover. Given that a rollover was not involved, 5.6% of all fatalities involved SUVs, vans, and pickups (A. Wilde Mathews, “Ford Ranger, Chevy Tracker Tilt in Test,” *The Wall Street Journal*, July 14, 1999, p. A2). Consider the following definitions:

$A$  = fatality involved an SUV, van, or pickup

$B$  = fatality involved a rollover

- Use Bayes’ theorem to find the probability that a fatality involved a rollover, given that the fatality involved an SUV, a van, or a pickup.
- Compare the result in (a) to the probability that a fatality involved a rollover and comment on whether SUVs, vans, and pickups are generally more prone to rollover accidents than other vehicles.

**4.55** Enzyme-linked immunosorbent assay (ELISA) is the most common type of screening test for detecting the HIV virus. A positive result from an ELISA indicates that the HIV virus is present. For most populations, ELISA has a high degree of sensitivity (to detect infection) and specificity (to detect noninfection). (See HIVInsite, at [HIVInsite.ucsf.edu](http://HIVInsite.ucsf.edu).) Suppose that the probability a person is infected with the HIV virus for a certain population is 0.015. If the HIV virus is actually present, the probability that the ELISA test will give a positive result is 0.995. If the HIV virus is not actually present, the probability of a positive result from an ELISA is 0.01. If the ELISA has given a positive result, use Bayes’ theorem to find the probability that the HIV virus is actually present.

### Team Project

The data file [Mutual Funds.xls](#) contains information regarding four categorical variables from a sample of 838 mutual funds. The variables are:

Category—Type of stocks comprising the mutual fund (small cap, mid cap, large cap)

Objective—Objective of stocks comprising the mutual fund (growth or value)

Fees—Sales charges (no or yes)

Risk—Risk-of-loss factor of the mutual fund (low, average, high)

**4.56** Construct contingency tables of category and objective, category and fees, category and risk, objective and fees, and fees and risk.

- For each of these contingency tables, compute all the conditional and marginal probabilities.
- Based on (a), what conclusions can you reach about the independence between these variables?

### Student Survey Data Base

**4.57** Problem 1.27 on page 15 describes a survey of 50 undergraduate students (see the file [undergradsurvey.xls](#)). For these data, construct contingency tables of gender and major, gender and graduate school intention, gender and employment status, class and graduate school intention, class and employment status, major and graduate school intention, and major and employment status.

- For each of these contingency tables, compute all the conditional and marginal probabilities.
- Based on (a), what conclusions can you reach about the independence between these variables?

**4.58** Problem 1.27 on page 15 describes a survey of 50 undergraduate students (see the file [undergradsurvey.xls](#)).

- Select a sample of 50 undergraduate students at your school and conduct a similar survey for those students.
- For these data, construct contingency tables of gender and major, gender and graduate school intention, gender and employment status, class and graduate school intention, class and employment status, major and graduate school intention, and major and employment status. For each of these contingency tables, compute all the conditional and marginal probabilities.
- Based on (b), what conclusions can you reach about the independence between these variables?
- Compare the results of (c) to those of Problem 4.57 (b).

**4.59** Problem 1.28 on page 15 describes a survey of 50 MBA students (see the file [gradsurvey.xls](#)). For these data, construct contingency tables of gender and graduate major, gender and undergraduate major, gender and employment status, graduate major and undergraduate major, and graduate major and employment status.

- For each of these contingency tables, compute all the conditional and marginal probabilities.
- Based on (b), what conclusions can you reach about the independence between these variables?

**4.60** Problem 1.28 on page 15 describes a survey of 50 MBA students (see the file [gradsurvey.xls](#)).

- Select a sample of 50 MBA students from your MBA program and conduct a similar survey for those students.
- For these data, construct contingency tables of gender and graduate major, gender and undergraduate major, gender and employment status, graduate major and undergraduate major, and graduate major and employment status. For each of these contingency tables, compute all the conditional and marginal probabilities.
- Based on (b), what conclusions can you reach about the independence between these variables?
- Compare the results of (c) to those of Problem 4.59 (b).

## Web Case

Apply your knowledge about contingency tables and the proper application of simple and joint probabilities in this continuing Web Case from Chapter 3.

Visit the EndRun Guaranteed Investment Package (GIP) Web page, at [www.prenhall.com/Springville/ER\\_Guaranteed.htm](http://www.prenhall.com/Springville/ER_Guaranteed.htm) or open this Web page file from the Student CD-ROM **Web Case** folder. Read the claims and examine the supporting data. Then answer the following:

1. How accurate is the claim of the probability of success for EndRun's GIP? In what ways is the claim mislead-

ing? How would you calculate and state the probability of having an annual rate of return not less than 15%?

2. What mistake was made in reporting the 7% probability claim? Using the table found on the "Winning Probabilities" Web page, [ER\\_Guaranteed3.htm](#), compute the proper probabilities for the group of investors.
3. Are there any probability calculations that would be appropriate for rating an investment service? Why or why not?

## REFERENCES

1. Bellhouse, D. R., "The Reverend Thomas Bayes, FRS: A Biography to Celebrate the Tercentenary of His Birth," *Statistical Science* 19 (2004), 3–43.
2. Lowd, D., and C. Meek, "Good Word Attacks on Statistical Spam Filters," presented at the Second Conference on Email and Anti-Spam, CEAS 2005.
3. *Microsoft Excel 2007* (Redmond, WA: Microsoft Corp., 2007).
4. Paulos, J. A., *Innumeracy* (New York: Hill and Wang, 1988).
5. Silberman, S., "The Quest for Meaning," *Wired* 8.02, February 2000.



# Excel Companion

## to Chapter 4

### E4.1 COMPUTING BASIC PROBABILITIES

You compute basic probabilities by either using the PHStat2 **Simple & Joint Probabilities** procedure or by making entries in the **Probabilities** worksheet of the **Probabilities.xls** workbook.

#### Using PHStat2 Simple & Joint Probabilities

Open the workbook in which you want your probabilities worksheet to be placed. Select **PHStat** → **Probability & Prob. Distributions** → **Simple & Joint Probabilities** and enter event labels and counts in the tinted cells in rows 3 through 6.

#### Using the Probabilities Worksheet

Open to the **Probabilities** worksheet of the **Probabilities.xls** workbook. Enter event labels and counts in the tinted cells in rows 3 through 6. Figure E4.1 shows a completed work-

	A	B	C	D	E
1	Probabilities for Purchase Behavior for Big-Screen Televisions				
2					
3	Sample Space				
4		Actually Purchased			
5	Planned to Purchase	Yes	200	50	250
6		No	100	650	750
7		Totals	300	700	1000
8					
9	Simple Probabilities				
10	P(Yes)	0.25			
11	P(No)	0.75			
12	P(Yes)	0.30			
13	P(No)	0.70			
14					
15	Joint Probabilities				
16	P(Yes and Yes)	0.20			
17	P(Yes and No)	0.05			
18	P(No and Yes)	0.10			
19	P(No and No)	0.65			
20					
21	Addition Rule				
22	P(Yes or Yes)	0.35			
23	P(Yes or No)	0.90			
24	P(No or Yes)	0.95			
25	P(No or No)	0.80			

Simple Probabilities	
"P(" & B5 & ")"	=E5 / E7
"P(" & B6 & ")"	=E6 / E7
"P(" & C4 & ")"	=C7 / E7
"P(" & D4 & ")"	=D7 / E7

Joint Probabilities	
"P(" & B5 & " and " & C4 & ")"	=C5 / E7
"P(" & B5 & " and " & D4 & ")"	=D5 / E7
"P(" & B6 & " and " & C4 & ")"	=C6 / E7
"P(" & B6 & " and " & D4 & ")"	=D6 / E7

Addition Rule	
"P(" & B5 & " or " & C4 & ")"	=B10 + B12 - B16
"P(" & B5 & " or " & D4 & ")"	=B10 + B13 - B17
"P(" & B6 & " or " & C4 & ")"	=B11 + B12 - B18
"P(" & B6 & " or " & D4 & ")"	=B11 + B13 - B19

FIGURE E4.1

Probabilities worksheet

sheet for the data on purchase behavior for big-screen televisions from Table 4.1 on page 151. Also shown offset in Figure E4.1 are the formulas this worksheet uses to compute the various probabilities.

The worksheet also “computes” the probabilities table labels, using a novel application of formulas that include the ampersand (&) operator. For example, the cell A10 formula =“P(" & B5 & ")” asks Excel to add the contents of B5 (Yes) to the label “P(” to get “P(Yes)” and then to “add” “)” to get “P(Yes)” and form the formula =“P(Yes)”. This is a novel but acceptable way of entering the label value P(Yes) in a cell.

### E4.2 USING BAYES' THEOREM

You use Bayes' theorem by making entries in the cell range B5:C6 of the **Bayes** worksheet of the **Bayes.xls** workbook. Figure E4.2 shows a completed worksheet for the television-marketing example shown in Table 4.4 on page 167.

	A	B	C	D	E
1	Bayes Theorem Calculations				
2					
3	Probabilities				
4	Event	Prior	Conditional	Joint	Revised
5	S	0.4	0.8	0.32	0.64
6	S'	0.6	0.3	0.18	0.36
7			Total:	0.5	

Joint	Revised
=B5 * C5	=D5 / \$D\$7
=B6 * C6	=D6 / \$D\$7
=D5 + D6	

FIGURE E4.2

Bayes' theorem worksheet



# CHAPTER 5

## Some Important Discrete Probability Distributions

USING STATISTICS @ Saxon Home Improvement

### 5.1 THE PROBABILITY DISTRIBUTION FOR A DISCRETE RANDOM VARIABLE

Expected Value of a Discrete Random Variable  
Variance and Standard Deviation of a Discrete Random Variable

### 5.2 COVARIANCE AND ITS APPLICATION IN FINANCE

Covariance  
Expected Value, Variance, and Standard Deviation of the Sum of Two Random Variables  
Portfolio Expected Return and Portfolio Risk

### 5.3 BINOMIAL DISTRIBUTION

### 5.4 POISSON DISTRIBUTION

### 5.5 HYPERGEOMETRIC DISTRIBUTION

### 5.6 (CD-ROM TOPIC) USING THE POISSON DISTRIBUTION TO APPROXIMATE THE BINOMIAL DISTRIBUTION

#### EXCEL COMPANION TO CHAPTER 5

- E5.1 Computing the Expected Value of a Discrete Random Variable
- E5.2 Computing Portfolio Expected Return and Portfolio Risk
- E5.3 Computing Binomial Probabilities
- E5.4 Computing Poisson Probabilities
- E5.5 Computing Hypergeometric Probabilities
- E5.6 Creating Histograms for Discrete Probability Distributions

## LEARNING OBJECTIVES

In this chapter, you learn:

- The properties of a probability distribution
- To compute the expected value and variance of a probability distribution
- To calculate the covariance and understand its use in finance
- To compute probabilities from binomial, hypergeometric, and Poisson distributions
- How to use the binomial, hypergeometric, and Poisson distributions to solve business problems

## USING STATISTICS @ Saxon Home Improvement



Accounting information systems collect, process, store, transform, and distribute financial information to decision makers both internal and external to a business organization (see reference 6). These systems continuously audit accounting information, looking for errors or incomplete or improbable information. For example, when customers of the Saxon Home Improvement Company submit online orders, the company's accounting information system reviews the order forms for possible mistakes. Any questionable invoices are *tagged* and included in a daily *exceptions report*. Recent data collected by the company show that the likelihood is 0.10 that an order form will be tagged. Saxon would like to determine the likelihood of finding a certain number of tagged forms in a sample of a specific size. For example, what would be the likelihood that none of the order forms are tagged in a sample of four forms? That one of the order forms is tagged?

How could the Saxon Home Improvement Company determine the solution to this type of probability problem? One way is to use a model, or small-scale representation, that approximates the process. By using such an approximation, Saxon managers could make inferences about the actual order process. Model building is a difficult task for some endeavors, and in this case, the Saxon managers can use *probability distributions*, mathematical models suited for solving the type of probability problems the managers are facing. Reading this chapter will help you learn about characteristics of a probability distribution and how to specifically apply the binomial, Poisson, and hypergeometric distributions to business problems.

### 5.1 THE PROBABILITY DISTRIBUTION FOR A DISCRETE RANDOM VARIABLE

In Section 1.5, a *numerical variable* was defined as a variable that yielded numerical responses, such as the number of magazines you subscribe to or your height, in inches. Numerical variables are classified as *discrete* or *continuous*. Continuous numerical variables produce outcomes that come from a measuring process (for example, your height). Discrete numerical variables produce outcomes that come from a counting process (for example, the number of magazines you subscribe to). This chapter deals with probability distributions that represent discrete numerical variables.

#### PROBABILITY DISTRIBUTION FOR A DISCRETE RANDOM VARIABLE

A **probability distribution for a discrete random variable** is a mutually exclusive listing of all possible numerical outcomes for that variable such that a particular probability of occurrence is associated with each outcome.

For example, Table 5.1 gives the distribution of the number of mortgages approved per week at the local branch office of a bank. The listing in Table 5.1 is collectively exhaustive because all possible outcomes are included. Thus, the probabilities must sum to 1. Figure 5.1 is a graphical representation of Table 5.1.

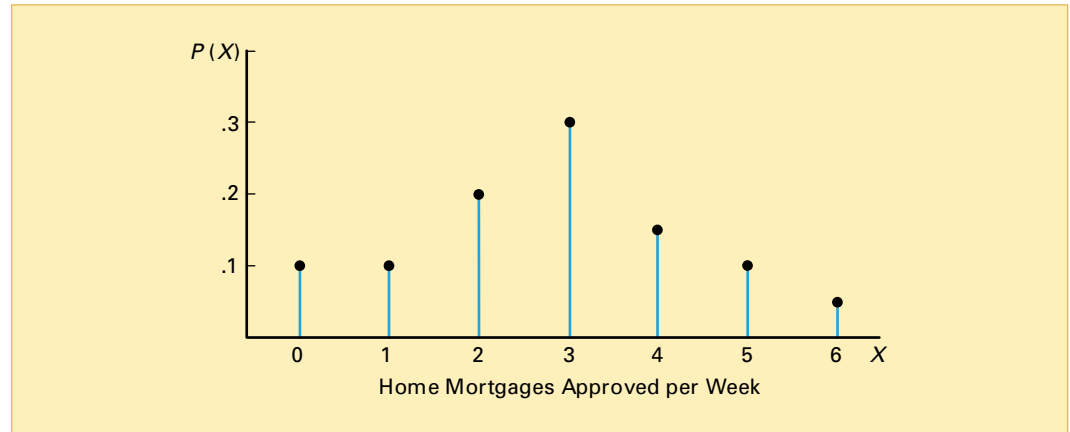
**TABLE 5.1**

Probability Distribution of the Number of Home Mortgages Approved per Week

Home Mortgages Approved per Week	Probability
0	0.10
1	0.10
2	0.20
3	0.30
4	0.15
5	0.10
6	0.05

**FIGURE 5.1**

Probability distribution of the number of home mortgages approved per week



### Expected Value of a Discrete Random Variable

The mean,  $\mu$ , of a probability distribution is the *expected value* of its random variable. To calculate the expected value, you multiply each possible outcome,  $X$ , by its corresponding probability,  $P(X)$ , and then sum these products.

#### EXPECTED VALUE, $\mu$ , OF A DISCRETE RANDOM VARIABLE

$$\mu = E(X) = \sum_{i=1}^N X_i P(X_i) \quad (5.1)$$

where

$X_i$  = the  $i$ th outcome of the discrete random variable  $X$

$P(X_i)$  = probability of occurrence of the  $i$ th outcome of  $X$

For the probability distribution of the number of home mortgages approved per week (Table 5.1), the expected value is computed using Equation (5.1) below and is also shown in Table 5.2.

$$\begin{aligned} \mu &= E(X) = \sum_{i=1}^N X_i P(X_i) \\ &= (0)(0.1) + (1)(0.1) + (2)(0.2) + (3)(0.3) + (4)(0.15) + (5)(0.1) + (6)(0.05) \\ &= 0 + 0.1 + 0.4 + 0.9 + 0.6 + 0.5 + 0.3 \\ &= 2.8 \end{aligned}$$

**TABLE 5.2**

Computing the Expected Value of the Number of Home Mortgages Approved per Week

Home Mortgages		
Approved per Week ( $X_i$ )	$P(X_i)$	$X_i P(X_i)$
0	0.10	(0)(0.10) = 0.0
1	0.10	(1)(0.10) = 0.1
2	0.20	(2)(0.20) = 0.4
3	0.30	(3)(0.30) = 0.9
4	0.15	(4)(0.15) = 0.6
5	0.10	(5)(0.10) = 0.5
6	0.05	(6)(0.05) = 0.3
	1.00	$\mu = E(X) = 2.8$

The expected value of 2.8 for the number of mortgages approved is not a possible outcome because the actual number of mortgages approved in a given week must be an integer value. The expected value represents the *mean* number of mortgages approved per week.

### Variance and Standard Deviation of a Discrete Random Variable

You compute the variance of a probability distribution by multiplying each possible squared difference  $[X_i - E(X)]^2$  by its corresponding probability,  $P(X_i)$ , and then summing the resulting products. Equation (5.2) defines the **variance of a discrete random variable**.

#### VARIANCE OF A DISCRETE RANDOM VARIABLE

$$\sigma^2 = \sum_{i=1}^N [X_i - E(X)]^2 P(X_i) \quad (5.2)$$

where

$X_i$  = the  $i$ th outcome of the discrete random variable  $X$   
 $P(X_i)$  = probability of occurrence of the  $i$ th outcome of  $X$

Equation (5.3) defines the **standard deviation of a discrete random variable**.

#### STANDARD DEVIATION OF A DISCRETE RANDOM VARIABLE

$$\sigma = \sqrt{\sigma^2} = \sqrt{\sum_{i=1}^N [X_i - E(X)]^2 P(X_i)} \quad (5.3)$$

In Table 5.3, the variance and the standard deviation of the number of home mortgages approved per week are computed using Equations (5.2) and (5.3):

**TABLE 5.3**

Computing the Variance and Standard Deviation of the Number of Home Mortgages Approved per Week

Home Mortgages Approved per Week ( $X_i$ )	$P(X_i)$	$X_i P(X_i)$	$[X_i - E(X)]^2 P(X_i)$
0	0.10	$(0)(0.10) = 0.0$	$(0 - 2.8)^2(0.10) = 0.784$
1	0.10	$(1)(0.10) = 0.1$	$(1 - 2.8)^2(0.10) = 0.324$
2	0.20	$(2)(0.20) = 0.4$	$(2 - 2.8)^2(0.20) = 0.128$
3	0.30	$(3)(0.30) = 0.9$	$(3 - 2.8)^2(0.30) = 0.012$
4	0.15	$(4)(0.15) = 0.6$	$(4 - 2.8)^2(0.15) = 0.216$
5	0.10	$(5)(0.10) = 0.5$	$(5 - 2.8)^2(0.10) = 0.484$
6	0.05	$(6)(0.05) = 0.3$	$(6 - 2.8)^2(0.05) = 0.512$
	1.00	$\mu = E(X) = 2.8$	$\sigma^2 = 2.46$

$$\begin{aligned} \sigma^2 &= \sum_{i=1}^N [X_i - E(X)]^2 P(X_i) \\ &= (0 - 2.8)^2(0.10) + (1 - 2.8)^2(0.10) + (2 - 2.8)^2(0.20) + (3 - 2.8)^2(0.30) \\ &\quad + (4 - 2.8)^2(0.15) + (5 - 2.8)^2(0.10) + (6 - 2.8)^2(0.05) \\ &= 0.784 + 0.324 + 0.128 + 0.012 + 0.216 + 0.484 + 0.512 \\ &= 2.46 \end{aligned}$$

and

$$\sigma = \sqrt{\sigma^2} = \sqrt{2.46} = 1.57$$

Thus, the mean number of mortgages approved per week is 2.8, the variance is 2.46, and the standard deviation is 1.57.

## PROBLEMS FOR SECTION 5.1

### Learning the Basics

**PH Grade ASSIST** **5.1** Given the following probability distributions:

Distribution A		Distribution B	
$X$	$P(X)$	$X$	$P(X)$
0	0.50	0	0.05
1	0.20	1	0.10
2	0.15	2	0.15
3	0.10	3	0.20
4	0.05	4	0.50

- Compute the expected value for each distribution.
- Compute the standard deviation for each distribution.
- Compare the results of distributions *A* and *B*.

**PH Grade ASSIST** **5.2** Given the following probability distributions:

Distribution C		Distribution D	
$X$	$P(X)$	$X$	$P(X)$
0	0.20	0	0.10
1	0.20	1	0.20
2	0.20	2	0.40
3	0.20	3	0.20
4	0.20	4	0.10

- Compute the expected value for each distribution.
- Compute the standard deviation for each distribution.
- Compare the results of distributions *C* and *D*.

## Applying the Concepts

**5.3** How many credit cards do you have in your wallet? According to a survey by Ipsos, a large survey research company, 26% of adults in the United States reported having no credit cards; 38% reported having one or two; 20% three or four; 15% five or more; and 1% reported “not sure” (“Snapshots,” *usatoday.com*, April 18, 2006). Suppose that the following table contains the complete probability distribution for the number of credit cards owned by adults in the United States:

Number of Credit Cards ( $X$ )	$P(X)$
0	0.26
1	0.22
2	0.16
3	0.12
4	0.08
5	0.06
6	0.04
7	0.03
8	0.02
9	0.01

- Compute the mean number of credit cards owned by a U.S. adult.
- Compute the standard deviation.



**5.4** The following table contains the probability distribution for the number of traffic accidents daily in a small city:

Number of Accidents Daily ( $X$ )	$P(X)$
0	0.10
1	0.20
2	0.45
3	0.15
4	0.05
5	0.05

- Compute the mean number of accidents per day.
- Compute the standard deviation.

**5.5** The manager of a large computer network has developed the following probability distribution of the number of interruptions per day:

Interruptions ( $X$ )	$P(X)$
0	0.32
1	0.35
2	0.18
3	0.08
4	0.04
5	0.02
6	0.01

- Compute the expected number of interruptions per day.
- Compute the standard deviation.

**5.6** In the carnival game Under-or-Over-Seven, a pair of fair dice is rolled once, and the resulting sum determines whether the player wins or loses his or her bet. For example, the player can bet \$1 that the sum will be under 7—that is, 2, 3, 4, 5, or 6. For this bet, the player loses \$1 if the outcome equals or exceeds 7 and wins \$1 if the result is under 7. Similarly, the player can bet \$1 that the sum will be over 7—that is, 8, 9, 10, 11, or 12. Here, the player wins \$1 if the result is over 7 but loses \$1 if the result is 7 or under. A third method of play is to bet \$1 on the outcome 7. For this bet, the player wins \$4 if the result of the roll is 7 and loses \$1 otherwise.

- Construct the probability distribution representing the different outcomes that are possible for a \$1 bet on being under 7.
- Construct the probability distribution representing the different outcomes that are possible for a \$1 bet on being over 7.
- Construct the probability distribution representing the different outcomes that are possible for a \$1 bet on 7.
- Show that the expected long-run profit (or loss) to the player is the same, no matter which method of play is used.

## 5.2 COVARIANCE AND ITS APPLICATION IN FINANCE

In Section 5.1, the expected value, variance, and standard deviation of a discrete random variable of a probability distribution were discussed. In this section, the covariance between two variables is introduced and applied to portfolio management, a topic of great interest to financial analysts.

### Covariance

The **covariance**,  $\sigma_{XY}$ , is a measure of the strength of the relationship between two discrete random variables,  $X$  and  $Y$ . A positive covariance indicates a positive relationship. A negative covariance indicates a negative relationship. A covariance of 0 indicates that the two variables are independent. Equation (5.4) defines the covariance.



## COVARIANCE

$$\sigma_{XY} = \sum_{i=1}^N [X_i - E(X)][Y_i - E(Y)]P(X_iY_i) \quad (5.4)$$

where

$X$  = discrete random variable  $X$

$X_i$  =  $i$ th outcome of  $X$

$P(X_iY_i)$  = probability of occurrence of the  $i$ th outcome of  $X$   
and the  $i$ th outcome of  $Y$

$Y$  = discrete random variable  $Y$

$Y_i$  =  $i$ th outcome of  $Y$

$i = 1, 2, \dots, N$

To illustrate the covariance, suppose that you are deciding between two alternative investments for the coming year. The first investment is a mutual fund that consists of the stocks that comprise the Dow Jones Industrial Average. The second investment is a mutual fund that is expected to perform best when economic conditions are weak. Your estimate of the returns for each investment (per \$1,000 investment) under three economic conditions, each with a given probability of occurrence, is summarized in Table 5.4.

**TABLE 5.4**

Estimated Returns  
for Each Investment  
Under Three Economic  
Conditions

$P(X_iY_i)$	Economic Condition	Investment	
		Dow Jones Fund	Weak-Economy Fund
0.2	Recession	-\$100	+\$200
0.5	Stable economy	+100	+50
0.3	Expanding economy	+250	-100

The expected value and standard deviation for each investment and the covariance of the two investments are computed as follows:

Let  $X$  = Dow Jones fund, and  $Y$  = weak-economy fund

$$E(X) = \mu_X = (-100)(0.2) + (100)(0.5) + (250)(0.3) = \$105$$

$$E(Y) = \mu_Y = (+200)(0.2) + (50)(0.5) + (-100)(0.3) = \$35$$

$$\begin{aligned} Var(X) &= \sigma_X^2 = (-100 - 105)^2(0.2) + (100 - 105)^2(0.5) + (250 - 105)^2(0.3) \\ &= 14,725 \end{aligned}$$

$$\sigma_X = \$121.35$$

$$\begin{aligned} Var(Y) &= \sigma_Y^2 = (200 - 35)^2(0.2) + (50 - 35)^2(0.5) + (-100 - 35)^2(0.3) \\ &= 11,025 \end{aligned}$$

$$\sigma_Y = \$105.00$$

$$\begin{aligned} \sigma_{XY} &= (-100 - 105)(200 - 35)(0.2) + (100 - 105)(50 - 35)(0.5) \\ &\quad + (250 - 105)(-100 - 35)(0.3) \\ &= -6,765 - 37.5 - 5,782.5 \\ &= -12,675 \end{aligned}$$

Thus, the Dow Jones fund has a higher expected value (that is, larger expected return) than the weak-economy fund but has a higher standard deviation (that is, more risk). The covariance of  $-12,675$  between the two investments indicates a negative relationship in which the two investments are varying in the *opposite* direction. Therefore, when the return on one investment is high, typically, the return on the other is low.

### Expected Value, Variance, and Standard Deviation of the Sum of Two Random Variables

Equations (5.1)–(5.3) defined the expected value, variance, and standard deviation of a probability distribution while Equation (5.4) defines the covariance between two variables,  $X$  and  $Y$ . The **expected value of the sum of two random variables** is equal to the sum of the expected values. The **variance of the sum of two random variables** is equal to the sum of the variances plus twice the covariance. The **standard deviation of the sum of two random variables** is the square root of the variance of the sum of two random variables.

#### EXPECTED VALUE OF THE SUM OF TWO RANDOM VARIABLES

$$E(X + Y) = E(X) + E(Y) \quad (5.5)$$

#### VARIANCE OF THE SUM OF TWO RANDOM VARIABLES

$$\text{Var}(X + Y) = \sigma_{X+Y}^2 = \sigma_X^2 + \sigma_Y^2 + 2\sigma_{XY} \quad (5.6)$$

#### STANDARD DEVIATION OF THE SUM OF TWO RANDOM VARIABLES

$$\sigma_{X+Y} = \sqrt{\sigma_{X+Y}^2} \quad (5.7)$$

To illustrate the expected value, variance, and standard deviation of the sum of two random variables, consider the two investments previously discussed. If  $X$  = Dow Jones fund and  $Y$  = weak-economy fund, using Equations (5.5), (5.6), and (5.7),

$$E(X + Y) = E(X) + E(Y) = 105 + 35 = \$140$$

$$\begin{aligned} \sigma_{X+Y}^2 &= \sigma_X^2 + \sigma_Y^2 + 2\sigma_{XY} \\ &= 14,725 + 11,025 + (2)(-12,675) \\ &= 400 \end{aligned}$$

$$\sigma_{X+Y} = \$20$$

The expected return of the sum of the Dow Jones fund and the weak-economy fund is \$140, with a standard deviation of \$20. The standard deviation of the sum of the two investments is much less than the standard deviation of either single investment because there is a large negative covariance between the investments.

### Portfolio Expected Return and Portfolio Risk

Now that the covariance and the expected return and standard deviation of the sum of two random variables have been defined, these concepts can be applied to the study of a group of assets referred to as a **portfolio**. Investors combine assets into portfolios to reduce their risk (see references 1 and 2). Often, the objective is to maximize the return while minimizing the risk. For such portfolios, rather than studying the sum of two random variables, each investment is weighted by the proportion of assets assigned to that investment. Equations (5.8) and (5.9) define the **portfolio expected return** and **portfolio risk**.

**PORTFOLIO EXPECTED RETURN**

The portfolio expected return for a two-asset investment is equal to the weight assigned to asset  $X$  multiplied by the expected return of asset  $X$  plus the weight assigned to asset  $Y$  multiplied by the expected return of asset  $Y$ .

$$E(P) = wE(X) + (1 - w)E(Y) \quad (5.8)$$

where

$E(P)$  = portfolio expected return

$w$  = portion of the portfolio value assigned to asset  $X$

$(1 - w)$  = portion of the portfolio value assigned to asset  $Y$

$E(X)$  = expected return of asset  $X$

$E(Y)$  = expected return of asset  $Y$

**PORTFOLIO RISK**

$$\sigma_p = \sqrt{w^2\sigma_X^2 + (1 - w)^2\sigma_Y^2 + 2w(1 - w)\sigma_{XY}} \quad (5.9)$$

In the previous example, you evaluated the expected return and risk of two different investments, a Dow Jones fund and a weak-economy fund. You also computed the covariance of the two investments. Now suppose that you wish to form a portfolio of these two investments that consists of an equal investment in each of these two funds. To compute the portfolio expected return and the portfolio risk, using Equations (5.8) and (5.9), with  $w = 0.50$ ,  $E(X) = \$105$ ,  $E(Y) = \$35$ ,  $\sigma_X^2 = 14,725$ ,  $\sigma_Y^2 = 11,025$ , and  $\sigma_{XY} = -12,675$ ,

$$E(P) = (0.5)(105) + (1 - 0.5)(35) = \$70$$

$$\begin{aligned} \sigma_p &= \sqrt{(0.5)^2(14,725) + (1 - 0.5)^2(11,025) + 2(0.5)(1 - 0.5)(-12,675)} \\ &= \sqrt{100} = \$10 \end{aligned}$$

Thus, the portfolio has an expected return of \$70 for each \$1,000 invested (a return of 7%) and has a portfolio risk of \$10. The portfolio risk here is small because there is a large negative covariance between the two investments. The fact that each investment performs best under different circumstances reduces the overall risk of the portfolio.

**PROBLEMS FOR SECTION 5.2****Learning the Basics**

PH Grade  
ASSIST

**5.7** Given the following probability distributions for variables  $X$  and  $Y$ :

$P(X_iY_i)$	$X$	$Y$
0.4	100	200
0.6	200	100

Compute

- $E(X)$  and  $E(Y)$ .
- $\sigma_X$  and  $\sigma_Y$ .
- $\sigma_{XY}$ .
- $E(X + Y)$ .

**PH Grade ASSIST** **5.8** Given the following probability distributions for variables  $X$  and  $Y$ :

$P(X_i; Y_j)$	$X$	$Y$
0.2	-100	50
0.4	50	30
0.3	200	20
0.1	300	20

Compute

- $E(X)$  and  $E(Y)$ .
- $\sigma_X$  and  $\sigma_Y$ .
- $\sigma_{XY}$ .
- $E(X + Y)$ .

**PH Grade ASSIST** **5.9** Two investments,  $X$  and  $Y$ , have the following characteristics:

$$E(X) = \$50, E(Y) = \$100, \sigma_X^2 = 9,000, \\ \sigma_Y^2 = 15,000, \text{ and } \sigma_{XY} = 7,500.$$

If the weight of portfolio assets assigned to investment  $X$  is 0.4, compute the

- portfolio expected return.
- portfolio risk.

### Applying the Concepts

**5.10** The process of being served at a bank consists of two independent parts—the time waiting in line and the time it takes to be served by the teller. Suppose that the time waiting in line has an expected value of 4 minutes, with a standard deviation of 1.2 minutes, and the time it takes to be served by the teller has an expected value of 5.5 minutes, with a standard deviation of 1.5 minutes. Compute the

- expected value of the total time it takes to be served at the bank.
- standard deviation of the total time it takes to be served at the bank.

**PH Grade ASSIST** **5.11** In the portfolio example in this section (see page 187), half the portfolio assets are invested in the Dow Jones fund and half in a weak-economy fund. Recalculate the portfolio expected return and the portfolio risk if

- 30% of the portfolio assets are invested in the Dow Jones fund and 70% in a weak-economy fund.
- 70% of the portfolio assets are invested in the Dow Jones fund and 30% in a weak-economy fund.
- Which of the three investment strategies (30%, 50%, or 70% in the Dow Jones fund) would you recommend? Why?

**SELF Test** **5.12** You are trying to develop a strategy for investing in two different stocks. The anticipated annual return for a \$1,000 investment in each

stock under four different economic conditions has the following probability distribution:

Probability	Economic Condition	Returns	
		Stock X	Stock Y
0.1	Recession	-\$100	\$50
0.3	Slow growth	0	150
0.3	Moderate growth	80	-20
0.3	Fast growth	150	-100

Compute the

- expected return for stock  $X$  and for stock  $Y$ .
- standard deviation for stock  $X$  and for stock  $Y$ .
- covariance of stock  $X$  and stock  $Y$ .
- Would you invest in stock  $X$  or stock  $Y$ ? Explain.

**5.13** Suppose that in Problem 5.12 you wanted to create a portfolio that consists of stock  $X$  and stock  $Y$ . Compute the portfolio expected return and portfolio risk for each of the following percentages invested in stock  $X$

- 30%
- 50%
- 70%
- On the basis of the results of (a) through (c), which portfolio would you recommend? Explain.

**5.14** You are trying to develop a strategy for investing in two different stocks. The anticipated annual return for a \$1,000 investment in each stock under four different economic conditions has the following probability distribution:

Probability	Economic Condition	Returns	
		Stock X	Stock Y
0.1	Recession	-\$50	-\$100
0.3	Slow growth	20	50
0.4	Moderate growth	100	130
0.2	Fast growth	150	200

Compute the

- expected return for stock  $X$  and for stock  $Y$ .
- standard deviation for stock  $X$  and for stock  $Y$ .
- covariance of stock  $X$  and stock  $Y$ .
- Would you invest in stock  $X$  or stock  $Y$ ? Explain.

**5.15** Suppose that in Problem 5.14 you wanted to create a portfolio that consists of stock  $X$  and stock  $Y$ . Compute the portfolio expected return and portfolio risk for each of the following percentages invested in stock  $X$

- 30%
- 50%
- 70%
- On the basis of the results of (a) through (c), which portfolio would you recommend? Explain.

**5.16** You are trying to set up a portfolio that consists of a corporate bond fund and a common stock fund. The fol-

lowing information about the annual return (per \$1,000) of each of these investments under different economic conditions is available, along with the probability that each of these economic conditions will occur:

Probability	Economic Conditions	Corporate Bond Fund	Common Stock Fund
0.10	Recession	−\$30	−\$150
0.15	Stagnation	50	−20
0.35	Slow growth	90	120
0.30	Moderate growth	100	160
0.10	High growth	110	250

Compute the

- expected return for the corporate bond fund and for the common stock fund.
  - standard deviation for the corporate bond fund and for the common stock fund.
  - covariance of the corporate bond fund and the common stock fund.
  - Would you invest in the corporate bond fund or the common stock fund? Explain.
- 5.17** Suppose that in Problem 5.16 you wanted to create a portfolio that consists of a corporate bond fund and a common stock fund. Compute the portfolio expected return and portfolio risk for each of the following percentages invested in a corporate bond fund:
- 30%
  - 50%
  - 70%
  - On the basis of the results of (a) through (c), which portfolio would you recommend? Explain.

## 5.3 BINOMIAL DISTRIBUTION

This section and the two that follow use mathematical models to solve business problems.

### MATHEMATICAL MODEL

A **mathematical model** is a mathematical expression that represents a variable of interest.

When a mathematical expression is available, you can compute the exact probability of occurrence of any particular outcome of the variable.

The **binomial distribution** is one of the most useful mathematical models. You use the binomial distribution when the discrete variable of interest is the number of successes in a sample of  $n$  observations. The binomial distribution has four essential properties:

- The sample consists of a fixed number of observations,  $n$ .
- Each observation is classified into one of two mutually exclusive and collectively exhaustive categories, usually called *success* and *failure*.
- The probability of an observation being classified as success,  $p$ , is constant from observation to observation. Thus, the probability of an observation being classified as failure,  $1 - p$ , is constant over all observations.
- The outcome (i.e., success or failure) of any observation is independent of the outcome of any other observation. To ensure independence, the observations can be randomly selected either from an *infinite population without replacement* or from a *finite population with replacement*.

Returning to the Using Statistics scenario presented on page 180 concerning the accounting information system, suppose *success* is defined as a tagged order form and *failure* as any other outcome. You are interested in the number of tagged order forms in a given sample of orders.

What results can occur? If the sample contains four orders, there could be none, one, two, three, or four tagged order forms. The binomial random variable, the number of tagged order forms, cannot take on any other value because the number of tagged order forms cannot be more than the sample size,  $n$ , and cannot be less than zero. Therefore, the binomial random variable has a range from 0 to  $n$ .

Suppose that you observe the following result in a sample of four orders:

First Order	Second Order	Third Order	Fourth Order
Tagged	Tagged	Not tagged	Tagged

What is the probability of having three successes (tagged order forms) in a sample of four orders in this particular sequence? Because the historical probability of a tagged order is 0.10, the probability that each order occurs in the sequence is

First Order	Second Order	Third Order	Fourth Order
$p = 0.10$	$p = 0.10$	$1 - p = 0.90$	$p = 0.10$

Each outcome is independent of the others because the order forms were selected from an extremely large or practically infinite population without replacement. Therefore, the probability of having this particular sequence is

$$\begin{aligned}
 p p (1 - p) p &= p^3 (1 - p)^1 \\
 &= (0.10)(0.10)(0.10)(0.90) \\
 &= (0.10)^3 (0.90)^1 \\
 &= 0.0009
 \end{aligned}$$

This result indicates only the probability of three tagged order forms (successes) from a sample of four order forms in a *specific sequence*. To find the number of ways of selecting  $X$  objects from  $n$  objects, *irrespective of sequence*, you use the **rule of combinations** given in Equation (5.10).

### COMBINATIONS

The number of combinations of selecting  $X$  objects out of  $n$  objects is given by

$${}_n C_X = \frac{n!}{X!(n - X)!} \quad (5.10)$$

where

$$n! = (n)(n - 1) \dots (1) \text{ is called } n \text{ factorial. By definition, } 0! = 1.$$

With  $n = 4$  and  $X = 3$ , there are

$${}_4 C_3 = \frac{4!}{3!(4 - 3)!} = \frac{4!}{3!(1)!} = \frac{4 \times 3 \times 2 \times 1}{(3 \times 2 \times 1)(1)} = 4$$

such sequences. The four possible sequences are:

Sequence 1 = *tagged, tagged, tagged, not tagged*, with probability

$$p p p (1 - p) = p^3 (1 - p)^1 = 0.0009$$

Sequence 2 = *tagged, tagged, not tagged, tagged*, with probability

$$p p (1 - p) p = p^3 (1 - p)^1 = 0.0009$$

Sequence 3 = *tagged, not tagged, tagged, tagged*, with probability

$$p (1 - p) p p = p^3 (1 - p)^1 = 0.0009$$

Sequence 4 = *not tagged, tagged, tagged, tagged*, with probability

$$(1 - p) p p p = p^3 (1 - p)^1 = 0.0009$$

Therefore, the probability of three tagged order forms is equal to

$$\begin{aligned} & (\text{Number of possible sequences}) \times (\text{Probability of a particular sequence}) \\ &= (4) \times (0.0009) = 0.0036 \end{aligned}$$

You can make a similar, intuitive derivation for the other possible outcomes of the random variable—zero, one, two, and four tagged order forms. However, as  $n$ , the sample size, gets large, the computations involved in using this intuitive approach become time-consuming. A mathematical model provides a general formula for computing any binomial probability. Equation (5.11) is the mathematical model representing the binomial probability distribution for computing the number of successes,  $X$ , given the values of  $n$  and  $p$ .

### BINOMIAL DISTRIBUTION

$$P(X) = \frac{n!}{X!(n-X)!} p^X (1-p)^{n-X} \quad (5.11)$$

where

$P(X)$  = probability of  $X$  successes, given  $n$  and  $p$

$n$  = number of observations

$p$  = probability of success

$1 - p$  = probability of failure

$X$  = number of successes in the sample ( $X = 0, 1, 2, \dots, n$ )

Equation (5.11) restates what you had intuitively derived. The binomial variable  $X$  can have any integer value  $X$  from 0 through  $n$ . In Equation (5.11), the product

$$p^X (1-p)^{n-X}$$

indicates the probability of exactly  $X$  successes from  $n$  observations in a *particular sequence*. The term

$$\frac{n!}{X!(n-X)!}$$

indicates *how many combinations* of the  $X$  successes from  $n$  observations are possible. Hence, given the number of observations,  $n$ , and the probability of success,  $p$ , the probability of  $X$  successes is:

$$\begin{aligned} P(X) &= (\text{Number of possible sequences}) \times (\text{Probability of a particular sequence}) \\ &= \frac{n!}{X!(n-X)!} p^X (1-p)^{n-X} \end{aligned}$$

Example 5.1 illustrates the use of Equation (5.11).

**EXAMPLE 5.1** DETERMINING  $P(X = 3)$ , GIVEN  $n = 4$  AND  $p = 0.1$ 

If the likelihood of a tagged order form is 0.1, what is the probability that there are three tagged order forms in the sample of four?

**SOLUTION** Using Equation (5.11), the probability of three tagged orders from a sample of four is

$$\begin{aligned} P(X = 3) &= \frac{4!}{3!(4-3)!} (0.1)^3 (1-0.1)^{4-3} \\ &= \frac{4!}{3!(4-3)!} (0.1)^3 (0.9)^1 \\ &= 4(0.1)(0.1)(0.1)(0.9) = 0.0036 \end{aligned}$$

Examples 5.2 and 5.3 show the computations for other values of  $X$ .

**EXAMPLE 5.2** DETERMINING  $P(X \geq 3)$ , GIVEN  $n = 4$  AND  $p = 0.1$ 

If the likelihood of a tagged order form is 0.1, what is the probability that there are three or more (that is, at least three) tagged order forms in the sample of four?

**SOLUTION** In Example 5.1, you found that the probability of *exactly* three tagged order forms from a sample of four is 0.0036. To compute the probability of *at least* three tagged order forms, you need to add the probability of three tagged order forms to the probability of four tagged order forms. The probability of four tagged order forms is

$$\begin{aligned} P(X = 4) &= \frac{4!}{4!(4-4)!} (0.1)^4 (1-0.1)^{4-4} \\ &= \frac{4!}{4!(0)!} (0.1)^4 (0.9)^0 \\ &= 1(0.1)(0.1)(0.1)(0.1) = 0.0001 \end{aligned}$$

Thus, the probability of at least three tagged order forms is

$$\begin{aligned} P(X \geq 3) &= P(X = 3) + P(X = 4) \\ &= 0.0036 + 0.0001 \\ &= 0.0037 \end{aligned}$$

There is a 0.37% chance that there will be at least three tagged order forms in a sample of four.

**EXAMPLE 5.3** DETERMINING  $P(X < 3)$ , GIVEN  $n = 4$  AND  $p = 0.1$ 

If the likelihood of a tagged order form is 0.1, what is the probability that there are less than three tagged order forms in the sample of four?

**SOLUTION** The probability that there are less than three tagged order forms is

$$P(X < 3) = P(X = 0) + P(X = 1) + P(X = 2)$$



Using Equation (5.11) on page 191, these probabilities are

$$P(X = 0) = \frac{4!}{0!(4-0)!} (0.1)^0 (1-0.1)^{4-0} = 0.6561$$

$$P(X = 1) = \frac{4!}{1!(4-1)!} (0.1)^1 (1-0.1)^{4-1} = 0.2916$$

$$P(X = 2) = \frac{4!}{2!(4-2)!} (0.1)^2 (1-0.1)^{4-2} = 0.0486$$

Therefore,  $P(X < 3) = 0.6561 + 0.2916 + 0.0486 = 0.9963$ .

$P(X < 3)$  could also be calculated from its complement,  $P(X \geq 3)$ , as follows:

$$\begin{aligned} P(X < 3) &= 1 - P(X \geq 3) \\ &= 1 - 0.0037 = 0.9963 \end{aligned}$$

Computations such as those in Example 5.3 can become tedious, especially as  $n$  gets large. To avoid computational drudgery, you can find many binomial probabilities directly from Table E.6, a portion of which is reproduced in Table 5.5. Table E.6 provides binomial probabilities for  $X = 0, 1, 2, \dots, n$  for various selected combinations of  $n$  and  $p$ . For example, to find the probability of exactly two successes in a sample of four when the probability of success is 0.1, you first find  $n = 4$  and then look in the row  $X = 2$  and column  $p = 0.10$ . The result is 0.0486.

**TABLE 5.5**

Finding a Binomial Probability for  $n = 4$ ,  $X = 2$ , and  $p = 0.1$

$n$	$X$	$p$			
		0.01	0.02	...	0.10
4	0	0.9606	0.9224	...	0.6561
	1	0.0388	0.0753	...	0.2916
	2	0.0006	0.0023	...	0.0486
	3	0.0000	0.0000	...	0.0036
	4	0.0000	0.0000	...	0.0001

Source: Table E.6.

You can also compute the binomial probabilities given in Table E.6 by using Microsoft Excel as shown in Figure 5.2.

**FIGURE 5.2**

Microsoft Excel worksheet for computing binomial probabilities



See Section E5.3 to create this.

	A	B	
1	Tagged Orders		
2			
3	Data		
4	Sample size	4	
5	Probability of success	0.1	
6			
7	Statistics		
8	Mean	0.4	=B4 * B5
9	Variance	0.36	=B8 * (1 - B5)
10	Standard deviation	0.6	=SQRT(B9)
11			
12	Binomial Probabilities Table		
13	X	P(X)	
14	0	0.6561	=BINOMDIST(A14, \$B\$4, \$B\$5, FALSE)
15	1	0.2916	=BINOMDIST(A15, \$B\$4, \$B\$5, FALSE)
16	2	0.0486	=BINOMDIST(A16, \$B\$4, \$B\$5, FALSE)
17	3	0.0036	=BINOMDIST(A17, \$B\$4, \$B\$5, FALSE)
18	4	0.0001	=BINOMDIST(A18, \$B\$4, \$B\$5, FALSE)

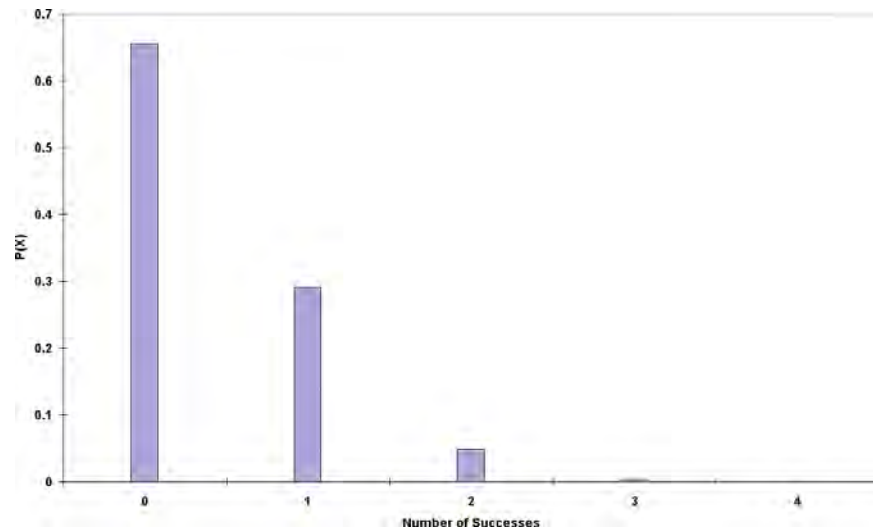
The shape of a binomial probability distribution depends on the values of  $n$  and  $p$ . Whenever  $p = 0.5$ , the binomial distribution is symmetrical, regardless of how large or small the value of  $n$ . When  $p \neq 0.5$ , the distribution is skewed. The closer  $p$  is to 0.5 and the larger the number of observations,  $n$ , the less skewed the distribution becomes. For example, the distribution of the number of tagged order forms is highly skewed to the right because  $p = 0.1$  and  $n = 4$  (see Figure 5.3).

**FIGURE 5.3**

Microsoft Excel histogram of the binomial probability distribution with  $n = 4$  and  $p = 0.1$



See Section E5.6 to create this.



The mean of the binomial distribution is equal to the product of  $n$  and  $p$ . Instead of using Equation (5.1) on page 181 to compute the mean of the probability distribution, you use Equation (5.12) to compute the mean for variables that follow the binomial distribution.

#### MEAN OF THE BINOMIAL DISTRIBUTION

The mean,  $\mu$ , of the binomial distribution is equal to the sample size,  $n$ , multiplied by the probability of success,  $p$ .

$$\mu = E(X) = np \quad (5.12)$$

On the average, over the long run, you theoretically expect  $\mu = E(X) = np = (4)(0.1) = 0.4$  tagged order form in a sample of four orders.

The standard deviation of the binomial distribution is calculated using Equation (5.13).

#### STANDARD DEVIATION OF THE BINOMIAL DISTRIBUTION

$$\sigma = \sqrt{\sigma^2} = \sqrt{Var(X)} = \sqrt{np(1-p)} \quad (5.13)$$

The standard deviation of the number of tagged order forms is

$$\sigma = \sqrt{4(0.1)(0.9)} = 0.60$$

You get the same result if you use Equation (5.3) on page 182.

Example 5.4 applies the binomial distribution to service at a fast-food restaurant.

**EXAMPLE 5.4**      **COMPUTING BINOMIAL PROBABILITIES**

Accuracy in taking orders at a drive-through window is an important feature for fast-food chains. Each month *QSR Magazine*, [www.qsrmagazine.com](http://www.qsrmagazine.com), publishes the results of its surveys. Accuracy is measured as the percentage of orders consisting of a main item, side item, and drink (but omitting one standard item, such as a pickle) that are filled correctly. In a recent month, suppose that the percentage of correct orders of this type filled at Burger King was approximately 88%. If a sample of three orders is taken, what are the mean and standard deviation of the binomial distribution for the number of orders filled accurately? Suppose that you and two friends go to the drive-through window at Burger King, and each of you places an order of the type just mentioned. What are the probabilities that all three, that none of the three, and that at least two of the three orders will be filled accurately?

**SOLUTION** Because there are three orders and the probability of an accurate order is 0.88,  $n = 3$  and  $p = 0.88$ . Using Equations (5.12) and (5.13),

$$\begin{aligned}\mu &= E(X) = np = 3(0.88) = 2.64 \\ \sigma &= \sqrt{\sigma^2} = \sqrt{\text{Var}(X)} = \sqrt{np(1-p)} \\ &= \sqrt{3(0.88)(0.12)} \\ &= \sqrt{0.3168} = 0.563\end{aligned}$$

Using Equation (5.11),

$$\begin{aligned}P(X = 3) &= \frac{3!}{3!(3-3)!} (0.88)^3 (1-0.88)^{3-3} \\ &= \frac{3!}{3!(3-3)!} (0.88)^3 (0.12)^0 \\ &= 1(0.88)(0.88)(0.88)(1) = 0.6815 \\ P(X = 0) &= \frac{3!}{0!(3-0)!} (0.88)^0 (1-0.88)^{3-0} \\ &= \frac{3!}{0!(3-0)!} (0.88)^0 (0.12)^3 \\ &= 1(1)(0.12)(0.12)(0.12) = 0.0017 \\ P(X = 2) &= \frac{3!}{2!(3-2)!} (0.88)^2 (1-0.88)^{3-2} \\ &= \frac{3!}{2!(3-2)!} (0.88)^2 (0.12)^1 \\ &= 3(0.88)(0.88)(0.12) = 0.2788 \\ P(X \geq 2) &= P(X = 2) + P(X = 3) \\ &= 0.2788 + 0.6815 \\ &= 0.9603\end{aligned}$$

The mean number of accurate orders filled in a sample of three orders is 2.64, and the standard deviation is 0.563. The probability that all three orders are filled accurately is 0.6815, or 68.15%. The probability that none of the orders are filled accurately is 0.0017, or 0.17%. The probability that at least two orders are filled accurately is 0.9603, or 96.03%.

In this section, you have been introduced to the binomial distribution. The binomial distribution is an important mathematical model in many business situations. It is also used to estimate or test hypotheses about proportions (see Chapters 8 and 9).

## PROBLEMS FOR SECTION 5.3

### Learning the Basics

**PH Grade** **5.18** Determine the following:

- ASSIST**
- For  $n = 4$  and  $p = 0.12$ , what is  $P(X = 0)$ ?
  - For  $n = 10$  and  $p = 0.40$ , what is  $P(X = 9)$ ?
  - For  $n = 10$  and  $p = 0.50$ , what is  $P(X = 8)$ ?
  - For  $n = 6$  and  $p = 0.83$ , what is  $P(X = 5)$ ?

**5.19** If  $n = 5$  and  $p = 0.40$ , what is the probability that

- $X = 4$ ?
- $X \leq 3$ ?
- $X < 2$ ?
- $X > 1$ ?

**PH Grade** **5.20** Determine the mean and standard deviation of the random variable  $X$  in each of the following binomial distributions:

- ASSIST**
- $n = 4$  and  $p = 0.10$
  - $n = 4$  and  $p = 0.40$
  - $n = 5$  and  $p = 0.80$
  - $n = 3$  and  $p = 0.50$

### Applying the Concepts

**5.21** The increase or decrease in the price of a stock between the beginning and the end of a trading day is assumed to be an equally likely random event. What is the probability that a stock will show an increase in its closing price on five consecutive days?

**5.22** Sixty percent of Americans read their employment contracts, including the fine print (“Snapshots,” [usatoday.com](http://usatoday.com), January 20, 2004). Assume that the number of employees who read every word of their contract can be modeled using the binomial distribution. For a group of five employees, what is the probability that

- all five will have read every word of their contracts?
- at least three will have read every word of their contracts?
- less than two will have read every word of their contracts?
- What are your answers in (a) through (c) if the probability is 0.80 that an employee reads every word of his or her contract?

**PH Grade** **5.23** A student is taking a multiple-choice exam in which each question has four choices.

**ASSIST** Assuming that she has no knowledge of the correct answers to any of the questions, she has decided on a strategy in which she will place four balls (marked  $A$ ,  $B$ ,  $C$ , and  $D$ ) into a box. She randomly selects one ball for each

question and replaces the ball in the box. The marking on the ball will determine her answer to the question. There are five multiple-choice questions on the exam. What is the probability that she will get

- five questions correct?
- at least four questions correct?
- no questions correct?
- no more than two questions correct?



**5.24** In Example 5.4 on page 195, you and two friends decided to go to Burger King. Instead, suppose that you went to McDonald’s, which last month filled 90% of the orders accurately. What is the probability that

- all three orders will be filled accurately?
- none of the three will be filled accurately?
- at least two of the three will be filled accurately?
- What are the mean and standard deviation of the binomial distribution used in (a) through (c)? Interpret these values.

**5.25** In April 2006, Gallup News Service reported that just 25% of U.S. adults view the country’s economic outlook as positive. Gallup further reports that this pessimistic view of the economy is relatively unchanged since early 2001. During this five-year period, Gallup has surveyed more than 100,000 people (F. Newport and J. Carroll, *Public’s View of Economy Has Never Recovered After Dot-Com Bust*, [galluppoll.com](http://galluppoll.com), April 18, 2006.). Thus the probability that a randomly selected adult in the United States views the economic outlook as positive is 0.25.

- You select a random sample of 10 adults in the United States. Assume that the number of the 10 adults having a positive outlook on the economy is distributed as a binomial random variable. What are the mean and standard deviation of this distribution?
- What assumptions are necessary in (a)?

**5.26** Referring to Problem 5.25, find the probability that of the 10 adults:

- 0 have a positive outlook on the economy.
- exactly 5 have a positive outlook on the economy.
- 5 or less have a positive outlook on the economy.
- 6 or more have a positive outlook on the economy.
- If you took a random sample of 10 adults from the state of California and found that 6 had a positive outlook on the economy, what could you infer about Californians’

views of the economy compared to those in the United States as a whole?

PH Grade  
ASSIST

**5.27** When a customer places an order with Rudy's On-Line Office Supplies, a computerized accounting information system (AIS) automatically checks to see if the customer has exceeded his or her credit limit. Past records indicate that the probability of customers exceeding their credit limit is 0.05. Suppose that, on a given day, 20 customers place orders. Assume that the number of customers that the AIS detects as having exceeded their credit limit is distributed as a binomial random variable.

- What are the mean and standard deviation of the number of customers exceeding their credit limits?
- What is the probability that 0 customers will exceed their limits?
- What is the probability that 1 customer will exceed his or her limit?
- What is the probability that 2 or more customers will exceed their limits?

**5.28** In a survey conducted by the Society for Human Resource Management, 68% of workers said that employers have the right to monitor their telephone use. ("Snapshots," *usatoday.com*, April 18, 2006). Suppose that a random sample of 20 workers is selected, and they are asked if employers have the right to monitor telephone use. What is the probability that:

- 5 or less of the workers agree?
- 10 or less of the workers agree?
- 15 or less of the workers agree?

**5.29** Referring to Problem 5.28, when the same workers were asked if employers have the right to monitor their cell phone use, the percentage dropped to 52%. Suppose that the 20 workers are asked if employers have the right to monitor cell phone use. What is the probability that:

- 5 or less of the workers agree?
- 10 or less of the workers agree?
- 15 or less of the workers agree?
- Compare the results of (a) through (c) to those for Problem 5.28.

## 5.4 POISSON DISTRIBUTION

Many studies are based on counts of the times a particular event occurs in a given *area of opportunity*. An **area of opportunity** is a continuous unit or interval of time, volume, or such area in which more than one occurrence of an event can occur. Examples are the surface defects on a new refrigerator, the number of network failures in a day, and the number of fleas on the body of a dog. When you have situations such as these, you can use the **Poisson distribution** to calculate probabilities if

- You are interested in counting the number of times a particular event occurs in a given area of opportunity. The area of opportunity is defined by time, length, surface area, and so forth.
- The probability that an event occurs in a given area of opportunity is the same for all the areas of opportunity.
- The number of events that occur in one area of opportunity is independent of the number of events that occur in any other area of opportunity.
- The probability that two or more events will occur in an area of opportunity approaches zero as the area of opportunity becomes smaller.

Consider the number of customers arriving during the lunch hour at a bank located in the central business district in a large city. You are interested in the number of customers that arrive each minute. Does this situation match the four properties of the Poisson distribution given above? First, the *event* of interest is a customer arriving, and the *given area of opportunity* is defined as a 1-minute interval. Will zero customers arrive, one customer arrive, two customers arrive, and so on? Second, it is reasonable to assume that the probability that a customer arrives during a 1-minute interval is the same as the probability for all the other 1-minute intervals. Third, the arrival of one customer in any 1-minute interval has no effect on (that is, is statistically independent of) the arrival of any other customer in any other 1-minute interval. Finally, the probability that two or more customers will arrive in a given time period approaches zero as the time interval becomes small. For example, the probability is virtually zero that two customers will arrive in a time interval with a width of 1/100 second. Thus, you can use the Poisson distribution to determine probabilities involving the number of customers arriving at the bank in a 1-minute time interval during the lunch hour.

The Poisson distribution has one parameter, called  $\lambda$  (the Greek lowercase letter *lambda*), which is the mean or expected number of events per unit. The variance of a Poisson distribution is also equal to  $\lambda$ , and the standard deviation is equal to  $\sqrt{\lambda}$ . The number of events,  $X$ , of the Poisson random variable ranges from 0 to infinity ( $\infty$ ).

Equation (5.14) presents the mathematical expression for the Poisson distribution for computing the probability of  $X$  events, given that  $\lambda$  events are expected.

### POISSON PROBABILITY DISTRIBUTION

$$P(X) = \frac{e^{-\lambda} \lambda^X}{X!} \quad (5.14)$$

where

$P(X)$  = the probability of  $X$  events in an area of opportunity

$\lambda$  = expected number of events

$e$  = mathematical constant approximated by 2.71828

$X$  = number of events

To demonstrate the Poisson distribution, suppose that the mean number of customers who arrive per minute at the bank during the noon-to-1 p.m. hour is equal to 3.0. What is the probability that in a given minute, exactly two customers will arrive? And what is the probability that more than two customers will arrive in a given minute?

Using Equation (5.14) and  $\lambda = 3$ , the probability that in a given minute exactly two customers will arrive is

$$P(X = 2) = \frac{e^{-3.0} (3.0)^2}{2!} = \frac{9}{(2.71828)^3 (2)} = 0.2240$$

To determine the probability that in any given minute more than two customers will arrive,

$$P(X > 2) = P(X = 3) + P(X = 4) + \cdots + P(X = \infty)$$

Because all the probabilities in a probability distribution must sum to 1, the terms on the right side of the equation  $P(X > 2)$  also represent the complement of the probability that  $X$  is less than or equal to 2 [that is,  $1 - P(X \leq 2)$ ]. Thus,

$$P(X > 2) = 1 - P(X \leq 2) = 1 - [P(X = 0) + P(X = 1) + P(X = 2)]$$

Now, using Equation (5.14),

$$\begin{aligned} P(X > 2) &= 1 - \left[ \frac{e^{-3.0} (3.0)^0}{0!} + \frac{e^{-3.0} (3.0)^1}{1!} + \frac{e^{-3.0} (3.0)^2}{2!} \right] \\ &= 1 - [0.0498 + 0.1494 + 0.2240] \\ &= 1 - 0.4232 = 0.5768 \end{aligned}$$

Thus, there is a 57.68% chance that more than two customers will arrive in the same minute.

To avoid computational drudgery involved in these computations, you can find Poisson probabilities directly from Table E.7, a portion of which is reproduced in Table 5.6. Table E.7 provides the probabilities that the Poisson random variable takes on values of  $X = 0, 1, 2, \dots$ ,

for selected values of the parameter  $\lambda$ . To find the probability that exactly two customers will arrive in a given minute when the mean number of customers arriving is 3.0 per minute, you can read the probability corresponding to the row  $X = 2$  and column  $\lambda = 3.0$  from the table. The result is 0.2240, as demonstrated in Table 5.6.

**TABLE 5.6**

Finding a Poisson Probability for  $\lambda = 3$

X	$\lambda$			
	2.1	2.2	...	3.0
0	.1225	.1108	....	.0498
1	.2572	.2438	....	.1494
2	.2700	.2681	....	.2240
3	.1890	.1966	....	.2240
4	.0992	.1082	....	.1680
5	.0417	.0476	....	.1008
6	.0146	.0174	....	.0504
7	.0044	.0055	....	.0216
8	.0011	.0015	....	.0081
9	.0003	.0004	....	.0027
10	.0001	.0001	....	.0008
11	.0000	.0000	....	.0002
12	.0000	.0000	....	.0001

Source: Table E.7.

You can also compute the Poisson probabilities given in Table E.7 by using Microsoft Excel, as illustrated by the worksheet in Figure 5.4.

**FIGURE 5.4**

Microsoft Excel worksheet for computing Poisson probabilities with  $\lambda = 3$



See Section E5.4 to create this.

	A	B	C	D	E
1	<b>Customer Arrivals Analysis</b>				
2					
3	<b>Data</b>				
4	<b>Average/Expected number of successes:</b>				3
5					
6	<b>Poisson Probabilities Table</b>				
7	<b>X</b>	<b>P(X)</b>			
8	0	0.049787	=	POISSON(A8, \$E\$4, FALSE)	
9	1	0.149361	=	POISSON(A9, \$E\$4, FALSE)	
10	2	0.224042	=	POISSON(A10, \$E\$4, FALSE)	
11	3	0.224042	=	POISSON(A11, \$E\$4, FALSE)	
12	4	0.168031	=	POISSON(A12, \$E\$4, FALSE)	
13	5	0.100819	=	POISSON(A13, \$E\$4, FALSE)	
14	6	0.050409	=	POISSON(A14, \$E\$4, FALSE)	
15	7	0.021604	=	POISSON(A15, \$E\$4, FALSE)	
16	8	0.008102	=	POISSON(A16, \$E\$4, FALSE)	
17	9	0.002701	=	POISSON(A17, \$E\$4, FALSE)	
18	10	0.000810	=	POISSON(A18, \$E\$4, FALSE)	
19	11	0.000221	=	POISSON(A19, \$E\$4, FALSE)	
20	12	0.000055	=	POISSON(A20, \$E\$4, FALSE)	
21	13	0.000013	=	POISSON(A21, \$E\$4, FALSE)	
22	14	0.000003	=	POISSON(A22, \$E\$4, FALSE)	
23	15	0.000001	=	POISSON(A23, \$E\$4, FALSE)	
24	16	0.000000	=	POISSON(A24, \$E\$4, FALSE)	
25	17	0.000000	=	POISSON(A25, \$E\$4, FALSE)	
26	18	0.000000	=	POISSON(A26, \$E\$4, FALSE)	
27	19	0.000000	=	POISSON(A27, \$E\$4, FALSE)	
28	20	0.000000	=	POISSON(A28, \$E\$4, FALSE)	

**EXAMPLE 5.5** COMPUTING POISSON PROBABILITIES

The number of work-related injuries per month in your manufacturing plant is known to follow a Poisson distribution with a mean of 2.5 work-related injuries a month. What is the probability that in a given month no work-related injuries occur? That at least one work-related injury occurs?

**SOLUTION** Using Equation (5.14) on page 198 with  $\lambda = 2.5$  (or using Table E.7 or Microsoft Excel), the probability that in a given month no work-related injuries occur is

$$P(X = 0) = \frac{e^{-2.5}(2.5)^0}{0!} = \frac{1}{(2.71828)^{2.5}(1)} = 0.0821$$

The probability that there will be no work-related injuries in a given month is 0.0821 or 8.21%. Thus,

$$\begin{aligned} P(X \geq 1) &= 1 - P(X = 0) \\ &= 1 - 0.0821 \\ &= 0.9179 \end{aligned}$$

The probability that there will be at least one work-related injury is 0.9179 or 91.79%.

## PROBLEMS FOR SECTION 5.4

## Learning the Basics

**PH Grade** **5.30** Assume a Poisson distribution.

- ASSIST**
- If  $\lambda = 2.5$ , find  $P(X = 2)$ .
  - If  $\lambda = 8.0$ , find  $P(X = 8)$ .
  - If  $\lambda = 0.5$ , find  $P(X = 1)$ .
  - If  $\lambda = 3.7$ , find  $P(X = 0)$ .

**PH Grade** **5.31** Assume a Poisson distribution.

- ASSIST**
- If  $\lambda = 2.0$ , find  $P(X \geq 2)$ .
  - If  $\lambda = 8.0$ , find  $P(X \geq 3)$ .
  - If  $\lambda = 0.5$ , find  $P(X \leq 1)$ .
  - If  $\lambda = 4.0$ , find  $P(X \geq 1)$ .
  - If  $\lambda = 5.0$ , find  $P(X \leq 3)$ .

**5.32** Assume a Poisson distribution with  $\lambda = 5.0$ . What is the probability that

- $X = 1$ ?
- $X < 1$ ?
- $X > 1$ ?
- $X \leq 1$ ?

## Applying the Concepts

**5.33** Assume that the number of network errors experienced in a day on a local area network (LAN) is distributed as a Poisson random variable. The mean number of network errors experienced in a day is 2.4. What is the probability that in any given day

- zero network errors will occur?
- exactly one network error will occur?

- two or more network errors will occur?
- less than three network errors will occur?

**SELF Test** **5.34** The quality control manager of Marilyn's Cookies is inspecting a batch of chocolate-chip cookies that has just been baked. If the production process is in control, the mean number of chip parts per cookie is 6.0. What is the probability that in any particular cookie being inspected

- less than five chip parts will be found?
- exactly five chip parts will be found?
- five or more chip parts will be found?
- either four or five chip parts will be found?

**5.35** Refer to Problem 5.34. How many cookies in a batch of 100 should the manager expect to discard if company policy requires that all chocolate-chip cookies sold have at least four chocolate-chip parts?

**5.36** The U.S. Department of Transportation maintains statistics for mishandled bags per 1,000 airline passengers. In 2005, Jet Blue had 4.06 mishandled bags per 1,000 passengers (extracted from M. Mullins, "Out of Place," *USA Today*, March 24, 2006, p. 10A). What is the probability that in the next 1,000 passengers, Jet Blue will have

- no mishandled bags?
- at least one mishandled bag?
- at least two mishandled bags?
- Compare the results in (a) through (c) to those of Delta in Problem 5.37 (a) through (c).



- 5.37** The U.S. Department of Transportation maintains statistics for mishandled bags per 1,000 airline passengers. In 2005, Delta had 7.09 mishandled bags per 1,000 passengers (extracted from M. Mullins, “Out of Place,” *USA Today*, March 24, 2006, p. 10A). What is the probability that in the next 1,000 passengers, Delta will have
- no mishandled bags?
  - at least one mishandled bag?
  - at least two mishandled bags?
  - Compare the results in (a) through (c) to those of Jet Blue in Problem 5.36 (a) through (c).

**PH Grade ASSIST** **5.38** Based on past experience, it is assumed that the number of flaws per foot in rolls of grade 2 paper follows a Poisson distribution with a mean of 1 flaw per 5 feet of paper (0.2 flaw per foot). What is the probability that in a

- 1-foot roll, there will be at least 2 flaws?
- 12-foot roll, there will be at least 1 flaw?
- 50-foot roll, there will be greater than or equal to 5 flaws and less than or equal to 15 flaws?

**5.39** J.D. Power and Associates calculates and publishes various statistics concerning car quality. The initial quality score measures the number of problems per new car sold. For 2004 model cars, the Lexus had 0.87 problems per car. Korea’s Kia had 1.53 problems per car (D. Hakim, “Hyundai Near Top of a Quality Ranking,” *The New York Times*, April 29, 2004, p. C8). Let the random variable  $X$  be equal to the number of problems with a newly purchased Lexus.

- What assumptions must be made in order for  $X$  to be distributed as a Poisson random variable? Are these assumptions reasonable?
- Making the assumptions as in (a), if you purchased a 2004 Lexus, what is the probability that the new car will have
- zero problems?
  - two or less problems?
  - Give an operational definition for *problem*. Why is the operational definition important in interpreting the initial quality score?

- 5.40** Refer to Problem 5.39. If you purchased a 2004 Kia, what is the probability that the new car will have
- zero problems?
  - two or less problems?
  - Compare your answers in (a) and (b) to those for the Lexus in Problem 5.39 (b) and (c).

**5.41** In 2005, both Lexus and Kia improved their performance. Lexus had 0.81 problems per car, and Korea’s Kia had 1.40 problems per car (S. S. Carty, “Toyota Comes Out on Top Again in Quality Study,” *USA Today*, May 19, 2005, p. 3B). If you purchased a 2005 Lexus, what is the probability that the new car will have

- zero problems?
- two or less problems?
- Compare your answers in (a) and (b) to those for the 2004 Lexus in Problem 5.39 (b) and (c).

- 5.42** Refer to Problem 5.41. If you purchased a 2005 Kia, what is the probability that the new car will have
- zero problems?
  - two or less problems?
  - Compare your answers in (a) and (b) to those for the 2004 Kia in Problem 5.40 (a) and (b).

**5.43** A toll-free phone number is available from 9 a.m. to 9 p.m. for your customers to register complaints about a product purchased from your company. Past history indicates that an average of 0.4 calls are received per minute.

- What properties must be true about the situation described here in order to use the Poisson distribution to calculate probabilities concerning the number of phone calls received in a 1-minute period?
- Assuming that this situation matches the properties discussed in (a), what is the probability that during a 1-minute period
- zero phone calls will be received?
  - three or more phone calls will be received?
  - What is the maximum number of phone calls that will be received in a 1-minute period 99.99% of the time?

## 5.5 HYPERGEOMETRIC DISTRIBUTION

Both the binomial distribution and the **hypergeometric distribution** are concerned with the number of successes in a sample containing  $n$  observations. One of the differences in these two probability distributions is in the way that the samples are selected. For the binomial distribution, the sample data are selected *with* replacement from a *finite* population or *without* replacement from an *infinite* population. Thus, the probability of success,  $p$ , is constant over all observations, and the outcome of any particular observation is independent of any other. For the hypergeometric distribution, the sample data are selected *without* replacement from a *finite* population. Thus, the outcome of one observation is dependent on the outcomes of the previous observations.

Consider a population of size  $N$ . Let  $A$  represent the total number of successes in the population. The hypergeometric distribution is then used to find the probability of  $X$  successes in a sample of size  $n$ , selected without replacement. Equation (5.15) presents the mathematical expression of the hypergeometric distribution for finding  $X$  successes, given a knowledge of  $n$ ,  $N$ , and  $A$ .

### HYPERGEOMETRIC DISTRIBUTION

$$P(X) = \frac{\binom{A}{X} \binom{N-A}{n-X}}{\binom{N}{n}} \quad (5.15)$$

where

$P(X)$  = the probability of  $X$  successes, given knowledge of  $n$ ,  $N$ , and  $A$

$n$  = sample size

$N$  = population size

$A$  = number of successes in the population

$N - A$  = number of failures in the population

$X$  = number of successes in the sample

$\binom{A}{X} = {}_A C_X$  (see Equation (5.10) on p. 190)

The number of successes in the sample, represented by  $X$ , cannot be greater than the number of successes in the population,  $A$ , or the sample size,  $n$ . Thus, the range of the hypergeometric random variable is limited to the sample size or to the number of successes in the population, whichever is smaller.

Equation (5.16) defines the mean of the hypergeometric distribution, and Equation (5.17) defines the standard deviation.

### MEAN OF THE HYPERGEOMETRIC DISTRIBUTION

$$\mu = E(X) = \frac{nA}{N} \quad (5.16)$$

### STANDARD DEVIATION OF THE HYPERGEOMETRIC DISTRIBUTION

$$\sigma = \sqrt{\frac{nA(N-A)}{N^2}} \sqrt{\frac{N-n}{N-1}} \quad (5.17)$$

In Equation (5.17), the expression  $\sqrt{\frac{N-n}{N-1}}$  is a **finite population correction factor** that results from sampling without replacement from a finite population.

To illustrate the hypergeometric distribution, suppose that you are forming a team of 8 executives from different departments within your company. Your company has a total of 30 executives, and 10 of these people are from the finance department. If members of the team are to be selected at random, what is the probability that the team will contain 2 executives from the finance department? Here, the population of  $N = 30$  executives within the company

is finite. In addition,  $A = 10$  are from the finance department. A team of  $n = 8$  members is to be selected.

Using Equation (5.15),

$$\begin{aligned}
 P(X = 2) &= \frac{\binom{10}{2} \binom{20}{6}}{\binom{30}{8}} \\
 &= \frac{10!}{2! (8)!} \frac{(20)!}{(6)! (14)!} \\
 &= \frac{30!}{8! (22)!} \\
 &= 0.298
 \end{aligned}$$

Thus, the probability that the team will contain two members from the finance department is 0.298, or 29.8%.

Such computations can become tedious, especially as  $N$  gets large. However, you can compute the probabilities by using Microsoft Excel. Figure 5.5 presents a Microsoft Excel worksheet for the team-formation example, where the number of executives from the finance department (that is, the number of successes in the sample) can be equal to 0, 1, 2, . . . , 8.

**FIGURE 5.5**

Microsoft Excel worksheet for the team member example



See Section E5.5 to create this.

A		B
1	Team Formation Analysis	
2		
3	Data	
4	Sample size	8
5	No. of successes in population	10
6	Population size	30
7		
8	Hypergeometric Probabilities Table	
9	X	P(X)
10	0	0.0215 =HYPGEOMDIST(A10, \$B\$4, \$B\$5, \$B\$6)
11	1	0.1324 =HYPGEOMDIST(A11, \$B\$4, \$B\$5, \$B\$6)
12	2	0.2980 =HYPGEOMDIST(A12, \$B\$4, \$B\$5, \$B\$6)
13	3	0.3179 =HYPGEOMDIST(A13, \$B\$4, \$B\$5, \$B\$6)
14	4	0.1738 =HYPGEOMDIST(A14, \$B\$4, \$B\$5, \$B\$6)
15	5	0.0491 =HYPGEOMDIST(A15, \$B\$4, \$B\$5, \$B\$6)
16	6	0.0068 =HYPGEOMDIST(A16, \$B\$4, \$B\$5, \$B\$6)
17	7	0.0004 =HYPGEOMDIST(A17, \$B\$4, \$B\$5, \$B\$6)
18	8	0.0000 =HYPGEOMDIST(A18, \$B\$4, \$B\$5, \$B\$6)

## PROBLEMS FOR SECTION 5.5

### Learning the Basics

- PH Grade ASSIST** **5.44** Determine the following:
- If  $n = 4$ ,  $N = 10$ , and  $A = 5$ , find  $P(X = 3)$ .
  - If  $n = 4$ ,  $N = 6$ , and  $A = 3$ , find  $P(X = 1)$ .
- 5.45** If  $n = 5$ ,  $N = 12$ , and  $A = 3$ , find  $P(X = 0)$ .
- 5.46** If  $n = 3$ ,  $N = 10$ , and  $A = 3$ , find  $P(X = 3)$ .

- PH Grade ASSIST** **5.45** Referring to Problem 5.44, compute the mean and standard deviation for the hypergeometric distributions described in (a) through (d).

### Applying the Concepts

- PH Grade ASSIST** **5.46** An auditor for the Internal Revenue Service is selecting a sample of 6 tax returns for an audit. If 2 or more of these returns are “improper,” the entire population of 100 tax returns will be audited. What is the probability that the entire population will be audited if the true number of improper returns in the population is
- 25?
  - 30?
  - 5?
- SELF Test**

d. 10?

e. Discuss the differences in your results, depending on the true number of improper returns in the population.

**5.47** The dean of a business school wishes to form an executive committee of 5 from among the 40 tenured faculty members at the school. The selection is to be random, and at the school there are 8 tenured faculty members in accounting. What is the probability that the committee will contain

a. none of them?

b. at least 1 of them?

c. not more than 1 of them?

d. What is your answer to (a) if the committee consists of 7 members?

**5.48** From an inventory of 30 cars being shipped to a local automobile dealer, 4 are SUVs. What is the probability that if 4 cars arrive at a particular dealership,

a. all 4 are SUVs?

b. none are SUVs?

c. at least 1 is an SUV?

d. What are your answers to (a) through (c) if 6 cars being shipped are SUVs?

**5.49** A state lottery is conducted in which 6 winning numbers are selected from a total of 54 numbers. What is the probability that if 6 numbers are randomly selected,

a. all 6 numbers will be winning numbers?

b. 5 numbers will be winning numbers?

c. none of the numbers will be winning numbers?

d. What are your answers to (a) through (c) if the 6 winning numbers are selected from a total of 40 numbers?

**5.50** In a shipment of 15 sets of golf clubs, 3 are left-handed. If 4 sets of golf clubs are selected, what is the probability that

a. exactly 1 is left-handed?

b. at least 1 is left-handed?

c. no more than 2 are left-handed?

d. What is the mean number of left-handed sets of golf clubs that you would expect to find in the sample of 4 sets of golf clubs?

## 5.6 (CD-ROM Topic) USING THE POISSON DISTRIBUTION TO APPROXIMATE THE BINOMIAL DISTRIBUTION

Under certain circumstances, the Poisson distribution can be used to approximate the binomial distribution. To study this topic, go to the [section5.6.pdf](#) file located on the CD-ROM that accompanies this book.

## SUMMARY

In this chapter, you have studied mathematical expectation, the covariance, and the development and application of the binomial, Poisson, and hypergeometric distributions. In the Using Statistics scenario, you learned how to calculate probabilities from the binomial distribution concerning the observation of tagged invoices in the accounting information system used by the Saxon Home Improvement Company. In the following chapter, you will study several important continuous distributions including the normal distribution.

To help decide what probability distribution to use for a particular situation, you need to ask the following questions:

- Is there a fixed number of observations,  $n$ , each of which is classified as success or failure? Or is there an area of opportunity? If there is a fixed number of observations,  $n$ , each of which is classified as success or failure, you use the binomial or hypergeometric distribution. If there is an area of opportunity, you use the Poisson distribution.
- In deciding whether to use the binomial or hypergeometric distribution, is the probability of success constant over all trials? If yes, you can use the binomial distribution. If no, you can use the hypergeometric distribution.

## KEY EQUATIONS

**Expected Value,  $\mu$ , of a Discrete Random Variable**

$$\mu = E(X) = \sum_{i=1}^N X_i P(X_i) \quad (5.1)$$

**Variance of a Discrete Random Variable**

$$\sigma^2 = \sum_{i=1}^N [X_i - E(X)]^2 P(X_i) \quad (5.2)$$

**Standard Deviation of a Discrete Random Variable**

$$\sigma = \sqrt{\sigma^2} = \sqrt{\sum_{i=1}^N [X_i - E(X)]^2 P(X_i)} \quad (5.3)$$

**Covariance**

$$\sigma_{XY} = \sum_{i=1}^N [X_i - E(X)][Y_i - E(Y)]P(X_i Y_i) \quad (5.4)$$

**Expected Value of the Sum of Two Random Variables**

$$E(X+Y) = E(X) + E(Y) \quad (5.5)$$

**Variance of the Sum of Two Random Variables**

$$Var(X+Y) = \sigma_{X+Y}^2 = \sigma_X^2 + \sigma_Y^2 + 2\sigma_{XY} \quad (5.6)$$

**Standard Deviation of the Sum of Two Random Variables**

$$\sigma_{X+Y} = \sqrt{\sigma_{X+Y}^2} \quad (5.7)$$

**Portfolio Expected Return**

$$E(P) = wE(X) + (1-w)E(Y) \quad (5.8)$$

**Portfolio Risk**

$$\sigma_p = \sqrt{w^2\sigma_X^2 + (1-w)^2\sigma_Y^2 + 2w(1-w)\sigma_{XY}} \quad (5.9)$$

**Combinations**

$${}^n C_X = \frac{n!}{X!(n-X)!} \quad (5.10)$$

**Binomial Distribution**

$$P(X) = \frac{n!}{X!(n-X)!} p^X (1-p)^{n-X} \quad (5.11)$$

**Mean of the Binomial Distribution**

$$\mu = E(X) = np \quad (5.12)$$

**Standard Deviation of the Binomial Distribution**

$$\sigma = \sqrt{\sigma^2} = \sqrt{Var(X)} = \sqrt{np(1-p)} \quad (5.13)$$

**Poisson Distribution**

$$P(X) = \frac{e^{-\lambda} \lambda^X}{X!} \quad (5.14)$$

**Hypergeometric Distribution**

$$P(X) = \frac{\binom{A}{X} \binom{N-A}{n-X}}{\binom{N}{n}} \quad (5.15)$$

**Mean of the Hypergeometric Distribution**

$$\mu = E(X) = \frac{nA}{N} \quad (5.16)$$

**Standard Deviation of the Hypergeometric Distribution**

$$\sigma = \sqrt{\frac{nA(N-A)}{N^2} \sqrt{\frac{N-n}{N-1}}} \quad (5.17)$$

**KEY TERMS**

area of opportunity 197

binomial distribution 189

covariance,  $\sigma_{XY}$  184expected value of the sum of two  
random variables 186expected value,  $\mu$ , of a discrete random  
variable 181

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variables 186

## CHAPTER REVIEW PROBLEMS

### Checking Your Understanding

- 5.51** What is the meaning of the expected value of a probability distribution?
- 5.52** What are the four properties that must be present in order to use the binomial distribution?
- 5.53** What are the four properties that must be present in order to use the Poisson distribution?
- 5.54** When do you use the hypergeometric distribution instead of the binomial distribution?

### Applying the Concepts

**PH Grade ASSIST** **5.55** Event insurance allows promoters of sporting and entertainment events to protect themselves from financial losses due to uncontrollable circumstances such as rain-outs. For example, each spring, Cincinnati's Downtown Council puts on the Taste of Cincinnati. This is a rainy time of year in Cincinnati, and the chance of receiving an inch or more of rain during a spring weekend is about one out of four. An article in the *Cincinnati Enquirer*, by Jim Knippenberg ("Chicken Pox Means 3 Dog Night Remedy," *Cincinnati Enquirer*, May 28, 1997, p. E1), gave the details for an insurance policy purchased by the Downtown Council. The policy would pay \$100,000 if it rained more than an inch during the weekend festival. The cost of the policy was reported to be \$6,500.

- a. Determine whether you believe that these dollar amounts are correct. (*Hint*: Calculate the expected value of the profit to be made by the insurance company.)
- b. Assume that the dollar amounts are correct. Is this policy a good deal for Cincinnati's Downtown Council?

**PH Grade ASSIST** **5.56** Between 1872 and 2000, stock prices rose in 74% of the years (M. Hulbert, "The Stock Market Must Rise in 2002? Think Again," *The New York Times*, December 6, 2001, Business, p. 6). Based on this information, and assuming a binomial distribution, what do you think the probability is that the stock market will rise

- a. next year?
- b. the year after next?
- c. in four of the next five years?
- d. in none of the next five years?
- e. For this situation, what assumption of the binomial distribution might not be valid?

**5.57** The mean cost of a phone call handled by an automated customer-service system is \$0.45. The mean cost of a phone call passed on to a "live" operator is \$5.50. However, as more and more companies have implemented automated systems, customer annoyance with such systems has grown. Many customers are quick to leave the automated system when given an option such as "Press zero to talk to a cus-

tommer-service representative." According to the Center for Client Retention, 40% of all callers to automated customer-service systems automatically opt to go to a live operator when given the chance (J. Spencer, "In Search of the Operator," *The Wall Street Journal*, May 8, 2002, p. D1).

If 10 independent callers contact an automated customer-service system, what is the probability that

- a. 0 will automatically opt to talk to a live operator?
- b. exactly 1 will automatically opt to talk to a live operator?
- c. 2 or less will automatically opt to talk to a live operator?
- d. all 10 will automatically opt to talk to a live operator?
- e. If all 10 automatically opt to talk to a live operator, do you think that the 40% figure given in the article applies to this particular system? Explain.

**5.58** One theory concerning the Dow Jones Industrial Average is that it is likely to increase during U.S. presidential election years. From 1964 through 2004, the Dow Jones Industrial Average increased in 9 of the 11 U.S. presidential election years. Assuming that this indicator is a random event with no predictive value, you would expect that the indicator would be correct 50% of the time.

- a. What is the probability of the Dow Jones Industrial Average increasing in 9 or more of the 11 U.S. presidential election years if the true probability of an increase in the Dow Jones Industrial Average is 0.50?
- b. Read Problem 5.56 and note that the Dow Jones Industrial Average increased in 74% of the years studied. What is the probability of the Dow Jones Industrial Average increasing in 9 of the 11 U.S. presidential election years if the probability of an increase in the Dow Jones Industrial Average is 0.74?

**5.59** Priority Mail is the U.S. Postal Service's alternative to commercial express mail companies such as FedEx. An article in *The Wall Street Journal* presented some interesting conclusions comparing Priority Mail shipments with the much less expensive first-class shipments (R. Brooks, "New Data Reveal 'Priority Mail' Is Slower Than a Stamp," *The Wall Street Journal*, May 29, 2002, p. D1). When comparing shipments intended for delivery in three days, first-class deliveries failed to deliver on time 19% of the time, while Priority Mail failed 33% of the time. Note that at the time of the article, first-class deliveries started as low as \$0.34, and Priority Mail started at \$3.50.

If 10 items are to be shipped first-class to 10 different destinations claimed to be in a three-day delivery location, what is the probability that

- a. 0 items will take more than three days?
- b. exactly 1 will take more than three days?
- c. 2 or more will take more than three days?
- d. What are the mean and the standard deviation of the probability distribution?

**5.60** Refer to Problem 5.59. If the shipments are made using Priority Mail, what is the probability that

- 0 items will take more than three days?
- exactly 1 will take more than three days?
- 2 or more will take more than three days?
- What are the mean and the standard deviation of the probability distribution?
- Compare the results of (a) through (c) to those of Problem 5.59 (a) through (c).

**5.61** Cinema advertising is increasing. Normally 60 to 90 seconds long, these advertisements are longer and more extravagant, and they tend to have more captive audiences than television advertisements. Thus, it is not surprising that the recall rates for viewers of cinema advertisements are higher than those for television advertisements. According to survey research conducted by the ComQUEST division of BBM Bureau of Measurement in Toronto, the probability a viewer will remember a cinema advertisement is 0.74, whereas the probability a viewer will remember a 30-second television advertisement is 0.37 (N. Hendley, "Cinema Advertising Comes of Age," *Marketing Magazine*, May 6, 2002, p. 16).

- Is the 0.74 probability reported by the BBM Bureau of Measurement best classified as *a priori* classical probability, empirical classical probability, or subjective probability?
- Suppose that 10 viewers of a cinema advertisement are randomly sampled. Consider the random variable defined by the number of viewers who recall the advertisement. What assumptions must be made in order to assume that this random variable is distributed as a binomial random variable?
- Assuming that the number of viewers who recall the cinema advertisement is a binomial random variable, what are the mean and standard deviation of this distribution?
- Based on your answer to (c), if none of the viewers can recall the ad, what can be inferred about the 0.74 probability given in the article?

**5.62** Refer to Problem 5.61. Compute the probability that of the 10 viewers of the cinema advertisement,

- exactly 0 can recall the advertisement.
- all 10 can recall the advertisement.
- more than half can recall the advertisement.
- 8 or more can recall the advertisement.

**5.63** Refer to Problem 5.61. For television advertisements, using the given probability of recall, 0.37, compute the probability that of the 10 viewers,

- exactly 0 can recall the advertisement.
- all 10 can recall the advertisement.
- more than half can recall the advertisement.
- 8 or more can recall the advertisement.
- Compare the results of (a) through (d) to those of Problem 5.62 (a) through (d).

**5.64** In a survey conducted by the Council for Marketing and Opinion Research (CMOR), a national nonprofit research industry trade group based in Cincinnati, 1,628 of 3,700 adults contacted in the United States refuse to participate in phone surveys (S. Jarvis, "CMOR Finds Survey Refusal Rate Still Rising," *Marketing News*, February 4, 2002, p. 4). Suppose that you are to randomly call 10 adults in the United States and ask them to participate in a phone survey. Using the results of the CMOR study, what is the probability that

- all 10 will refuse?
- exactly 5 will refuse?
- at least 5 will refuse?
- less than 5 will refuse?
- less than 5 will agree to be surveyed?
- What is the expected number of people who will refuse to participate? Explain the practical meaning of this number.

**5.65** Credit card companies are increasing their revenues by raising the late fees charged to their customers. According to a study by **cardweb.com**, late fees represent the third largest revenue source for card companies, after interest charges and payments from the merchants who accept their cards. In the preceding year, 58% of all credit card customers had had to pay late fees (R. Lieber, "Credit-Card Firms Collect Record Levels of Late Fees," *The Wall Street Journal*, May 21, 2002, p. D1).

If a random sample of 20 credit card holders is selected, what is the probability that

- 0 had to pay a late fee?
- no more than 5 had to pay late fees?
- more than 10 had to pay late fees?
- What assumptions did you have to make to answer (a) through (c)?

**5.66** One of the retail industry's biggest frustrations is customers who abuse the return and exchange policies (S. Kang, "New Return Policy: Retailers Say 'No' to Serial Exchangers," *The Wall Street Journal*, November 29, 2004, pp. B1, B3). In a recent year, returns were 13% of sales in department stores. Consider a sample of 20 customers who make a purchase at a department store. Use the binomial model to answer the following questions:

- What is the expected value, or mean, of the binomial distribution?
- What is the standard deviation of the binomial distribution?
- What is the probability that none of the 20 customers will make a return?
- What is the probability that no more than 2 of the customers will make a return?
- What is the probability that 3 or more of the customers will make a return?

**5.67** Refer to Problem 5.66. In the same year, returns were 1% of sales in grocery stores.

- What is the expected value, or mean, of the binomial distribution?
- What is the standard deviation of the binomial distribution?
- What is the probability that none of the 20 customers will make a return?
- What is the probability that no more than 2 of the customers will make a return?
- What is the probability that 3 or more of the customers will make a return?
- Compare the results of (a) through (e) to those of Problem 5.66 (a) through (e).

**5.68** One theory concerning the S&P 500 index is that if it increases during the first five trading days of the year, it is likely to increase during the entire year. From 1950 through 2005, the S&P 500 index had these early gains in 35 years. In 30 of these 35 years, the S&P 500 index increased. Assuming that this indicator is a random event with no predictive value, you would expect that the indicator would be correct 50% of the time. What is the probability of the S&P 500 index increasing in 30 or more years if the true probability of an increase in the S&P 500 index is

- 0.50?
- 0.70?
- 0.90?
- Based on the results of (a) through (c), what do you think is the probability that the S&P 500 index will increase if there is an early gain in the first five trading days of the year? Explain.

**5.69** *Spurious correlation* refers to the apparent relationship between variables that either have no true relationship or are related to other variables that have not been measured. One widely publicized stock market indicator in the United States that is an example of spurious correlation is the relationship between the winner of the National Football League Super Bowl and the performance of the Dow Jones Industrial Average in that year. The indicator states that when a team representing the National Football Conference wins the Super Bowl, the Dow Jones Industrial Average will increase in that year. When a team representing the American Football Conference wins the Super Bowl, the Dow Jones Industrial Average will decline in that year. Since the first Super Bowl was held in 1967 through 2005, the indicator has been correct 32 out of 39 times. Assuming that this indicator is a random event with no predictive value, you would expect that the indicator would be correct 50% of the time.

- What is the probability that the indicator would be correct 32 or more times in 39 years?
- What does this tell you about the usefulness of this indicator?

**5.70** Worldwide golf ball sales total more than \$1 billion annually. One reason for such a large number of golf ball purchases is that golfers lose them at a rate of 4.5 per 18-hole round (“Snapshots,” *usatoday.com*, January 29, 2004). Assume that the number of golf balls lost in an 18-hole round is distributed as a Poisson random variable.

- What assumptions need to be made so that the number of golf balls lost in an 18-hole round is distributed as a Poisson random variable?

Making the assumptions given in (a), what is the probability that

- 0 balls will be lost in an 18-hole round?
- 5 or less balls will be lost in an 18-hole round?
- 6 or more balls will be lost in an 18-hole round?

**5.71** A study of the home pages of Fortune 500 companies reports that the mean number of bad links per home page is 0.4 and the mean number of spelling errors per home page is 0.16 (N. Tamimi, M. Rajan, and R. Sebastianella, “Benchmarking the Home Pages of ‘Fortune’ 500 Companies,” *Quality Progress*, July 2000). Use the Poisson distribution to find the probability that a randomly selected home page will contain

- exactly 0 bad links.
- 5 or more bad links.
- exactly 0 spelling errors.
- 10 or more spelling errors.

**5.72** Mega Millions is one of the most popular lottery games in the United States. Participating states in Mega Millions are Georgia, Illinois, Maryland, Massachusetts, Michigan, New Jersey, New York, Ohio, and Virginia. Rules for playing and the list of prizes are given below (“Win Megamoney Playing Ohio’s Biggest Jackpot Game,” Ohio Lottery Headquarters, 2002):

*Rules:*

- Select five numbers from a pool of numbers from 1 to 52 and one Mega Ball number from a second pool of numbers from 1 to 52.
- Each wager costs \$1.

*Prizes:*

- Match all five numbers + Mega Ball—win jackpot (minimum of \$10,000,000)
- Match all five numbers—win \$175,000
- Match four numbers + Mega Ball—win \$5,000
- Match four numbers—win \$150
- Match three numbers + Mega Ball—win \$150
- Match two numbers + Mega Ball—win \$10
- Match three numbers—win \$7
- Match one number + Mega Ball—win \$3
- Match Mega Ball—win \$2



Find the probability of winning

- a. the jackpot.
  - b. the \$175,000 prize. (Note that this requires matching all five numbers but not matching the Mega Ball.)
  - c. \$5,000.
  - d. \$150.
  - e. \$10.
  - f. \$7.
  - g. \$3.
  - h. \$2.
- i. nothing.
  - j. All stores selling Mega Millions tickets are required to have a brochure that gives complete game rules and probabilities of winning each prize (the probability of having a losing ticket is not given). The slogan for all lottery games in the state of Ohio is “Play Responsibly. Odds Are, You’ll Have Fun.” Do you think Ohio’s slogan and the requirement of making available complete game rules and probabilities of winning is an ethical approach to running the lottery system?

## Managing the Springville Herald

The *Herald* marketing department is seeking to increase home-delivery sales through an aggressive direct-marketing campaign that includes mailings, discount coupons, and telephone solicitations. Feedback from these efforts indicates that getting their newspapers delivered early in the morning is a very important factor for both prospective as well as existing subscribers. After several brainstorming sessions, a team consisting of members from the marketing and circulation departments decided that guaranteeing newspaper delivery by a specific time could be an important selling point in retaining and getting new subscribers. The team concluded that the *Herald* should offer a guarantee that customers will receive their newspapers by a certain time or else that day’s issue is free.

To assist the team in setting a guaranteed delivery time, Al Leslie, the research director, noted that the circulation department had data that showed the percentage of newspapers yet undelivered every quarter hour from 6 a.m. to 8 a.m. Jan Shapiro remembered that customers were asked on their subscription forms at what time they would be looking for their copy of the *Herald* to be delivered. These data were subsequently combined and posted on an internal *Herald* Web page. (See *Circulation\_Data.htm* in the **HeraldCase** folder on the Student CD or go to [www.prenhall.com/HeraldCase/Circulation\\_Data.htm](http://www.prenhall.com/HeraldCase/Circulation_Data.htm)).

### EXERCISES

Review the internal data and propose a reasonable time (to the nearest quarter hour) to guarantee delivery. To help explore the effects of your choice, calculate the following probabilities:

- SH5.1 If a sample of 50 customers is selected on a given day, what is the probability, given your selected delivery time, that
- a. less than 3 customers will receive a free newspaper?
  - b. 2, 3, or 4 customers will receive a free newspaper?
  - c. more than 5 customers will receive a free newspaper?
- SH5.2 Consider the effects of improving the newspaper delivery process so that the percentage of newspapers that go undelivered by your guaranteed delivery time decreases by 2%. If a sample of 50 customers is selected on a given day, what is the probability, given your selected delivery time (and the delivery improvement), that
- a. less than 3 customers will receive a free newspaper?
  - b. 2, 3, or 4 customers will receive a free newspaper?
  - c. more than 5 customers will receive a free newspaper?

## Web Case

Apply your knowledge about expected value and the covariance in this continuing Web Case from Chapters 3 and 4.

Visit the EndRun Bulls and Bears Web page, at [www.prenhall.com/Springville/ER\\_BullsandBears.htm](http://www.prenhall.com/Springville/ER_BullsandBears.htm) (or open the Web page file from the Student CD-ROM **Web Case** folder), read the claims, and examine the supporting data. Then answer the following:

1. Are there any “catches” about the claims the Web site makes for the rate of return of Happy Bull and Worried Bear Funds?
2. What subjective data influence the rate-of-return analyses of these funds? Could EndRun be accused of making false and misleading statements? Why or why not?
3. The expected-return analysis seems to show that the Worried Bear Fund has a greater expected return than the Happy Bull Fund. Should a rational investor then never invest in the Happy Bull Fund? Why or why not?

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# Excel Companion

## to Chapter 5

### E5.1 COMPUTING THE EXPECTED VALUE OF A DISCRETE RANDOM VARIABLE

You compute the expected value of a discrete random variable by making entries in the **Discrete** worksheet of the **Expected Value.xls** workbook. This worksheet uses the SUM and SQRT (square root) functions to calculate its statistics.

Figure E5.1 shows a completed worksheet using the mortgage probability distribution of Table 5.1 on page 181. To adapt this worksheet to other problems that have more or less than seven outcomes, first select the cell range **A5:E5**. To add table rows, right-click and select **Insert**. (If a box of options appears, click **Shift cells down** and then click **OK**.) Then, copy the formulas in cell range C4:E4 down through the new table rows and enter the new  $X$  and  $P(X)$  values in columns A and B.

To delete table rows, right-click and select **Delete**. (If a box of options appears, click **Shift cells up** and then click **OK**.) Enter a corrected list of  $X$  values, starting with **1** in cell A5 in column A, and enter the new  $P(X)$  values in column B.

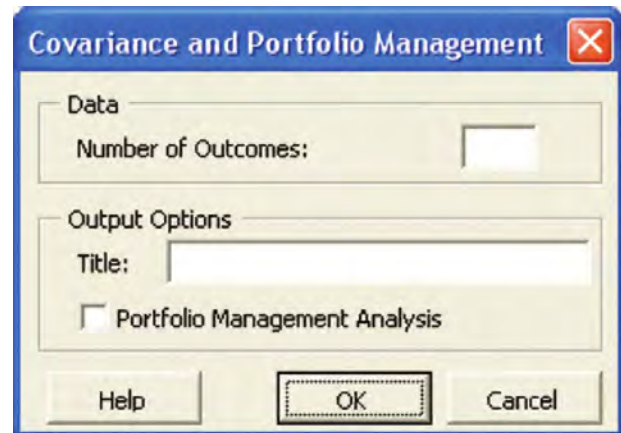
### E5.2 COMPUTING PORTFOLIO EXPECTED RETURN AND PORTFOLIO RISK

You compute the portfolio expected return and the portfolio risk of two investments by either using the PHStat2

Covariance and Portfolio Management procedure or by making entries in the **Portfolio.xls** workbook.

### Using PHStat2 Covariance and Portfolio Management

Open the workbook in which you want your probabilities worksheet to be placed. Select **PHStat** → **Probability & Prob. Distributions** → **Covariance and Portfolio Management**. In the Covariance and Portfolio Management dialog box (shown below), enter a value for the **Number of Outcomes**, enter a title as the **Title**, click **Portfolio Management Analysis**, and click **OK**. In the worksheet created by PHStat2, enter the values for the probabilities and



	A	B	C	D	E	F	G	H
1	<b>Discrete Random Variable Probability Distribution</b>							
2							<b>Statistics</b>	
3	<b>X</b>	<b>P(X)</b>	<b>X*P(X)</b>	<b>[X-E(X)]^2</b>	<b>[X-E(X)]^2*P(X)</b>		<b>Expected value</b>	<b>=SUM(C:C)</b>
4	0	0.10	0	7.84	0.784		<b>Variance</b>	<b>=SUM(E:E)</b>
5	1	0.10	0.1	3.24	0.324		<b>Standard deviation</b>	<b>=SQRT(H4)</b>
6	2	0.20	0.4	0.64	0.128			
7	3	0.30	0.9	0.04	0.012			
8	4	0.15	0.6	1.44	0.216			
9	5	0.10	0.5	4.84	0.484			
10	6	0.05	0.3	10.24	0.512			
			<b>X*P(X)</b>	<b>[X - E(X)]^2</b>	<b>[X - E(X)]^2*P(X)</b>			
			=A4 * B4	=(A4 - \$H\$3)^2	=D4 * B4			
			=A5 * B5	=(A5 - \$H\$3)^2	=D5 * B5			
			=A6 * B6	=(A6 - \$H\$3)^2	=D6 * B6			
			=A7 * B7	=(A7 - \$H\$3)^2	=D7 * B7			
			=A8 * B8	=(A8 - \$H\$3)^2	=D8 * B8			
			=A9 * B9	=(A9 - \$H\$3)^2	=D9 * B9			
			=A10 * B10	=(A10 - \$H\$3)^2	=D10 * B10			

FIGURE E5.1 Discrete worksheet

outcomes in the cell range **B4:D6** and the weight ( $w$ ) assigned to X in cell **B8**.

## Using the Portfolio Worksheet

Open to the **Portfolio** worksheet of the **Portfolio.xls** workbook (see Figure E5.2). This worksheet already contains the entries for the estimated returns for the investment data of Table 5.4. To adapt this worksheet to other problems, change the probabilities and outcomes data and the **Weight Assigned to X** value. If your problem has more or fewer than three outcomes, first select row 5 and then add or delete rows one at a time by right-clicking row 5 and clicking either **Insert** or **Delete**. (If you inserted rows, you also have to copy formulas down to those new rows.)

The worksheet features the **SUMPRODUCT** function, which computes the sum of the products of corresponding elements of two cell ranges. In this worksheet, the cell range is always the set of  $P(X_i Y_j)$  probabilities. To compute the variance, standard deviation, and covariance, the worksheet uses a calculation area in columns F through J, shown in Figure E5.3. This area was created by entering the following row 4 formulas: F4: **=C4 - \$B\$11**, G4: **=D4 - \$B\$12**, H4: **=F4^2**, I4: **=G4^2**, and J4: **=F4\*G4** and then copying them down to rows 5 and 6.

	F	G	H	I	J
1	Calculations Area				
2	For variance and standard deviation:			For covariance	
3	X-mu	Y-mu	(X-mu)^2	(Y-mu)^2	(X-mu)(Y-mu)
4	-205	165	42025	27225	-33825
5	-5	15	25	225	-75
6	145	-135	21025	18225	-19575

FIGURE E5.3 Portfolio worksheet calculations area

## E5.3 COMPUTING BINOMIAL PROBABILITIES

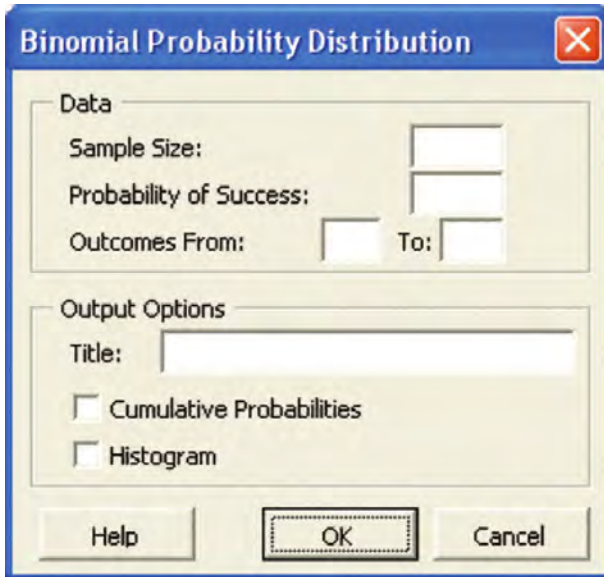
You compute binomial probabilities either by using the PHStat2 **Binomial** procedure or by making entries in the **Binomial.xls** workbook.

### Using PHStat2 Binomial

Open the workbook in which you want your probabilities worksheet to be placed. Select **PHStat** → **Probability & Prob. Distributions** → **Binomial**. In the Binomial Probability Distribution dialog box (shown at the top of page 213), enter the **Sample Size**, the **Probability of Success**, and the range of the outcomes. Enter a title as the **Title**, click **Cumulative Probabilities**, and then click **OK**.

	A	B	C	D
1	Portfolio Expected Return and Risk			
2				
3	Probabilities & Outcomes:	P	X	Y
4		0.2	-100	200
5		0.5	100	50
6		0.3	250	-100
7				
8	Weight Assigned to X	0.5		
9				
10	Statistics			
11	E(X)	105	=SUMPRODUCT(B4:B6, C4:C6)	
12	E(Y)	35	=SUMPRODUCT(B4:B6, D4:D6)	
13	Variance(X)	14725	=SUMPRODUCT(B4:B6, H4:H6)	
14	Standard Deviation(X)	121.3466	=SQRT(B13)	
15	Variance(Y)	11025	=SUMPRODUCT(B4:B6, I4:I6)	
16	Standard Deviation(Y)	105	=SQRT(B15)	
17	Covariance(XY)	-12675	=SUMPRODUCT(B4:B6, J4:J6)	
18	Variance(X+Y)	400	=B13 + B15 + 2 * B17	
19	Standard Deviation(X+Y)	20	=SQRT(B18)	
20				
21	Portfolio Management			
22	Weight Assigned to X	0.5	=B8	
23	Weight Assigned to Y	0.5	=1 - B22	
24	Portfolio Expected Return	70	=B22 * B11 + B23 * B12	
25	Portfolio Risk	10	=SQRT(B22^2 * B13 + B23^2 * B15 + 2 * B22 * B23 * B17)	

FIGURE E5.2 Portfolio worksheet



You can create your version of the Binomial worksheet shown in Figure E5.4 by entering **0** as the **Outcomes From** value and **4** as the **Outcomes To** value.

### Using the Binomial Worksheet

Open to the **Binomial** worksheet of the [Binomial.xls](#) workbook (see Figure E5.4). This worksheet already contains the entries for the tagged orders example of Section 5.3. To adapt this worksheet to other problems, change the **Sample size** and **Probability of success** values in cells B4 and B5. If your problem has a sample size other than 4, first select row **15** and then add or delete rows one at a time by right-clicking row 15 and clicking either **Insert** or **Delete** and adjusting the  $X$  values in column A. (If you inserted rows, you also have to copy formulas down to those new rows.)

	A	B	
1	Tagged Orders		
2			
3	Data		
4	Sample size	4	
5	Probability of success	0.1	
6			
7	Statistics		
8	Mean	0.4	=B4 * B5
9	Variance	0.36	=B8 * (1 - B5)
10	Standard deviation	0.6	=SQRT(B9)
11			
12	Binomial Probabilities Table		
13	X	P(X)	
14	0	0.6561	=BINOMDIST(A14, \$B\$4, \$B\$5, FALSE)
15	1	0.2916	=BINOMDIST(A15, \$B\$4, \$B\$5, FALSE)
16	2	0.0486	=BINOMDIST(A16, \$B\$4, \$B\$5, FALSE)
17	3	0.0036	=BINOMDIST(A17, \$B\$4, \$B\$5, FALSE)
18	4	0.0001	=BINOMDIST(A18, \$B\$4, \$B\$5, FALSE)

FIGURE E5.4 Binomial worksheet

The worksheet features the **BINOMDIST( $X, n, p, cumulative$ )** function, in which  $X$  is the number of successes,  $n$  is the sample size,  $p$  is the probability of success, and *cumulative* is True or False. When *cumulative* is **True**, the function computes the probability of  $X$  or fewer successes; when *cumulative* is **False**, the function computes the probability of exactly  $X$  successes.

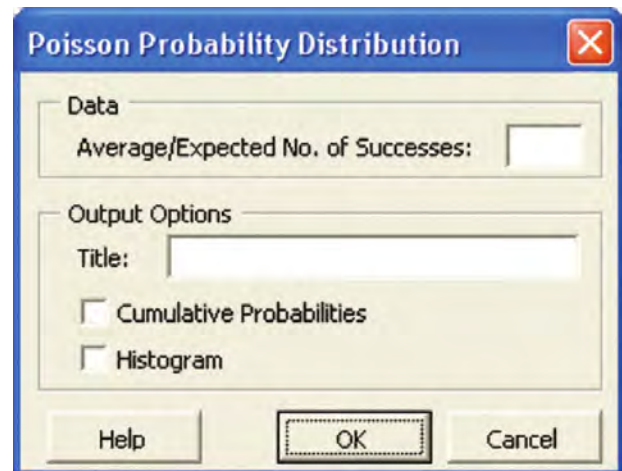
**Forum** Click the **ALTERNATE TECHNIQUES** link to learn how to create a table of cumulative probabilities, similar to the PHStat2 output option.

## E5.4 COMPUTING POISSON PROBABILITIES

You compute Poisson probabilities either by using the PHStat2 **Poisson** procedure or by making entries in the [Poisson.xls](#) workbook.

### Using PHStat2 Poisson

Open the workbook in which you want your probabilities worksheet to be placed. Select **PHStat** → **Probability & Prob. Distributions** → **Poisson**. In the Poisson Probability Distribution dialog box (shown below), enter the **Average/Expected No. of Successes**, enter a title as the **Title**, click **Cumulative Probabilities**, and then click **OK**.



### Using the Poisson Worksheet

Open to the **Poisson** worksheet of the [Poisson.xls](#) workbook (see Figure E5.5 on page 214). This worksheet already contains the entries for the bank customer arrivals problem of Section 5.4. To adapt this worksheet to other problems, change the **Average/Expected number of successes** value in cell E4.

	A	B	C	D	E
1	Customer Arrivals Analysis				
2					
3	Data				
4	Average/Expected number of successes:				3
5					
6	Poisson Probabilities Table				
7	X	P(X)			
8	0	0.049787	=POISSON(A8, \$E\$4, FALSE)		
9	1	0.149361	=POISSON(A9, \$E\$4, FALSE)		
10	2	0.224042	=POISSON(A10, \$E\$4, FALSE)		
11	3	0.224042	=POISSON(A11, \$E\$4, FALSE)		
12	4	0.168031	=POISSON(A12, \$E\$4, FALSE)		
13	5	0.100819	=POISSON(A13, \$E\$4, FALSE)		
14	6	0.050409	=POISSON(A14, \$E\$4, FALSE)		
15	7	0.021604	=POISSON(A15, \$E\$4, FALSE)		
16	8	0.008102	=POISSON(A16, \$E\$4, FALSE)		
17	9	0.002701	=POISSON(A17, \$E\$4, FALSE)		
18	10	0.000810	=POISSON(A18, \$E\$4, FALSE)		
19	11	0.000221	=POISSON(A19, \$E\$4, FALSE)		
20	12	0.000055	=POISSON(A20, \$E\$4, FALSE)		
21	13	0.000013	=POISSON(A21, \$E\$4, FALSE)		
22	14	0.000003	=POISSON(A22, \$E\$4, FALSE)		
23	15	0.000001	=POISSON(A23, \$E\$4, FALSE)		
24	16	0.000000	=POISSON(A24, \$E\$4, FALSE)		
25	17	0.000000	=POISSON(A25, \$E\$4, FALSE)		
26	18	0.000000	=POISSON(A26, \$E\$4, FALSE)		
27	19	0.000000	=POISSON(A27, \$E\$4, FALSE)		
28	20	0.000000	=POISSON(A28, \$E\$4, FALSE)		

FIGURE E5.5 Poisson worksheet

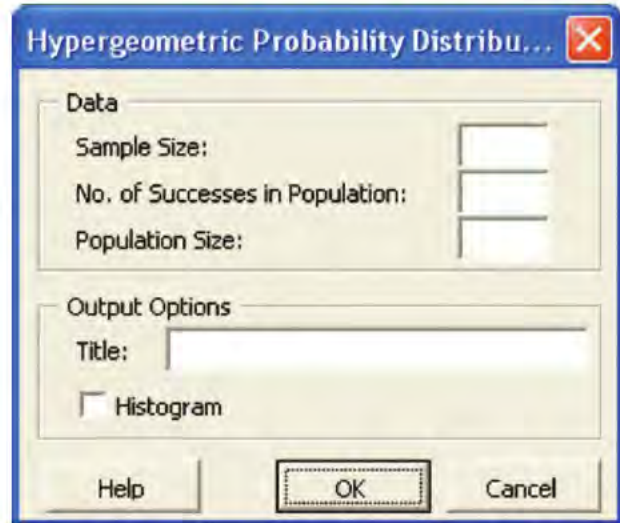
The worksheet features the **POISSON(*X*, *lambda*, *cumulative*)** function, in which *X* is the number of successes, *lambda* is the average or expected number of successes, and *cumulative* is True or False. When *cumulative* is **True**, the function computes the probability of *X* or fewer successes; when *cumulative* is **False**, the function computes the probability of exactly *X* successes.

### E5.5 COMPUTING HYPERGEOMETRIC PROBABILITIES

You compute hypergeometric probabilities either by using the PHStat2 **Hypergeometric** procedure or by making entries in the **Hypergeometric.xls** workbook.

#### Using PHStat2 Hypergeometric

Open the workbook in which you want your probabilities worksheet to be placed. Select **PHStat** → **Probability & Prob. Distributions** → **Hypergeometric**. In the Hypergeometric Probability Distribution dialog box (shown at right), enter the **Sample Size**, the **No. of Successes in Population**, and the **Population Size**. Enter a title as the **Title**, and then click **OK**.



You can create your version of the Hypergeometric worksheet shown in Figure E5.6 by entering **8** as the **Sample Size**, **10** as the **No. of Successes in Population**, and **30** as the **Population Size**.

#### Using the Hypergeometric Worksheet

Open to the **Hypergeometric** worksheet of the **Hypergeometric.xls** workbook (see Figure E5.6). This worksheet already contains the entries for the team formation problem of Section 5.5. To adapt this worksheet to other problems, change the **Sample size**, **No. of successes in population**, and **Population size** in the cell range **B4:B6**. If your problem has a sample size other than 8, first select row **11** and then add or delete rows one at a time by right-clicking and clicking either **Insert** or **Delete** and adjusting the *X* values in column A. (If you inserted rows, you also have to copy formulas down to those new rows.)

The worksheet features the **HYPERGEOMDIST(*X*, *n*, *A*, *N*)** function, in which *X* is the number of successes, *n* is the sample size, *A* is the number of successes in the population, and *N* is the population size.

	A	B
1	Team Formation Analysis	
2		
3	Data	
4	Sample size	8
5	No. of successes in population	10
6	Population size	30
7		
8	Hypergeometric Probabilities Table	
9	X	P(X)
10	0	0.0215 =HYPERGEOMDIST(A10, \$B\$4, \$B\$5, \$B\$6)
11	1	0.1324 =HYPERGEOMDIST(A11, \$B\$4, \$B\$5, \$B\$6)
12	2	0.2980 =HYPERGEOMDIST(A12, \$B\$4, \$B\$5, \$B\$6)
13	3	0.3179 =HYPERGEOMDIST(A13, \$B\$4, \$B\$5, \$B\$6)
14	4	0.1738 =HYPERGEOMDIST(A14, \$B\$4, \$B\$5, \$B\$6)
15	5	0.0491 =HYPERGEOMDIST(A15, \$B\$4, \$B\$5, \$B\$6)
16	6	0.0068 =HYPERGEOMDIST(A16, \$B\$4, \$B\$5, \$B\$6)
17	7	0.0004 =HYPERGEOMDIST(A17, \$B\$4, \$B\$5, \$B\$6)
18	8	0.0000 =HYPERGEOMDIST(A18, \$B\$4, \$B\$5, \$B\$6)

FIGURE E5.6 Hypergeometric worksheet

## E5.6 CREATING HISTOGRAMS FOR DISCRETE PROBABILITY DISTRIBUTIONS

You can create histograms for discrete probability distributions using Excel charting features and your copy of the Binomial, Poisson, or Hypergeometric worksheets discussed in the previous three sections.

*The PHStat2 Binomial, Poisson, and Hypergeometric procedures can create a histogram for you when you click the Histogram output option of those procedures.*

To create a histogram, open to your discrete probability worksheet and use the appropriate set of instructions.

### Creating Histograms (97–2003)

Begin the Chart Wizard (see Section E2.2) and make the following entries and choices in the step dialog boxes:

**Step 1** Click the **Standard Types** tab and then click **Column** as the **Chart type**. Select the first **Chart subtype**, labeled **Clustered Column** when selected.

**Step 2** Click the **Data Range** tab. Enter the cell range of the  $P(X)$  values as the **Data range** and click the **Columns** option in the **Series in** group. Click the **Series** tab. Enter the cell range of the  $X$  values as a formula in the **Category (X) axis labels** box, using the *Sheetname!CellRange* format. For the **Binomial** worksheet of Figure E5.4, enter **B14:B18** as the **Data range** and **=Binomial!A14:A18** as the **Category (X) axis labels**. For the **Poisson** worksheet of Figure E5.5, enter **B8:B28** as the **Data range** and **=Poisson!A8:A28** as the **Category (X) axis labels**. For the **Hypergeometric** worksheet of Figure E5.6, enter **B10:B18** as the **Data range** and **=Hypergeometric!A10:A18** as the **Category (X) axis labels**.

**Step 3** Click the **Titles** tab. Enter a title as the **Chart title** edit box, enter **Number of Successes (X)** as the **Category**

**(X) axis** title, and enter **P(X)** as the **Value (Y) axis** title. Use the formatting settings for the **Axes**, **Gridlines**, **Legend**, **Data Labels**, and **Data Table** tabs that are given in the “Creating Charts (97–2003)” part of Section E2.2 on page 79.

Because the histogram is for a discrete probability distribution, the histogram bars should appear as spikes. To narrow the histogram bars to approximate spikes, right-click one of the histogram bars. (You will see a ToolTip that begins with “Series ‘Frequency’ ” when your mouse is properly positioned.) Click **Format Data Series** in the shortcut menu. In the Format Data Series dialog box, click the **Options** tab, change the value of **Gap width** to **500**, and click **OK**.

### Creating Histograms (2007)

Select the cell range that contains the  $P(X)$  values (exclusive of the  $P(X)$  label). Then select **Insert** → **Column** → **Clustered Column**. Right-click the chart that appears and click **Edit Data Source** in the shortcut menu. In the Edit Data Source dialog box, click the **Edit** button under the **Horizontal (Categories) Axis Labels** heading, enter the cell range of the  $X$  values (exclusive of the  $X$  label) as a formula in the form *=SheetName!CellRange*, and click **OK**. Click **OK** (in the Edit Data Source dialog box) to complete this task. Relocate your chart to a chart sheet and customize your chart by using the instructions in “Creating Charts (2007)” in Section E2.2 on page 80.

For the **Binomial** worksheet of Figure E5.4, initially select **B14:B18** and then enter **=Binomial!A14:A18** as the **Horizontal (Categories) Axis Labels**. For the **Poisson** worksheet of Figure E5.5, select **B8:B28** and then enter **=Poisson!A8:A28**. For the **Hypergeometric** worksheet of Figure E5.6, select **B10:B18** and then enter **=Hypergeometric!A10:A18**.





# CHAPTER 6

## The Normal Distribution and Other Continuous Distributions

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USING STATISTICS @ OurCampus!

**6.1 CONTINUOUS PROBABILITY DISTRIBUTIONS**

**6.2 THE NORMAL DISTRIBUTION**

**6.3 EVALUATING NORMALITY**

Comparing Data Characteristics to Theoretical Properties

Constructing the Normal Probability Plot

**6.4 THE UNIFORM DISTRIBUTION**

**6.5 THE EXPONENTIAL DISTRIBUTION**

**6.6 (CD-ROM TOPIC) THE NORMAL APPROXIMATION TO THE BINOMIAL DISTRIBUTION**

**EXCEL COMPANION TO CHAPTER 6**

E6.1 Computing Normal Probabilities

E6.2 Creating Normal Probability Plots

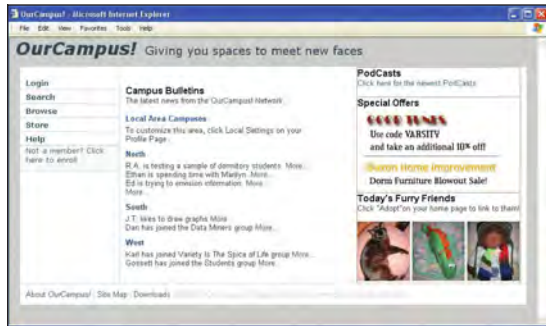
E6.3 Computing Exponential Probabilities

### LEARNING OBJECTIVES

In this chapter, you learn:

- To compute probabilities from the normal distribution
- To use the normal probability plot to determine whether a set of data is approximately normally distributed
- To compute probabilities from the uniform distribution
- To compute probabilities from the exponential distribution

# USING STATISTICS @ OurCampus!



You are a designer for the OurCampus! Web site, which targets college students. To attract and retain users, you need to make sure that the home page downloads quickly. Both the design of the home page and the load on the company's Web servers affect the download time. To check how fast the home page loads, you open a Web browser on a PC at the corporate offices of OurCampus! and measure the download time—the number of seconds that pass from first linking to the Web site until the home page is fully displayed.

Past data indicate that the mean download time is 7 seconds and that the standard deviation is 2 seconds. Approximately two-thirds of the download times are between 5 and 9 seconds, and about 95% of the download times are between 3 and 11 seconds. In other words, the download times are distributed as a bell-shaped curve, with a clustering around the mean of 7 seconds. How could you use this information to answer questions about the download times of the current home page?

In Chapter 5, Saxon Home Improvement Company managers wanted to be able to solve problems about the number of occurrences of a certain type of outcome in a given sample size. As an OurCampus! Web designer, you face a different task, one that involves a continuous measurement because a download time could be any value and not just a whole number. How can you answer questions about this *continuous numerical variable*, such as:

- What proportion of the home page downloads take more than 10 seconds?
- How many seconds elapse before 10% of the downloads are complete?
- How many seconds elapse before 99% of the downloads are complete?
- How would redesigning the home page to download faster affect the answers to these questions?

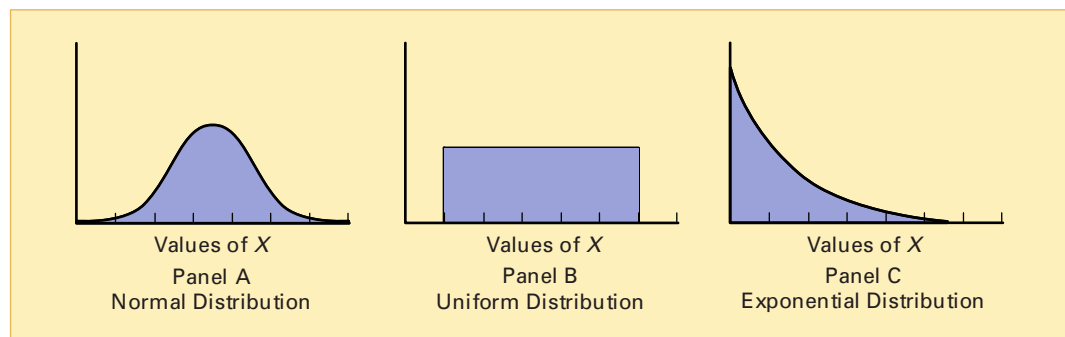
As in Chapter 5, you can use a probability distribution as a model. Reading this chapter will help you learn about characteristics of a continuous probability distribution and how to use the normal, uniform, and exponential distributions to solve business problems.

## 6.1 CONTINUOUS PROBABILITY DISTRIBUTIONS

A **continuous probability density function** is the mathematical expression that defines the distribution of the values for a continuous random variable. Figure 6.1 graphically displays the three continuous probability density functions discussed in this chapter.

**FIGURE 6.1**

Three continuous distributions



Panel A depicts a normal distribution. The normal distribution is symmetrical and bell shaped, implying that most values tend to cluster around the mean, which, due to the distribution's symmetrical shape, is equal to the median. Although the values in a normal distribution can range from negative infinity to positive infinity, the shape of the distribution makes it very unlikely that extremely large or extremely small values will occur.

Panel B depicts a uniform distribution where each value has an equal probability of occurrence anywhere in the range between the smallest value,  $a$ , and the largest value,  $b$ . Sometimes referred to as the rectangular distribution, the uniform distribution is symmetrical and therefore the mean equals the median.

Panel C illustrates an exponential distribution. This distribution is skewed to the right, making the mean larger than the median. The range for an exponential distribution is zero to positive infinity, but the distribution's shape makes the occurrence of extremely large values unlikely.

## 6.2 THE NORMAL DISTRIBUTION

The **normal distribution** (sometimes referred to as the *Gaussian distribution*) is the most common continuous distribution used in statistics. The normal distribution is vitally important in statistics for three main reasons:

- Numerous continuous variables common in business have distributions that closely resemble the normal distribution.
- The normal distribution can be used to approximate various discrete probability distributions.
- The normal distribution provides the basis for *classical statistical inference* because of its relationship to the *Central Limit Theorem* (which is discussed in Section 7.4).

The normal distribution is represented by the classic bell shape depicted in Panel A of Figure 6.1. In the normal distribution, you can calculate the probability that various values occur within certain ranges or intervals. However, the *exact* probability of a *particular value* from a continuous distribution such as the normal distribution is zero. This property distinguishes continuous variables, which are measured, from discrete variables, which are counted. As an example, time (in seconds) is measured and not counted. Therefore, you can determine the probability that the download time for a home page on a Web browser is between 7 and 10 seconds, or the probability that the download time is between 8 and 9 seconds, or the probability that the download time is between 7.99 and 8.01 seconds. However, the probability that the download time is *exactly* 8 seconds is zero.

The normal distribution has several important theoretical properties:

- It is bell shaped (and thus symmetrical) in its appearance.
- Its measures of central tendency (mean, median, and mode) are equal.
- Its interquartile range is equal to 1.33 standard deviations. This means that the middle 50% of the values are contained within an interval of two-thirds of a standard deviation below the mean and two-thirds of a standard deviation above the mean.
- It has an infinite range ( $-\infty < X < \infty$ ).

In practice, many variables have distributions that closely resemble the theoretical properties of the normal distribution. The data in Table 6.1 represent the thickness (in inches) of 10,000 brass washers manufactured by a large company. The continuous variable of interest, thickness, can be approximated by the normal distribution. The measurements of the thickness of the 10,000 brass washers cluster in the interval 0.0190 to 0.0192 inch and distribute symmetrically around that grouping, forming a bell-shaped pattern.

**TABLE 6.1**

Thickness of 10,000  
Brass Washers

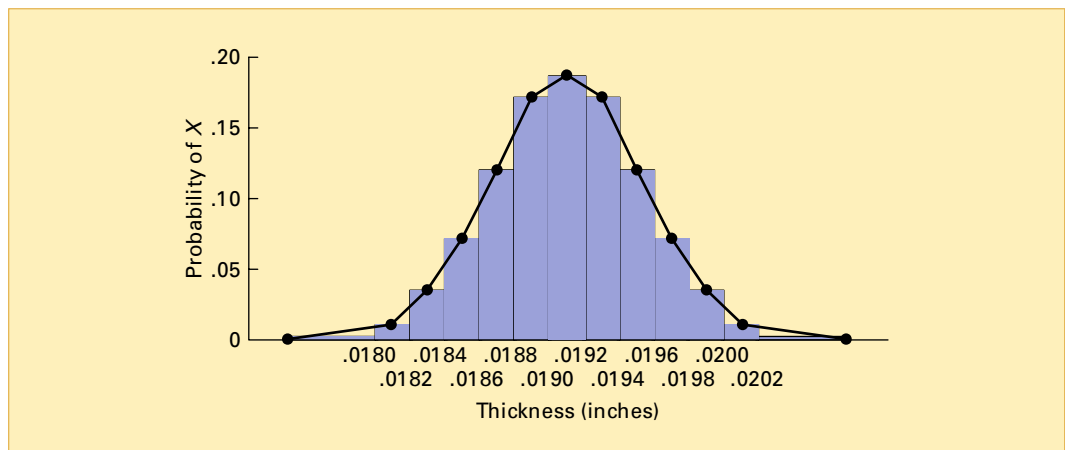
Thickness (inches)	Relative Frequency
< 0.0180	48/10,000 = 0.0048
0.0180 < 0.0182	122/10,000 = 0.0122
0.0182 < 0.0184	325/10,000 = 0.0325
0.0184 < 0.0186	695/10,000 = 0.0695
0.0186 < 0.0188	1,198/10,000 = 0.1198
0.0188 < 0.0190	1,664/10,000 = 0.1664
0.0190 < 0.0192	1,896/10,000 = 0.1896
0.0192 < 0.0194	1,664/10,000 = 0.1664
0.0194 < 0.0196	1,198/10,000 = 0.1198
0.0196 < 0.0198	695/10,000 = 0.0695
0.0198 < 0.0200	325/10,000 = 0.0325
0.0200 < 0.0202	122/10,000 = 0.0122
0.0202 or above	48/10,000 = 0.0048
Total	1.0000

Figure 6.2 shows the relative frequency histogram and polygon for the distribution of the thickness of 10,000 brass washers. For these data, the first three theoretical properties of the normal distribution are approximately satisfied; however, the fourth one, having an infinite range, does not hold. The thickness of the washer cannot possibly be zero or below, nor can a washer be so thick that it becomes unusable. From Table 6.1, you see that only 48 out of every 10,000 brass washers manufactured are expected to have a thickness of 0.0202 inch or more, whereas an equal number are expected to have a thickness under 0.0180 inch. Thus, the chance of randomly getting a washer so thin or so thick is  $0.0048 + 0.0048 = 0.0096$ , less than 1 in 100.

**FIGURE 6.2**

Relative frequency  
histogram and polygon  
of the thickness of  
10,000 brass washers

Source: Data are taken  
from Table 6.1.



The mathematical expression representing a continuous probability density function is denoted by the symbol  $f(X)$ . For the normal distribution, the **normal probability density function** is given in Equation (6.1).

**NORMAL PROBABILITY DENSITY FUNCTION**

$$f(X) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(1/2)[(X-\mu)/\sigma]^2} \quad (6.1)$$

where

$e$  = the mathematical constant approximated by 2.71828

$\pi$  = the mathematical constant approximated by 3.14159

$\mu$  = the mean

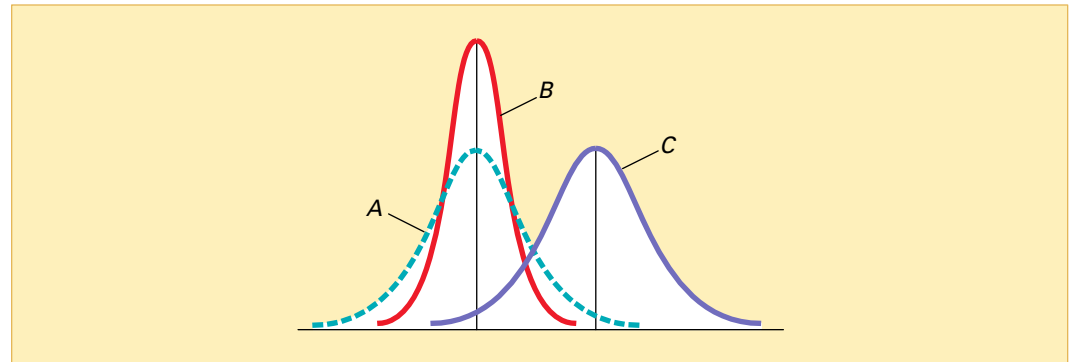
$\sigma$  = the standard deviation

$X$  = any value of the continuous variable, where  $-\infty < X < \infty$

Because  $e$  and  $\pi$  are mathematical constants, the probabilities of the random variable  $X$  are dependent only on the two parameters of the normal distribution—the mean,  $\mu$ , and the standard deviation,  $\sigma$ . Every time you specify a *particular combination* of  $\mu$  and  $\sigma$ , a *different* normal probability distribution is generated. Figure 6.3 illustrates three different normal distributions.

**FIGURE 6.3**

Three normal distributions



Distributions  $A$  and  $B$  have the same mean ( $\mu$ ) but have different standard deviations. Distributions  $A$  and  $C$  have the same standard deviation ( $\sigma$ ) but have different means. Distributions  $B$  and  $C$  differ with respect to both  $\mu$  and  $\sigma$ .

The mathematical expression in Equation (6.1) is computationally tedious and requires integral calculus. Fortunately, normal probability tables are available, and you can avoid the complicated computations of Equation (6.1). The first step in finding normal probabilities is to use the **transformation formula**, given in Equation (6.2), to convert any normal random variable,  $X$ , to a **standardized normal random variable**,  $Z$ .

**TRANSFORMATION FORMULA**

The  $Z$  value is equal to the difference between  $X$  and the mean,  $\mu$ , divided by the standard deviation,  $\sigma$ .

$$Z = \frac{X - \mu}{\sigma} \quad (6.2)$$

Although the original data for the random variable  $X$  had mean  $\mu$  and standard deviation  $\sigma$ , the standardized random variable,  $Z$ , will always have mean  $\mu = 0$  and standard deviation  $\sigma = 1$ . By substituting  $\mu = 0$  and  $\sigma = 1$  in Equation (6.1), the probability density function of a standardized normal variable  $Z$  is given in Equation (6.3).

## STANDARDIZED NORMAL PROBABILITY DENSITY FUNCTION

$$f(Z) = \frac{1}{\sqrt{2\pi}} e^{-(1/2)Z^2} \quad (6.3)$$

Any set of normally distributed values can be converted to its standardized form. Then you can determine the desired probabilities by using Table E.2, the **cumulative standardized normal distribution**.

To see how the transformation formula is applied and the results are used to find probabilities from Table E.2, recall from the Using Statistics scenario on page 218 that past data indicate that the time to download the Web page is normally distributed, with a mean,  $\mu$ , of 7 seconds and a standard deviation,  $\sigma$ , of 2 seconds. From Figure 6.4, you see that every measurement,  $X$ , has a corresponding standardized measurement,  $Z$ , computed from the transformation formula [Equation (6.2)]. Therefore, a download time of 9 seconds is equivalent to 1 standardized unit (that is, 1 standard deviation above the mean) because

$$Z = \frac{9 - 7}{2} = +1$$

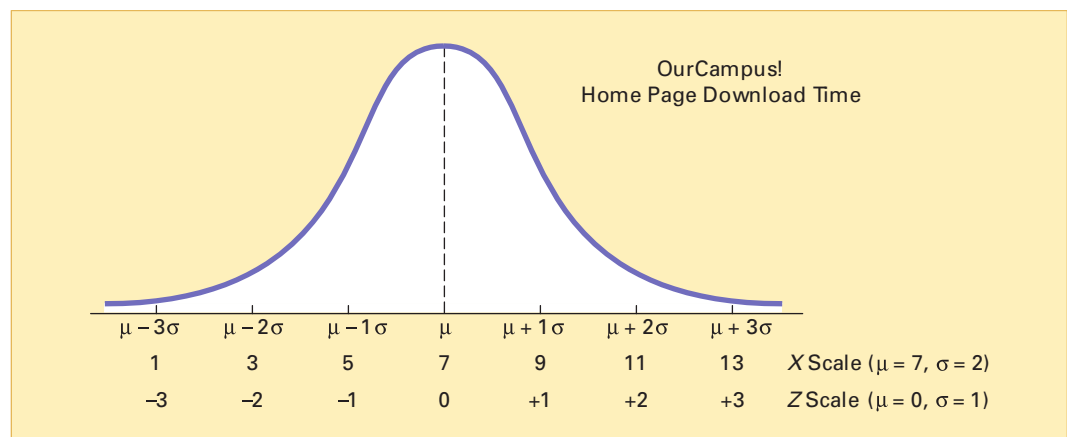
A download time of 1 second is equivalent to 3 standardized units (3 standard deviations) below the mean because

$$Z = \frac{1 - 7}{2} = -3$$

Thus, the standard deviation is the unit of measurement. In other words, a time of 9 seconds is 2 seconds (that is, 1 standard deviation) higher, or *slower*, than the mean time of 7 seconds. Similarly, a time of 1 second is 6 seconds (that is, 3 standard deviations) lower, or *faster*, than the mean time.

FIGURE 6.4

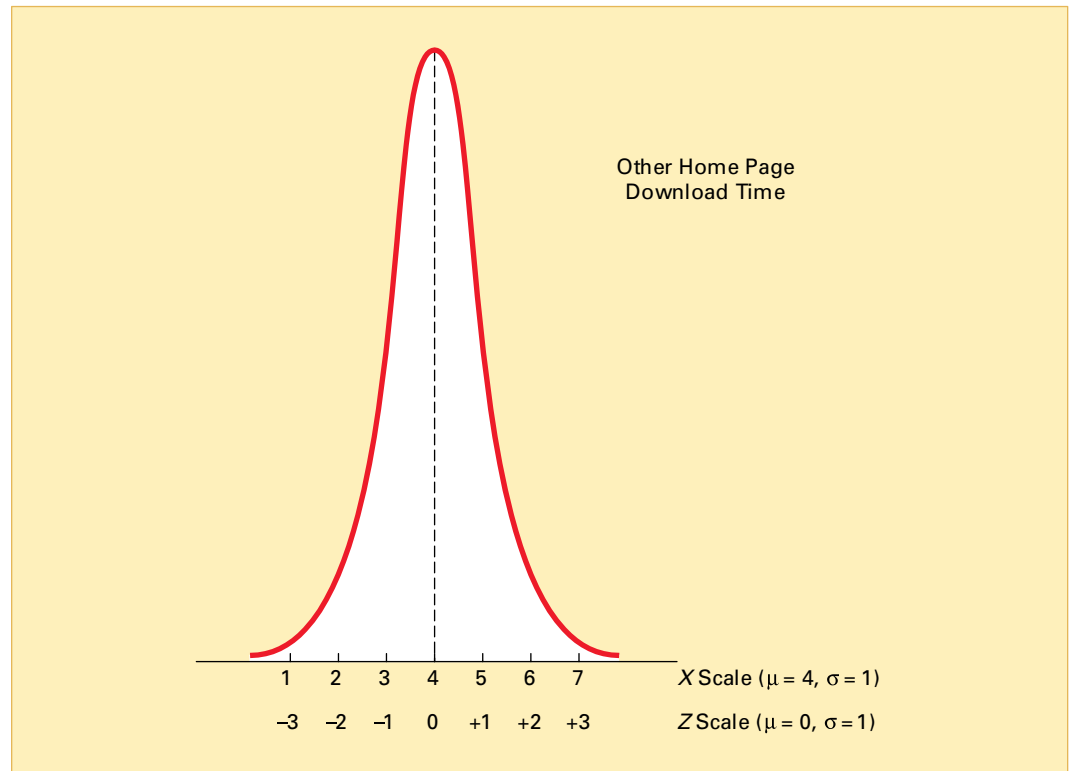
Transformation of scales



To further illustrate the transformation formula, suppose that the home page of another Web site has a download time that is normally distributed, with a mean,  $\mu$ , of 4 seconds and a standard deviation,  $\sigma$ , of 1 second. This distribution is illustrated in Figure 6.5.

**FIGURE 6.5**

A different transformation of scales



Comparing these results with those of the OurCampus! Web site, you see that a download time of 5 seconds is 1 standard deviation above the mean download time because

$$Z = \frac{5 - 4}{1} = +1$$

A time of 1 second is 3 standard deviations below the mean download time because

$$Z = \frac{1 - 4}{1} = -3$$

Suppose you wanted to find the probability that the download time for the OurCampus! site is less than 9 seconds. First, you use Equation (6.2) on page 221 to transform  $X = 9$  to standardized  $Z$  units. Because  $X = 9$  is one standard deviation above the mean,  $Z = +1.00$ . Next, you use Table E.2 to find the cumulative area under the normal curve less than (that is, to the left of)  $Z = +1.00$ . To read the probability or area under the curve less than  $Z = +1.00$ , you scan down the  $Z$  column from Table E.2 until you locate the  $Z$  value of interest (in 10ths) in the  $Z$  row for 1.0. Next, you read across this row until you intersect the column that contains the 100ths place of the  $Z$  value. Therefore, in the body of the table, the tabulated probability for  $Z = 1.00$  corresponds to the intersection of the row  $Z = 1.0$  with the column  $Z = .00$ , as shown in Table 6.2, which is extracted from Table E.2. This probability is 0.8413. As illustrated in Figure 6.6, there is an 84.13% chance that the download time will be less than 9 seconds.

**TABLE 6.2**

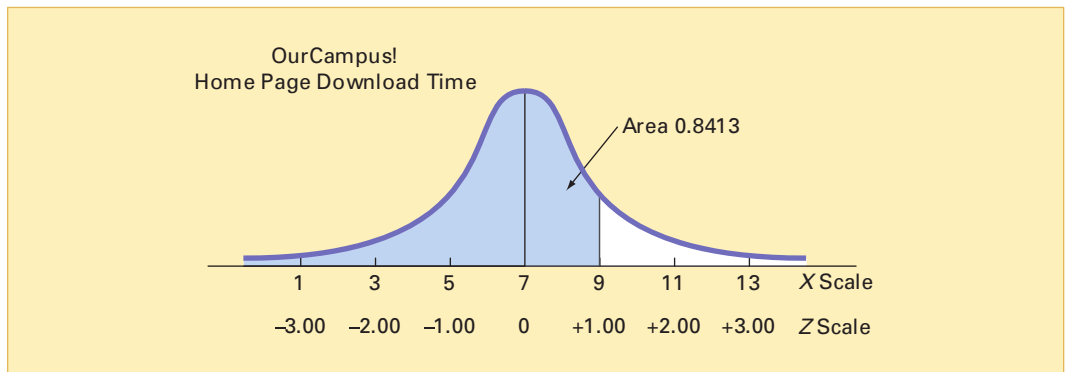
Finding a Cumulative Area Under the Normal Curve

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7518	.7549
0.7	.7580	.7612	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621

Source: Extracted from Table E.2.

**FIGURE 6.6**

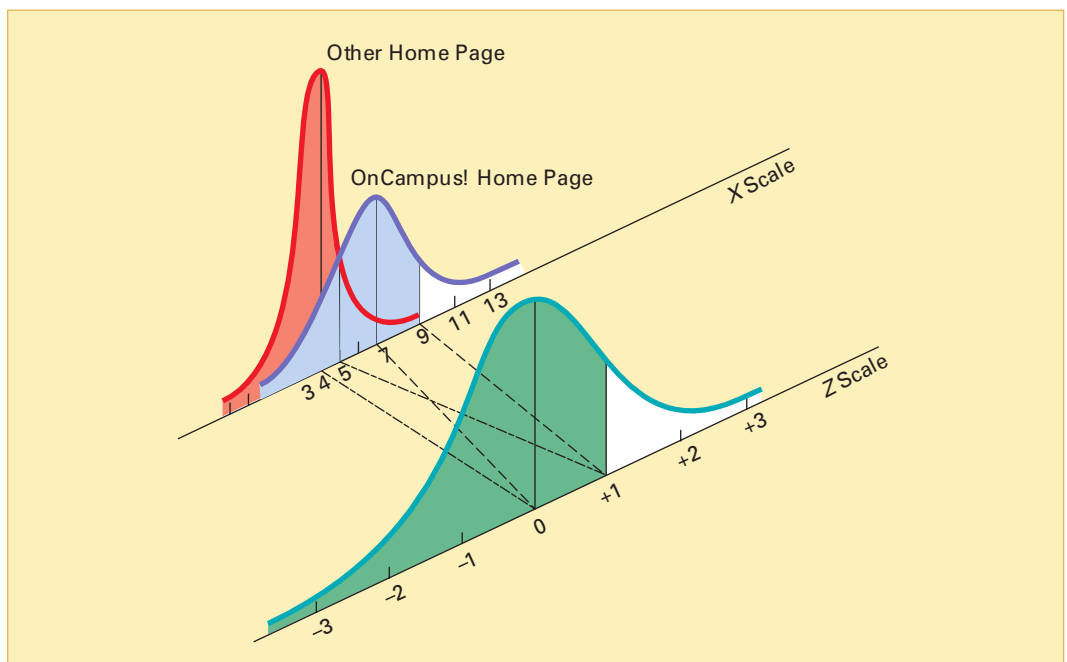
Determining the area less than Z from a cumulative standardized normal distribution



However, for the other home page, you see that a time of 5 seconds is 1 standardized unit above the mean time of 4 seconds. Thus, the probability that the download time will be less than 5 seconds is also 0.8413. Figure 6.7 shows that regardless of the value of the mean,  $\mu$ , and standard deviation,  $\sigma$ , of a normally distributed variable, Equation (6.2) can transform the problem to Z values.

**FIGURE 6.7**

Demonstrating a transformation of scales for corresponding cumulative portions under two normal curves





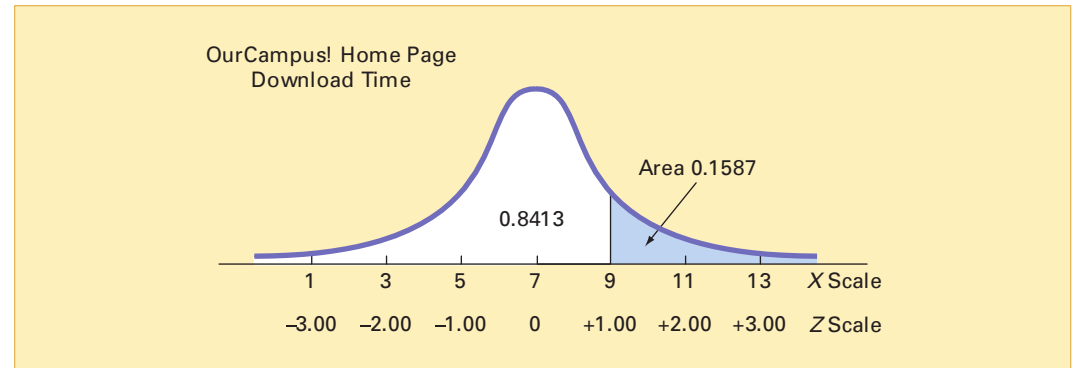
Now that you have learned to use Table E.2 with Equation (6.2), you can answer many questions related to the OurCampus! home page, using the normal distribution.

### EXAMPLE 6.1 FINDING $P(X > 9)$

What is the probability that the download time will be more than 9 seconds?

**SOLUTION** The probability that the download time will be less than 9 seconds is 0.8413 (see Figure 6.6 on page 224). Thus, the probability that the download time will be more than 9 seconds is the *complement* of less than 9 seconds,  $1 - 0.8413 = 0.1587$ . Figure 6.8 illustrates this result.

**FIGURE 6.8**  
Finding  $P(X > 9)$

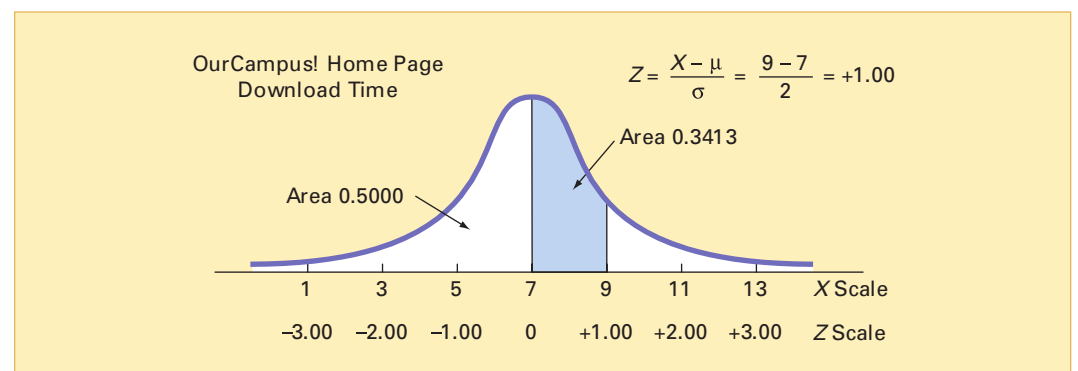


### EXAMPLE 6.2 FINDING $P(7 < X < 9)$

What is the probability that the download time will be between 7 and 9 seconds?

**SOLUTION** From Figure 6.6 on page 224, you already determined that the probability that a download time will be less than 9 seconds is 0.8413. Now you must determine the probability that the download time will be under 7 seconds and subtract this from the probability that the download time is under 9 seconds. This is shown in Figure 6.9.

**FIGURE 6.9**  
Finding  $P(7 < X < 9)$



Using Equation (6.2) on page 221,

$$Z = \frac{7 - 7}{2} = 0.00$$

Using Table E.2, the area under the normal curve less than the mean of  $Z = 0.00$  is 0.5000. Hence, the area under the curve between  $Z = 0.00$  and  $Z = 1.00$  is  $0.8413 - 0.5000 = 0.3413$ .

**EXAMPLE 6.3** FINDING  $P(X < 7 \text{ OR } X > 9)$

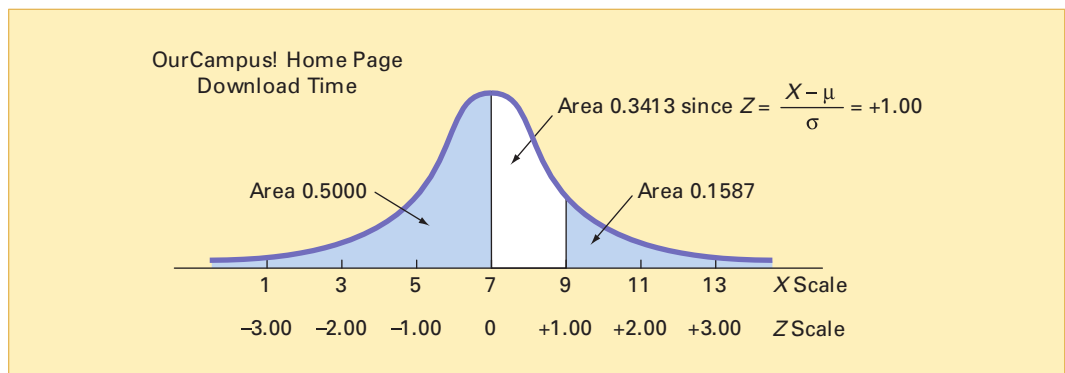
What is the probability that the download time is under 7 seconds or over 9 seconds?

**SOLUTION** From Figure 6.9, the probability that the download time is between 7 and 9 seconds is 0.3413. The probability that the download time is under 7 seconds or over 9 seconds is its complement,  $1 - 0.3413 = 0.6587$ .

Another way to view this problem, however, is to separately calculate the probability of a download time of less than 7 seconds and the probability of a download time of greater than 9 seconds and then add these two probabilities together. Figure 6.10 illustrates this result. Because the mean and median are the same for normally distributed data, 50% of download times are under 7 seconds. From Example 6.1, the probability that the download time is greater than 9 seconds is 0.1587. Hence, the probability that a download time is under 7 or over 9 seconds,  $P(X < 7 \text{ or } X > 9)$ , is  $0.5000 + 0.1587 = 0.6587$ .

**FIGURE 6.10**

Finding  $P(X < 7 \text{ or } X > 9)$



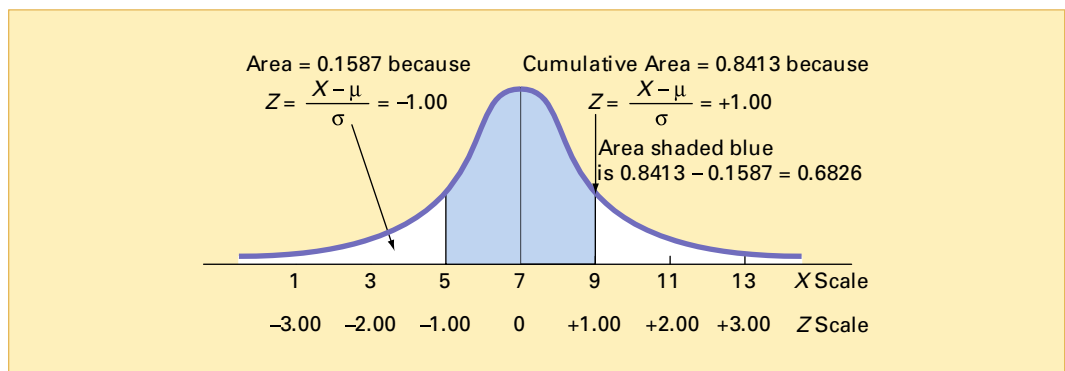
**EXAMPLE 6.4** FINDING  $P(5 < X < 9)$

What is the probability that the download time will be between 5 and 9 seconds—that is,  $P(5 < X < 9)$ ?

**SOLUTION** In Figure 6.11, you can see that the area of interest is located between two values, 5 and 9.

**FIGURE 6.11**

Finding  $P(5 < X < 9)$



In Example 6.1 on page 225, you already found that the area under the normal curve less than 9 seconds is 0.8413. To find the area under the normal curve less than 5 seconds,

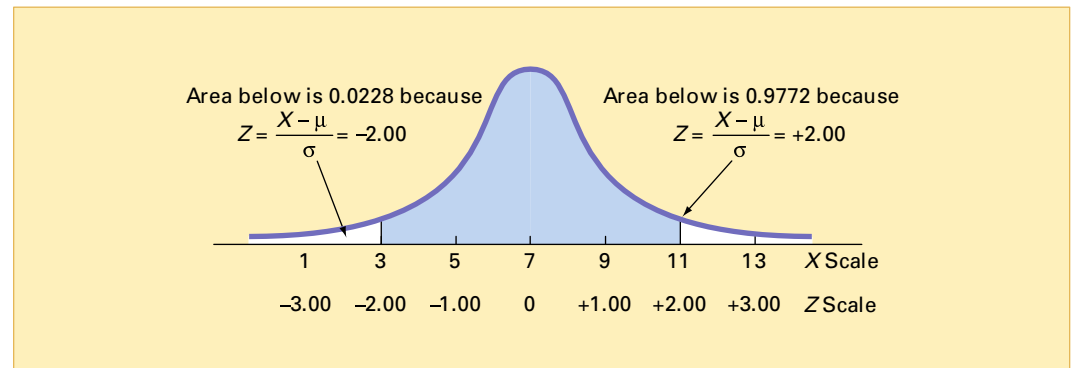
$$Z = \frac{5 - 7}{2} = -1.00$$

Using Table E.2, you look up  $Z = -1.00$  and find 0.1587. Thus, the probability that the download time will be between 5 and 9 seconds is  $0.8413 - 0.1587 = 0.6826$ , as displayed in Figure 6.11.

The result of Example 6.4 enables you to state that for any normal distribution, 68.26% of the values will fall within  $\pm 1$  standard deviation of the mean. From Figure 6.12, slightly more than 95% of the values will fall within  $\pm 2$  standard deviations of the mean. Thus, 95.44% of the download times are between 3 and 11 seconds. From Figure 6.13, 99.73% of the values will fall within  $\pm 3$  standard deviations above or below the mean. Thus, 99.73% of the download times are between 1 and 13 seconds. Therefore, it is unlikely (0.0027, or only 27 in 10,000) that a download time will be so fast or so slow that it will take under 1 second or more than 13 seconds. This is why  $6\sigma$  (that is, 3 standard deviations above the mean to 3 standard deviations below the mean) is often used as a *practical approximation of the range* for normally distributed data.

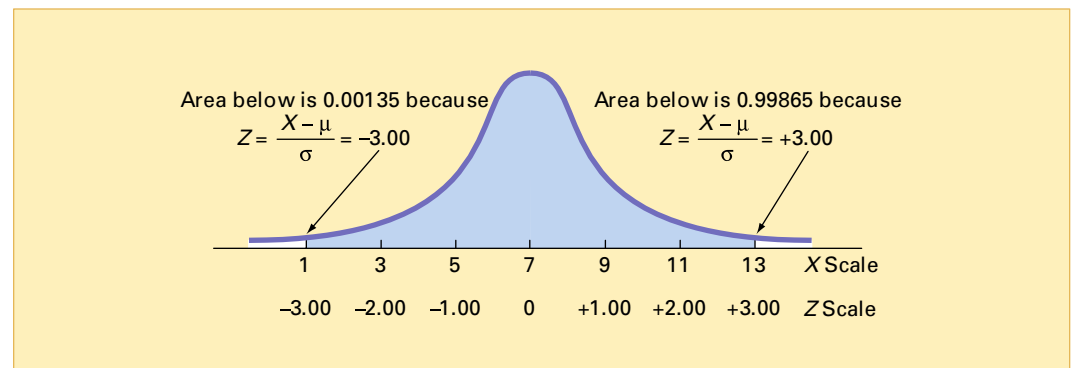
**FIGURE 6.12**

Finding  $P(3 < X < 11)$



**FIGURE 6.13**

Finding  $P(1 < X < 13)$



Figures 6.11, 6.12, and 6.13 illustrate how the values of a normal distribution cluster near the mean. For any normal distribution:

- Approximately 68.26% of the values fall within  $\pm 1$  standard deviation of the mean.
- Approximately 95.44% of the values fall within  $\pm 2$  standard deviations of the mean.
- Approximately 99.73% of the values fall within  $\pm 3$  standard deviations of the mean.

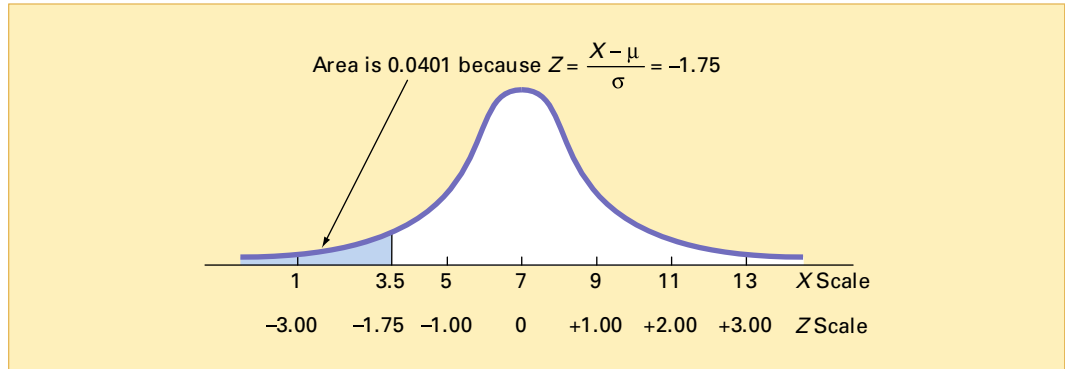
This result is the justification for the empirical rule presented on page 120. The accuracy of the empirical rule improves as a data set follows the normal distribution more closely.

**EXAMPLE 6.5 FINDING  $P(X < 3.5)$**

What is the probability that a download time will be under 3.5 seconds?

**SOLUTION** To calculate the probability that a download time will be under 3.5 seconds, you need to examine the shaded lower-left tail region of Figure 6.14.

**FIGURE 6.14**  
Finding  $P(X < 3.5)$



To determine the area under the curve below 3.5 seconds, you first calculate

$$Z = \frac{X - \mu}{\sigma} = \frac{3.5 - 7}{2} = -1.75$$

You then look up the  $Z$  value of  $-1.75$  in Table E.2 by matching the appropriate  $Z$  row ( $-1.7$ ) with the appropriate  $Z$  column ( $.05$ ), as shown in Table 6.3 (which is extracted from Table E.2). The resulting probability or area under the curve less than  $-1.75$  standard deviations below the mean is  $0.0401$ .

**TABLE 6.3**  
Finding a Cumulative Area Under the Normal Curve

$Z$	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.
<b>-1.7</b>	.0446	.0436	.0427	.0418	.0409	<b>.0401</b>	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455

Source: Extracted from Table E.2.

Examples 6.1 through 6.5 require you to use the normal table to find an area under the normal curve that corresponds to a specific  $X$  value. There are many circumstances under which you want to find the  $X$  value that corresponds to a specific area. Examples 6.6 and 6.7 illustrate such situations.

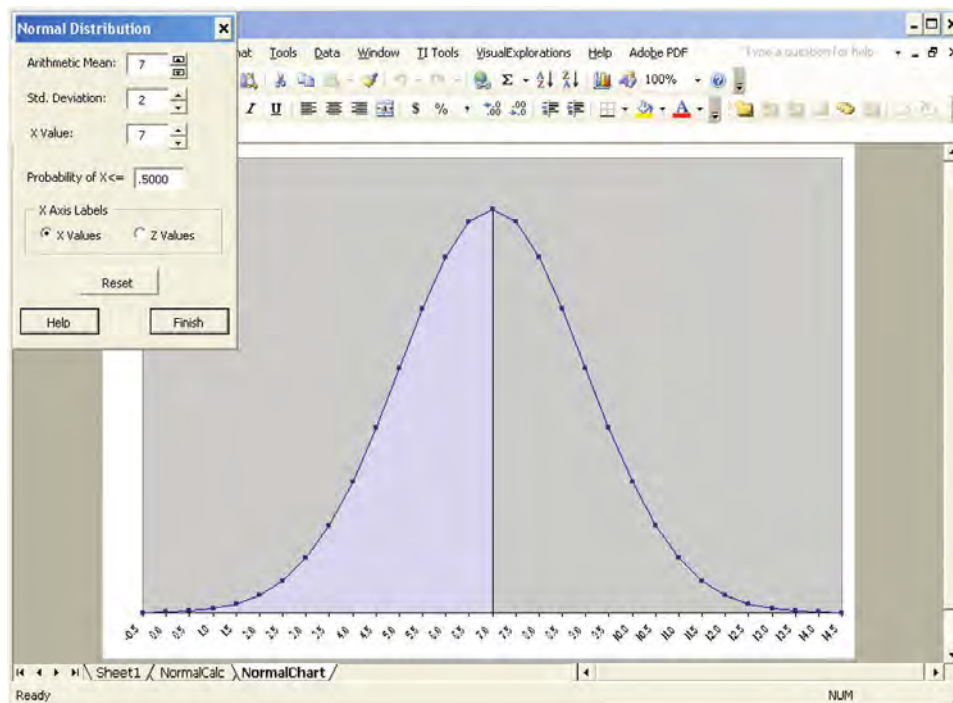
## VISUAL EXPLORATIONS Exploring the Normal Distribution

You use the Normal Distribution command of Visual Explorations to see the effects of changes in the mean and standard deviation on the area under a normal distribution curve.

Open the **Visual explorations.xls** add-in workbook and select **VisualExplorations → Normal** (Excel 97–2003) or **Add-Ins → Visual Explorations → Normal Distribution** (Excel 2007). You will see a normal curve for the Using Statistics home page download example and a floating control panel that allows you to adjust the shape of the curve

and the shaded area under the curve (see illustration below). Use the control panel spinner buttons to change the values for the mean, standard deviation, and  $X$  value, while noting their effects on the probability of  $X \leq$  value and the corresponding shaded area under the curve (see illustration below). If you prefer, you can select the **Z Values** option button to see the normal curve labeled with  $Z$  values.

Click the **Reset** button to reset the control panel values or click **Help** for additional information about the problem. Click **Finish** when you are done exploring.



### EXAMPLE 6.6

#### FINDING THE $X$ VALUE FOR A CUMULATIVE PROBABILITY OF 0.10

How much time (in seconds) will elapse before 10% of the downloads are complete?

**SOLUTION** Because 10% of the home pages are expected to download in under  $X$  seconds, the area under the normal curve less than this value is 0.1000. Using the body of Table E.2, you search for the area or probability of 0.1000. The closest result is 0.1003, as shown in Table 6.4 (which is extracted from Table E.2).

**TABLE 6.4**  
Finding a Z Value Corresponding to a Particular Cumulative Area (0.10) Under the Normal Curve

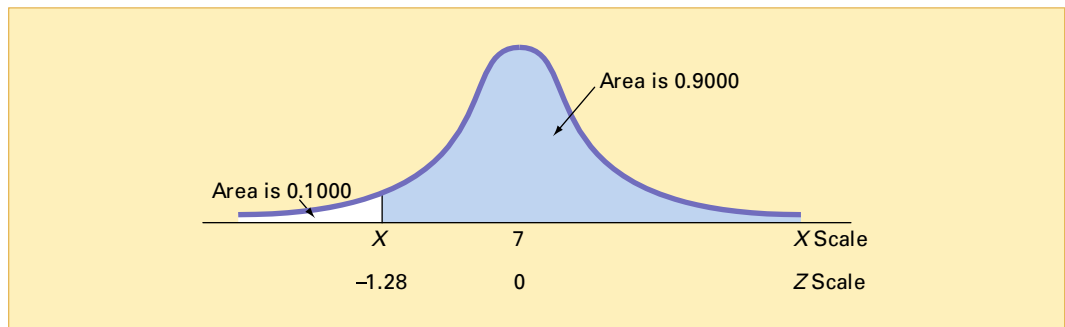
Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
·	·	·	·	·	·	·	·	·	·	·
·	·	·	·	·	·	·	·	·	·	·
·	·	·	·	·	·	·	·	·	·	·
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985

Source: Extracted from Table E.2.

Working from this area to the margins of the table, the Z value corresponding to the particular Z row (-1.2) and Z column (.08) is -1.28 (see Figure 6.15).

**FIGURE 6.15**

Finding Z to determine X



Once you find Z, use the transformation formula Equation (6.2) on page 221 to determine the X value, as follows. Let

$$Z = \frac{X - \mu}{\sigma}$$

then

$$X = \mu + Z\sigma$$

Substituting  $\mu = 7$ ,  $\sigma = 2$ , and  $Z = -1.28$ ,

$$X = 7 + (-1.28)(2) = 4.44 \text{ seconds}$$

Thus, 10% of the download times are 4.44 seconds or less.

Equation (6.4) is used for finding an X value.

**FINDING AN X VALUE ASSOCIATED WITH KNOWN PROBABILITY**

The X value is equal to the mean  $\mu$  plus the product of the Z value and the standard deviation  $\sigma$ .

$$X = \mu + Z\sigma \tag{6.4}$$

To find a *particular* value associated with a known probability, follow these steps:

1. Sketch the normal curve and then place the values for the mean and  $X$  value on the  $X$  and  $Z$  scales.
2. Find the cumulative area less than  $X$ .
3. Shade the area of interest.
4. Using Table E.2, determine the  $Z$  value corresponding to the area under the normal curve less than  $X$ .
5. Using Equation (6.4), solve for  $X$ :

$$X = \mu + Z\sigma$$

**EXAMPLE 6.7**

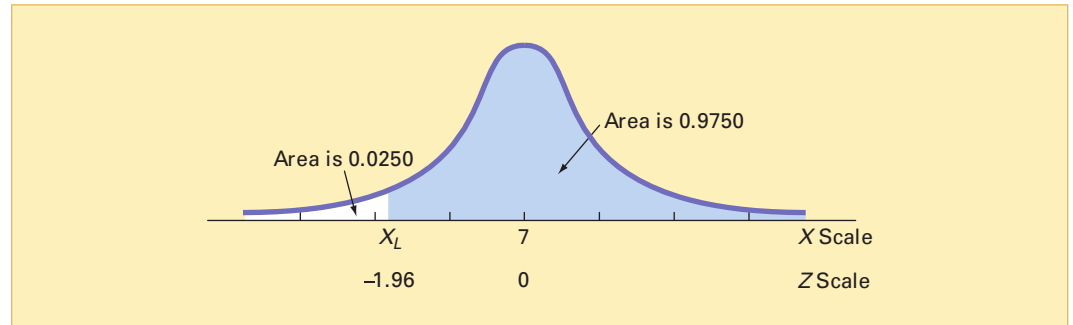
**FINDING THE  $X$  VALUES THAT INCLUDE 95% OF THE DOWNLOAD TIMES**

What are the lower and upper values of  $X$ , symmetrically distributed around the mean, that include 95% of the download times?

**SOLUTION** First, you need to find the lower value of  $X$  (called  $X_L$ ). Then you find the upper value of  $X$  (called  $X_U$ ). Because 95% of the values are between  $X_L$  and  $X_U$ , and because  $X_L$  and  $X_U$  are equally distant from the mean, 2.5% of the values are below  $X_L$  (see Figure 6.16).

**FIGURE 6.16**

Finding  $Z$  to determine  $X_L$



Although  $X_L$  is not known, you can find the corresponding  $Z$  value because the area under the normal curve less than this  $Z$  is 0.0250. Using the body of Table 6.5, you search for the probability 0.0250.

**TABLE 6.5**

Finding a  $Z$  Value Corresponding to a Cumulative Area of 0.025 Under the Normal Curve

$Z$	.00	.01	.02	.03	.04	.05	<b>.06</b>	.07	.08	.09
·	·	·	·	·	·	·	·	·	·	·
·	·	·	·	·	·	·	·	·	·	·
·	·	·	·	·	·	·	·	·	·	·
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
<b>-1.9</b>	<b>.0287</b>	.0281	.0274	.0268	.0262	.0256	<b>.0250</b>	.0244	.0239	.0233
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294

Source: Extracted from Table E.2.

Working from the body of the table to the margins of the table, you see that the  $Z$  value corresponding to the particular  $Z$  row (-1.9) and  $Z$  column (.06) is -1.96.

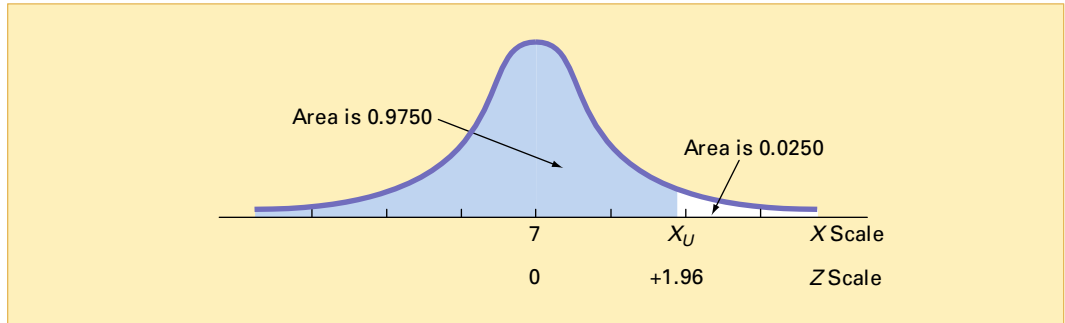
Once you find  $Z$ , the final step is to use Equation (6.4) on page 230 as follows:

$$\begin{aligned}
 X &= \mu + Z\sigma \\
 &= 7 + (-1.96)(2) \\
 &= 7 - 3.92 \\
 &= 3.08 \text{ seconds}
 \end{aligned}$$

You use a similar process to find  $X_U$ . Because only 2.5% of the home page downloads take longer than  $X_U$  seconds, 97.5% of the home page downloads take less than  $X_U$  seconds. From the symmetry of the normal distribution, the desired Z value, as shown in Figure 6.17, is +1.96 (because Z lies to the right of the standardized mean of 0). You can also extract this Z value from Table 6.6. You can see that 0.975 is the area under the normal curve less than the Z value of +1.96.

**FIGURE 6.17**

Finding Z to determine  $X_U$



**TABLE 6.6**

Finding a Z Value Corresponding to a Cumulative Area of 0.975 Under the Normal Curve

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .
+1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
<b>+1.9</b>	<b>.9713</b>	.9719	.9726	.9732	.9738	.9744	<b>.9750</b>	.9756	.9761	.9767
+2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817

Source: Extracted from Table E.2.

Using Equation (6.4) on page 230,

$$\begin{aligned}
 X &= \mu + Z\sigma \\
 &= 7 + (+1.96)(2) \\
 &= 7 + 3.92 \\
 &= 10.92 \text{ seconds}
 \end{aligned}$$

Therefore, 95% of the download times are between 3.08 and 10.92 seconds.

You can also use Microsoft Excel to compute normal probabilities. Figure 6.18 illustrates a Microsoft Excel worksheet for Examples 6.5 and 6.6.

**FIGURE 6.18**

Microsoft Excel worksheet for computing normal probabilities



See Section E6.1 to create this.

	A	B
1	<b>Normal Probabilities</b>	
2		
3	<b>Common Data</b>	
4	Mean	7
5	Standard Deviation	2
6		
7	<b>Probability for X &lt;=</b>	
8	X Value	3.5
9	Z Value	-1.75
10	P(X<=3.5)	0.0401
11		
12	<b>Find X and Z Given Cum. Pctage.</b>	
13	Cumulative Percentage	10.00%
14	Z Value	-1.2816
15	X Value	4.4369

=STANDARDIZE(B8, B4, B5)  
 =NORMDIST(B8, B4, B5, TRUE)  
 =NORMSINV(B13)  
 =NORMINV(B13, B4, B5)



## PROBLEMS FOR SECTION 6.2

### Learning the Basics

**PH Grade ASSIST** **6.1** Given a standardized normal distribution (with a mean of 0 and a standard deviation of 1, as in Table E.2), what is the probability that

- $Z$  is less than 1.57?
- $Z$  is greater than 1.84?
- $Z$  is between 1.57 and 1.84?
- $Z$  is less than 1.57 or greater than 1.84?

**PH Grade ASSIST** **6.2** Given a standardized normal distribution (with a mean of 0 and a standard deviation of 1, as in Table E.2), what is the probability that

- $Z$  is between  $-1.57$  and  $1.84$ ?
- $Z$  is less than  $-1.57$  or greater than  $1.84$ ?
- What is the value of  $Z$  if only 2.5% of all possible  $Z$  values are larger?
- Between what two values of  $Z$  (symmetrically distributed around the mean) will 68.26% of all possible  $Z$  values be contained?

**PH Grade ASSIST** **6.3** Given a standardized normal distribution (with a mean of 0 and a standard deviation of 1, as in Table E.2), what is the probability that

- $Z$  is less than 1.08?
- $Z$  is greater than  $-0.21$ ?
- $Z$  is less than  $-0.21$  or greater than the mean?
- $Z$  is less than  $-0.21$  or greater than 1.08?

**6.4** Given a standardized normal distribution (with a mean of 0 and a standard deviation of 1, as in Table E.2), determine the following probabilities:

- $P(Z > 1.08)$
- $P(Z < -0.21)$
- $P(-1.96 < Z < -0.21)$
- What is the value of  $Z$  if only 15.87% of all possible  $Z$  values are larger?

**PH Grade ASSIST** **6.5** Given a normal distribution with  $\mu = 100$  and  $\sigma = 10$ , what is the probability that

- $X > 75$ ?
- $X < 70$ ?
- $X < 80$  or  $X > 110$ ?
- 80% of the values are between what two  $X$  values (symmetrically distributed around the mean)?

**PH Grade ASSIST** **6.6** Given a normal distribution with  $\mu = 50$  and  $\sigma = 4$ , what is the probability that

- $X > 43$ ?
- $X < 42$ ?
- 5% of the values are less than what  $X$  value?
- 60% of the values are between what two  $X$  values (symmetrically distributed around the mean)?

### Applying the Concepts

**6.7** In a recent year, about two-thirds of U.S. households purchased ground coffee. Consider the annual ground coffee expenditures for households purchasing ground coffee, assuming that these expenditures are approximately distributed as a normal random variable with a mean of \$45.16 and a standard deviation of \$10.00.

- Find the probability that a household spent less than \$25.00.
- Find the probability that a household spent more than \$50.00.
- What proportion of the households spent between \$30.00 and \$40.00?
- 99% of the households spent less than what amount?

**PH Grade ASSIST** **6.8** Toby's Trucking Company determined that the distance traveled per truck per year is normally distributed, with a mean of 50.0 thousand miles and a standard deviation of 12.0 thousand miles.

- What proportion of trucks can be expected to travel between 34.0 and 50.0 thousand miles in the year?
- What percentage of trucks can be expected to travel either below 30.0 or above 60.0 thousand miles in the year?
- How many miles will be traveled by at least 80% of the trucks?
- What are your answers to (a) through (c) if the standard deviation is 10.0 thousand miles?

**PH Grade ASSIST** **6.9** The breaking strength of plastic bags used for packaging produce is normally distributed, with a mean of 5 pounds per square inch and a standard deviation of 1.5 pounds per square inch. What proportion of the bags have a breaking strength of

- less than 3.17 pounds per square inch?
- at least 3.6 pounds per square inch?
- between 5 and 5.5 pounds per square inch?
- 95% of the breaking strengths will be contained between what two values symmetrically distributed around the mean?

**6.10** A set of final examination grades in an introductory statistics course is normally distributed, with a mean of 73 and a standard deviation of 8.

- What is the probability of getting a grade below 91 on this exam?
- What is the probability that a student scored between 65 and 89?
- The probability is 5% that a student taking the test scores higher than what grade?
- If the professor grades on a curve (that is, gives A's to the top 10% of the class, regardless of the score), are you

better off with a grade of 81 on this exam or a grade of 68 on a different exam, where the mean is 62 and the standard deviation is 3? Show your answer statistically and explain.

**6.11** A statistical analysis of 1,000 long-distance telephone calls made from the headquarters of the Bricks and Clicks Computer Corporation indicates that the length of these calls is normally distributed, with  $\mu = 240$  seconds and  $\sigma = 40$  seconds.

- What is the probability that a call lasted less than 180 seconds?
- What is the probability that a call lasted between 180 and 300 seconds?
- What is the probability that a call lasted between 110 and 180 seconds?
- What is the length of a call if only 1% of all calls are shorter?

**6.12** The number of shares traded daily on the New York Stock Exchange (NYSE) is referred to as the *volume* of trading. During the first three months of 2006, daily volume ranged from 1.424 billion to 2.170 billion (NYSE Group, [www.nyse.com](http://www.nyse.com), April 28, 2006). Assume that the number of shares traded on the NYSE is a normally distrib-

uted random variable, with a mean of 1.8 billion and a standard deviation of 0.15 billion. For a randomly selected day, what is the probability that the volume is

- below 1.5 billion?
- below 1.7 billion?
- above 2.0 billion?
- above 2.3 billion?

**6.13** Many manufacturing problems involve the matching of machine parts, such as shafts that fit into a valve hole. A particular design requires a shaft with a diameter of 22.000 mm, but shafts with diameters between 21.900 mm and 22.010 mm are acceptable. Suppose that the manufacturing process yields shafts with diameters normally distributed, with a mean of 22.002 mm and a standard deviation of 0.005 mm. For this process, what is

- the proportion of shafts with a diameter between 21.90 mm and 22.00 mm?
- the probability that a shaft is acceptable?
- the diameter that will be exceeded by only 2% of the shafts?
- What would be your answers in (a) through (c) if the standard deviation of the shaft diameters was 0.004 mm?

### 6.3 EVALUATING NORMALITY

As discussed in Section 6.2, many continuous variables used in business closely follow a normal distribution. This section presents two approaches for evaluating whether a set of data can be approximated by the normal distribution:

- Compare the characteristics of the data with the theoretical properties of the normal distribution.
- Construct a normal probability plot.

#### Comparing Data Characteristics to Theoretical Properties

The normal distribution has several important theoretical properties:

- It is symmetrical; thus, the mean and median are equal.
- It is bell shaped; thus, the empirical rule applies.
- The interquartile range equals 1.33 standard deviations.

In actual practice, a continuous variable may have characteristics that approximate these theoretical properties. However, many continuous variables are neither normally distributed nor approximately normally distributed. For such variables, the descriptive characteristics of the data do not match well with the properties of a normal distribution. One approach to determining whether a data set follows a normal distribution is to compare the characteristics of the data with the corresponding properties from an underlying normal distribution, as follows:

- Construct charts and observe their appearance. For small- or moderate-sized data sets, construct a stem-and-leaf display or a box-and-whisker plot. For large data sets, construct the frequency distribution and plot the histogram or polygon.
- Compute descriptive numerical measures and compare the characteristics of the data with the theoretical properties of the normal distribution. Compare the mean and median. Is the

interquartile range approximately 1.33 times the standard deviation? Is the range approximately 6 times the standard deviation?

- Evaluate how the values in the data are distributed. Determine whether approximately two-thirds of the values lie between the mean and  $\pm 1$  standard deviation. Determine whether approximately four-fifths of the values lie between the mean and  $\pm 1.28$  standard deviations. Determine whether approximately 19 out of every 20 values lie between the mean  $\pm 2$  standard deviations.

Do the three-year returns discussed in Chapters 2 and 3 (see the [Mutual Funds.xls](#) file) contain the properties of the normal distribution? Figure 6.19 displays descriptive statistics for these data, and Figure 6.20 presents a box-and-whisker plot.

**FIGURE 6.19**

Microsoft Excel descriptive statistics for the three-year returns



See Section E3.1 to create this.

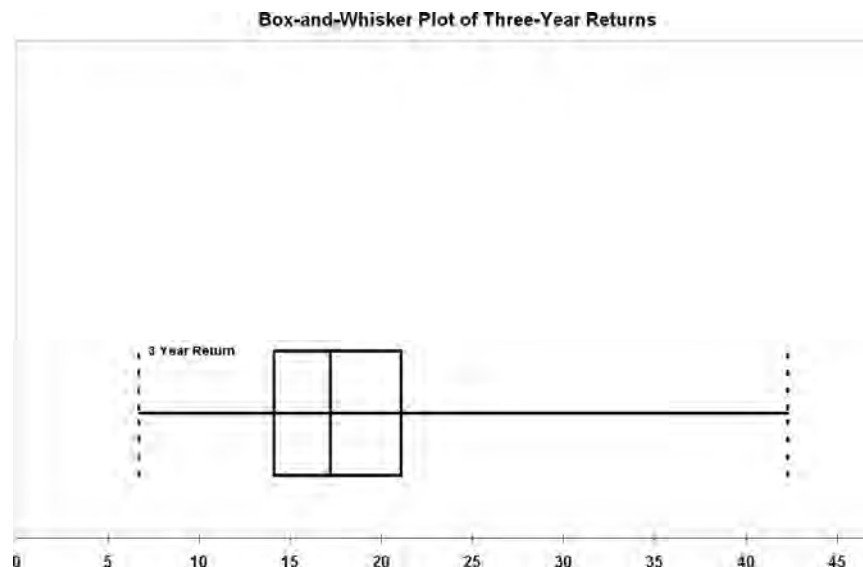
	A	B
1	<b>3 Year Return</b>	
2		
3	<b>Mean</b>	17.8
4	<b>Standard Error</b>	0.17099
5	<b>Median</b>	17.2
6	<b>Mode</b>	15.1
7	<b>Standard Deviation</b>	4.94991
8	<b>Sample Variance</b>	24.5016
9	<b>Kurtosis</b>	1.03812
10	<b>Skewness</b>	0.66073
11	<b>Range</b>	35.6
12	<b>Minimum</b>	6.7
13	<b>Maximum</b>	42.3
14	<b>Sum</b>	14916.4
15	<b>Count</b>	838
16	<b>Largest(1)</b>	42.3
17	<b>Smallest(1)</b>	6.7
18	<b>Confidence Level(95.0%)</b>	0.33562

**FIGURE 6.20**

Microsoft Excel box-and-whisker plot for the three-year returns



See Section E3.4 to create this.



From Figures 6.19 and 6.20 and from an ordered array of the three-year returns (not shown here), you can make the following statements:

1. The mean of 17.8 is slightly higher than the median of 17.2. (In a normal distribution, the mean and median are equal.)
2. The box-and-whisker plot appears right-skewed, with an outlier at the maximum value of 42.3. (The normal distribution is symmetrical.)

3. The interquartile range of 7.0 is approximately 1.41 standard deviations. (In a normal distribution, the interquartile range is 1.33 standard deviations.)
4. The range of 35.6 is equal to 7.19 standard deviations. (In a normal distribution, the range is approximately six standard deviations.)
5. 74.2% of the returns are within  $\pm 1$  standard deviation of the mean. (In a normal distribution, 68.26% of the values lie between the mean  $\pm 1$  standard deviation.)
6. 83.3% of the returns are within  $\pm 1.28$  standard deviations of the mean. (In a normal distribution, 80% of the values lie between the mean  $\pm 1.28$  standard deviations.)

Based on these statements and the criteria given on pages 234–235, you can conclude that the three-year returns are right-skewed and are not normally distributed.

### Constructing the Normal Probability Plot

A **normal probability plot** is a graphical approach for evaluating whether data are normally distributed. One common approach is called the **quantile–quantile plot**. In this method, you transform each ordered value to a  $Z$  value and then plot the data values versus the  $Z$  values. For example, if you have a sample of  $n = 19$ , the  $Z$  value for the smallest value corresponds to a

cumulative area of  $\frac{1}{n+1} = \frac{1}{19+1} = \frac{1}{20} = 0.05$ . The  $Z$  value for a cumulative area of 0.05

(from Table E.2) is  $-1.65$ . Table 6.7 illustrates the entire set of  $Z$  values for a sample of  $n = 19$ .

**TABLE 6.7**

Ordered Values and Corresponding  $Z$  Values for a Sample of  $n = 19$

Ordered Value	$Z$ Value	Ordered Value	$Z$ Value
1	-1.65	11	0.13
2	-1.28	12	0.25
3	-1.04	13	0.39
4	-0.84	14	0.52
5	-0.67	15	0.67
6	-0.52	16	0.84
7	-0.39	17	1.04
8	-0.25	18	1.28
9	-0.13	19	1.65
10	0.00		

The  $Z$  values are plotted on the  $X$  axis, and the corresponding values of the variable are plotted on the  $Y$  axis.

Figure 6.21 illustrates the typical shape of normal probability plots for a left-skewed distribution (Panel A), a normal distribution (Panel B), and a right-skewed distribution (Panel C). If the data are left-skewed, the curve will rise more rapidly at first and then level off. If the data are normally distributed, the points will plot along an approximately straight line. If the data are right-skewed, the data will rise more slowly at first and then rise at a faster rate for higher values of the variable being plotted.

**FIGURE 6.21**

Normal probability plots for a left-skewed distribution, a normal distribution, and a right-skewed distribution

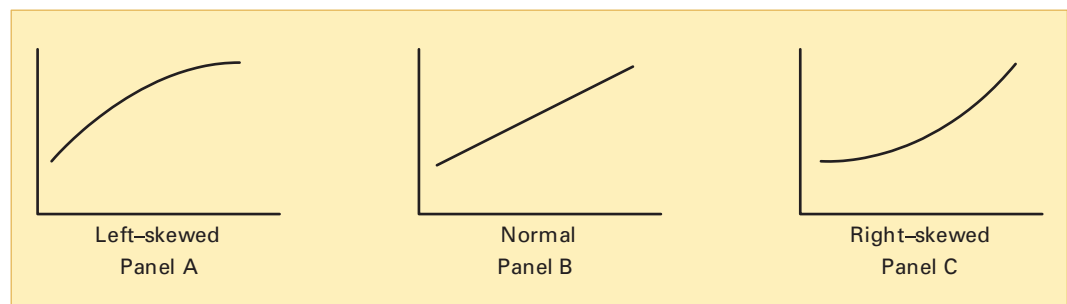


Figure 6.22 shows a Microsoft Excel quantile–quantile normal probability plot for the three-year returns.

**FIGURE 6.22**

Microsoft Excel normal probability plot for three-year returns



See Section E6.2 to create this.

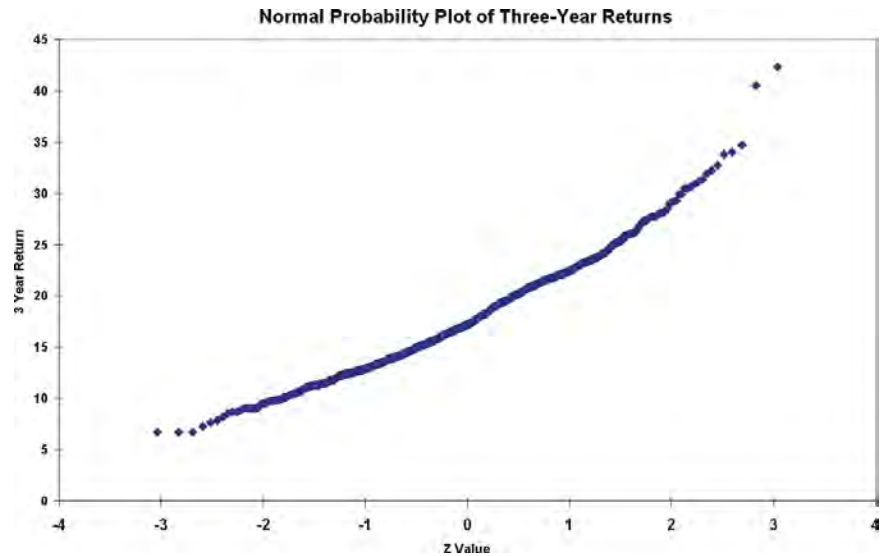


Figure 6.22 shows that the three-year returns rise more slowly at first and subsequently rise more rapidly. This occurs since the data are right-skewed. Thus, the data are not normally distributed.

## PROBLEMS FOR SECTION 6.3

### Learning the Basics

**PH Grade ASSIST** **6.14** Show that for a sample of  $n = 39$ , the smallest and largest  $Z$  values are  $-1.96$  and  $+1.96$ , and the middle (that is, 20th)  $Z$  value is  $0.00$ .

**6.15** For a sample of  $n = 6$ , list the six  $Z$  values.

### Applying the Concepts

**SELF Test** **6.16** The data in the file [chicken.xls](#) contains the total fat, in grams per serving, for a sample of 20 chicken sandwiches from fast-food chains.

The data are as follows:

7 8 4 5 16 20 20 24 19 30 23 30 25 19 29 29 30 30 40 56

Source: Extracted from “Fast Food: Adding Health to the Menu,” Consumer Reports, September 2004, pp. 28–31.

Decide whether the data appear to be approximately normally distributed by

- comparing data characteristics to theoretical properties.
- constructing a normal probability plot.

**6.17** A problem with a telephone line that prevents a customer from receiving or making calls is disconcerting to both the customer and the telephone company. The follow-

ing data (stored in the file [phone.xls](#)) represent two samples of 20 problems reported to two different offices of a telephone company. The time to clear these problems from the customers’ lines is recorded, in minutes.

#### Central Office I Time to Clear Problems (Minutes)

1.48 1.75 0.78 2.85 0.52 1.60 4.15 3.97 1.48 3.10  
1.02 0.53 0.93 1.60 0.80 1.05 6.32 3.93 5.45 0.97

#### Central Office II Time to Clear Problems (Minutes)

7.55 3.75 0.10 1.10 0.60 0.52 3.30 2.10 0.58 4.02  
3.75 0.65 1.92 0.60 1.53 4.23 0.08 1.48 1.65 0.72

For each of the two central office locations, decide whether the data appear to be approximately normally distributed by

- comparing data characteristics to theoretical properties.
- constructing a normal probability plot.

**6.18** Many manufacturing processes use the term *work-in-process* (often abbreviated as *WIP*). In a book manufacturing plant, the WIP represents the time it takes for sheets from a press to be folded, gathered, sewn, tipped on end sheets, and bound. The following data (stored in the file [wip.xls](#)) represent samples of 20 books at each of two production plants and the processing time (operationally

defined as the time, in days, from when the books came off the press to when they were packed in cartons) for these jobs:

#### Plant A

5.62 5.29 16.25 10.92 11.46 21.62 8.45 8.58 5.41 11.42  
11.62 7.29 7.50 7.96 4.42 10.50 7.58 9.29 7.54 18.92

#### Plant B

9.54 11.46 16.62 12.62 25.75 15.41 14.29 13.13 13.71 10.04  
5.75 12.46 9.17 13.21 6.00 2.33 14.25 5.37 6.25 9.71

For each of the two plants, decide whether the data appear to be approximately normally distributed by

- comparing data characteristics to theoretical properties.
- constructing a normal probability plot.

**6.19** The data in the file [spending.xls](#) represent the per capita spending, in thousands of dollars, for each state in 2004. Decide whether the data appear to be approximately normally distributed by

- comparing data characteristics to theoretical properties.
- constructing a normal probability plot.

**6.20** One operation of a mill is to cut pieces of steel into parts that will later be used as the frame for front seats in an automotive plant. The steel is cut with a diamond saw and requires the resulting parts to be within  $\pm 0.005$  inch of the length specified by the automobile company. The data come from a sample of 100 steel parts and are stored in the file [steel.xls](#). The measurement reported is the difference, in inches, between the actual length of the steel part, as

measured by a laser measurement device, and the specified length of the steel part.

Decide whether the data appear to be approximately normally distributed by

- comparing data characteristics to theoretical properties.
- constructing a normal probability plot.

**6.21** The data in the file [savings.xls](#) are the yields for a money market account, a one-year certificate of deposit (CD), and a five-year CD for 40 banks in south Florida as of December 20, 2005 (extracted from [Bankrate.com](#), December 20, 2005). For each of the three types of investments, decide whether the data appear to be approximately normally distributed by

- comparing data characteristics to theoretical properties.
- constructing a normal probability plot.

**6.22** The following data, stored in the file [utility.xls](#), represent the electricity costs in dollars, during July 2006 for a random sample of 50 two-bedroom apartments in a large city:

96 171 202 178 147 102 153 197 127 82  
157 185 90 116 172 111 148 213 130 165  
141 149 206 175 123 128 144 168 109 167  
95 163 150 154 130 143 187 166 139 149  
108 119 183 151 114 135 191 137 129 158

Decide whether the data appear to be approximately normally distributed by:

- comparing data characteristics to theoretical properties.
- constructing a normal probability plot.

## 6.4 THE UNIFORM DISTRIBUTION

In the **uniform distribution**, a value has the same probability of occurrence anywhere in the range between the smallest value,  $a$ , and the largest value,  $b$ . Because of its shape, the uniform distribution is sometimes called the **rectangular distribution** (see Panel B of Figure 6.1 on page 218). Equation (6.5) defines the continuous probability density function for the uniform distribution.

### UNIFORM DISTRIBUTION

$$f(X) = \frac{1}{b-a} \text{ if } a \leq X \leq b \text{ and } 0 \text{ elsewhere} \quad (6.5)$$

where

$a$  = the minimum value of  $X$

$b$  = the maximum value of  $X$

Equation (6.6) defines the mean of the uniform distribution.

### MEAN OF THE UNIFORM DISTRIBUTION

$$\mu = \frac{a + b}{2} \quad (6.6)$$

Equation (6.7) defines the variance and standard deviation of the uniform distribution.

### VARIANCE AND STANDARD DEVIATION OF THE UNIFORM DISTRIBUTION

$$\sigma^2 = \frac{(b - a)^2}{12} \quad (6.7a)$$

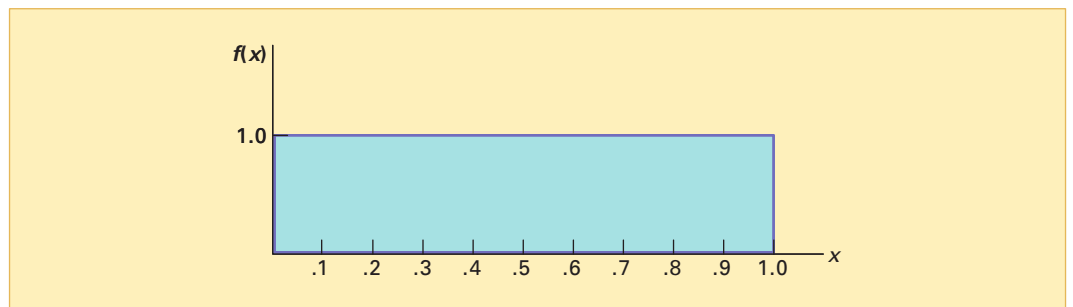
$$\sigma = \sqrt{\frac{(b - a)^2}{12}} \quad (6.7b)$$

One of the most common uses of the uniform distribution is in the selection of random numbers. When you use simple random sampling (see Section 7.1), you assume that each value comes from a uniform distribution that has a minimum value of 0 and a maximum value of 1.

Figure 6.23 illustrates the uniform distribution with  $a = 0$  and  $b = 1$ . The total area inside the rectangle is equal to the base (1.0) times the height (1.0). Thus, the resulting area of 1.0 satisfies the requirement that the area under any probability density function equals 1.0.

**FIGURE 6.23**

Probability density function for a uniform distribution with  $a = 0$  and  $b = 1$

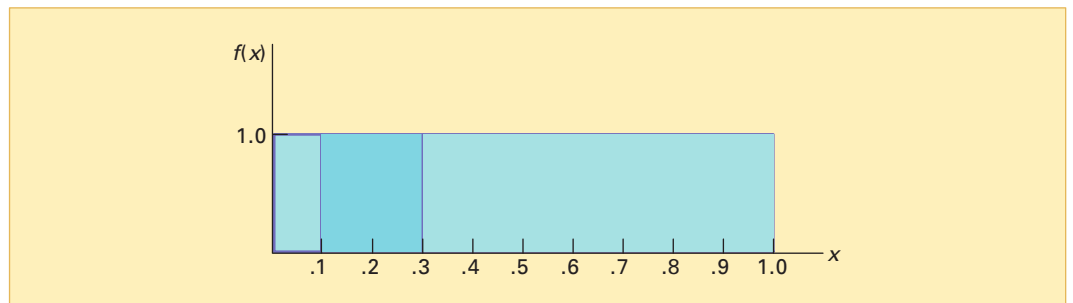


In such a distribution, what is the probability of getting a random number between 0.10 and 0.30? The area between 0.10 and 0.30, depicted in Figure 6.24, is equal to the base (which is  $0.30 - 0.10 = 0.20$ ) times the height (1.0). Therefore,

$$P(0.10 < X < 0.30) = (\text{Base})(\text{Height}) = (0.20)(1.0) = 0.20$$

**FIGURE 6.24**

Finding  $P(0.10 < X < 0.30)$  for a uniform distribution with  $a = 0$  and  $b = 1$



From Equations (6.6) and (6.7), the mean and standard deviation of the uniform distribution for  $a = 0$  and  $b = 1$  are computed as follows:

$$\begin{aligned}\mu &= \frac{a + b}{2} \\ &= \frac{0 + 1}{2} = 0.5\end{aligned}$$

and

$$\begin{aligned}\sigma^2 &= \frac{(b - a)^2}{12} \\ &= \frac{(1 - 0)^2}{12} \\ &= \frac{1}{12} = 0.0833 \\ \sigma &= \sqrt{0.0833} = 0.2887\end{aligned}$$

Thus, the mean is 0.5 and the standard deviation is 0.2887.

## PROBLEMS FOR SECTION 6.4

### Learning the Basics

**PH Grade ASSIST** **6.23** Suppose you sample one value from a uniform distribution with  $a = 0$  and  $b = 10$ . What is the probability that the value will be

- between 5 and 7?
- between 2 and 3?
- What is the mean?
- What is the standard deviation?

### Applying the Concepts

**PH Grade ASSIST** **6.24** The time between arrivals of customers at a bank during the noon-to-1 p.m. hour has a uniform distribution between 0 to 120 seconds. What is the probability that the time between the

- arrival of two customers will be
- less than 20 seconds?
  - between 10 and 30 seconds?
  - more than 35 seconds?
  - What are the mean and standard deviation of the time between arrivals?

**6.25** A study of the time spent shopping in a supermarket for a market basket of 20 specific items showed an approximately uniform distribution between 20 minutes and 40 minutes. What is the probability that the shopping time will be

- between 25 and 30 minutes?
- less than 35 minutes?
- What are the mean and standard deviation of the shopping time?

**PH Grade ASSIST** **6.26** The time to failure for a continuous-operation monitoring device of air quality has a uniform distribution over a 24-hour day.

- If a failure occurs on a day when daylight is between 5:55 a.m. and 7:38 p.m., what is the probability that the failure will occur during daylight hours?
- If the device is in secondary mode from 10 p.m. to 5 a.m., what is the probability that if a failure occurs, it will happen during secondary mode?
- If the device has a self-checking computer chip that determines whether the device is operational every hour on the hour, what is the probability that a failure will be detected within 10 minutes of its occurrence?
- If the device has a self-checking computer chip that determines whether the device is operational every hour on the hour, what is the probability that it will take at least 40 minutes to detect that a failure has occurred?

**6.27** The scheduled commuting time on the Long Island Rail Road from Glen Cove to New York City is 65 minutes. Suppose that the actual commuting time is uniformly distributed between 64 and 74 minutes. What is the probability that the commuting time will be

- less than 70 minutes?
- between 65 and 70 minutes?
- greater than 65 minutes?
- What are the mean and standard deviation of the commuting time?



## 6.5 THE EXPONENTIAL DISTRIBUTION

The **exponential distribution** is a continuous distribution that is right-skewed and ranges from zero to positive infinity (see Panel C of Figure 6.1 on page 218). The exponential distribution is widely used in waiting-line (or queuing) theory to model the length of time between arrivals in processes such as customers at a bank's ATM, patients entering a hospital emergency room, and hits on a Web site.

The exponential distribution is defined by a single parameter, its mean,  $\lambda$ , the mean number of arrivals per unit of time. The value  $1/\lambda$  is equal to the mean time between arrivals. For example, if the mean number of arrivals in a minute is  $\lambda = 4$ , then the mean time between arrivals is  $1/\lambda = 0.25$  minutes, or 15 seconds. Equation (6.8) defines the probability that the length of time before the next arrival is less than  $X$ .

### EXPONENTIAL DISTRIBUTION

$$P(\text{Arrival time} < X) = 1 - e^{-\lambda X} \quad (6.8)$$

where

$e$  = the mathematical constant approximated by 2.71828

$\lambda$  = the mean number of arrivals per unit

$X$  = any value of the continuous variable where  $0 < X < \infty$

To illustrate the exponential distribution, suppose that customers arrive at a bank's ATM at a rate of 20 per hour. If a customer has just arrived, what is the probability that the next customer will arrive within 6 minutes (that is, 0.1 hour)? For this example,  $\lambda = 20$  and  $X = 0.1$ . Using Equation (6.8),

$$\begin{aligned} P(\text{Arrival time} < 0.1) &= 1 - e^{-20(0.1)} \\ &= 1 - e^{-2} \\ &= 1 - 0.1353 = 0.8647 \end{aligned}$$

Thus, the probability that a customer will arrive within 6 minutes is 0.8647, or 86.47%. You can also use Microsoft Excel to compute this probability (see Figure 6.25).

**FIGURE 6.25**

Microsoft Excel worksheet for finding exponential probabilities (mean =  $\lambda$ )



See Section E6.3 to create this.

	A	B
1	<b>Exponential Probability</b>	
2		
3	<b>Data</b>	
4	Mean	20
5	X Value	0.1
6		
7	<b>Results</b>	
8	P(<=X)	0.8647

=EXPONDIST(B5, B4, TRUE)

### EXAMPLE 6.8

### COMPUTING EXPONENTIAL PROBABILITIES

In the ATM example, what is the probability that the next customer will arrive within 3 minutes (that is, 0.05 hour)?

**SOLUTION** For this example,  $\lambda = 20$  and  $X = 0.05$ . Using Equation (6.8),

$$\begin{aligned} P(\text{Arrival time} < 0.05) &= 1 - e^{-20(0.05)} \\ &= 1 - e^{-1} \\ &= 1 - 0.3679 = 0.6321 \end{aligned}$$

Thus, the probability that a customer will arrive within 3 minutes is 0.6321, or 63.21%.

## PROBLEMS FOR SECTION 6.5

### Learning the Basics

**6.28** Given an exponential distribution with  $\lambda = 10$ , what is the probability that the arrival time is

- less than  $X = 0.1$ ?
- greater than  $X = 0.1$ ?
- between  $X = 0.1$  and  $X = 0.2$ ?
- less than  $X = 0.1$  or greater than  $X = 0.2$ ?

**6.29** Given an exponential distribution with  $\lambda = 30$ , what is the probability that the arrival time is

- less than  $X = 0.1$ ?
- greater than  $X = 0.1$ ?
- between  $X = 0.1$  and  $X = 0.2$ ?
- less than  $X = 0.1$  or greater than  $X = 0.2$ ?

**6.30** Given an exponential distribution with  $\lambda = 20$ , what is the probability that the arrival time is

- less than  $X = 4$ ?
- greater than  $X = 0.4$ ?
- between  $X = 0.4$  and  $X = 0.5$ ?
- less than  $X = 0.4$  or greater than  $X = 0.5$ ?

### Applying the Concepts

**PH Grade ASSIST** **6.31** Autos arrive at a tollplaza located at the entrance to a bridge at the rate of 50 per minute during the 5:00–6:00 p.m. hour. If an auto has just arrived,

- what is the probability that the next auto will arrive within 3 seconds (0.05 minute)?
- what is the probability that the next auto will arrive within 1 second (0.0167 minute)?
- What are your answers to (a) and (b) if the rate of arrival of autos is 60 per minute?
- What are your answers to (a) and (b) if the rate of arrival of autos is 30 per minute?

**SELF Test** **6.32** Customers arrive at the drive-up window of a fast-food restaurant at a rate of 2 per minute during the lunch hour.

- What is the probability that the next customer will arrive within 1 minute?
- What is the probability that the next customer will arrive within 5 minutes?

c. During the dinner time period, the arrival rate is 1 per minute. What are your answers to (a) and (b) for this period?

**6.33** Telephone calls arrive at the information desk of a large computer software company at a rate of 15 per hour.

- What is the probability that the next call will arrive within 3 minutes (0.05 hour)?
- What is the probability that the next call will arrive within 15 minutes (0.25 hour)?
- Suppose the company has just introduced an updated version of one of its software programs, and telephone calls are now arriving at a rate of 25 per hour. Given this information, redo (a) and (b).

**PH Grade ASSIST** **6.34** An on-the-job injury occurs once every 10 days on average at an automobile plant. What is the probability that the next on-the-job injury will occur within

- 10 days?
- 5 days?
- 1 day?

**PH Grade ASSIST** **6.35** The time between unplanned shutdowns of a power plant has an exponential distribution with a mean of 20 days. Find the probability that the time between two unplanned shutdowns is

- less than 14 days.
- more than 21 days.
- less than 7 days.

**PH Grade ASSIST** **6.36** Golfers arrive at the starter's booth of a public golf course at a rate of 8 per hour during the Monday-to-Friday midweek period. If a golfer has just arrived,

- what is the probability that the next golfer will arrive within 15 minutes (0.25 hour)?
- what is the probability that the next golfer will arrive within 3 minutes (0.05 hour)?
- The actual arrival rate on Fridays is 15 per hour. What are your answers to (a) and (b) for Fridays?

**6.37** TrafficWeb.org claims that it can deliver 10,000 hits to a Web site in the next 60 days for only \$21.95 ([www.trafficweb.org](http://www.trafficweb.org), April 26, 2004). If this amount of

Web site traffic is experienced, then the time between hits has as a mean of 8.64 minutes (or 0.116 per minute). Assume that your Web site does get 10,000 hits in the next 60 days and that the time between hits has an exponential distribution. What is the probability that the time between two hits is

- less than 5 minutes?
- less than 10 minutes?
- more than 15 minutes?
- Do you think it is reasonable to assume that the time between hits has an exponential distribution?

## 6.6 (CD-ROM Topic) THE NORMAL APPROXIMATION TO THE BINOMIAL DISTRIBUTION

In many circumstances, the normal distribution can be used to approximate the binomial distribution. For further discussion, see [section 6.6.pdf](#) on the Student CD-ROM that accompanies this text.

## SUMMARY

In this chapter, you used the normal distribution in the Using Statistics scenario to study the time to download a Web page. In addition, you studied the uniform distribu-

tion, the exponential distribution, and the normal probability plot. In Chapter 7, the normal distribution is used in developing the subject of statistical inference.

## KEY EQUATIONS

### Normal Probability Density Function

$$f(X) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(1/2)[(X-\mu)/\sigma]^2} \quad (6.1)$$

### Transformation Formula

$$Z = \frac{X - \mu}{\sigma} \quad (6.2)$$

### Standardized Normal Probability Density Function

$$f(Z) = \frac{1}{\sqrt{2\pi}} e^{-(1/2)Z^2} \quad (6.3)$$

### Finding an $X$ Value Associated with Known Probability

$$X = \mu + Z\sigma \quad (6.4)$$

### Uniform Distribution

$$f(X) = \frac{1}{b - a} \quad (6.5)$$

### Mean of the Uniform Distribution

$$\mu = \frac{a + b}{2} \quad (6.6)$$

### Variance and Standard Deviation of the Uniform Distribution

$$\sigma^2 = \frac{(b - a)^2}{12} \quad (6.7a)$$

$$\sigma = \sqrt{\frac{(b - a)^2}{12}} \quad (6.7b)$$

### Exponential Distribution

$$P(\text{Arrival time} < X) = 1 - e^{-\lambda X} \quad (6.8)$$

## KEY TERMS

continuous probability density function 218  
cumulative standardized normal distribution 222  
exponential distribution 241

normal distribution 219  
normal probability density function 220  
normal probability plot 236  
quantile-quantile plot 236

rectangular distribution 238  
standardized normal random variable 221  
transformation formula 221  
uniform distribution 238

# CHAPTER REVIEW PROBLEMS

## Checking Your Understanding

- 6.38** Why is it that only one normal distribution table such as Table E.2 is needed to find any probability under the normal curve?
- 6.39** How do you find the area between two values under the normal curve?
- 6.40** How do you find the  $X$  value that corresponds to a given percentile of the normal distribution?
- 6.41** What are some of the distinguishing properties of a normal distribution?
- 6.42** How does the shape of the normal distribution differ from those of the uniform and exponential distributions?
- 6.43** How can you use the normal probability plot to evaluate whether a set of data is normally distributed?
- 6.44** Under what circumstances can you use the exponential distribution?

## Applying the Concepts

- 6.45** An industrial sewing machine uses ball bearings that are targeted to have a diameter of 0.75 inch. The lower and upper specification limits under which the ball bearings can operate are 0.74 inch and 0.76 inch, respectively. Past experience has indicated that the actual diameter of the ball bearings is approximately normally distributed, with a mean of 0.753 inch and a standard deviation of 0.004 inch. What is the probability that a ball bearing is
- between the target and the actual mean?
  - between the lower specification limit and the target?
  - above the upper specification limit?
  - below the lower specification limit?
  - 93% of the diameters are greater than what value?
- 6.46** The fill amount of soft drink bottles is normally distributed, with a mean of 2.0 liters and a standard deviation of 0.05 liter. If bottles contain less than 95% of the listed net content (1.90 liters, in this case), the manufacturer may be subject to penalty by the state office of consumer affairs. Bottles that have a net content above 2.10 liters may cause excess spillage upon opening. What proportion of the bottles will contain
- between 1.90 and 2.0 liters?
  - between 1.90 and 2.10 liters?
  - below 1.90 liters or above 2.10 liters?
  - 99% of the bottles contain at least how much soft drink?
  - 99% of the bottles contain an amount that is between which two values (symmetrically distributed) around the mean?
- 6.47** In an effort to reduce the number of bottles that contain less than 1.90 liters, the bottler in Problem 6.46 sets the filling machine so that the mean is 2.02 liters. Under these circumstances, what are your answers in (a) through (e)?

PH Grade  
ASSIST

- 6.48** An orange juice producer buys all his oranges from a large orange grove. The amount of juice squeezed from each of these oranges is approximately normally distributed, with a mean of 4.70 ounces and a standard deviation of 0.40 ounce.
- What is the probability that a randomly selected orange will contain between 4.70 and 5.00 ounces?
  - What is the probability that a randomly selected orange will contain between 5.00 and 5.50 ounces?
  - 77% of the oranges will contain at least how many ounces of juice?
  - 80% of the oranges contain between what two values (in ounces), symmetrically distributed around the population mean?

**6.49** Data concerning 58 of the best-selling domestic beers in the United States are located in the file [domesticbeer.xls](#). The values for three variables are included: percentage alcohol, number of calories per 12 ounces, and number of carbohydrates (in grams) per 12 ounces. For each of the three variables, decide whether the data appear to be approximately normally distributed. Support your decision through the use of appropriate statistics and graphs.

Source: Extracted from [www.Beer100.com](#), March 31, 2006.

- 6.50** The evening manager of a restaurant was very concerned about the length of time some customers were waiting in line to be seated. She also had some concern about the seating times—that is, the length of time between when a customer is seated and the time he or she leaves the restaurant. Over the course of one week, 100 customers (no more than 1 per party) were randomly selected, and their waiting and seating times (in minutes) were recorded in the file [wait.xls](#).
- Think about your favorite restaurant. Do you think waiting times more closely resemble a uniform, exponential, or normal distribution?
  - Again, think about your favorite restaurant. Do you think seating times more closely resemble a uniform, exponential, or normal distribution?
  - Construct a histogram and a normal probability plot of the waiting-times. Do you think these waiting times more closely resemble a uniform, exponential, or normal distribution?
  - Construct a histogram and a normal probability plot of the seating times. Do you think these seating times more closely resemble a uniform, exponential, or normal distribution?

**6.51** At the end of the first quarter of 2006, all the major stock market indexes had posted strong gains in the past 12 months. Mass Mutual Financial Group credited the increases to solid growth in corporate profits (“Market Commentary: Economic Growth Characterizes Q1 2006,”

**www.massmutual.com**, May 1, 2006). The mean one-year return for stocks in the S&P 500, a group of 500 very large companies, was approximately 12%. The mean one-year return for companies in the Russell 2000, a group of 2000 small companies, was approximately 26%. Historically, the one-year returns are approximately normal, the standard deviation in the S&P 500 is approximately 20%, and the standard deviation in the Russell 200 is approximately 35%.

- What is the probability that a stock in the S&P 500 gained 25% or more in the last year? gained 50% or more?
- What is the probability that a stock in the S&P 500 lost money in the last year? Lost 25% or more? lost 50% or more?
- Repeat (a) and (b) for a stock in the Russell 2000.
- Write a short summary on your findings. Be sure to include a discussion of the risks associated with a large standard deviation.

**6.52** The *New York Times* reported (L. J. Flynn, “Tax Surfing,” *The New York Times*, March 25, 2002, p. C10) that the mean time to download the home page for the Internal Revenue Service, **www.irs.gov**, is 0.8 second. Suppose that the download time is normally distributed with a standard deviation of 0.2 seconds. What is the probability that a download time is

- less than 1 second?
- between 0.5 and 1.5 seconds?
- above 0.5 second?
- 99% of the download times are above how many seconds?
- 95% of the download times are between what two values, symmetrically distributed around the mean?

**6.53** The same article mentioned in Problem 6.52 also reported that the mean download time for the H&R Block Web site, **www.hrblock.com**, is 2.5 seconds. Suppose that the download time is normally distributed with a standard deviation of 0.5 second. What is the probability that a download time is

- less than 1 second?
- between 0.5 and 1.5 seconds?
- above 0.5 second?
- 99% of the download times are above how many seconds?
- Compare the results for the IRS site computed in Problem 6.52 to those of the H&R Block site.

**6.54 (Class Project)** According to Burton G. Malkiel, the daily changes in the closing price of stock follow a *random walk*—that is, these daily events are independent of each other and move upward or downward in a random manner—and can be approximated by a normal distribution. To test this theory, use either a newspaper or the Internet to select one company traded on the NYSE, one company traded on the American Stock Exchange, and one company traded “over the counter” (that is, on the NASDAQ national market) and then do the following:

- Record the daily closing stock price of each of these companies for six consecutive weeks (so that you have 30 values per company).
- Record the daily changes in the closing stock price of each of these companies for six consecutive weeks (so that you have 30 values per company).

For each of your six data sets, decide whether the data are approximately normally distributed by

- examining the stem-and-leaf display, histogram or polygon, and box-and-whisker plot.
- comparing data characteristics to theoretical properties.
- constructing a normal probability plot.
- Discuss the results of (a) through (c). What can you say about your three stocks with respect to daily closing prices and daily changes in closing prices? Which, if any, of the data sets are approximately normally distributed?

*Note:* The random-walk theory pertains to the daily changes in the closing stock price, not the daily closing stock price.

## Team Projects

The data file **Mutual Funds.xls** contains information regarding nine variables from a sample of 838 mutual funds. The variables are:

Category—Type of stocks comprising the mutual fund (small cap, mid cap, large cap)

Objective—Objective of stocks comprising the mutual fund (growth or value)

Assets—In millions of dollars

Fees—Sales charges (no or yes)

Expense ratio—Ratio of expenses to net assets, in percentage

Risk—Risk-of-loss factor of the mutual fund (low, average, high)

2005 return—Twelve-month return in 2005

Three-year return—Annualized return, 2003–2005

Five-year return—Annualized return, 2001–2005

**6.55** For the expense ratio in percentage, 2005 return, and five-year return, decide whether the data are approximately normally distributed by

- comparing data characteristics to theoretical properties.
- constructing a normal probability plot.

## Student Survey Data Base

**6.56** Problem 1.27 on page 15 describes a survey of 50 undergraduate students (see the file **undergradsurvey.xls**). For these data, for each numerical variable, decide whether the data are approximately normally distributed by

- comparing data characteristics to theoretical properties.
- constructing a normal probability plot.

**6.57** Problem 1.27 on page 15 describes a survey of 50 undergraduate students (see the file **undergradsurvey.xls**).

- Select a sample of 50 undergraduate students and conduct a similar survey for those students.

- b. For the data collected in (a), repeat (a) and (b) of Problem 6.56.
- c. Compare the results of (b) to those of Problem 6.56.

**6.58** Problem 1.28 on page 15 describes a survey of 50 MBA students (see the file [gradsurvey.xls](#)). For these data, for each numerical variable, decide whether the data are approximately normally distributed by

- a. comparing data characteristics to theoretical properties.
- b. constructing a normal probability plot.

**6.59** Problem 1.28 on page 15 describes a survey of 50 MBA students (see the file [gradsurvey.xls](#)).

- a. Select a sample of 50 undergraduate students and conduct a similar survey for those students.
- b. For the data collected in (a), repeat (a) and (b) of Problem 6.58.
- c. Compare the results of (b) to those of Problem 6.58.

## Managing the Springville Herald

The production department of the newspaper has embarked on a quality improvement effort. Its first project relates to the blackness of the newspaper print. Each day, a determination needs to be made concerning how black the newspaper is printed. Blackness is measured on a standard scale in which the target value is 1.0. Data collected over the past year indicate that the blackness is normally distributed, with a mean of 1.005 and a standard deviation of 0.10. Each day, one spot on the first newspaper printed is chosen, and the blackness of the spot is measured. The blackness of the newspaper is considered acceptable if the blackness of the spot is between 0.95 and 1.05.

### EXERCISES

- SH6.1 Assuming that the distribution has not changed from what it was in the past year, what is the probability that the blackness of the spot is
- less than 1.0?
  - between 0.95 and 1.0?
  - between 1.0 and 1.05?
  - less than 0.95 or greater than 1.05?
- SH6.2 The objective of the production team is to reduce the probability that the blackness is below 0.95 or above 1.05. Should the team focus on process improvement that lowers the mean to the target value of 1.0 or on process improvement that reduces the standard deviation to 0.075? Explain.

## Web Case

*Apply your knowledge about the normal distribution in this Web Case, which extends the Using Statistics scenario from this chapter.*

To satisfy concerns of potential advertisers, the management of OurCampus! has undertaken a research project to learn the amount of time it takes users to download a complex video features page. The marketing department has collected data and has made some claims based on the assertion that the data follow a normal distribution. These data and conclusions can be found in a report located on the internal Web page [www.prenhall.com/Springville/Our\\_DownloadResearch.htm](http://www.prenhall.com/Springville/Our_DownloadResearch.htm) (or in the file with the same name in the Student CD-ROM Web Case folder).

Read this marketing report and then answer the following:

- Can the collected data be approximated by the normal distribution?
- Review and evaluate the conclusions made by the OurCampus! marketing department. Which conclusions are correct? Which ones are incorrect?
- If OurCampus! could improve the mean time by five minutes, how would the probabilities change?

## REFERENCES

- Gunter, B., "Q-Q Plots," *Quality Progress* (February 1994), 81–86.
- Levine, D. M., P. Ramsey, and R. Smidt, *Applied Statistics for Engineers and Scientists Using Microsoft Excel and Minitab* (Upper Saddle River, NJ: Prentice Hall, 2001).
- Microsoft Excel 2007* (Redmond, WA: Microsoft Corp., 2007).

# Excel Companion

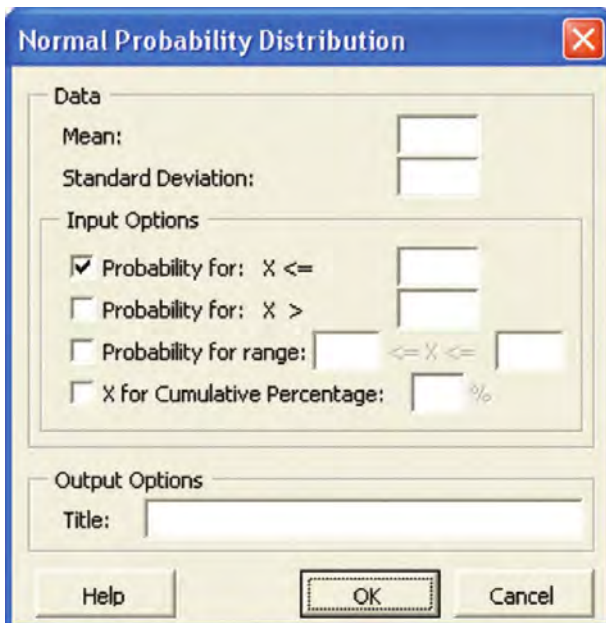
## to Chapter 6

### E6.1 COMPUTING NORMAL PROBABILITIES

You compute normal probabilities by either using the PHStat2 Normal procedure or by making entries in the **Normal.xls** workbook.

#### Using PHStat2 Normal

Select **PHStat** → **Probability & Prob. Distributions** → **Normal**. In the Normal Probability Distribution dialog box (shown below), enter values for the mean and standard deviation. Click one or more of the input options and enter values. Enter a title as the **Title** and click **OK**.



For Example 6.1, click **Probability for: X >** and enter 9 in its box. For Example 6.2, click **Probability for range** and enter 7 as the “from” value and 9 as the “to” value. For Example 6.5, click **Probability for: X <=** and enter 3.5. For Example 6.6, click **X for Cumulative Percentage** and enter 10 in its box.

For problems such as Example 6.3 that require a probability of  $X$  less than a value or  $X$  greater than a value, click the first two input options and enter the tail values. Clicking all the input options creates a worksheet identical to the **Normal Expanded** worksheet described next.

#### Using the Normal Workbook

You open and use either the **Normal** or **Normal Expanded** worksheets of the **Normal.xls** workbook to compute normal probabilities. The **Normal** worksheet, shown in Figure 6.18, computes probabilities for problems similar to Examples 6.3 and 6.6. The **Normal Expanded** worksheet (see Figures E6.1 and E6.2) expands the Normal worksheet to compute all types of normal probabilities. Both worksheets use the **STANDARDIZE**, **NORMDIST**, **NORMSINV**, and **NORMINV** functions in computing normal probabilities and use the **ABS** (absolute value) function in cell E13 to compute the probability for a range as a positive value.

The **STANDARDIZE( $X$ , mean, standard deviation)** function returns the  $Z$  value for a specific  $X$  value, mean, and standard deviation. The **NORMDIST( $X$ , mean, standard deviation, True)** function returns the area or probability of less than a given  $X$  value for a specific mean and standard deviation. The **NORMSINV( $P < X$ )** function returns the  $Z$  value corresponding to the probability of less than a given  $X$ . The **NORMINV( $P < X$ , mean, standard deviation)** function returns the  $X$  value for a given probability, mean, and standard deviation.

To use either worksheet for similar probability problems, change the entries in the turquoise tinted cells to match the details of your problem. Because cells A10, A15, A17, A18, and D9 through D13 contain formulas that use the ampersand operator (see the Excel Companion to

	A	B	
1	<b>Normal Probabilities</b>		
2			
3	<b>Common Data</b>		
4	Mean	7	
5	Standard Deviation	2	
6			
7	<b>Probability for X &lt;=</b>		
8	X Value	7	
9	Z Value	0	=STANDARDIZE(B8, B4, B5)
10	P(X<=7)	0.5	=NORMDIST(B8, B4, B5, TRUE)
11			
12	<b>Probability for X &gt;</b>		
13	X Value	9	
14	Z Value	1	=STANDARDIZE(B13, B4, B5)
15	P(X>9)	0.1587	=1 - NORMDIST(B13, B4, B5, TRUE)
16			
17	<b>Probability for X&lt;7 or X&gt;9</b>		
18	P(X<7 or X>9)	0.6587	=B10 + B15

**FIGURE E6.1**

Normal expanded columns A and B

Chapter 4) to create labels from the  $X$  values you enter, you do not need to edit any labels. (These nine formulas are not shown in Figures E6.1 and E6.2.)

	D	E
1		
2		
3		
4		
5		
6	<b>Probability for a Range</b>	
7	From X Value	7
8	To X Value	9
9	Z Value for 7	0 =STANDARDIZE(E7, B4, B5)
10	Z Value for 9	1 =STANDARDIZE(E8, B4, B5)
11	P(X<=7)	0.5000 =NORMDIST(E7, B4, B5, TRUE)
12	P(X<=9)	0.8413 =NORMDIST(E8, B4, B5, TRUE)
13	P(7<=X<=9)	0.3413 =ABS(E12 - E11)
14		
15	<b>Find X and Z Given Cum. Pctage.</b>	
16	Cumulative Percentage	10.00%
17	Z Value	-1.281552 =NORMSINV(E16)
18	X Value	4.436897 =NORMINV(E16, B4, B5)

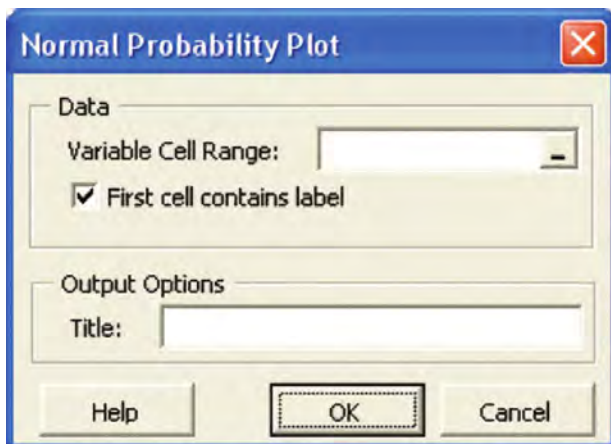
**FIGURE E6.2** Normal expanded columns D and E

## E6.2 CREATING NORMAL PROBABILITY PLOTS

You create normal probability plots by using the PHStat2 Normal Probability Plot procedure (recommended) or by using Excel charting features with a worksheet of  $Z$  values that you create.

### Using PHStat2 Normal Probability Plot

Open to the worksheet that contains the data to be plotted. Select **PHStat** → **Probability & Prob. Distributions** → **Normal Probability Plot**. In the Normal Probability Plot dialog box (shown below), enter the cell range of the data to be plotted as the **Variable Cell Range** and click **First cell contains label**. Enter a title as the **Title** and click **OK**. PHStat2 creates a chart sheet containing the plot and a worksheet of  $Z$  values on which the plot is based.



## Creating a Z Values Worksheet

To use Excel charting features to create a normal probability plot, you first create a worksheet that computes  $Z$  values. This worksheet contains four columns in ascending order: rank, proportion,  $Z$  value, and the data to be evaluated for normality.

Table E6.1 shows the entries of the first three rows and the last row of the **Plot** worksheet of the **Npp.xls** workbook. This worksheet contains the source data for the Figure 6.22 plot of the three-year rate of returns. Column B formulas are the rank divided by  $n + 1$  (839, for the sample of 838 mutual funds). Column C formulas use **NORMSINV** (see Section E6.1) to compute  $Z$  values based on the proportions computed in column B.

To create normal probability plots for another variable, you adjust the worksheet design by either adding or deleting rows and adjusting the divisor in the column B formulas. (The divisor should always be 1 more than the sample size.)

**TABLE E6.1** Z Values Worksheet Design for Three-Year Returns Variable

	A	B	C	D
1	Rank	Proportion	Z Value	3-Year Return
2	1	=A2/839	=NORMSINV(B2)	6.7
3	2	=A3/839	=NORMSINV(B3)	6.7
:	:	:	:	:
839	838	=A839/839	=NORMSINV(B839)	42.3

## Creating a Normal Probability Plot (97–2003)

Open to your copy of the **Z Values** worksheet. Begin the Chart Wizard and make these entries:

**Step 1.** Click the **Standard Types** tab and then click **XY (Scatter)** from the **Standard Types Chart type** box. Click the first **Chart sub-type**, labeled “**Scatter. Compares pairs of values**” when selected.

**Step 2.** Click the **Data Range** tab. Enter the columns C and D cell range as the **Data range** and click the **Columns** option in the **Series in** group.

**Step 3.** Click the **Titles** tab. Enter a title as the **Chart title** and enter **Z Value** as the **Category (X) axis** title. Enter a title for the **Value (Y) axis** title. Click, in turn, the **Axes**, **Gridlines**, **Data Labels**, tabs and adjust the settings, as discussed in the “**Creating Charts (97–2003)**” part of Section E2.2 on page 79. Click the **Legend** tab and clear **Show legend**.



## Creating a Normal Probability Plot (2007)

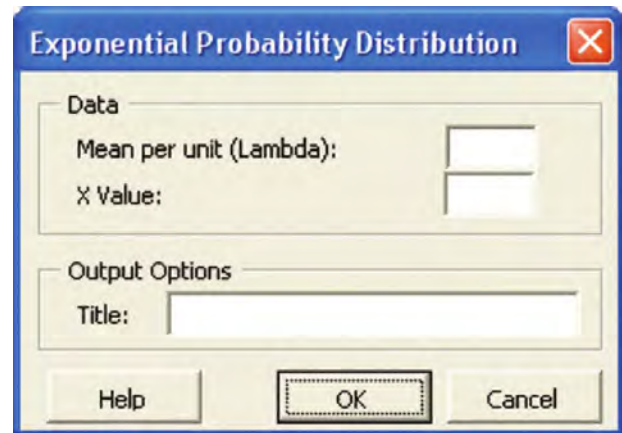
Open to your copy of the **Z Values** worksheet. Select the columns C and D cell range that contains the Z values and the data whose normality is being tested. Click **Insert** → **X Y (Scatter)** → **Scatter with only Markers**. Finish by relocating your chart to a chart sheet and customizing your chart using the instructions in “Creating Charts (2007)” in Section E2.2 on page 80.

## E6.3 COMPUTING EXPONENTIAL PROBABILITIES

You compute exponential probabilities by either using the PHStat2 Exponential procedure or by making entries in the **Exponential.xls** workbook.

### Using PHStat2 Exponential

Open to the worksheet that contains the data to be plotted. Select **PHStat** → **Probability & Prob. Distributions** → **Exponential**. In the Exponential Probability Distribution dialog box (shown at right), enter values for **Mean per unit (Lambda)** and the **X Value**. Enter a title as the **Title** and click **OK**.



### Using the Exponential Worksheet

Open to the **Exponential** worksheet of the **Exponential.xls** workbook. This worksheet (see Figure 6.25 on page 241) uses the **EXPONDIST (X value, mean, True)** function to compute the exponential probability for the bank ATM problem of Section 6.5. To adapt this worksheet for other problems, change the **Mean** and **X Value** values in cells B4 and B5.



# CHAPTER 7

## Sampling and Sampling Distributions

### USING STATISTICS @ Oxford Cereals

#### 7.1 TYPES OF SAMPLING METHODS

Simple Random Samples  
Systematic Samples  
Stratified Samples  
Cluster Samples

#### 7.2 EVALUATING SURVEY WORTHINESS

Survey Errors  
Ethical Issues

#### 7.3 SAMPLING DISTRIBUTIONS

#### 7.4 SAMPLING DISTRIBUTION OF THE MEAN

The Unbiased Property of the Sample Mean  
Standard Error of the Mean

Sampling from Normally Distributed Populations  
Sampling from Non-Normally Distributed Populations—The Central Limit Theorem

#### 7.5 SAMPLING DISTRIBUTION OF THE PROPORTION

#### 7.6 (CD-ROM TOPIC) SAMPLING FROM FINITE POPULATIONS

#### EXCEL COMPANION TO CHAPTER 7

E7.1 Creating Simple Random Samples (Without Replacement)

E7.2 Creating Simulated Sampling Distributions

### LEARNING OBJECTIVES

In this chapter, you learn:

- To distinguish between different sampling methods
- The concept of the sampling distribution
- To compute probabilities related to the sample mean and the sample proportion
- The importance of the Central Limit Theorem

## Using Statistics @ Oxford Cereals



Oxford Cereals fills thousands of boxes of cereal during an eight-hour shift. As the plant operations manager, you are responsible for monitoring the amount of cereal placed in each box. To be consistent with package labeling, boxes should contain a mean of 368 grams of cereal. Because of the speed of the process, the cereal weight varies from box to box, causing some boxes to be underfilled and others overfilled. If the process is not working properly, the mean weight in the boxes could vary too much from the label weight of 368 grams to be acceptable.

Because weighing every single box is too time-consuming, costly, and inefficient, you must take a sample of boxes. For each sample you select, you plan to weigh the individual boxes and calculate a sample mean. You need to determine the probability that such a sample mean could have been randomly selected from a population whose mean is

368 grams. Based on your analysis, you will have to decide whether to maintain, alter, or shut down the process.

In Chapter 6, you used the normal distribution to study the distribution of download times for the OurCampus! Web site. In this chapter, you need to make a decision about the cereal-filling process, based on a *sample* of cereal boxes. You will learn different methods of sampling and about *sampling distributions* and how to use them to solve business problems.

### 7.1 TYPES OF SAMPLING METHODS

In Section 1.3, a *sample* was defined as the portion of a population that has been selected for analysis. Rather than selecting every item in the population, statistical sampling procedures focus on collecting a small representative group of the larger population. The results of the sample are then used to estimate characteristics of the entire population. There are three main reasons for selecting a sample:

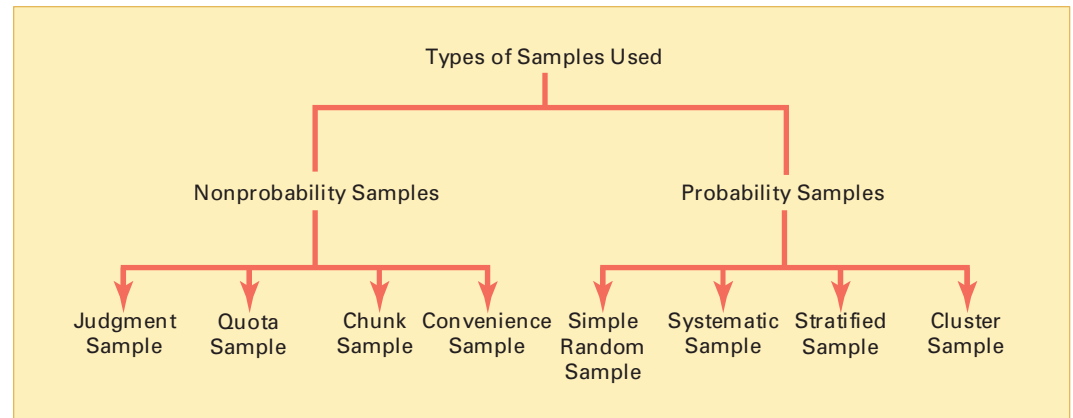
- Selecting a sample is less time-consuming than selecting every item in the population.
- Selecting a sample is less costly than selecting every item in the population.
- An analysis of a sample is less cumbersome and more practical than an analysis of the entire population.

The sampling process begins by defining the **frame**. The frame is a listing of items that make up the population. Frames are data sources such as population lists, directories, or maps. Samples are drawn from frames. Inaccurate or biased results can result if a frame excludes certain portions of the population. Using different frames to generate data can lead to opposite conclusions.

After you select a frame, you draw a sample from the frame. As illustrated in Figure 7.1, there are two kinds of samples: the nonprobability sample and the probability sample.

**FIGURE 7.1**

Types of samples



In a **nonprobability sample**, you select the items or individuals without knowing their probabilities of selection. Thus, the theory that has been developed for probability sampling cannot be applied to nonprobability samples. A common type of nonprobability sampling is **convenience sampling**. In convenience sampling, items are selected based only on the fact that they are easy, inexpensive, or convenient to sample. In many cases, participants are self-selected. For example, many companies conduct surveys by giving visitors to their Web site the opportunity to complete survey forms and submit them electronically. The responses to these surveys can provide large amounts of data quickly and inexpensively, but the sample consists of self-selected Web users (see p. 8). For many studies, only a nonprobability sample such as a judgment sample is available. In a **judgment sample**, you get the opinions of preselected experts in the subject matter. Some other common procedures of nonprobability sampling are quota sampling and chunk sampling. These are discussed in detail in specialized books on sampling methods (see reference 1).

Nonprobability samples can have certain advantages, such as convenience, speed, and low cost. However, their lack of accuracy due to selection bias and the fact that the results cannot be generalized more than offset these advantages. Therefore, you should use nonprobability sampling methods only for small-scale studies that precede larger investigations.

In a **probability sample**, you select the items based on known probabilities. Whenever possible, you should use probability sampling methods. Probability samples allow you to make unbiased inferences about the population of interest. In practice, it is often difficult or impossible to take a probability sample. However, you should work toward achieving a probability sample and acknowledge any potential biases that might exist. The four types of probability samples most commonly used are simple random, systematic, stratified, and cluster samples. These sampling methods vary in their cost, accuracy, and complexity.

### Simple Random Samples

In a **simple random sample**, every item from a frame has the same chance of selection as every other item. In addition, every sample of a fixed size has the same chance of selection as every other sample of that size. Simple random sampling is the most elementary random sampling technique. It forms the basis for the other random sampling techniques.

With simple random sampling, you use  $n$  to represent the sample size and  $N$  to represent the frame size. You number every item in the frame from 1 to  $N$ . The chance that you will select any particular member of the frame on the first selection is  $1/N$ .

You select samples with replacement or without replacement. **Sampling with replacement** means that after you select an item, you return it to the frame, where it has the same probability of being selected again. Imagine that you have a fishbowl containing  $N$  business cards.

On the first selection, you select the card for Judy Craven. You record pertinent information and replace the business card in the bowl. You then mix up the cards in the bowl and select the second card. On the second selection, Judy Craven has the same probability of being selected again,  $1/N$ . You repeat this process until you have selected the desired sample size,  $n$ . However, usually you do not want the same item to be selected again.

**Sampling without replacement** means that once you select an item, you cannot select it again. The chance that you will select any particular item in the frame—for example, the business card for Judy Craven—on the first draw is  $1/N$ . The chance that you will select any card not previously selected on the second draw is now 1 out of  $N - 1$ . This process continues until you have selected the desired sample of size  $n$ .

Regardless of whether you have sampled with or without replacement, “fishbowl” methods of sample selection have a major drawback—the ability to thoroughly mix the cards and randomly select the sample. As a result, fishbowl methods are not very useful. You need to use less cumbersome and more scientific methods of selection.

One such method uses a **table of random numbers** (see Table E.1) for selecting the sample. A table of random numbers consists of a series of digits listed in a randomly generated sequence (see reference 8). Because the numeric system uses 10 digits (0, 1, 2, . . . , 9), the chance that you will randomly generate any particular digit is equal to the probability of generating any other digit. This probability is 1 out of 10. Hence, if you generate a sequence of 800 digits, you would expect about 80 to be the digit 0, 80 to be the digit 1, and so on. In fact, those who use tables of random numbers usually test the generated digits for randomness prior to using them. Table E.1 has met all such criteria for randomness. Because every digit or sequence of digits in the table is random, the table can be read either horizontally or vertically. The margins of the table designate row numbers and column numbers. The digits themselves are grouped into sequences of five in order to make reading the table easier.

To use such a table instead of a fishbowl for selecting the sample, you first need to assign code numbers to the individual members of the frame. Then you generate the random sample by reading the table of random numbers and selecting those individuals from the frame whose assigned code numbers match the digits found in the table. You can better understand the process of sample selection by examining Example 7.1.

### EXAMPLE 7.1

#### SELECTING A SIMPLE RANDOM SAMPLE BY USING A TABLE OF RANDOM NUMBERS

A company wants to select a sample of 32 full-time workers from a population of 800 full-time employees in order to collect information on expenditures concerning a company-sponsored dental plan. How do you select a simple random sample?

**SOLUTION** The company decides to conduct an email survey. Assuming that not everyone will respond to the survey, you need to send more than 32 surveys to get the necessary 32 responses. Assuming that 8 out of 10 full-time workers will respond to such a survey (that is, a response rate of 80%), you decide to send 40 surveys.

The frame consists of a listing of the names and email addresses of all  $N = 800$  full-time employees taken from the company personnel files. Thus, the frame is an accurate and complete listing of the population. To select the random sample of 40 employees from this frame, you use a table of random numbers. Because the population size (800) is a three-digit number, each assigned code number must also be three digits so that every full-time worker has an equal chance of selection. You assign a code of 001 to the first full-time employee in the population listing, a code of 002 to the second full-time employee in the population listing, and so on, until a code of 800 is assigned to the  $N$ th full-time worker in the listing. Because  $N = 800$  is the largest possible coded value, you discard all three-digit code sequences greater than 800 (that is, 801 through 999 and 000).

To select the simple random sample, you choose an arbitrary starting point from the table of random numbers. One method you can use is to close your eyes and strike the table of random numbers with a pencil. Suppose you used this procedure and you selected row 06, column 05, of Table 7.1 (which is extracted from Table E.1) as the starting point. Although you can go in any direction, in this example, you read the table from left to right, in sequences of three digits, without skipping.

**TABLE 7.1**

Using a Table of Random Numbers

Row	Column								
	00000 12345	00001 67890	11111 12345	11112 67890	22222 12345	22223 67890	33333 12345	33334 67890	
01	49280	88924	35779	00283	81163	07275	89863	02348	
02	61870	41657	07468	08612	98083	97349	20775	45091	
03	43898	65923	25078	86129	78496	97653	91550	08078	
04	62993	93912	30454	84598	56095	20664	12872	64647	
05	33850	58555	51438	85507	71865	79488	76783	31708	
<b>Begin selection (row 06, column 5)</b>	06	97340	03364	88472	04334	63919	36394	11095	92470
	07	70543	29776	10087	10072	55980	64688	68239	20461
	08	89382	93809	00796	95945	34101	81277	66090	88872
	09	37818	72142	67140	50785	22380	16703	53362	44940
	10	60430	22834	14130	96593	23298	56203	92671	15925
	11	82975	66158	84731	19436	55790	69229	28661	13675
	12	39087	71938	40355	54324	08401	26299	49420	59208
	13	55700	24586	93247	32596	11865	63397	44251	43189
	14	14756	23997	78643	75912	83832	32768	18928	57070
	15	32166	53251	70654	92827	63491	04233	33825	69662
	16	23236	73751	31888	81718	06546	83246	47651	04877
	17	45794	26926	15130	82455	78305	55058	52551	47182
	18	09893	20505	14225	68514	46427	56788	96297	78822
	19	54382	74598	91499	14523	68479	27686	46162	83554
	20	94750	89923	37089	20048	80336	94598	26940	36858
	21	70297	34135	53140	33340	42050	82341	44104	82949
	22	85157	47954	32979	26575	57600	40881	12250	73742
	23	11100	02340	12860	74697	96644	89439	28707	25815
	24	36871	50775	30592	57143	17381	68856	25853	35041
	25	23913	48357	63308	16090	51690	54607	72407	55538

Source: Partially extracted from The Rand Corporation, A Million Random Digits with 100,000 Normal Deviates (Glencoe, IL: The Free Press, 1955) and displayed in Table E.1 in Appendix E.

The individual with code number 003 is the first full-time employee in the sample (row 06 and columns 05–07), the second individual has code number 364 (row 06 and columns 08–10), and the third individual has code number 884. Because the highest code for any employee is 800, you discard the number 884. Individuals with code numbers 720, 433, 463, 363, 109, 592, 470, and 705 are selected third through tenth, respectively.

You continue the selection process until you get the required sample size of 40 full-time employees. During the selection process, if any three-digit coded sequence repeats, you include the employee corresponding to that coded sequence again as part of the sample if you are sampling with replacement. You discard the repeating coded sequence if you are sampling without replacement.

## Systematic Samples

In a **systematic sample**, you partition the  $N$  items in the frame into  $n$  groups of  $k$  items, where

$$k = \frac{N}{n}$$

You round  $k$  to the nearest integer. To select a systematic sample, you choose the first item to be selected at random from the first  $k$  items in the frame. Then, you select the remaining  $n - 1$  items by taking every  $k$ th item thereafter from the entire frame.

If the frame consists of a listing of prenumbered checks, sales receipts, or invoices, a systematic sample is faster and easier to take than a simple random sample. A systematic sample is also a convenient mechanism for collecting data from telephone books, class rosters, and consecutive items coming off an assembly line.

To take a systematic sample of  $n = 40$  from the population of  $N = 800$  employees, you partition the frame of 800 into 40 groups, each of which contains 20 employees. You then select a random number from the first 20 individuals and include every twentieth individual after the first selection in the sample. For example, if the first random number you select is 008, your subsequent selections are 028, 048, 068, 088, 108, . . . , 768, and 788.

Although they are simpler to use, simple random sampling and systematic sampling are generally less efficient than other, more sophisticated, probability sampling methods. Even greater possibilities for selection bias and lack of representation of the population characteristics occur when using systematic samples than with simple random samples. If there is a pattern in the frame, you could have severe selection biases. To overcome the potential problem of disproportionate representation of specific groups in a sample, you can use either stratified sampling methods or cluster sampling methods.

## Stratified Samples

In a **stratified sample**, you first subdivide the  $N$  items in the frame into separate subpopulations, or **strata**. A strata is defined by some common characteristic, such as gender or year in school. You select a simple random sample within each of the strata and combine the results from the separate simple random samples. Stratified sampling is more efficient than either simple random sampling or systematic sampling because you are ensured of the representation of items across the entire population. The homogeneity of items within each stratum provides greater precision in the estimates of underlying population parameters.

### EXAMPLE 7.2

#### SELECTING A STRATIFIED SAMPLE

A company wants to select a sample of 32 full-time workers from a population of 800 full-time employees in order to estimate expenditures from a company-sponsored dental plan. Of the full-time employees, 25% are managers and 75% are nonmanagerial workers. How do you select the stratified sample in order for the sample to represent the correct percentage of managers and nonmanagerial workers?

**SOLUTION** If you assume an 80% response rate, you need to send 40 surveys to get the necessary 32 responses. The frame consists of a listing of the names and email addresses of all  $N = 800$  full-time employees included in the company personnel files. Because 25% of the full-time employees are managers, you first separate the population frame into two strata: a subpopulation listing of all 200 managerial-level personnel and a separate subpopulation listing of all 600 full-time nonmanagerial workers. Because the first stratum consists of a listing of 200 managers, you assign three-digit code numbers from 001 to 200. Because the second stra-



tum contains a listing of 600 nonmanagerial workers, you assign three-digit code numbers from 001 to 600.

To collect a stratified sample proportional to the sizes of the strata, you select 25% of the overall sample from the first stratum and 75% of the overall sample from the second stratum. You take two separate simple random samples, each of which is based on a distinct random starting point from a table of random numbers (Table E.1). In the first sample, you select 10 managers from the listing of 200 in the first stratum, and in the second sample, you select 30 nonmanagerial workers from the listing of 600 in the second stratum. You then combine the results to reflect the composition of the entire company.

## Cluster Samples

In a **cluster sample**, you divide the  $N$  items in the frame into several clusters so that each cluster is representative of the entire population. **Clusters** are naturally occurring designations, such as counties, election districts, city blocks, households, or sales territories. You then take a random sample of one or more clusters and study all items in each selected cluster. If clusters are large, a probability-based sample taken from a single cluster is all that is needed.

Cluster sampling is often more cost-effective than simple random sampling, particularly if the population is spread over a wide geographic region. However, cluster sampling often requires a larger sample size to produce results as precise as those from simple random sampling or stratified sampling. A detailed discussion of systematic sampling, stratified sampling, and cluster sampling procedures can be found in reference 1.

## PROBLEMS FOR SECTION 7.1

### Learning the Basics

- PH Grade ASSIST** **7.1** For a population containing  $N = 902$  individuals, what code number would you assign for
- the first person on the list?
  - the fortieth person on the list?
  - the last person on the list?

**7.2** For a population of  $N = 902$ , verify that by starting in row 05, column 1 of the table of random numbers (Table E.1), you need only six rows to select a sample of  $n = 60$  without replacement.

- PH Grade ASSIST** **7.3** Given a population of  $N = 93$ , starting in row 29 of the table of random numbers (Table E.1), and reading across the row, select a sample of  $n = 15$
- without replacement.
  - with replacement.

### Applying the Concepts

**7.4** For a study that consists of personal interviews with participants (rather than mail or phone surveys), explain why simple random sampling might be less practical than some other sampling methods.

- PH Grade ASSIST** **7.5** You want to select a random sample of  $n = 1$  from a population of three items (which are called  $A$ ,  $B$ , and  $C$ ). The rule for selecting the sample is: Flip a coin; if it is heads, pick item  $A$ ; if it is tails, flip the coin again; this time, if it is heads, choose  $B$ ; if it is tails, choose  $C$ . Explain why this is a probability sample but not a simple random sample.

- SELF Test** **7.6** A population has four members (called  $A$ ,  $B$ ,  $C$ , and  $D$ ). You would like to select a random sample of  $n = 2$ , which you decide to do in the following way: Flip a coin; if it is heads, the sample will be items  $A$  and  $B$ ; if it is tails, the sample will be items  $C$  and  $D$ . Although this is a random sample, it is not a simple random sample. Explain why. (If you did Problem 7.5, compare the procedure described there with the procedure described in this problem.)

**7.7** The registrar of a college with a population of  $N = 4,000$  full-time students is asked by the president to conduct a survey to measure satisfaction with the quality of life on campus. The table at the top of page 258 contains a breakdown of the 4,000 registered full-time students, by gender and class designation:

Gender	Class Designation				Total
	Fr.	So.	Jr.	Sr.	
Female	700	520	500	480	2,200
Male	560	460	400	380	1,800
Total	1,260	980	900	860	4,000

The registrar intends to take a probability sample of  $n = 200$  students and project the results from the sample to the entire population of full-time students.

- If the frame available from the registrar's files is an alphabetical listing of the names of all  $N = 4,000$  registered full-time students, what type of sample could you take? Discuss.
- What is the advantage of selecting a simple random sample in (a)?
- What is the advantage of selecting a systematic sample in (a)?
- If the frame available from the registrar's files is a listing of the names of all  $N = 4,000$  registered full-time students compiled from eight separate alphabetical lists, based on the gender and class designation breakdowns shown in the class designation table, what type of sample should you take? Discuss.
- Suppose that each of the  $N = 4,000$  registered full-time students lived in one of the 20 campus dormitories. Each dormitory contains four floors, with 50 beds per floor,

and therefore accommodates 200 students. It is college policy to fully integrate students by gender and class designation on each floor of each dormitory. If the registrar is able to compile a frame through a listing of all student occupants on each floor within each dormitory, what type of sample should you take? Discuss.

**7.8** Prenumbered sales invoices are kept in a sales journal. The invoices are numbered from 0001 to 5,000.

- Beginning in row 16, column 1, and proceeding horizontally in Table E.1, select a simple random sample of 50 invoice numbers.
- Select a systematic sample of 50 invoice numbers. Use the random numbers in row 20, columns 5–7, as the starting point for your selection.
- Are the invoices selected in (a) the same as those selected in (b)? Why or why not?

**PH Grade ASSIST** **7.9** Suppose that 5,000 sales invoices are separated into four strata. Stratum 1 contains 50 invoices, stratum 2 contains 500 invoices, stratum 3 contains 1,000 invoices, and stratum 4 contains 3,450 invoices. A sample of 500 sales invoices is needed.

- What type of sampling should you do? Why?
- Explain how you would carry out the sampling according to the method stated in (a).
- Why is the sampling in (a) not simple random sampling?

## 7.2 EVALUATING SURVEY WORTHINESS

Surveys are often used to collect samples. Nearly every day, you read or hear about survey or opinion poll results in newspapers, on the Internet, or on radio or television. To identify surveys that lack objectivity or credibility, you must critically evaluate what you read and hear by examining the worthiness of the survey. First, you must evaluate the purpose of the survey, why it was conducted, and for whom it was conducted. An opinion poll or a survey conducted to satisfy curiosity is mainly for entertainment. Its result is an end in itself rather than a means to an end. You should be skeptical of such a survey because the result should not be put to further use.

The second step in evaluating the worthiness of a survey is to determine whether it was based on a probability or nonprobability sample (as discussed in Section 7.1). You need to remember that the only way to make correct statistical inferences from a sample to a population is through the use of a probability sample. Surveys that use nonprobability sampling methods are subject to serious, perhaps unintentional, biases that may render the results meaningless, as illustrated in the following example from the 1948 U.S. presidential election.

In 1948, major pollsters predicted the outcome of the U.S. presidential election between Harry S. Truman, the incumbent president, and Thomas E. Dewey, then governor of New York, as going to Dewey. The *Chicago Tribune* was so confident of the polls' predictions that it printed its early edition based on the predictions rather than waiting for the ballots to be counted.

An embarrassed newspaper and the pollsters it had relied on had a lot of explaining to do. Why were the pollsters so wrong? Intent on discovering the source of the error, the pollsters found that their use of a nonprobability sampling method was the culprit (see reference 7). As a result, polling organizations adopted probability sampling methods for future elections.

## Survey Error

Even when surveys use random probability sampling methods, they are subject to potential errors. There are four types of survey errors:

- Coverage error
- Nonresponse error
- Sampling error
- Measurement error

Good survey research design attempts to reduce or minimize these various types of survey error, often at considerable cost.

**Coverage Error** The key to proper sample selection is an adequate frame. Remember, a frame is an up-to-date list of all the items from which you will select the sample. **Coverage error** occurs if certain groups of items are excluded from this frame so that they have no chance of being selected in the sample. Coverage error results in a **selection bias**. If the frame is inadequate because certain groups of items in the population were not properly included, any random probability sample selected will provide an estimate of the characteristics of the frame, not the *actual* population.

**Nonresponse Error** Not everyone is willing to respond to a survey. In fact, research has shown that individuals in the upper and lower economic classes tend to respond less frequently to surveys than do people in the middle class. **Nonresponse error** arises from the failure to collect data on all items in the sample and results in a **nonresponse bias**. Because you cannot always assume that persons who do not respond to surveys are similar to those who do, you need to follow up on the nonresponses after a specified period of time. You should make several attempts to convince such individuals to complete the survey. The follow-up responses are then compared to the initial responses in order to make valid inferences from the survey (reference 1).

The mode of response you use affects the rate of response. The personal interview and the telephone interview usually produce a higher response rate than does the mail survey—but at a higher cost. The following is a famous example of coverage error and nonresponse error.

In 1936, the magazine *Literary Digest* predicted that Governor Alf Landon of Kansas would receive 57% of the votes in the U.S. presidential election and overwhelmingly defeat President Franklin D. Roosevelt's bid for a second term. However, Landon was soundly defeated when he received only 38% of the vote. Such a large error in a poll conducted by a well-known source had never occurred before. As a result, the prediction devastated the magazine's credibility with the public, eventually causing it to go bankrupt. *Literary Digest* thought it had done everything right. It had based its prediction on a huge sample size, 2.4 million respondents, out of a survey sent to 10 million registered voters. What went wrong? There are two answers: selection bias and nonresponse bias.

To understand the role of selection bias, you need some historical background. In 1936, the United States was still suffering from the Great Depression. Not accounting for this, *Literary Digest* compiled its frame from such sources as telephone books, club membership lists, magazine subscriptions, and automobile registrations (reference 2). Inadvertently, it chose a frame primarily composed of the rich and excluded the majority of the voting population, who, during the Great Depression, could not afford telephones, club memberships, magazine subscriptions, and automobiles. Thus, the 57% estimate for the Landon vote may have been very close to the frame but certainly not the total U.S. population.

Nonresponse error produced a possible bias when the huge sample of 10 million registered voters produced only 2.4 million responses. A response rate of only 24% is far too low to yield accurate estimates of the population parameters without some way of ensuring that the 7.6 million individual nonrespondents have similar opinions. However, the problem of nonresponse bias was secondary to the problem of selection bias. Even if all 10 million registered voters in the sample had responded, this would not have compensated for the fact that the composition of the frame differed substantially from that of the actual voting population.

**Sampling Error** A sample is selected because it is simpler, less costly, and more efficient. However, chance dictates which individuals or items will or will not be included in the sample. **Sampling error** reflects the variation, or “chance differences,” from sample to sample, based on the probability of particular individuals or items being selected in the particular samples.

When you read about the results of surveys or polls in newspapers or magazines, there is often a statement regarding a margin of error, such as “the results of this poll are expected to be within  $\pm 4$  percentage points of the actual value.” This margin of error is the sampling error. You can reduce sampling error by taking larger sample sizes, although this also increases the cost of conducting the survey.

**Measurement Error** In the practice of good survey research, you design a questionnaire with the intention of gathering meaningful information. But you have a dilemma here: Getting meaningful measurements is often easier said than done. Consider the following proverb:

A person with one watch always knows what time it is;  
 A person with two watches always searches to identify the correct one;  
 A person with ten watches is always reminded of the difficulty in measuring time.

Unfortunately, the process of measurement is often governed by what is convenient, not what is needed. The measurements you get are often only a proxy for the ones you really desire. Much attention has been given to measurement error that occurs because of a weakness in question wording (reference 3). A question should be clear, not ambiguous. Furthermore, in order to avoid *leading questions*, you need to present them in a neutral manner.

Three sources of **measurement error** are ambiguous wording of questions, the halo effect, and respondent error. As an example of ambiguous wording, in November 1993, the U.S. Department of Labor reported that the unemployment rate in the United States had been underestimated for more than a decade because of poor questionnaire wording in the Current Population Survey. In particular, the wording had led to a significant undercount of women in the labor force. Because unemployment rates are tied to benefit programs such as state unemployment compensation, survey researchers had to rectify the situation by adjusting the questionnaire wording.

The “halo effect” occurs when the respondent feels obligated to please the interviewer. Proper interviewer training can minimize the halo effect.

Respondent error occurs as a result of an overzealous or underzealous effort by the respondent. You can minimize this error in two ways: (1) by carefully scrutinizing the data and calling back those individuals whose responses seem unusual and (2) by establishing a program of random callbacks in order to determine the reliability of the responses.

## Ethical Issues

Ethical considerations arise with respect to the four types of potential errors that can occur when designing surveys that use probability samples: coverage error, nonresponse error, sampling error, and measurement error. Coverage error can result in selection bias and becomes an ethical issue if particular groups or individuals are *purposely* excluded from the frame so that the survey results are more favorable to the survey’s sponsor. Nonresponse error can lead to nonresponse bias and becomes an ethical issue if the sponsor knowingly designs the survey so that particular groups or individuals are less likely than others to respond. Sampling error becomes an ethical issue if the findings are purposely presented without reference to sample size and margin of error so that the sponsor can promote a viewpoint that might otherwise be truly insignificant. Measurement error becomes an ethical issue in one of three ways: (1) a survey sponsor chooses leading questions that guide the responses in a particular direction; (2) an interviewer, through mannerisms and tone, purposely creates a halo effect or otherwise guides the responses in a particular direction; or (3) a respondent willfully provides false information.

Ethical issues also arise when the results of nonprobability samples are used to form conclusions about the entire population. When you use a nonprobability sampling method, you need to explain the sampling procedures and state that the results cannot be generalized beyond the sample.

## PROBLEMS FOR SECTION 7.2

### Applying the Concepts

**7.10** “A survey indicates that the vast majority of college students own their own personal computers.” What information would you want to know before you accepted the results of this survey?

**7.11** A simple random sample of  $n = 300$  full-time employees is selected from a company list containing the names of all  $N = 5,000$  full-time employees in order to evaluate job satisfaction.

- Give an example of possible coverage error.
- Give an example of possible nonresponse error.
- Give an example of possible sampling error.
- Give an example of possible measurement error.

**7.12** Business professor Thomas Callarman traveled to China more than a dozen times from 2000 to 2005. He warns people about believing everything they read about surveys conducted in China and gives two specific reasons. Callarman stated, “First, things are changing so rapidly that what you hear today may not be true tomorrow. Second, the people who answer the surveys may tell you what they think you want to hear, rather than what they really believe” (T. E. Callarman, “Some Thoughts on China,” *Decision Line*, March, 2006, pp. 1, 43–44).

- List the four types (or categories) of survey error discussed in this section.
- Which categories best describe the types of survey error discussed by Professor Callarman?

**7.13** The gourmet foods industry is expected to exceed \$62 billion in sales by the year 2009. A survey conducted by Packaged Facts indicates that one-fifth of American adults consider themselves “gourmet consumers” (“Galloping Gourmet,” *The Progressive Grocer*, January 1, 2006, pp. 80–81). What additional information would you want to know before you accepted the results of the survey?

**7.14** Only 10% of Americans rated their financial situation as “excellent,” according to a Gallup Poll taken April 10–13, 2006. However, 41% rated their financial situation as “good,” while 37% said “only fair” and 12% “poor” (J. M. Jones, “Americans More Worried About Meeting Basic Financial Needs,” *The Gallup Poll*, [galluppoll.com](http://galluppoll.com), April 25, 2006). What additional information would you want to know before you accepted the results of the survey?

**7.15** Researchers studied repeat purchases from two online grocers. Valid responses from 1,150 customers indicated that 28.7% placed no further orders in the following 12 months, 35.4% placed 1–10 orders, and 35.8% placed 11 or more orders (K. K. Boyer and G. T. M. Hult, “Customer Behavior in an Online Ordering Application: A Decision Scoring Model,” *Decision Sciences*, December, 2005, pp. 569–598). What additional information would you want to know before you accepted the results of this study?

**7.16** A study investigating the effects of CEO succession on the stock performance of large publicly held corporations also investigated the demographics of the newly announced CEOs. The mean and standard deviation of the new CEO’s age were 53.3 and 5.97, respectively. The mean and standard deviation of the number of years the new CEO had been with the firm were 20.1 and 12.6, respectively. 93.6% of the new CEOs held college degrees, 30.4% held MBAs, and 3.2% held doctorates (J. C. Rhim, J. V. Peluchette, and I. Song, “Stock Market Reactions and Firm Performance Surrounding CEO Succession: Antecedents of Succession and Successor Origin,” *Mid-American Journal of Business*, Spring 2006, pp. 21–30). What additional information would you want to know before you accepted the results of this study?

## 7.3 SAMPLING DISTRIBUTIONS

In many applications, you want to make statistical inferences that use statistics calculated from samples to estimate the values of population parameters. In the next two sections, you will learn about how the sample mean (the statistic) is used to estimate a population mean (a parameter) and how the sample proportion (the statistic) is used to estimate the population proportion (a parameter). Your main concern when making a statistical inference is drawing conclusions about a population, *not* about a sample. For example, a political pollster is interested in the sample results only as a way of estimating the actual proportion of the votes that each candidate will receive from the population of voters. Likewise, as plant operations manager for Oxford Cereals, you are only interested in using the sample mean calculated from a sample of cereal boxes for estimating the mean weight contained in a population of boxes.

In practice, you select a single random sample of a predetermined size from the population. The items included in the sample are determined through the use of a random number

generator, such as a table of random numbers (see Section 7.1 and Table E.1) or by using Microsoft Excel (see pages 281–282).

Hypothetically, to use the sample statistic to estimate the population parameter, you should examine *every* possible sample of a given size that could occur. A **sampling distribution** is the distribution of the results if you actually selected all possible samples.

## 7.4 SAMPLING DISTRIBUTION OF THE MEAN

In Chapter 3, several measures of central tendency, including the mean, median, and mode, were discussed. Undoubtedly, the mean is the most widely used measure of central tendency. The sample mean is often used to estimate the population mean. The **sampling distribution of the mean** is the distribution of all possible sample means if you select all possible samples of a certain size.

### The Unbiased Property of the Sample Mean

The sample mean is **unbiased** because the mean of all the possible sample means (of a given sample size,  $n$ ) is equal to the population mean,  $\mu$ . A simple example concerning a population of four administrative assistants demonstrates this property. Each assistant is asked to type the same page of a manuscript. Table 7.2 presents the number of errors. This population distribution is shown in Figure 7.2.

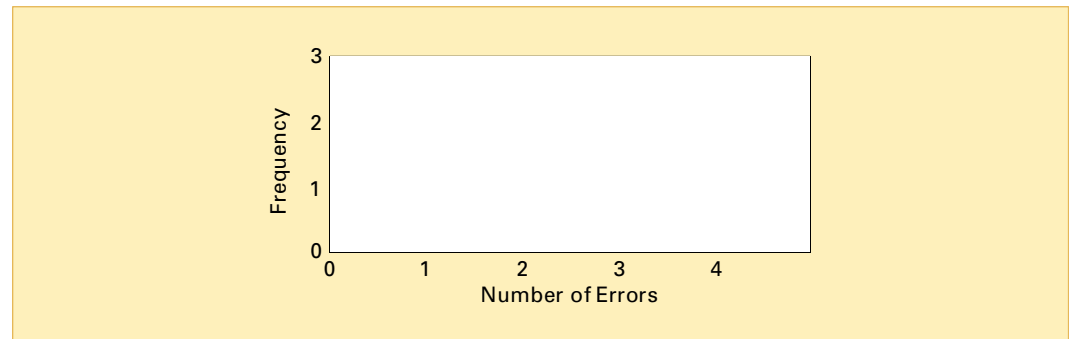
**TABLE 7.2**

Number of Errors Made by Each of Four Administrative Assistants

Administrative Assistant	Number of Errors
Ann	$X_1 = 3$
Bob	$X_2 = 2$
Carla	$X_3 = 1$
Dave	$X_4 = 4$

**FIGURE 7.2**

Number of errors made by a population of four administrative assistants



When you have the data from a population, you compute the mean by using Equation (7.1).

### POPULATION MEAN

The population mean is the sum of the values in the population divided by the population size,  $N$ .

$$\mu = \frac{\sum_{i=1}^N X_i}{N} \quad (7.1)$$

You compute the population standard deviation,  $\sigma$ , using Equation (7.2):

### POPULATION STANDARD DEVIATION

$$\sigma = \sqrt{\frac{\sum_{i=1}^N (X_i - \mu)^2}{N}} \quad (7.2)$$

Thus, for the data of Table 7.1,

$$\mu = \frac{3 + 2 + 1 + 4}{4} = 2.5 \text{ errors}$$

and

$$\sigma = \sqrt{\frac{(3 - 2.5)^2 + (2 - 2.5)^2 + (1 - 2.5)^2 + (4 - 2.5)^2}{4}} = 1.12 \text{ errors}$$

If you select samples of two administrative assistants *with* replacement from this population, there are 16 possible samples ( $N^n = 4^2 = 16$ ). Table 7.3 lists the 16 possible sample outcomes. If you average all 16 of these sample means, the mean of these values,  $\mu_{\bar{X}}$ , is equal to 2.5, which is also the mean of the population  $\mu$ .

**TABLE 7.3**

All 16 Samples of  $n = 2$  Administrative Assistants from a Population of  $N = 4$  Administrative Assistants When Sampling *with* Replacement

Sample	Administrative Assistants	Sample Outcomes	Sample Mean
1	Ann, Ann	3, 3	$\bar{X}_1 = 3$
2	Ann, Bob	3, 2	$\bar{X}_2 = 2.5$
3	Ann, Carla	3, 1	$\bar{X}_3 = 2$
4	Ann, Dave	3, 4	$\bar{X}_4 = 3.5$
5	Bob, Ann	2, 3	$\bar{X}_5 = 2.5$
6	Bob, Bob	2, 2	$\bar{X}_6 = 2$
7	Bob, Carla	2, 1	$\bar{X}_7 = 1.5$
8	Bob, Dave	2, 4	$\bar{X}_8 = 3$
9	Carla, Ann	1, 3	$\bar{X}_9 = 2$
10	Carla, Bob	1, 2	$\bar{X}_{10} = 1.5$
11	Carla, Carla	1, 1	$\bar{X}_{11} = 1$
12	Carla, Dave	1, 4	$\bar{X}_{12} = 2.5$
13	Dave, Ann	4, 3	$\bar{X}_{13} = 3.5$
14	Dave, Bob	4, 2	$\bar{X}_{14} = 3$
15	Dave, Carla	4, 1	$\bar{X}_{15} = 2.5$
16	Dave, Dave	4, 4	$\bar{X}_{16} = 4$
			$\mu_{\bar{X}} = 2.5$

Because the mean of the 16 sample means is equal to the population mean, the sample mean is an unbiased estimator of the population mean. Therefore, although you do not know how close the sample mean of any particular sample selected comes to the population mean, you are at least assured that the mean of all the possible sample means that could have been selected is equal to the population mean.

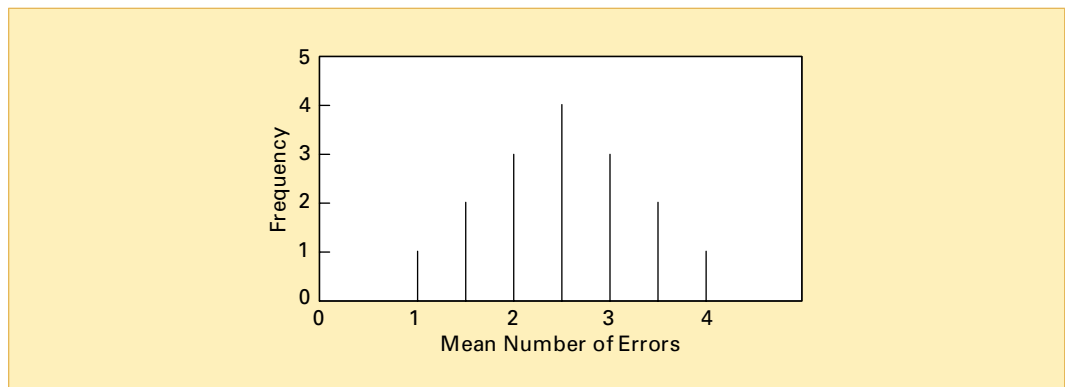
## Standard Error of the Mean

Figure 7.3 illustrates the variation in the sample means when selecting all 16 possible samples. In this small example, although the sample means vary from sample to sample, depending on which two administrative assistants are selected, the sample means do not vary as much as the individual values in the population. That the sample means are less variable than the individual values in the population follows directly from the fact that each sample mean averages together all the values in the sample. A population consists of individual outcomes that can take on a wide range of values, from extremely small to extremely large. However, if a sample contains an extreme value, although this value will have an effect on the sample mean, the effect is reduced because the value is averaged with all the other values in the sample. As the sample size increases, the effect of a single extreme value becomes smaller because it is averaged with more values.

**FIGURE 7.3**

Sampling distribution of the mean, based on all possible samples containing two administrative assistants

Source: Data are from Table 7.3.



The value of the standard deviation of all possible sample means, called the **standard error of the mean**, expresses how the sample means vary from sample to sample. Equation (7.3) defines the standard error of the mean when sampling *with* replacement or *without* replacement (see page 254) from large or infinite populations.

### STANDARD ERROR OF THE MEAN

The standard error of the mean,  $\sigma_{\bar{X}}$ , is equal to the standard deviation in the population,  $\sigma$ , divided by the square root of the sample size,  $n$ .

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} \quad (7.3)$$

Therefore, as the sample size increases, the standard error of the mean decreases by a factor equal to the square root of the sample size.

You can also use Equation (7.3) as an approximation of the standard error of the mean when the sample is selected without replacement if the sample contains less than 5% of the entire population. Example 7.3 computes the standard error of the mean for such a situation. (See the **section 7.6.pdf** file on the Student CD-ROM that accompanies this book for the case in which more than 5% of the population is contained in a sample selected without replacement.)



**EXAMPLE 7.3****COMPUTING THE STANDARD ERROR OF THE MEAN**

Returning to the cereal-filling process described in the Using Statistics scenario on page 252, if you randomly select a sample of 25 boxes without replacement from the thousands of boxes filled during a shift, the sample contains far less than 5% of the population. Given that the standard deviation of the cereal-filling process is 15 grams, compute the standard error of the mean.

**SOLUTION** Using Equation (7.3) with  $n = 25$  and  $\sigma = 15$ , the standard error of the mean is

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{15}{\sqrt{25}} = \frac{15}{5} = 3$$

The variation in the sample means for samples of  $n = 25$  is much less than the variation in the individual boxes of cereal (that is,  $\sigma_{\bar{X}} = 3$  while  $\sigma = 15$ ).

**Sampling from Normally Distributed Populations**

Now that the concept of a sampling distribution has been introduced and the standard error of the mean has been defined, what distribution will the sample mean,  $\bar{X}$ , follow? If you are sampling from a population that is normally distributed with mean,  $\mu$ , and standard deviation,  $\sigma$ , regardless of the sample size,  $n$ , the sampling distribution of the mean is normally distributed, with mean,  $\mu_{\bar{X}} = \mu$ , and standard error of the mean,  $\sigma_{\bar{X}}$ .

In the simplest case, if you take samples of size  $n = 1$ , each possible sample mean is a single value from the population because

$$\bar{X} = \frac{\sum_{i=1}^n X_i}{n} = \frac{X_i}{1} = X_i$$

Therefore, if the population is normally distributed, with mean,  $\mu$ , and standard deviation,  $\sigma$ , the sampling distribution of  $\bar{X}$  for samples of  $n = 1$  must also follow the normal distribution, with mean  $\mu_{\bar{X}} = \mu$  and standard error of the mean  $\sigma_{\bar{X}} = \sigma/\sqrt{1} = \sigma$ . In addition, as the sample size increases, the sampling distribution of the mean still follows a normal distribution, with  $\mu_{\bar{X}} = \mu$ , but the standard error of the mean decreases, so that a larger proportion of sample means are closer to the population mean. Figure 7.4 on page 266 illustrates this reduction in variability in which 500 samples of 1, 2, 4, 8, 16, and 32 were randomly selected from a normally distributed population. From the polygons in Figure 7.4, you can see that, although the sampling distribution of the mean is approximately<sup>1</sup> normal for each sample size, the sample means are distributed more tightly around the population mean as the sample size increases.

To further examine the concept of the sampling distribution of the mean, consider the Using Statistics scenario described on page 252. The packaging equipment that is filling 368-gram boxes of cereal is set so that the amount of cereal in a box is normally distributed, with a mean of 368 grams. From past experience, you know the population standard deviation for this filling process is 15 grams.

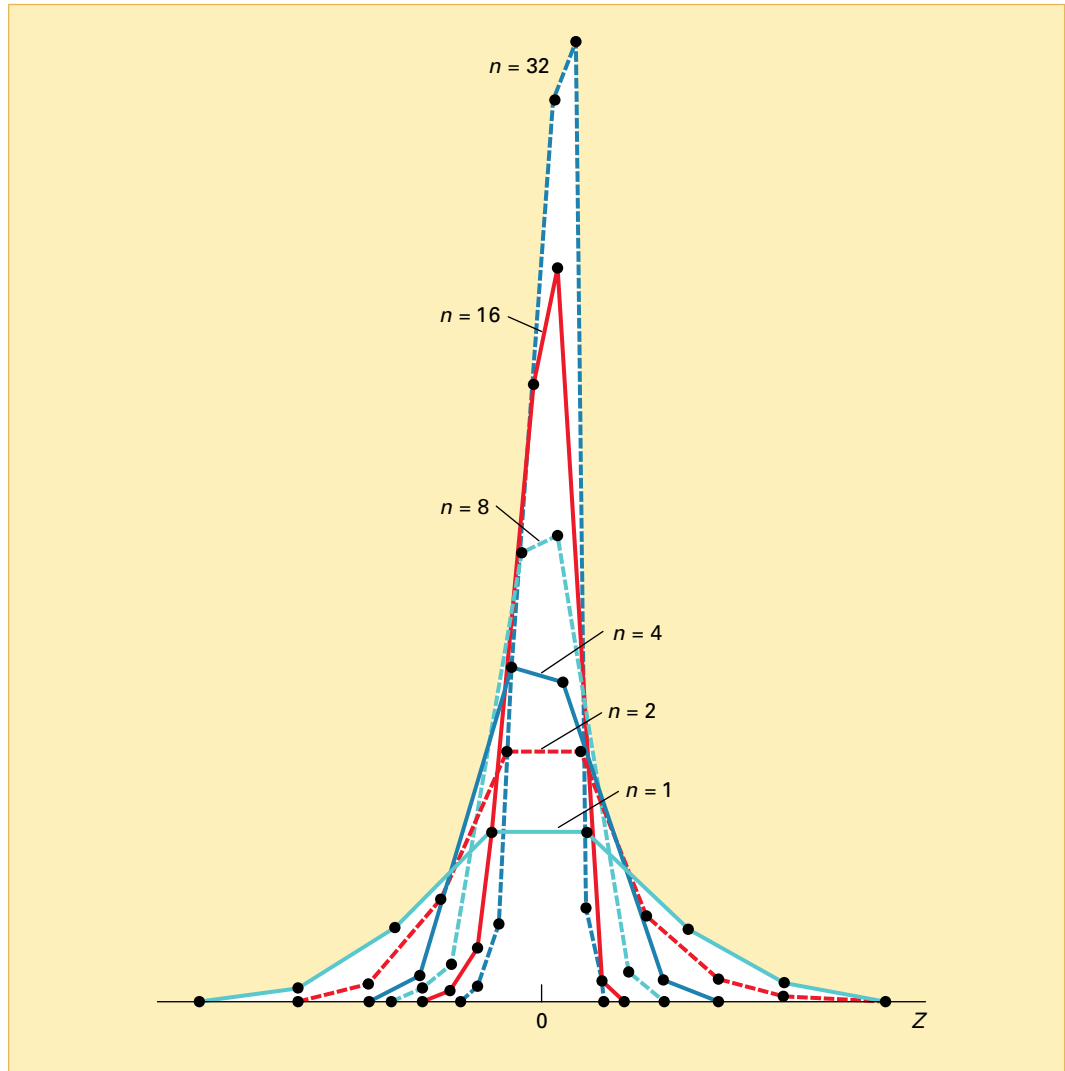
If you randomly select a sample of 25 boxes from the many thousands that are filled in a day and the mean weight is computed for this sample, what type of result could you expect? For example, do you think that the sample mean could be 368 grams? 200 grams? 365 grams?

The sample acts as a miniature representation of the population, so if the values in the population are normally distributed, the values in the sample should be approximately normally distributed. Thus, if the population mean is 368 grams, the sample mean has a good chance of being close to 368 grams.

<sup>1</sup>Remember that “only” 500 samples out of an infinite number of samples have been selected, so that the sampling distributions shown are only approximations of the true distributions.

**FIGURE 7.4**

Sampling distributions of the mean from 500 samples of sizes  $n = 1, 2, 4, 8, 16,$  and  $32$  selected from a normal population



How can you determine the probability that the sample of 25 boxes will have a mean below 365 grams? From the normal distribution (Section 6.2), you know that you can find the area below any value  $X$  by converting to standardized  $Z$  values:

$$Z = \frac{X - \mu}{\sigma}$$

In the examples in Section 6.2, you studied how any single value,  $X$ , differs from the mean. Now, in this example, the value involved is a sample mean,  $\bar{X}$ , and you want to determine the likelihood that a sample mean is below 365. In Equation (7.4), to find the  $Z$  value, you substitute  $\bar{X}$  for  $X$ ,  $\mu_{\bar{X}}$  for  $\mu$ , and  $\sigma_{\bar{X}}$  for  $\sigma$ .

#### FINDING $Z$ FOR THE SAMPLING DISTRIBUTION OF THE MEAN

The  $Z$  value is equal to the difference between the sample mean,  $\bar{X}$ , and the population mean,  $\mu$ , divided by the standard error of the mean,  $\sigma_{\bar{X}}$ .

$$Z = \frac{\bar{X} - \mu_{\bar{X}}}{\sigma_{\bar{X}}} = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} \quad (7.4)$$

To find the area below 365 grams, from Equation (7.4),

$$Z = \frac{\bar{X} - \mu_{\bar{X}}}{\sigma_{\bar{X}}} = \frac{365 - 368}{\frac{15}{\sqrt{25}}} = \frac{-3}{3} = -1.00$$

The area corresponding to  $Z = -1.00$  in Table E.2 is 0.1587. Therefore, 15.87% of all the possible samples of 25 boxes have a sample mean below 365 grams.

The preceding statement is not the same as saying that a certain percentage of *individual* boxes will have less than 365 grams of cereal. You compute that percentage as follows:

$$Z = \frac{X - \mu}{\sigma} = \frac{365 - 368}{15} = \frac{-3}{15} = -0.20$$

The area corresponding to  $Z = -0.20$  in Table E.2 is 0.4207. Therefore, 42.07% of the *individual* boxes are expected to contain less than 365 grams. Comparing these results, you see that many more *individual boxes* than *sample means* are below 365 grams. This result is explained by the fact that each sample consists of 25 different values, some small and some large. The averaging process dilutes the importance of any individual value, particularly when the sample size is large. Thus, the chance that the sample mean of 25 boxes is far away from the population mean is less than the chance that a *single* box is far away.

Examples 7.4 and 7.5 show how these results are affected by using different sample sizes.

#### EXAMPLE 7.4

#### THE EFFECT OF SAMPLE SIZE $n$ ON THE COMPUTATION OF $\sigma_{\bar{X}}$

How is the standard error of the mean affected by increasing the sample size from 25 to 100 boxes?

**SOLUTION** If  $n = 100$  boxes, then using Equation (7.3) on page 264:

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{15}{\sqrt{100}} = \frac{15}{10} = 1.5$$

The fourfold increase in the sample size from 25 to 100 reduces the standard error of the mean by half—from 3 grams to 1.5 grams. This demonstrates that taking a larger sample results in less variability in the sample means from sample to sample.

#### EXAMPLE 7.5

#### THE EFFECT OF SAMPLE SIZE $n$ ON THE CLUSTERING OF MEANS IN THE SAMPLING DISTRIBUTION

If you select a sample of 100 boxes, what is the probability that the sample mean is below 365 grams?

**SOLUTION** Using Equation (7.4) on page 266,

$$Z = \frac{\bar{X} - \mu_{\bar{X}}}{\sigma_{\bar{X}}} = \frac{365 - 368}{\frac{15}{\sqrt{100}}} = \frac{-3}{1.5} = -2.00$$

From Table E.2, the area less than  $Z = -2.00$  is 0.0228. Therefore, 2.28% of the samples of 100 boxes have means below 365 grams, as compared with 15.87% for samples of 25 boxes.

Sometimes you need to find the interval that contains a fixed proportion of the sample means. You need to determine a distance below and above the population mean containing a specific area of the normal curve. From Equation (7.4) on page 266,

$$Z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

Solving for  $\bar{X}$  results in Equation (7.5).

#### FINDING $\bar{X}$ FOR THE SAMPLING DISTRIBUTION OF THE MEAN

$$\bar{X} = \mu + Z \frac{\sigma}{\sqrt{n}} \quad (7.5)$$

Example 7.6 illustrates the use of Equation (7.5).

#### EXAMPLE 7.6

#### DETERMINING THE INTERVAL THAT INCLUDES A FIXED PROPORTION OF THE SAMPLE MEANS

In the cereal-fill example, find an interval symmetrically distributed around the population mean that will include 95% of the sample means based on samples of 25 boxes.

**SOLUTION** If 95% of the sample means are in the interval, then 5% are outside the interval. Divide the 5% into two equal parts of 2.5%. The value of  $Z$  in Table E.2 corresponding to an area of 0.0250 in the lower tail of the normal curve is  $-1.96$ , and the value of  $Z$  corresponding to a cumulative area of 0.975 (that is, 0.025 in the upper tail of the normal curve) is  $+1.96$ . The lower value of  $\bar{X}$  (called  $\bar{X}_L$ ) and the upper value of  $\bar{X}$  (called  $\bar{X}_U$ ) are found by using Equation (7.5):

$$\bar{X}_L = 368 + (-1.96) \frac{15}{\sqrt{25}} = 368 - 5.88 = 362.12$$

$$\bar{X}_U = 368 + (1.96) \frac{15}{\sqrt{25}} = 368 + 5.88 = 373.88$$

Therefore, 95% of all sample means based on samples of 25 boxes are between 362.12 and 373.88 grams.

### Sampling from Non-Normally Distributed Populations—The Central Limit Theorem

Thus far in this section, only the sampling distribution of the mean for a normally distributed population has been considered. However, in many instances, either you know that the population is not normally distributed or it is unrealistic to assume that the population is normally distributed. An important theorem in statistics, the Central Limit Theorem, deals with this situation.

#### THE CENTRAL LIMIT THEOREM

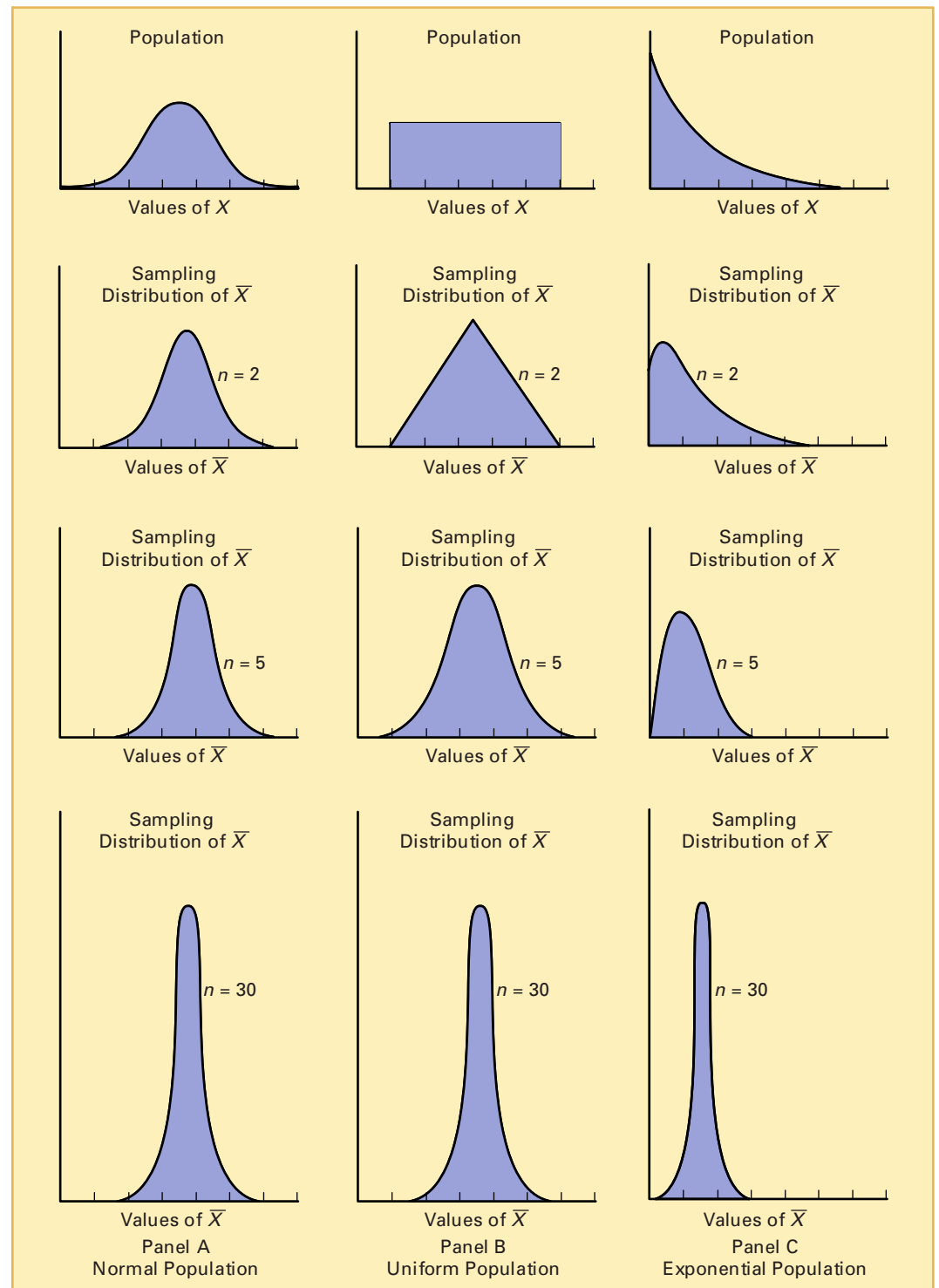
The **Central Limit Theorem** states that as the sample size (that is, the number of values in each sample) gets *large enough*, the sampling distribution of the mean is approximately normally distributed. This is true regardless of the shape of the distribution of the individual values in the population.

What sample size is large enough? A great deal of statistical research has gone into this issue. As a general rule, statisticians have found that for many population distributions, when the sample size is at least 30, the sampling distribution of the mean is approximately normal. However, you can apply the Central Limit Theorem for even smaller sample sizes if the population distribution is approximately bell shaped. In the uncommon case in which the distribution is extremely skewed or has more than one mode, you may need sample sizes larger than 30 to ensure normality.

Figure 7.5 illustrates the application of the Central Limit Theorem to different populations. The sampling distributions from three different continuous distributions (normal, uniform, and exponential) for varying sample sizes ( $n = 2, 5, 30$ ) are displayed.

**FIGURE 7.5**

Sampling distribution of the mean for different populations for samples of  $n = 2, 5,$  and  $30$



In each of the panels, because the sample mean has the property of being unbiased, the mean of any sampling distribution is always equal to the mean of the population.

Panel A of Figure 7.5 shows the sampling distribution of the mean selected from a normal population. As mentioned earlier in this section, when the population is normally distributed, the sampling distribution of the mean is normally distributed for any sample size. (You can measure the variability by using the standard error of the mean, Equation 7.3, on page 264.)

Panel B of Figure 7.5 depicts the sampling distribution from a population with a uniform (or rectangular) distribution (see Section 6.4). When samples of size  $n = 2$  are selected, there is a peaking, or *central limiting*, effect already working. For  $n = 5$ , the sampling distribution is bell shaped and approximately normal. When  $n = 30$ , the sampling distribution looks very similar to a normal distribution. In general, the larger the sample size, the more closely the sampling distribution will follow a normal distribution. As with all cases, the mean of each sampling distribution is equal to the mean of the population, and the variability decreases as the sample size increases.

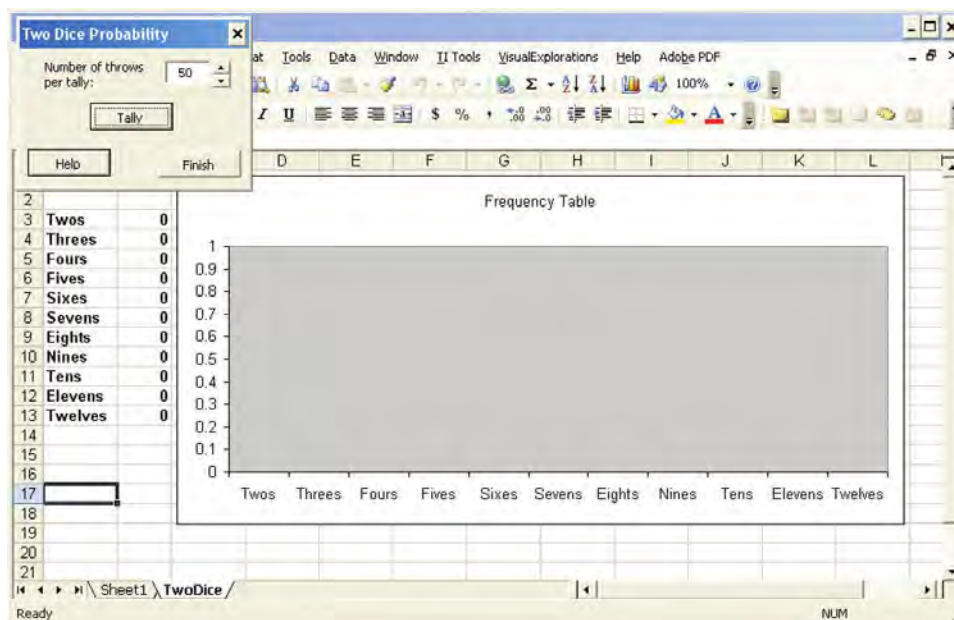
Panel C of Figure 7.5 presents an exponential distribution (see Section 6.5). This population is extremely right-skewed. When  $n = 2$ , the sampling distribution is still highly right-skewed but less so than the distribution of the population. For  $n = 5$ , the sampling distribution is slightly right-skewed. When  $n = 30$ , the sampling distribution looks approximately normal. Again, the mean of each sampling distribution is equal to the mean of the population, and the variability decreases as the sample size increases.

## VISUAL EXPLORATIONS Exploring Sampling Distributions

Use the Visual Explorations **Two Dice Probability** procedure to observe the effects of simulated throws on the frequency distribution of the sum of the two dice. Open the **Visual Explorations.xla** macro workbook on the text's CD (see Appendix D) and select **VisualExplorations** → **Two Dice Probability** (Excel 97–2003) or **Add-Ins** → **VisualExplorations** → **Two Dice Probability** (Excel 2007). The procedure produces a worksheet that contains

an empty frequency distribution table and histogram and a floating control panel (see below).

Click the **Tally** button to tally a set of throws in the frequency distribution table and histogram. Optionally, use the spinner buttons to adjust the number of throws per tally (round). Click the **Help** button for more information about this simulation. Click **Finish** when you are done with this exploration.



Using the results from the normal, uniform, and exponential statistical distributions, you can reach the following conclusions regarding the Central Limit Theorem:

- For most population distributions, regardless of shape, the sampling distribution of the mean is approximately normally distributed if samples of at least 30 are selected.
- If the population distribution is fairly symmetric, the sampling distribution of the mean is approximately normal for samples as small as 5.
- If the population is normally distributed, the sampling distribution of the mean is normally distributed, regardless of the sample size.

The Central Limit Theorem is of crucial importance in using statistical inference to draw conclusions about a population. It allows you to make inferences about the population mean without having to know the specific shape of the population distribution.

## PROBLEMS FOR SECTION 7.4

### Learning the Basics

**PH Grade ASSIST** **7.17** Given a normal distribution with  $\mu = 100$  and  $\sigma = 10$ , if you select a sample of  $n = 25$ , what is the probability that  $\bar{X}$  is

- a. less than 95?
- b. between 95 and 97.5?
- c. above 102.2?
- d. There is a 65% chance that  $\bar{X}$  is above what value?

**PH Grade ASSIST** **7.18** Given a normal distribution with  $\mu = 50$  and  $\sigma = 5$ , if you select a sample of  $n = 100$ , what is the probability that  $\bar{X}$  is

- a. less than 47?
- b. between 47 and 49.5?
- c. above 51.1?
- d. There is a 35% chance that  $\bar{X}$  is above what value?

### Applying the Concepts

**7.19** For each of the following three populations, indicate what the sampling distribution for samples of 25 would consist of:

- a. Travel expense vouchers for a university in an academic year
- b. Absentee records (days absent per year) in 2006 for employees of a large manufacturing company
- c. Yearly sales (in gallons) of unleaded gasoline at service stations located in a particular county

**7.20** The following data represent the number of days absent per year in a population of six employees of a small company:

1 3 6 7 9 10

- a. Assuming that you sample without replacement, select all possible samples of  $n = 2$  and construct the sampling distribution of the mean. Compute the mean of all the

sample means and also compute the population mean. Are they equal? What is this property called?

- b. Repeat (a) for all possible samples of  $n = 3$ .
- c. Compare the shape of the sampling distribution of the mean in (a) and (b). Which sampling distribution has less variability? Why?
- d. Assuming that you sample with replacement, repeat (a) through (c) and compare the results. Which sampling distributions have the least variability—those in (a) or (b)? Why?

**PH Grade ASSIST** **7.21** The diameter of Ping-Pong balls manufactured at a large factory is approximately normally distributed, with a mean of 1.30 inches and a standard deviation of 0.04 inch. If you select a random sample of 16 Ping-Pong balls,

- a. what is the sampling distribution of the mean?
- b. what is the probability that the sample mean is less than 1.28 inches?
- c. what is the probability that the sample mean is between 1.31 and 1.33 inches?
- d. The probability is 60% that the sample mean will be between what two values, symmetrically distributed around the population mean?

**7.22** The U.S. Census Bureau announced that the median sales price of new houses sold in March 2006 was \$224,200, while the mean sales price was \$279,100 ([www.census.gov/newhomesales](http://www.census.gov/newhomesales), April 26, 2006). Assume that the standard deviation of the prices is \$90,000.

- a. If you select samples of  $n = 2$ , describe the shape of the sampling distribution of  $\bar{X}$ .
- b. If you select samples of  $n = 100$ , describe the shape of the sampling distribution of  $\bar{X}$ .
- c. If you select a random sample of  $n = 100$ , what is the probability that the sample mean will be less than \$250,000?

PH Grade  
ASSIST

**7.23** Time spent using email per session is normally distributed, with  $\mu = 8$  minutes and  $\sigma = 2$  minutes. If you select a random sample of 25 sessions,

- what is the probability that the sample mean is between 7.8 and 8.2 minutes?
- what is the probability that the sample mean is between 7.5 and 8 minutes?
- If you select a random sample of 100 sessions, what is the probability that the sample mean is between 7.8 and 8.2 minutes?
- Explain the difference in the results of (a) and (c).

PH Grade  
ASSIST

**7.24** The amount of time a bank teller spends with each customer has a population mean,  $\mu$ , of 3.10 minutes and standard deviation,  $\sigma$ , of 0.40 minute.

SELF  
Test

If you select a random sample of 16 customers,

- what is the probability that the mean time spent per customer is at least 3 minutes?
- there is an 85% chance that the sample mean is less than how many minutes?
- What assumption must you make in order to solve (a) and (b)?
- If you select a random sample of 64 customers, there is an 85% chance that the sample mean is less than how many minutes?

**7.25** The *New York Times* reported (L. J. Flynn, “Tax Surfing,” *The New York Times*, March 25, 2002, p. C10) that the mean time to download the home page for the

Internal Revenue Service (IRS), [www.irs.gov](http://www.irs.gov), was 0.8 second. Suppose that the download time was normally distributed, with a standard deviation of 0.2 second. If you select a random sample of 30 download times,

- what is the probability that the sample mean is less than 0.75 second?
- what is the probability that the sample mean is between 0.70 and 0.90 second?
- the probability is 80% that the sample mean is between what two values, symmetrically distributed around the population mean?
- the probability is 90% that the sample mean is less than what value?

**7.26** The article discussed in Problem 7.25 also reported that the mean download time for the H&R Block Web site, [www.hrblock.com](http://www.hrblock.com), was 2.5 seconds. Suppose that the download time for the H&R Block Web site was normally distributed, with a standard deviation of 0.5 second. If you select a random sample of 30 download times,

- what is the probability that the sample mean is less than 2.75 seconds?
- what is the probability that the sample mean is between 2.70 and 2.90 seconds?
- the probability is 80% that the sample mean is between what two values symmetrically distributed around the population mean?
- the probability is 90% that the sample mean is less than what value?

## 7.5 SAMPLING DISTRIBUTION OF THE PROPORTION

Consider a categorical variable that has only two categories, such as the customer prefers your brand or the customer prefers the competitor’s brand. Of interest is the proportion of items belonging to one of the categories—for example, the proportion of customers that prefers your brand. The population proportion, represented by  $\pi$ , is the proportion of items in the entire population with the characteristic of interest. The sample proportion, represented by  $p$ , is the proportion of items in the sample with the characteristic of interest. The sample proportion, a statistic, is used to estimate the population proportion, a parameter. To calculate the sample proportion, you assign the two possible outcomes scores of 1 or 0 to represent the presence or absence of the characteristic. You then sum all the 1 and 0 scores and divide by  $n$ , the sample size. For example, if, in a sample of five customers, three preferred your brand and two did not, you have three 1s and two 0s. Summing the three 1s and two 0s and dividing by the sample size of 5 gives you a sample proportion of 0.60.

### SAMPLE PROPORTION

$$p = \frac{X}{n} = \frac{\text{Number of items having the characteristic of interest}}{\text{Sample size}} \quad (7.6)$$

The sample proportion,  $p$ , takes on values between 0 and 1. If all individuals possess the characteristic, you assign each a score of 1, and  $p$  is equal to 1. If half the individuals possess



the characteristic, you assign half a score of 1 and assign the other half a score of 0, and  $p$  is equal to 0.5. If none of the individuals possesses the characteristic, you assign each a score of 0, and  $p$  is equal to 0.

While the sample mean,  $\bar{X}$ , is an unbiased estimator of the population mean,  $\mu$ , the statistic  $p$  is an unbiased estimator of the population proportion,  $\pi$ . By analogy to the sampling distribution of the mean, the **standard error of the proportion**,  $\sigma_p$  is given in Equation (7.7).

### STANDARD ERROR OF THE PROPORTION

$$\sigma_p = \sqrt{\frac{\pi(1 - \pi)}{n}} \quad (7.7)$$

If you select all possible samples of a certain size, the distribution of all possible sample proportions is referred to as the **sampling distribution of the proportion**. The sampling distribution of the proportion follows the binomial distribution, as discussed in Section 5.3. However, you can use the normal distribution to approximate the binomial distribution when  $n\pi$  and  $n(1 - \pi)$  are each at least 5 (see Section 6.6 on the CD-ROM). In most cases in which inferences are made about the proportion, the sample size is substantial enough to meet the conditions for using the normal approximation (see reference 1). Therefore, in many instances, you can use the normal distribution to estimate the sampling distribution of the proportion.

Substituting  $p$  for  $\bar{X}$ ,  $\pi$  for  $\mu$ , and  $\sqrt{\frac{\pi(1 - \pi)}{n}}$  for  $\frac{\sigma}{\sqrt{n}}$  in Equation (7.4) on page 266, results in Equation (7.8).

### FINDING Z FOR THE SAMPLING DISTRIBUTION OF THE PROPORTION

$$Z = \frac{p - \pi}{\sqrt{\frac{\pi(1 - \pi)}{n}}} \quad (7.8)$$

To illustrate the sampling distribution of the proportion, suppose that the manager of the local branch of a savings bank determines that 40% of all depositors have multiple accounts at the bank. If you select a random sample of 200 depositors, the probability that the sample proportion of depositors with multiple accounts is less than 0.30 is calculated as follows: Because  $n\pi = 200(0.40) = 80 \geq 5$  and  $n(1 - \pi) = 200(0.60) = 120 \geq 5$ , the sample size is large enough to assume that the sampling distribution of the proportion is approximately normally distributed. Using Equation (7.8),

$$\begin{aligned} Z &= \frac{p - \pi}{\sqrt{\frac{\pi(1 - \pi)}{n}}} \\ &= \frac{0.30 - 0.40}{\sqrt{\frac{(0.40)(0.60)}{200}}} = \frac{-0.10}{\sqrt{\frac{0.24}{200}}} = \frac{-0.10}{0.0346} \\ &= -2.89 \end{aligned}$$

Using Table E.2, the area under the normal curve less than  $-2.89$  is 0.0019. Therefore, the probability that the sample proportion is less than 0.30 is 0.0019—a highly unlikely event. This means that if the true proportion of successes in the population is 0.40, less than one-fifth of 1% of the samples of  $n = 200$  would be expected to have sample proportions of less than 0.30.

## PROBLEMS FOR SECTION 7.5

### Learning the Basics

**PH Grade ASSIST** **7.27** In a random sample of 64 people, 48 are classified as “successful.”

- Determine the sample proportion,  $p$ , of “successful” people.
- If the population proportion is 0.70, determine the standard error of the proportion.

**PH Grade ASSIST** **7.28** A random sample of 50 households was selected for a telephone survey. The key question asked was, “Do you or any member of your household own a cellular telephone with a built-in camera?” Of the 50 respondents, 15 said yes and 35 said no.

- Determine the sample proportion,  $p$ , of households with cellular telephones with built-in cameras.
- If the population proportion is 0.40, determine the standard error of the proportion.

**7.29** The following data represent the responses ( $Y$  for yes and  $N$  for no) from a sample of 40 college students to the question “Do you currently own shares in any stocks?”

N N Y N N Y N Y N Y N N Y N Y Y N N N Y  
N Y N N N N Y N N Y Y N N N Y N N Y N N

- Determine the sample proportion,  $p$ , of college students who own shares of stock.
- If the population proportion is 0.30, determine the standard error of the proportion.

### Applying the Concepts

**PH Grade ASSIST** **7.30** A political pollster is conducting an analysis of sample results in order to make predictions on election night. Assuming a two-candidate election, if a specific candidate receives at least 55% of the vote in the sample, then that candidate will be forecast as the winner of the election. If you select a random sample of 100 voters, what is the probability that a candidate will be forecast as the winner when

- the true percentage of her vote is 50.1%?
- the true percentage of her vote is 60%?
- the true percentage of her vote is 49% (and she will actually lose the election)?
- If the sample size is increased to 400, what are your answers to (a) through (c)? Discuss.

**PH Grade ASSIST** **7.31** You plan to conduct a marketing experiment in which students are to taste one of two different brands of soft drink. Their task is to correctly identify the brand tasted. You select a random sample of 200 students and assume that the students have no abil-

ity to distinguish between the two brands. (Hint: If an individual has no ability to distinguish between the two soft drinks, then each brand is equally likely to be selected.)

- What is the probability that the sample will have between 50% and 60% of the identifications correct?
- The probability is 90% that the sample percentage is contained within what symmetrical limits of the population percentage?
- What is the probability that the sample percentage of correct identifications is greater than 65%?
- Which is more likely to occur—more than 60% correct identifications in the sample of 200 or more than 55% correct identifications in a sample of 1,000? Explain.

**7.32** A study of women in corporate leadership was conducted by Catalyst, a New York research organization. The study concluded that slightly more than 15% of corporate officers at Fortune 500 companies are women (C. Hymowitz, “Women Put Noses to the Grindstone, and Miss Opportunities,” *The Wall Street Journal*, February 3, 2004, p. B1). Suppose that you select a random sample of 200 corporate officers, and the true proportion held by women is 0.15.

- What is the probability that in the sample, less than 15% of the corporate officers will be women?
- What is the probability that in the sample, between 13% and 17% of the corporate officers will be women?
- What is the probability that in the sample, between 10% and 20% of the corporate officers will be women?
- If a sample of 100 is taken, how does this change your answers to (a) through (c)?

**7.33** The NBC hit comedy *Friends* was TiVo’s most popular show during the week of April 18–24, 2004. According to the Nielsen ratings, 29.7% of TiVo owners in the United States either recorded *Friends* or watched it live (“Prime-Time Nielsen Ratings,” *USA Today*, April 28, 2004, p. 3D). Suppose you select a random sample of 50 TiVo owners.

- What is the probability that more than half the people in the sample watched or recorded *Friends*?
- What is the probability that less than 25% of the people in the sample watched or recorded *Friends*?
- If a random sample of size 500 is taken, how does this change your answers to (a) and (b)?

**7.34** According to Gallup’s annual poll on personal finances, while most U.S. workers reported living comfortably now, many expected a downturn in their lifestyle when they stop working. Approximately half said they have enough money to live comfortably now and expected to do so in the future (J. M. Jones, “Only Half of Non-Retirees Expect to be Comfortable in Retirement,” *The Gallup Poll*,

**galluppoll.com**, May 2, 2006). If you select a random sample of 200 U.S. workers,

- what is the probability that the sample will have between 45% and 55% who say they have enough money to live comfortably now and expect to do so in the future?
- the probability is 90% that the sample percentage will be contained within what symmetrical limits of the population percentage?
- the probability is 95% that the sample percentage will be contained within what symmetrical limits of the population percentage?

**7.35** According to the National Restaurant Association, 20% of fine-dining restaurants have instituted policies restricting the use of cell phones (“Business Bulletin,” *The Wall Street Journal*, June 1, 2000, p. A1). If you select a random sample of 100 fine-dining restaurants,

- what is the probability that the sample has between 15% and 25% that have established policies restricting cell phone use?
- the probability is 90% that the sample percentage will be contained within what symmetrical limits of the population percentage?
- the probability is 95% that the sample percentage will be contained within what symmetrical limits of the population percentage?
- Suppose that in January 2007, you selected a random sample of 100 fine-dining restaurants and found that 31 had policies restricting the use of cell phones. Do you think that the population percentage has changed?

**7.36** An article (P. Kitchen, “Retirement Plan: To Keep Working,” *Newsday*, September 24, 2003) discussed the retirement plans of Americans ages 50 to 70 who were employed full time or part time. Twenty-nine percent of the respondents said that they did not intend to work for pay at all. If you select a random sample of 400 Americans ages 50 to 70 employed full time or part time,

- what is the probability that the sample has between 25% and 30% who do not intend to work for pay at all?
- If a current sample of 400 Americans ages 50 to 70 employed full time or part time has 35% who do not intend to work for pay at all, what can you infer about the population estimate of 29%? Explain.
- If a current sample of 100 Americans ages 50 to 70 employed full time or part time has 35% who do not intend to work for pay at all, what can you infer about the population estimate of 29%? Explain.
- Explain the difference in the results in (b) and (c).

**7.37** The IRS discontinued random audits in 1988. Instead, the IRS conducts audits on returns deemed questionable by its Discriminant Function System (DFS), a complicated and highly secretive computerized analysis system. In an attempt to reduce the proportion of “no-change” audits (that is, audits that uncover that no additional taxes are due), the IRS only audits returns the DFS scores as highly questionable. The proportion of no-change audits has risen over the years and is currently approximately 0.25 (T. Herman, “Unhappy Returns: IRS Moves to Bring Back Random Audits,” *The Wall Street Journal*, June 20, 2002, p. A1). Suppose that you select a random sample of 100 audits. What is the probability that the sample has

- between 24% and 26% no-change audits?
- between 20% and 30% no-change audits?
- more than 30% no-change audits?

**7.38** Referring to Problem 7.37, the IRS announced that it planned to resume totally random audits in 2002. Suppose that you select a random sample of 200 totally random audits and that 90% of all the returns filed would result in no-change audits. What is the probability that the sample has

- between 89% and 91% no-change audits?
- between 85% and 95% no-change audits?
- more than 95% no-change audits?

## 7.6 (CD-ROM Topic) SAMPLING FROM FINITE POPULATIONS

In this section, sampling without replacement from finite populations is considered. For further discussion, see **section 7.6.pdf** on the Student CD-ROM that accompanies this book.

## SUMMARY

In this chapter, you studied four common sampling methods—simple random, systematic, stratified, and cluster. You also studied the sampling distribution of the sample mean, the Central Limit Theorem, and the sampling distribution of the sample proportion. You learned that the sample mean is an unbiased estimator of the population mean, and the sample proportion is an unbiased estimator of the

population proportion. By observing the mean weight in a sample of cereal boxes filled by Oxford Cereals, you were able to reach conclusions concerning the mean weight in the population of cereal boxes. In the next five chapters, the techniques of confidence intervals and tests of hypotheses commonly used for statistical inference are discussed.

## KEY EQUATIONS

### Population Mean

$$\mu = \frac{\sum_{i=1}^N X_i}{N} \quad (7.1)$$

### Population Standard Deviation

$$\sigma = \sqrt{\frac{\sum_{i=1}^N (X_i - \mu)^2}{N}} \quad (7.2)$$

### Standard Error of the Mean

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} \quad (7.3)$$

### Finding $Z$ for the Sampling Distribution of the Mean

$$Z = \frac{\bar{X} - \mu_{\bar{X}}}{\sigma_{\bar{X}}} = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} \quad (7.4)$$

### Finding $\bar{X}$ for the Sampling Distribution of the Mean

$$\bar{X} = \mu + Z \frac{\sigma}{\sqrt{n}} \quad (7.5)$$

### Sample Proportion

$$p = \frac{X}{n} \quad (7.6)$$

### Standard Error of the Proportion

$$\sigma_p = \sqrt{\frac{\pi(1 - \pi)}{n}} \quad (7.7)$$

### Finding $Z$ for the Sampling Distribution of the Proportion

$$Z = \frac{p - \pi}{\sqrt{\frac{\pi(1 - \pi)}{n}}} \quad (7.8)$$

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## CHAPTER REVIEW PROBLEMS

### Checking Your Understanding

**PH Grade ASSIST** 7.39 Why is the sample mean an unbiased estimator of the population mean?

**PH Grade ASSIST** 7.40 Why does the standard error of the mean decrease as the sample size,  $n$ , increases?

7.41 Why does the sampling distribution of the mean follow a normal distribution for a large enough sample size, even though the population may not be normally distributed?

7.42 What is the difference between a probability distribution and a sampling distribution?

7.43 Under what circumstances does the sampling distribution of the proportion approximately follow the normal distribution?

7.44 What is the difference between probability and non-probability sampling?

**7.45** What are some potential problems with using “fish-bowl” methods to select a simple random sample?

**7.46** What is the difference between sampling *with* replacement versus *without* replacement?

**7.47** What is the difference between a simple random sample and a systematic sample?

**PH Grade** **7.48** What is the difference between a simple **ASSIST** random sample and a stratified sample?

**7.49** What is the difference between a stratified sample and a cluster sample?

### Applying the Concepts

**7.50** An industrial sewing machine uses ball bearings that are targeted to have a diameter of 0.75 inch. The lower and upper specification limits under which the ball bearing can operate are 0.74 inch (lower) and 0.76 inch (upper). Past experience has indicated that the actual diameter of the ball bearings is approximately normally distributed, with a mean of 0.753 inch and a standard deviation of 0.004 inch. If you select a random sample of 25 ball bearings, what is the probability that the sample mean is

- between the target and the population mean of 0.753?
- between the lower specification limit and the target?
- greater than the upper specification limit?
- less than the lower specification limit?
- The probability is 93% that the sample mean diameter will be greater than what value?

**PH Grade** **7.51** The fill amount of bottles of a soft drink is **ASSIST** normally distributed, with a mean of 2.0 liters and a standard deviation of 0.05 liter. If you select a random sample of 25 bottles, what is the probability that the sample mean will be

- between 1.99 and 2.0 liters?
- below 1.98 liters?
- greater than 2.01 liters?
- The probability is 99% that the sample mean will contain at least how much soft drink?
- The probability is 99% that the sample mean will contain an amount that is between which two values (symmetrically distributed around the mean)?

**PH Grade** **7.52** An orange juice producer buys all his **ASSIST** oranges from a large orange grove that has one variety of orange. The amount of juice squeezed from each of these oranges is approximately normally distributed, with a mean of 4.70 ounces and a standard deviation of 0.40 ounce. Suppose that you select a sample of 25 oranges.

- What is the probability that the sample mean will be at least 4.60 ounces?
- The probability is 70% that the sample mean will be contained between what two values symmetrically distributed around the population mean?
- The probability is 77% that the sample mean will be greater than what value?

**7.53** In his management information systems textbook, Professor David Kroenke raises an interesting point: “If 98% of our market has Internet access, do we have a responsibility to provide non-Internet materials to that other 2%?” (D. M. Kroenke, *Using MIS*, Upper Saddle River, NJ: Prentice Hall, 2007, p. 29a.) Suppose that 98% of the customers in your market have Internet access and you select a random sample of 500 customers. What is the probability that the sample has

- greater than 99% with Internet access?
- between 97% and 99% with Internet access?
- less than 97% with Internet access?

**7.54** Mutual funds reported strong earnings in the first quarter of 2006. Especially strong growth occurred in mutual funds consisting of companies focusing on Latin America. This population of mutual funds earned a mean return of 15.9% in the first quarter (M. Skala, “Banking on the World,” *Christian Science Monitor*, [www.csmonitor.com](http://www.csmonitor.com), April 10, 2006.) Assume that the returns for the Latin America mutual funds were distributed as a normal random variable, with a mean of 15.9 and a standard deviation of 20. If you selected a random sample of 10 funds from this population, what is the probability that the sample would have a mean return

- less than 0—that is, a loss?
- between 0 and 6?
- greater than 10?

**7.55** Mutual funds reported strong earnings in the first quarter of 2006. The population of mutual funds focusing on Europe had a mean return of 13.3% during this time. Assume that the returns for the Europe mutual funds were distributed as a normal random variable, with a mean of 13.3 and a standard deviation of 12. If you select an individual fund from this population, what is the probability that it would have a return

- less than 0—that is, a loss?
- between 0 and 6?
- greater than 10?

If you selected a random sample of 10 funds from this population, what is the probability that the sample would have a mean return

- less than 0—that is, a loss?
- between 0 and 6?
- greater than 10?
- Compare your results in parts (d) through (f) to (a) through (c).
- Compare your results in parts (d) through (f) to Problem 7.54 (a) through (c).

**7.56** Political polling has traditionally used telephone interviews. Researchers at Harris Black International Ltd. have argued that Internet polling is less expensive, faster, and offers higher response rates than telephone surveys. Critics are concerned about the scientific reliability of this approach (*The Wall Street Journal*, April 13, 1999). Even

amid this strong criticism, Internet polling is becoming more and more common. What concerns, if any, do you have about Internet polling?

**7.57** A survey sponsored by The American Dietetic Association and the agri-business giant ConAgra found that 53% of office workers take 30 minutes or less for lunch each day. Approximately 37% take 30 to 60 minutes, and 10% take more than an hour. (“Snapshots,” *usatoday.com*, April 26, 2006.)

- What additional information would you want to know before you accepted the results of the survey?
- Discuss the four types of survey errors in the context of this survey.
- One of the types of survey errors discussed in part (b) should have been measurement error. Explain how the root cause of measurement error in this survey could be the halo effect.

**7.58** As part of a mediation process overseen by a federal judge to end a lawsuit that accuses Cincinnati, Ohio, of decades of discrimination against African Americans, surveys were done on how to improve Cincinnati police–community relations. One survey was sent to the 1,020 members of the Cincinnati police force. The survey included a cover in which the chief of police and president of the Fraternal Order of Police encouraged participation. Respondents could either return a hard copy of the survey or complete the survey online. To the researchers’ dismay, only 158 surveys were completed (“Few Cops Fill Out Survey,” *The Cincinnati Enquirer*, August 22, 2001, p. B3).

- What type of errors or biases should the researchers be especially concerned with?
- What step(s) should the researchers take to try to overcome the problems noted in (a)?
- What could have been done differently to improve the survey’s worthiness?

**7.59** Connecticut shoppers spend more on women’s clothing than do shoppers in any other state, according to a survey conducted by MapInfo. The mean spending per household in Connecticut was \$975 annually (“Snapshots,” *usatoday.com*, April 17, 2006).

- What other information would you want to know before you accepted the results of this survey?
- Suppose that you wished to conduct a similar survey for the geographic region you live in. Describe the population for your survey.
- Explain how you could minimize the chance of coverage error in this type of survey.
- Explain how you could minimize the chance of nonresponse error in this type of survey.
- Explain how you could minimize the chance of sampling error in this type of survey.
- Explain how you could minimize the chance of measurement error in this type of survey.

**7.60** According to Dr. Sarah Beth Estes, sociology professor at the University of Cincinnati, and Dr. Jennifer Glass, sociology professor at the University of Iowa, working women who take advantage of family-friendly schedules can fall behind in wages. More specifically, the sociologists report that in a study of 300 working women who had children and returned to work and opted for flextime, telecommuting, and so on, these women had pay raises that averaged between 16% and 26% less than other workers (“Study: ‘Face Time’ Can Affect Moms’ Raises,” *The Cincinnati Enquirer*, August 28, 2001, p. A1).

- What other information would you want to know before you accepted the results of this study?
- If you were to perform a similar study in the geographic area where you live, define a population, frame, and sampling method you could use.

**7.61 (Class Project)** The table of random numbers is an example of a uniform distribution because each digit is equally likely to occur. Starting in the row corresponding to the day of the month in which you were born, use the table of random numbers (Table E.1) to take one digit at a time.

Select five different samples each of  $n = 2$ ,  $n = 5$ , and  $n = 10$ . Compute the sample mean of each sample. Develop a frequency distribution of the sample means for the results of the entire class, based on samples of sizes  $n = 2$ ,  $n = 5$ , and  $n = 10$ .

What can be said about the shape of the sampling distribution for each of these sample sizes?

**7.62 (Class Project)** Toss a coin 10 times and record the number of heads. If each student performs this experiment five times, a frequency distribution of the number of heads can be developed from the results of the entire class. Does this distribution seem to approximate the normal distribution?

**7.63 (Class Project)** The number of cars waiting in line at a car wash is distributed as follows:

Number of Cars	Probability
0	0.25
1	0.40
2	0.20
3	0.10
4	0.04
5	0.01

You can use the table of random numbers (Table E.1) to select samples from this distribution by assigning numbers as follows:

- Start in the row corresponding to the day of the month in which you were born.
- Select a two-digit random number.
- If you select a random number from 00 to 24, record a length of 0; if from 25 to 64, record a length of 1; if from

65 to 84, record a length of 2; if from 85 to 94, record a length of 3; if from 95 to 98, record a length of 4; if 99, record a length of 5.

Select samples of  $n = 2$ ,  $n = 5$ , and  $n = 10$ . Compute the mean for each sample. For example, if a sample of size 2 results in the random numbers 18 and 46, these would correspond to lengths of 0 and 1, respectively, producing a sample mean of 0.5. If each student selects five different samples for each sample size, a frequency distribution of the sample means (for each sample size) can be developed from the results of the entire class. What conclusions can you reach concerning the sampling distribution of the mean as the sample size is increased?

**7.64 (Class Project)** Using Table E.1, simulate the selection of different-colored balls from a bowl as follows:

1. Start in the row corresponding to the day of the month in which you were born.
2. Select one-digit numbers.

3. If a random digit between 0 and 6 is selected, consider the ball white; if a random digit is a 7, 8, or 9, consider the ball red.

Select samples of  $n = 10$ ,  $n = 25$ , and  $n = 50$  digits. In each sample, count the number of white balls and compute the proportion of white balls in the sample. If each student in the class selects five different samples for each sample size, a frequency distribution of the proportion of white balls (for each sample size) can be developed from the results of the entire class. What conclusions can you reach about the sampling distribution of the proportion as the sample size is increased?

**7.65 (Class Project)** Suppose that step 3 of Problem 7.64 uses the following rule: “If a random digit between 0 and 8 is selected, consider the ball to be white; if a random digit of 9 is selected, consider the ball to be red.” Compare and contrast the results in this problem and those in Problem 7.64.

## Managing the Springville Herald

Continuing its quality improvement effort first described in the Chapter 6 “Managing the *Springville Herald*” case, the production department of the newspaper has been monitoring the blackness of the newspaper print. As before, blackness is measured on a standard scale in which the target value is 1.0. Data collected over the past year indicate that the blackness is normally distributed, with a mean of 1.005 and a standard deviation of 0.10.

### EXERCISE

SH7.1 Each day, 25 spots on the first newspaper printed are chosen, and the blackness of the spots is mea-

sured. Assuming that the distribution has not changed from what it was in the past year, what is the probability that the mean blackness of the spots is

- a. less than 1.0?
- b. between 0.95 and 1.0?
- c. between 1.0 and 1.05?
- d. less than 0.95 or greater than 1.05?
- e. Suppose that the mean blackness of today’s sample of 25 spots is 0.952. What conclusion can you make about the blackness of the newspaper based on this result? Explain.

## Web Case

*Apply your knowledge about sampling distributions in this Web Case, which reconsiders the Oxford Cereals Using Statistics scenario.*

The advocacy group Consumers Concerned About Cereal Cheaters (CCACC) suspects that cereal companies, including Oxford Cereals, are cheating consumers by packaging cereals at less than labeled weights. Visit the organization’s home page at [www.prenhall.com/Springville/ConsumersConcerned.htm](http://www.prenhall.com/Springville/ConsumersConcerned.htm) (or open the **ConsumersConcerned.htm** file in the text

CD’s WebCase folder), examine their claims and supporting data, and then answer the following:

1. Are the data collection procedures that the CCACC uses to form its conclusions flawed? What procedures could the group follow to make their analysis more rigorous?
2. Assume that the two samples of five cereal boxes (one sample for each of two cereal varieties) listed on the CCACC Web site were collected randomly by organization members. For each sample, do the following:

- a. Calculate the sample mean.
  - b. Assume that the standard deviation of the process is 15 grams and a population mean of 368 grams. Calculate the percentage of all samples for each process that would have a sample mean less than the value you calculated in (a).
  - c. Again, assuming that the standard deviation is 15 grams, calculate the percentage of individual boxes of cereal that would have a weight less than the value you calculated in (a).
3. What, if any, conclusions can you form by using your calculations about the filling processes of the two different cereals?
  4. A representative from Oxford Cereals has asked that the CCACC take down its page discussing shortages in Oxford Cereals boxes. Is that request reasonable? Why or why not?
  5. Can the techniques discussed in this chapter be used to prove cheating in the manner alleged by the CCACC? Why or why not?

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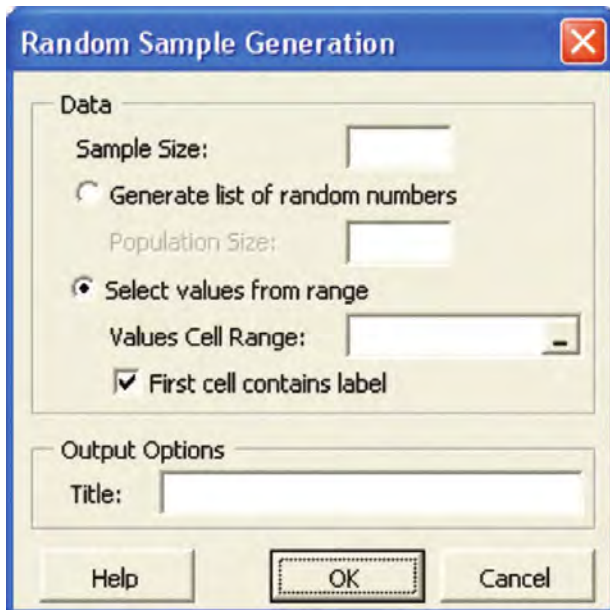
# Excel Companion

## to Chapter 7

### E7.1 CREATING SIMPLE RANDOM SAMPLES (WITHOUT REPLACEMENT)

You create simple random samples (without replacement) by using the PHStat2 Random Sample Generation procedure. (There are no basic Excel commands or features to create a simple random sample.)

Open to the worksheet that contains the data to be sampled and select **PHStat** → **Sampling** → **Random Sample Generation**. In the Random Sample Generation dialog box (shown below), enter the **Sample Size** and click **Select values from range**. Enter the cell range of the data to be sampled as the **Values Cell Range**, click **First cell contains label**, and click **OK**. A new simple random sample appears on a new worksheet.



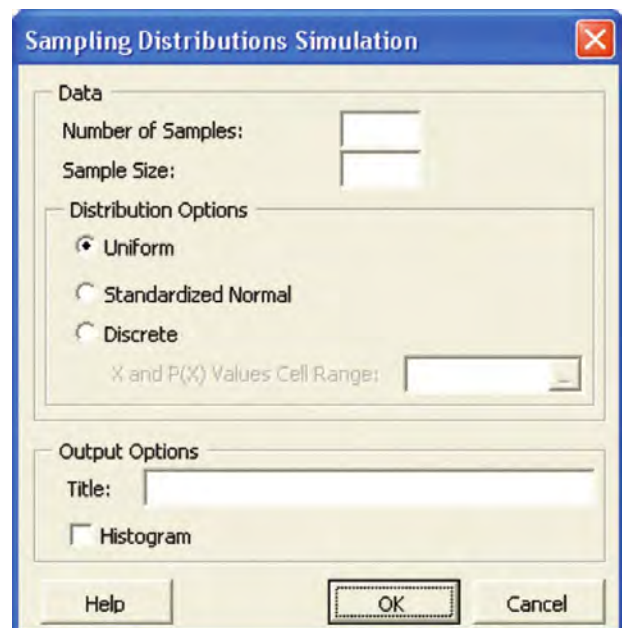
### E7.2 CREATING SIMULATED SAMPLING DISTRIBUTIONS

You create simulated sampling distributions by first using the ToolPak Random Number Generation procedure to create a worksheet of all the random samples. Then you add formulas to compute the sample means and other appropriate measures for each sample. You can also use the PHStat2 Sampling

Distributions Simulation procedure, which does both of these tasks for you and optionally creates a histogram.

#### Using PHStat2 Sampling Distributions Simulation

Select **PHStat** → **Sampling** → **Sampling Distributions Simulation**. In the Sampling Distributions Simulation dialog box (shown below), enter values for the **Number of Samples** and the **Sample Size**. Click one of the distribution options and then enter a title as the **Title** and click **OK**. To create a histogram of the sample means, click **Histogram** before clicking **OK**.

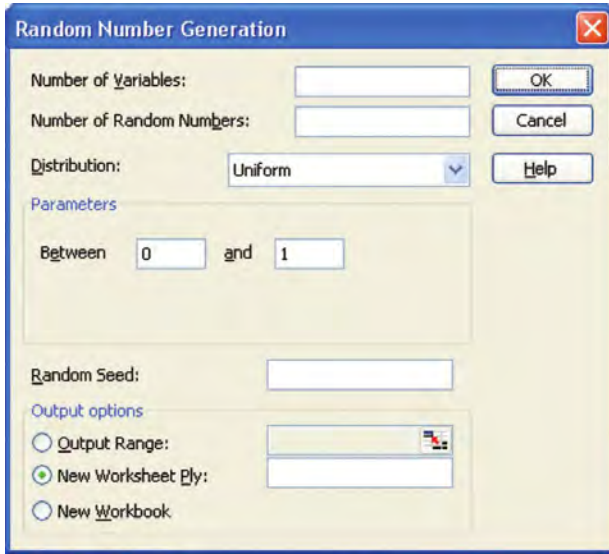


If you want to use the **Discrete** option, first open to a worksheet that contains a table of  $X$  and  $P(X)$  values and then select this procedure. Then select **Discrete** and enter that table range as the **X and P(X) Values Cell Range**.

#### Using ToolPak Random Number Generation

Select **Tools** → **Data Analysis**. From the list that appears in the Data Analysis dialog box, select **Random Number**

**Generation** and click **OK**. In the Random Number Generation dialog box (shown below), enter the number of samples as the **Number of Variables** and enter the sample size of each sample as the **Number of Random Numbers**. Select the type of distribution from the **Distribution** drop-down list and make entries in the Parameters area, as necessary. (The contents of this area vary according to the distribution chosen.) Click **New Worksheet Ply** and then click **OK**.



To create a histogram from the set of sample means for your simulation, enter a formula that uses the AVERAGE function in a row below the cell range that contains the samples created by the procedure. Then use the techniques for creating frequency distributions and histograms discussed in the Excel Companion to Chapter 2 to create your histogram.

---

### EXAMPLE 100 Samples of Sample Size 30 from a Uniformly Distributed Population

**Basic Excel Select Tools → Data Analysis.** From the list that appears in the Data Analysis dialog box, select **Random Number Generation** and click **OK**. In the Random Number Generation dialog box (shown at left), enter **100** as the **Number of Variables** and enter **30** as the **Number of Random Numbers**. Select **Uniform** from the **Distribution** drop-down list, click **New Worksheet Ply**, and then click **OK**.

**PHStat2 Select PHStat → Sampling → Sampling Distributions Simulation.** In the procedure dialog box, enter **100** as the **Number of Samples** and **30** as the **Sample Size**. Click **Uniform** and then enter a title as the **Title** and click **OK**.

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# CHAPTER 8

## Confidence Interval Estimation

USING STATISTICS @ Saxon Home Improvement

### 8.1 CONFIDENCE INTERVAL ESTIMATION FOR THE MEAN ( $\sigma$ KNOWN)

### 8.2 CONFIDENCE INTERVAL ESTIMATION FOR THE MEAN ( $\sigma$ UNKNOWN)

Student's  $t$  Distribution  
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### 8.6 CONFIDENCE INTERVAL ESTIMATION AND ETHICAL ISSUES

### 8.7 (CD-ROM TOPIC) ESTIMATION AND SAMPLE SIZE DETERMINATION FOR FINITE POPULATIONS

#### EXCEL COMPANION TO CHAPTER 8

- E8.1 Computing the Confidence Interval Estimate for the Mean ( $\sigma$  Known)
- E8.2 Computing the Confidence Interval Estimate for the Mean ( $\sigma$  Unknown)
- E8.3 Computing the Confidence Interval Estimate for the Proportion
- E8.4 Computing the Sample Size Needed for Estimating the Mean
- E8.5 Computing the Sample Size Needed for Estimating the Proportion
- E8.6 Computing the Confidence Interval Estimate for the Population Total
- E8.7 Computing the Confidence Interval Estimate for the Total Difference
- E8.8 Computing Finite Population Correction Factors

### LEARNING OBJECTIVES

In this chapter, you learn:

- To construct and interpret confidence interval estimates for the mean and the proportion
- How to determine the sample size necessary to develop a confidence interval for the mean or proportion
- How to use confidence interval estimates in auditing

## Using Statistics @ Saxon Home Improvement



Saxon Home Improvement distributes home improvement supplies in the northeastern United States. As a company accountant, you are responsible for the accuracy of the integrated inventory management and sales information system. You could review the contents of each and every record to check the accuracy of this system, but such a detailed review would be time-consuming and costly. A better approach would be to use statistical inference techniques to draw conclusions about the population of all records from a relatively small sample collected during an audit. At the end of each month, you could select a sample of the sales invoices to determine the following:

- The mean dollar amount listed on the sales invoices for the month.
  - The total dollar amount listed on the sales invoices for the month.
  - Any differences between the dollar amounts on the sales invoices and the amounts entered into the sales information system.
- The frequency of occurrence of errors that violate the internal control policy of the warehouse. Such errors include making a shipment when there is no authorized warehouse removal slip, failure to include the correct account number, and shipping the incorrect home improvement item.

How accurate are the results from the samples and how do you use this information? Are the sample sizes large enough to give you the information you need?

Statistical inference is the process of using sample results to draw conclusions about the characteristics of a population. Inferential statistics enables you to *estimate* unknown population characteristics such as a population mean or a population proportion. Two types of estimates are used to estimate population parameters: point estimates and interval estimates. A **point estimate** is the value of a single sample statistic. A **confidence interval estimate** is a range of numbers, called an interval, constructed around the point estimate. The confidence interval is constructed such that the probability that the population parameter is located somewhere within the interval is known.

Suppose you would like to estimate the mean GPA of all the students at your university. The mean GPA for all the students is an unknown population mean, denoted by  $\mu$ . You select a sample of students and find that the sample mean is 2.80. The sample mean,  $\bar{X} = 2.80$ , is a point estimate of the population mean,  $\mu$ . How accurate is 2.80? To answer this question, you must construct a confidence interval estimate.

In this chapter, you will learn how to construct and interpret confidence interval estimates. Recall that the sample mean,  $\bar{X}$ , is a point estimate of the population mean,  $\mu$ . However, the sample mean varies from sample to sample because it depends on the items selected in the sample. By taking into account the known variability from sample to sample (see Section 7.4 on the sampling distribution of the mean), you can develop the interval estimate for the population mean. The interval constructed should have a specified confidence of correctly estimating the value of the population parameter  $\mu$ . In other words, there is a specified confidence that  $\mu$  is somewhere in the range of numbers defined by the interval.

Suppose that after studying this chapter, you find that a 95% confidence interval for the mean GPA at your university is  $(2.75 \leq \mu \leq 2.85)$ . You can interpret this interval estimate by stating that you are 95% confident that the mean GPA at your university is between 2.75 and 2.85. There is a 5% chance that the mean GPA is below 2.75 or above 2.85.

After learning about the confidence interval for the mean, you will learn how to develop an interval estimate for the population proportion. Then you will learn how large a sample to select when constructing confidence intervals and how to perform several important estimation procedures accountants use when performing audits.

## 8.1 CONFIDENCE INTERVAL ESTIMATION FOR THE MEAN ( $\sigma$ KNOWN)

In Section 7.4, you used the Central Limit Theorem and knowledge of the population distribution to determine the percentage of sample means that fall within certain distances of the population mean. For instance, in the cereal-fill example used throughout Chapter 7 (see Example 7.6 on page 268), 95% of all sample means are between 362.12 and 373.88 grams. This statement is based on *deductive reasoning*. However, *inductive reasoning* is what you need here.

You need inductive reasoning because, in statistical inference, you use the results of a single sample to draw conclusions about the population, not vice versa. Suppose that in the cereal-fill example, you wish to estimate the unknown population mean, using the information from only a sample. Thus, rather than take  $\mu \pm (1.96)(\sigma/(\sqrt{n}))$  to find the upper and lower limits around  $\mu$ , as in Section 7.4, you substitute the sample mean,  $\bar{X}$ , for the unknown  $\mu$  and use  $\bar{X} \pm (1.96)(\sigma/(\sqrt{n}))$  as an interval to estimate the unknown  $\mu$ . Although in practice you select a single sample of size  $n$  and compute the mean,  $\bar{X}$ , in order to understand the full meaning of the interval estimate, you need to examine a hypothetical set of all possible samples of  $n$  values.

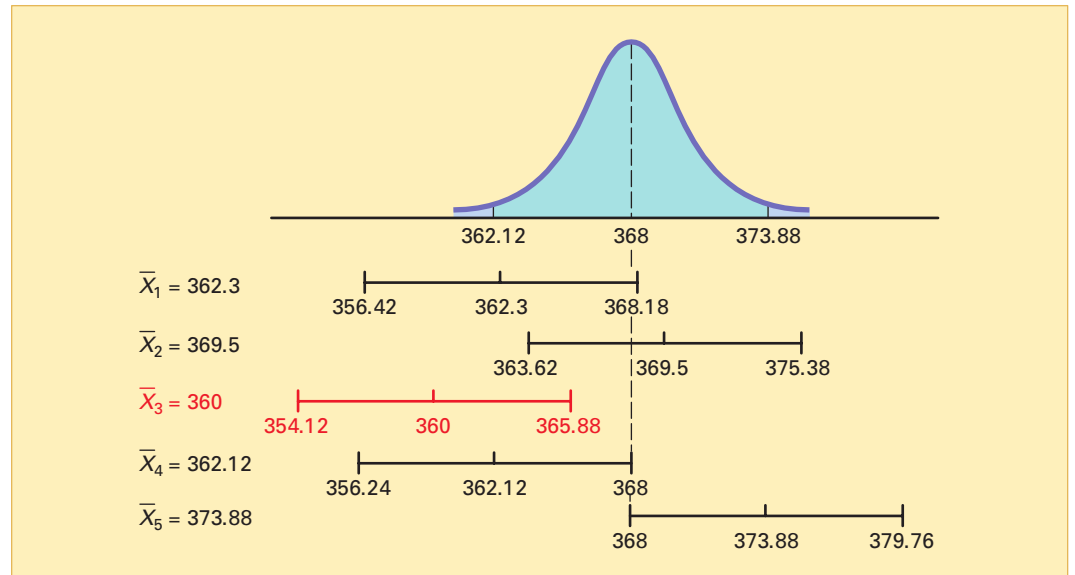
Suppose that a sample of  $n = 25$  boxes has a mean of 362.3 grams. The interval developed to estimate  $\mu$  is  $362.3 \pm (1.96)(15)/(\sqrt{25})$ , or  $362.3 \pm 5.88$ . The estimate of  $\mu$  is

$$356.42 \leq \mu \leq 368.18$$

Because the population mean,  $\mu$  (equal to 368), is included within the interval, this sample results in a correct statement about  $\mu$  (see Figure 8.1).

**FIGURE 8.1**

Confidence interval estimates for five different samples of  $n = 25$  taken from a population where  $\mu = 368$  and  $\sigma = 15$



To continue this hypothetical example, suppose that for a different sample of  $n = 25$  boxes, the mean is 369.5. The interval developed from this sample is

$$369.5 \pm (1.96)(15)/(\sqrt{25})$$

or  $369.5 \pm 5.88$ . The estimate is

$$363.62 \leq \mu \leq 375.38$$

Because the population mean,  $\mu$  (equal to 368), is also included within this interval, this statement about  $\mu$  is correct.

Now, before you begin to think that correct statements about  $\mu$  are always made by developing a confidence interval estimate, suppose a third hypothetical sample of  $n = 25$  boxes is selected and the sample mean is equal to 360 grams. The interval developed here is  $360 \pm (1.96)(15)/(\sqrt{25})$ , or  $360 \pm 5.88$ . In this case, the estimate of  $\mu$  is

$$354.12 \leq \mu \leq 365.88$$

This estimate is *not* a correct statement because the population mean,  $\mu$ , is not included in the interval developed from this sample (see Figure 8.1). Thus, for some samples, the interval estimate of  $\mu$  is correct, but for others it is incorrect. In practice, only one sample is selected, and because the population mean is unknown, you cannot determine whether the interval estimate is correct.

To resolve this dilemma of sometimes having an interval that provides a correct estimate and sometimes having an interval that provides an incorrect estimate, you need to determine the proportion of samples producing intervals that result in correct statements about the population mean,  $\mu$ . To do this, consider two other hypothetical samples: the case in which  $\bar{X} = 362.12$  grams and the case in which  $\bar{X} = 373.88$  grams. If  $\bar{X} = 362.12$ , the interval is  $362.12 \pm (1.96)(15)/(\sqrt{25})$ , or  $362.12 \pm 5.88$ . This leads to the following interval:

$$356.24 \leq \mu \leq 368.00$$

Because the population mean of 368 is at the upper limit of the interval, the statement is a correct one (see Figure 8.1).

When  $\bar{X} = 373.88$ , the interval is  $373.88 \pm (1.96)(15)/(\sqrt{25})$ , or  $373.88 \pm 5.88$ . The interval for the sample mean is

$$368.00 \leq \mu \leq 379.76$$

In this case, because the population mean of 368 is included at the lower limit of the interval, the statement is correct.

In Figure 8.1, you see that when the sample mean falls anywhere between 362.12 and 373.88 grams, the population mean is included *somewhere* within the interval. In Example 7.6 on page 268, you found that 95% of the sample means fall between 362.12 and 373.88 grams. Therefore, 95% of all samples of  $n = 25$  boxes have sample means that include the population mean within the interval developed.

Because, in practice, you select only one sample and  $\mu$  is unknown, you never know for sure whether your specific interval includes the population mean. However, if you take all possible samples of  $n$  and compute their sample means, 95% of the intervals will include the population mean, and only 5% of them will not. In other words, you have 95% confidence that the population mean is somewhere in your interval.

Consider once again, the first sample discussed in this section. A sample of  $n = 25$  boxes had a sample mean of 362.3 grams. The interval constructed to estimate  $\mu$  is:

$$\begin{aligned} 362.3 \pm (1.96)(15)/(\sqrt{25}) \\ 362.3 \pm 5.88 \\ 356.42 \leq \mu \leq 368.18 \end{aligned}$$

The interval from 356.42 to 368.18 is referred to as a 95% confidence interval.

“I am 95% confident that the mean amount of cereal in the population of boxes is somewhere between 356.42 and 368.18 grams.”

In some situations, you might want a higher degree of confidence (such as 99%) of including the population mean within the interval. In other cases, you might accept less confidence (such as 90%) of correctly estimating the population mean. In general, the **level of confidence** is symbolized by  $(1 - \alpha) \times 100\%$ , where  $\alpha$  is the proportion in the tails of the distribution that is outside the confidence interval. The proportion in the upper tail of the distribution is  $\alpha/2$ , and the proportion in the lower tail of the distribution is  $\alpha/2$ . You use Equation (8.1) to construct a  $(1 - \alpha) \times 100\%$  confidence interval estimate of the mean with  $\sigma$  known.

### CONFIDENCE INTERVAL FOR THE MEAN ( $\sigma$ KNOWN)

$$\bar{X} \pm Z \frac{\sigma}{\sqrt{n}}$$

or

$$\bar{X} - Z \frac{\sigma}{\sqrt{n}} \leq \mu \leq \bar{X} + Z \frac{\sigma}{\sqrt{n}} \quad (8.1)$$

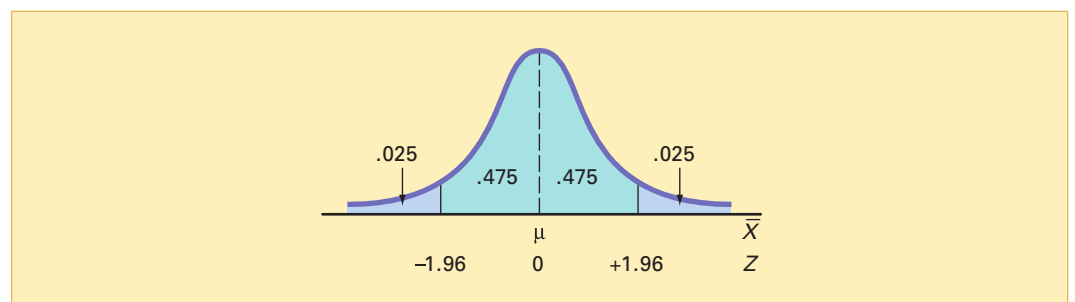
where  $Z$  = the value corresponding to a cumulative area of  $1 - \alpha/2$  from the standardized normal distribution (that is, an upper-tail probability of  $\alpha/2$ ).

The value of  $Z$  needed for constructing a confidence interval is called the **critical value** for the distribution. 95% confidence corresponds to an  $\alpha$  value of 0.05. The critical  $Z$  value corresponding to a cumulative area of 0.9750 is 1.96 because there is 0.025 in the upper tail of the distribution and the cumulative area less than  $Z = 1.96$  is 0.975.

There is a different critical value for each level of confidence,  $1 - \alpha$ . A level of confidence of 95% leads to a  $Z$  value of 1.96 (see Figure 8.2). 99% confidence corresponds to an  $\alpha$  value of 0.01. The  $Z$  value is approximately 2.58 because the upper-tail area is 0.005 and the cumulative area less than  $Z = 2.58$  is 0.995 (see Figure 8.3).

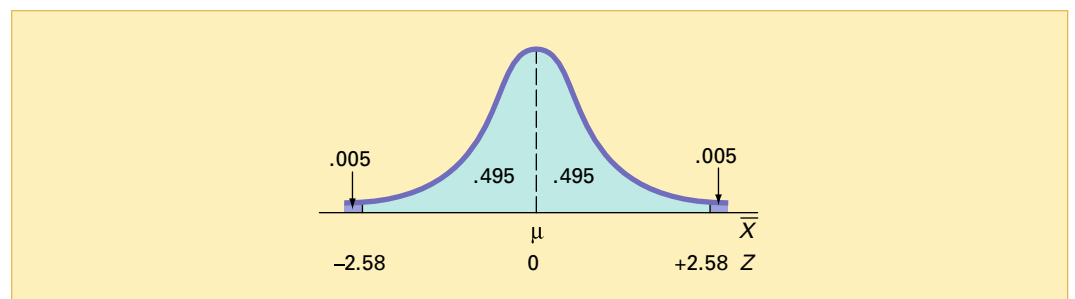
**FIGURE 8.2**

Normal curve for determining the  $Z$  value needed for 95% confidence



**FIGURE 8.3**

Normal curve for determining the  $Z$  value needed for 99% confidence



Now that various levels of confidence have been considered, why not make the confidence level as close to 100% as possible? Before doing so, you need to realize that any increase in the level of confidence is achieved only by widening (and making less precise) the confidence interval. There is no “free lunch” here. You would have more confidence that the population mean is within a broader range of values; however, this might make the interpretation of the confidence interval less useful. The trade-off between the width of the confidence interval and the level of confidence is discussed in greater depth in the context of determining the sample size in Section 8.4. Example 8.1 illustrates the application of the confidence interval estimate.

### EXAMPLE 8.1 ESTIMATING THE MEAN PAPER LENGTH WITH 95% CONFIDENCE

A paper manufacturer has a production process that operates continuously throughout an entire production shift. The paper is expected to have a mean length of 11 inches, and the standard deviation of the length is 0.02 inch. At periodic intervals, a sample is selected to determine whether the mean paper length is still equal to 11 inches or whether something has gone wrong in the production process to change the length of the paper produced. You select a random sample of 100 sheets, and the mean paper length is 10.998 inches. Construct a 95% confidence interval estimate for the population mean paper length.

**SOLUTION** Using Equation (8.1) on page 287, with  $Z = 1.96$  for 95% confidence,

$$\begin{aligned}\bar{X} \pm Z \frac{\sigma}{\sqrt{n}} &= 10.998 \pm (1.96) \frac{0.02}{\sqrt{100}} \\ &= 10.998 \pm 0.00392 \\ 10.99408 &\leq \mu \leq 11.00192\end{aligned}$$

Thus, with 95% confidence, you conclude that the population mean is between 10.99408 and 11.00192 inches. Because the interval includes 11, the value indicating that the production process is working properly, you have no reason to believe that anything is wrong with the production process.

To see the effect of using a 99% confidence interval, examine Example 8.2.

### EXAMPLE 8.2 ESTIMATING THE MEAN PAPER LENGTH WITH 99% CONFIDENCE

Construct a 99% confidence interval estimate for the population mean paper length.

**SOLUTION** Using Equation (8.1) on page 287, with  $Z = 2.58$  for 99% confidence,

$$\begin{aligned}\bar{X} \pm Z \frac{\sigma}{\sqrt{n}} &= 10.998 \pm (2.58) \frac{0.02}{\sqrt{100}} \\ &= 10.998 \pm 0.00516 \\ 10.99284 &\leq \mu \leq 11.00316\end{aligned}$$

Once again, because 11 is included within this wider interval, you have no reason to believe that anything is wrong with the production process.



As discussed in section 7.4, the sampling distribution of  $\bar{X}$  is normally distributed if the population of  $X$  is a normal distribution. And, if the population of  $X$  is not a normal distribution, the Central Limit Theorem ensures that  $\bar{X}$  is normally distributed when  $n$  is large. However, when dealing with a small sample size and a population of  $X$  that is not a normal distribution, the sampling distribution of  $\bar{X}$  is not normally distributed and therefore the confidence interval discussed in this section is inappropriate. In practice, however, as long as the sample size is large enough and the population is not very skewed, you can use the confidence interval defined in Equation 8.1 to estimate the population mean when  $\sigma$  is known. To assess the assumption of normality, you can evaluate the shape of the sample data by using a histogram, stem-and-leaf display, box-and-whisker plot, or normal probability plot.

## PROBLEMS FOR SECTION 8.1

### Learning the Basics

**PH Grade ASSIST** **8.1** If  $\bar{X} = 85$ ,  $\sigma = 8$ , and  $n = 64$ , construct a 95% confidence interval estimate of the population mean,  $\mu$ .

**PH Grade ASSIST** **8.2** If  $\bar{X} = 125$ ,  $\sigma = 24$ , and  $n = 36$ , construct a 99% confidence interval estimate of the population mean,  $\mu$ .

**8.3** A market researcher states that she has 95% confidence that the mean monthly sales of a product are between \$170,000 and \$200,000. Explain the meaning of this statement.

**8.4** Why is it not possible in Example 8.1 on page 288 to have 100% confidence? Explain.

**8.5** From the results of Example 8.1 on page 288 regarding paper production, is it true that 95% of the sample means will fall between 10.99408 and 11.00192 inches? Explain.

**8.6** Is it true in Example 8.1 on page 288 that you do not know for sure whether the population mean is between 10.99408 and 11.00192 inches? Explain.

### Applying the Concepts

**PH Grade ASSIST** **8.7** The manager of a paint supply store wants to estimate the actual amount of paint contained in 1-gallon cans purchased from a nationally known manufacturer. The manufacturer's specifications state that the standard deviation of the amount of paint is equal to 0.02 gallon. A random sample of 50 cans is selected, and the sample mean amount of paint per 1-gallon can is 0.995 gallon.

- Construct a 99% confidence interval estimate of the population mean amount of paint included in a 1-gallon can.
- On the basis of these results, do you think the manager has a right to complain to the manufacturer? Why?

- Must you assume that the population amount of paint per can is normally distributed here? Explain.
- Construct a 95% confidence interval estimate. How does this change your answer to (b)?

**PH Grade ASSIST** **8.8** The quality control manager at a light bulb factory needs to estimate the mean life of a large shipment of light bulbs. The standard deviation is 100 hours. A random sample of 64 light bulbs indicated a sample mean life of 350 hours.

- Construct a 95% confidence interval estimate of the population mean life of light bulbs in this shipment.
- Do you think that the manufacturer has the right to state that the light bulbs last an average of 400 hours? Explain.
- Must you assume that the population of light bulb life is normally distributed? Explain.
- Suppose that the standard deviation changes to 80 hours. What are your answers in (a) and (b)?

**PH Grade ASSIST** **8.9** The inspection division of the Lee County Weights and Measures Department wants to estimate the actual amount of soft drink in 2-liter bottles at the local bottling plant of a large nationally known soft-drink company. The bottling plant has informed the inspection division that the population standard deviation for 2-liter bottles is 0.05 liter. A random sample of 100 2-liter bottles at this bottling plant indicates a sample mean of 1.99 liters.

- Construct a 95% confidence interval estimate of the population mean amount of soft drink in each bottle.
- Must you assume that the population of soft-drink fill is normally distributed? Explain.
- Explain why a value of 2.02 liters for a single bottle is not unusual, even though it is outside the confidence interval you calculated.
- Suppose that the sample mean is 1.97 liters. What is your answer to (a)?

## 8.2 CONFIDENCE INTERVAL ESTIMATION FOR THE MEAN ( $\sigma$ UNKNOWN)

Just as the mean of the population,  $\mu$ , is usually unknown, you rarely know the actual standard deviation of the population,  $\sigma$ . Therefore, you often need to construct a confidence interval estimate of  $\mu$ , using only the sample statistics  $\bar{X}$  and  $S$ .

### Student's $t$ Distribution

At the beginning of the twentieth century, William S. Gosset, a statistician for Guinness Breweries in Ireland (see reference 3) wanted to make inferences about the mean when  $\sigma$  was unknown. Because Guinness employees were not permitted to publish research work under their own names, Gosset adopted the pseudonym “Student.” The distribution that he developed is known as **Student's  $t$  distribution** and is commonly referred to as the  $t$  distribution.

If the random variable  $X$  is normally distributed, then the following statistic has a  $t$  distribution with  $n - 1$  **degrees of freedom**:

$$t = \frac{\bar{X} - \mu}{\frac{S}{\sqrt{n}}}$$

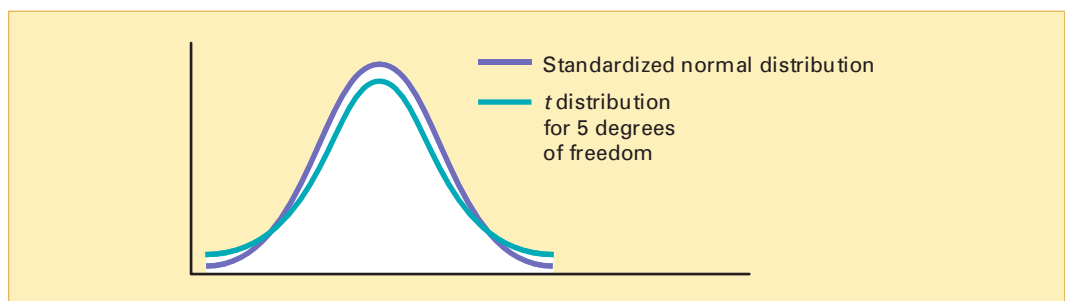
This expression has the same form as the  $Z$  statistic in Equation (7.4) on page 266, except that  $S$  is used to estimate the unknown  $\sigma$ . The concept of *degrees of freedom* is discussed further on pages 291–292.

### Properties of the $t$ Distribution

In appearance, the  $t$  distribution is very similar to the standardized normal distribution. Both distributions are bell shaped. However, the  $t$  distribution has more area in the tails and less in the center than does the standardized normal distribution (see Figure 8.4). Because  $S$  is used to estimate the unknown  $\sigma$ , the values of  $t$  are more variable than those for  $Z$ .

**FIGURE 8.4**

Standardized normal distribution and  $t$  distribution for 5 degrees of freedom



The degrees of freedom,  $n - 1$ , are directly related to the sample size,  $n$ . As the sample size and degrees of freedom increase,  $S$  becomes a better estimate of  $\sigma$ , and the  $t$  distribution gradually approaches the standardized normal distribution, until the two are virtually identical. With a sample size of about 120 or more,  $S$  estimates  $\sigma$  precisely enough that there is little difference between the  $t$  and  $Z$  distributions.

As stated earlier, the  $t$  distribution assumes that the random variable  $X$  is normally distributed. In practice, however, as long as the sample size is large enough and the population is not very skewed, you can use the  $t$  distribution to estimate the population mean when  $\sigma$  is unknown. When dealing with a small sample size and a skewed population distribution, the validity of the confidence interval is a concern. To assess the assumption of normality, you can

evaluate the shape of the sample data by using a histogram, stem-and-leaf display, box-and-whisker plot, or normal probability plot.

You find the critical values of  $t$  for the appropriate degrees of freedom from the table of the  $t$  distribution (see Table E.3). The columns of the table represent the area in the upper tail of the  $t$  distribution. The rows of the table represent the degrees of freedom. The cells of the table represent the particular  $t$  value for each specific degree of freedom. For example, with 99 degrees of freedom, if you want 95% confidence, you find the appropriate value of  $t$ , as shown in Table 8.1. The 95% confidence level means that 2.5% of the values (an area of 0.025) are in each tail of the distribution. Looking in the column for an upper-tail area of 0.025 and in the row corresponding to 99 degrees of freedom gives you a critical value for  $t$  of 1.9842. Because  $t$  is a symmetrical distribution with a mean of 0, if the upper-tail value is +1.9842, the value for the lower-tail area (lower 0.025) is -1.9842. A  $t$  value of -1.9842 means that the probability that  $t$  is less than -1.9842 is 0.025, or 2.5% (see Figure 8.5). Note that for a 95% confidence interval, you will always use an upper-tail area of 0.025. Similarly, for a 99% confidence interval, use 0.005, for 98% use 0.01, 90% use 0.05, and 80% use 0.10.

**TABLE 8.1**

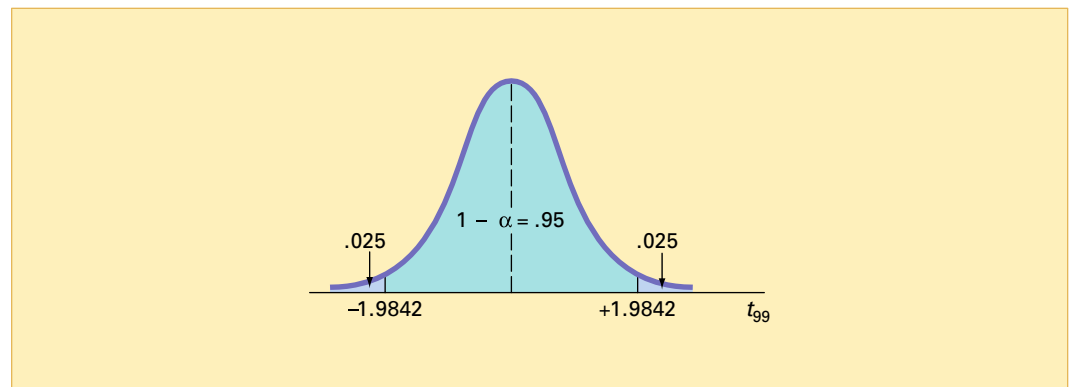
Determining the Critical Value from the  $t$  Table for an Area of 0.025 in Each Tail with 99 Degrees of Freedom

Degrees of Freedom	Upper-Tail Areas					
	.25	.10	.05	.025	.01	.005
1	1.0000	3.0777	6.3138	12.7062	31.8207	63.6574
2	0.8165	1.8856	2.9200	4.3027	6.9646	9.9248
3	0.7649	1.6377	2.3534	3.1824	4.5407	5.8409
4	0.7407	1.5332	2.1318	2.7764	3.7469	4.6041
5	0.7267	1.4759	2.0150	2.5706	3.3649	4.0322
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.
96	0.6771	1.2904	1.6609	1.9850	2.3658	2.6280
97	0.6770	1.2903	1.6607	1.9847	2.3654	2.6275
98	0.6770	1.2902	1.6606	1.9845	2.3650	2.6269
99	0.6770	1.2902	1.6604	1.9842	2.3646	2.6264
100	0.6770	1.2901	1.6602	1.9840	2.3642	2.6259

Source: Extracted from Table E.3.

**FIGURE 8.5**

$t$  distribution with 99 degrees of freedom



### The Concept of Degrees of Freedom

In Chapter 3 you learned that the numerator of the sample variance,  $S^2$  [see Equation (3.9) on page 107], requires the computation of

$$\sum_{i=1}^n (X_i - \bar{X})^2$$

In order to compute  $S^2$ , you first need to know  $\bar{X}$ . Therefore, only  $n - 1$  of the sample values are free to vary. This means that you have  $n - 1$  degrees of freedom. For example, suppose a sample of five values has a mean of 20. How many values do you need to know before you can determine the remainder of the values? The fact that  $n = 5$  and  $\bar{X} = 20$  also tells you that

$$\sum_{i=1}^n X_i = 100$$

because

$$\frac{\sum_{i=1}^n X_i}{n} = \bar{X}$$

Thus, when you know four of the values, the fifth one is *not* free to vary because the sum must add to 100. For example, if four of the values are 18, 24, 19, and 16, the fifth value must be 23 so that the sum equals 100.

### The Confidence Interval Statement

Equation (8.2) defines the  $(1 - \alpha) \times 100\%$  confidence interval estimate for the mean with  $\sigma$  unknown.

#### CONFIDENCE INTERVAL FOR THE MEAN ( $\sigma$ UNKNOWN)

$$\bar{X} \pm t_{n-1} \frac{S}{\sqrt{n}}$$

or

$$\bar{X} - t_{n-1} \frac{S}{\sqrt{n}} \leq \mu \leq \bar{X} + t_{n-1} \frac{S}{\sqrt{n}} \quad (8.2)$$

where  $t_{n-1}$  is the critical value of the  $t$  distribution, with  $n - 1$  degrees of freedom for an area of  $\alpha/2$  in the upper tail.

To illustrate the application of the confidence interval estimate for the mean when the standard deviation,  $\sigma$ , is unknown, recall the Saxon Home Improvement Company Using Statistics scenario presented on page 284. You wanted to estimate the mean dollar amount listed on the sales invoices for the month. You select a sample of 100 sales invoices from the population of sales invoices during the month, and the sample mean of the 100 sales invoices is \$110.27, with a sample standard deviation of \$28.95. For 95% confidence, the critical value from the  $t$  distribution (as shown in Table 8.1) is 1.9842. Using Equation (8.2),

$$\begin{aligned} & \bar{X} \pm t_{n-1} \frac{S}{\sqrt{n}} \\ & = 110.27 \pm (1.9842) \frac{28.95}{\sqrt{100}} \\ & = 110.27 \pm 5.74 \\ & \$104.53 \leq \mu \leq \$116.01 \end{aligned}$$

A Microsoft Excel worksheet for these data is presented in Figure 8.6.

**FIGURE 8.6**

Microsoft Excel worksheet to compute a confidence interval estimate for the mean sales invoice amount for the Saxon Home Improvement Company



See Section E8.2 to create this.

	A	B	
1	<b>Estimate for the Mean Sales Invoice Amount</b>		
2			
3	<b>Data</b>		
4	<b>Sample Standard Deviation</b>	28.95	
5	<b>Sample Mean</b>	110.27	
6	<b>Sample Size</b>	100	
7	<b>Confidence Level</b>	95%	
8			
9	<b>Intermediate Calculations</b>		
10	Standard Error of the Mean	2.8950	=B4/SQRT(B6)
11	Degrees of Freedom	99	=B6 - 1
12	t Value	1.9842	=TINV(1 - B7, B11)
13	Interval Half Width	5.7443	=B12 * B10
14			
15	<b>Confidence Interval</b>		
16	<b>Interval Lower Limit</b>	104.53	=B5 - B13
17	<b>Interval Upper Limit</b>	116.01	=B5 + B13

Thus, with 95% confidence, you conclude that the mean amount of all the sales invoices is between \$104.53 and \$116.01. The 95% confidence level indicates that if you selected all possible samples of 100 (something that is never done in practice), 95% of the intervals developed would include the population mean somewhere within the interval. The validity of this confidence interval estimate depends on the assumption of normality for the distribution of the amount of the sales invoices. With a sample of 100, the normality assumption is not overly restrictive, and the use of the  $t$  distribution is likely appropriate. Example 8.3 further illustrates how you construct the confidence interval for a mean when the population standard deviation is unknown.

**EXAMPLE 8.3****ESTIMATING THE MEAN FORCE REQUIRED TO BREAK ELECTRIC INSULATORS**

A manufacturing company produces electric insulators. If the insulators break when in use, a short circuit is likely. To test the strength of the insulators, you carry out destructive testing to determine how much *force* is required to break the insulators. You measure force by observing how many pounds are applied to the insulator before it breaks. Table 8.2 lists 30 values from this experiment, which are located in the file [force.xls](#). Construct a 95% confidence interval estimate for the population mean force required to break the insulator.

**TABLE 8.2**

Force (in Pounds) Required to Break the Insulator

1,870	1,728	1,656	1,610	1,634	1,784	1,522	1,696	1,592	1,662
1,866	1,764	1,734	1,662	1,734	1,774	1,550	1,756	1,762	1,866
1,820	1,744	1,788	1,688	1,810	1,752	1,680	1,810	1,652	1,736

**SOLUTION** Figure 8.7 shows that the sample mean is  $\bar{X} = 1,723.4$  pounds and the sample standard deviation is  $S = 89.55$  pounds. Using Equation (8.2) on page 292 to construct the confidence interval, you need to determine the critical value from the  $t$  table for an area of 0.025 in

**FIGURE 8.7**

Microsoft Excel confidence interval estimate for the mean amount of force required to break electric insulators



See Section E8.2 to create this.

	A	B	
1	<b>Estimate for the Mean Amount of Force Required</b>		
2			
3	<b>Data</b>		
4	<b>Sample Standard Deviation</b>	89.55	
5	<b>Sample Mean</b>	1723.4	
6	<b>Sample Size</b>	30	
7	<b>Confidence Level</b>	95%	
8			
9	<b>Intermediate Calculations</b>		
10	Standard Error of the Mean	16.3495	=B4/SQRT(B6)
11	Degrees of Freedom	29	=B6 - 1
12	t Value	2.0452	=TINV(1 - B7, B11)
13	Interval Half Width	33.4385	=B12 * B10
14			
15	<b>Confidence Interval</b>		
16	<b>Interval Lower Limit</b>	1689.96	=B5 - B13
17	<b>Interval Upper Limit</b>	1756.84	=B5 + B13

each tail, with 29 degrees of freedom. From Table E.3, you see that  $t_{29} = 2.0452$ . Thus, using  $\bar{X} = 1,723.4$ ,  $S = 89.55$ ,  $n = 30$ , and  $t_{29} = 2.0452$ ,

$$\begin{aligned} & \bar{X} \pm t_{n-1} \frac{S}{\sqrt{n}} \\ &= 1,723.4 \pm (2.0452) \frac{89.55}{\sqrt{30}} \\ &= 1,723.4 \pm 33.44 \\ & 1,689.96 \leq \mu \leq 1,756.84 \end{aligned}$$

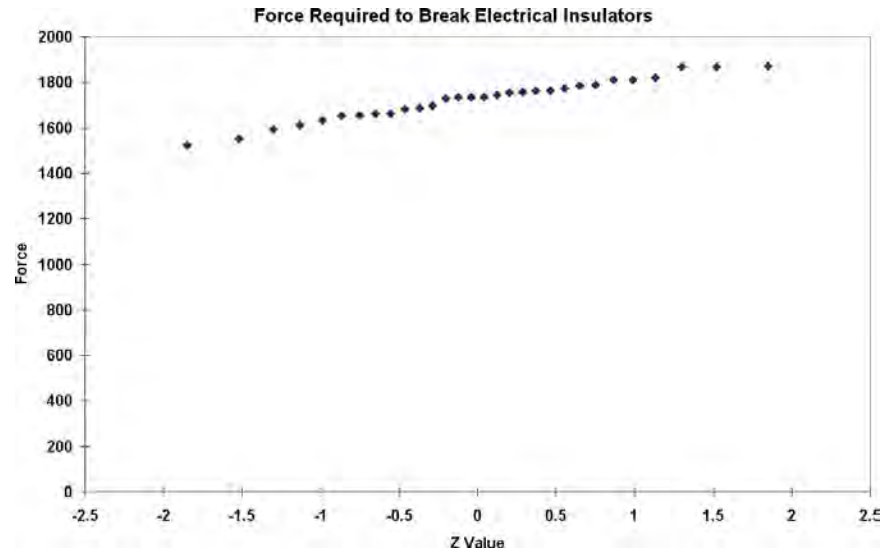
You conclude with 95% confidence that the mean breaking force required for the population of insulators is between 1,689.96 and 1,756.84 pounds. The validity of this confidence interval estimate depends on the assumption that the force required is normally distributed. Remember, however, that you can slightly relax this assumption for large sample sizes. Thus, with a sample of 30, you can use the  $t$  distribution even if the amount of force required is only slightly left skewed. From the normal probability plot displayed in Figure 8.8 or the box-and-whisker plot displayed in Figure 8.9, the amount of force required appears slightly left skewed. Thus, the  $t$  distribution is appropriate for these data.

**FIGURE 8.8**

Microsoft Excel normal probability plot for the amount of force required to break electric insulators



See Section E6.2 to create this.

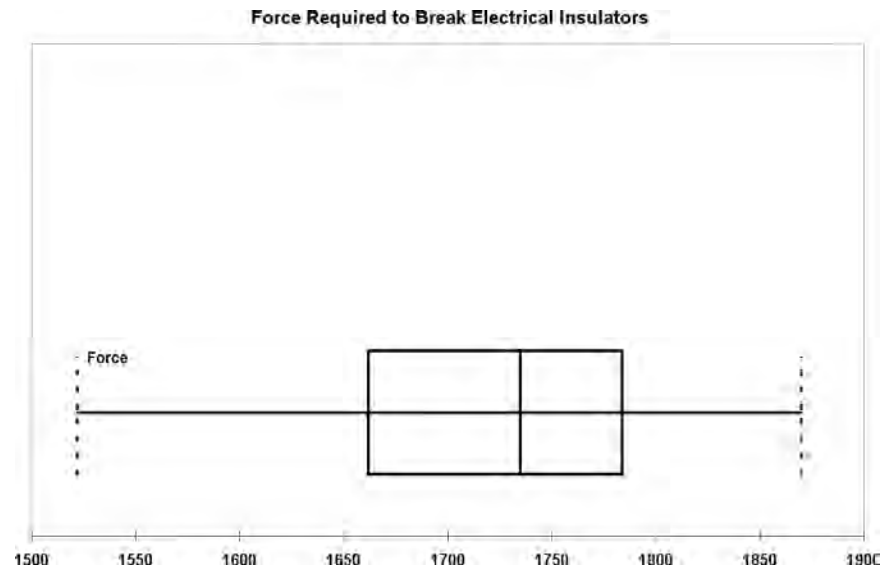


**FIGURE 8.9**

Microsoft Excel box-and-whisker plot for the amount of force required to break electric insulators



See Section E3.4 to create this.



## PROBLEMS FOR SECTION 8.2

### Learning the Basics

**PH Grade ASSIST** **8.10** Determine the critical value of  $t$  in each of the following circumstances:

- $1 - \alpha = 0.95, n = 10$
- $1 - \alpha = 0.99, n = 10$
- $1 - \alpha = 0.95, n = 32$
- $1 - \alpha = 0.95, n = 65$
- $1 - \alpha = 0.90, n = 16$

**PH Grade ASSIST** **8.11** If  $\bar{X} = 75, S = 24,$  and  $n = 36,$  and assuming that the population is normally distributed, construct a 95% confidence interval estimate of the population mean,  $\mu.$

**PH Grade ASSIST** **8.12** If  $\bar{X} = 50, S = 15,$  and  $n = 16,$  and assuming that the population is normally distributed, construct a 99% confidence interval estimate of the population mean,  $\mu.$

**8.13** Construct a 95% confidence interval estimate for the population mean, based on each of the following sets of data, assuming that the population is normally distributed:

Set 1 1, 1, 1, 1, 8, 8, 8, 8

Set 2 1, 2, 3, 4, 5, 6, 7, 8

Explain why these data sets have different confidence intervals even though they have the same mean and range.

**8.14** Construct a 95% confidence interval for the population mean, based on the numbers 1, 2, 3, 4, 5, 6, and 20. Change the number 20 to 7 and recalculate the confidence interval. Using these results, describe the effect of an outlier (that is, an extreme value) on the confidence interval.

### Applying the Concepts

**8.15** A stationery store wants to estimate the mean retail value of greeting cards that it has in its inventory. A random sample of 100 greeting cards indicates a mean value of \$2.55 and a standard deviation of \$0.44.

- Assuming a normal distribution, construct a 95% confidence interval estimate of the mean value of all greeting cards in the store's inventory.
- Suppose there were 2,500 greeting cards in the store's inventory. How are the results in (a) useful in assisting the store owner to estimate the total value of her inventory?

**SELF Test** **8.16** Southside Hospital in Bay Shore, New York, commonly conducts stress tests to study the heart muscle after a person has a heart attack.

Members of the diagnostic imaging department conducted a quality improvement project to try to reduce the turn-

around time for stress tests. Turnaround time is defined as the time from when the test is ordered to when the radiologist signs off on the test results. Initially, the mean turnaround time for a stress test was 68 hours. After incorporating changes into the stress-test process, the quality improvement team collected a sample of 50 turnaround times. In this sample, the mean turnaround time was 32 hours, with a standard deviation of 9 hours (Extracted from E. Godin, D. Raven, C. Sweetapple, and F. R. Del Guidice, "Faster Test Results," *Quality Progress*, January 2004, 37(1), pp. 33–39).

- Construct a 95% confidence interval for the population mean turnaround time.
- Interpret the interval constructed in (a).
- Do you think the quality improvement project was a success? Explain.

**PH Grade ASSIST** **8.17** The U.S. Department of Transportation requires tire manufacturers to provide tire performance information on the sidewall of the tire to better inform prospective customers when making purchasing decisions. One very important measure of tire performance is the tread wear index, which indicates the tire's resistance to tread wear compared with a tire graded with a base of 100. This means that a tire with a grade of 200 should last twice as long, on average, as a tire graded with a base of 100. A consumer organization wants to estimate the actual tread wear index of a brand name of tires graded 200 that are produced by a certain manufacturer. A random sample of  $n = 18$  indicates a sample mean tread wear index of 195.3 and a sample standard deviation of 21.4.

- Assuming that the population of tread wear indices is normally distributed, construct a 95% confidence interval estimate of the population mean tread wear index for tires produced by this manufacturer under this brand name.
- Do you think that the consumer organization should accuse the manufacturer of producing tires that do not meet the performance information provided on the sidewall of the tire? Explain.
- Explain why an observed tread wear index of 210 for a particular tire is not unusual, even though it is outside the confidence interval developed in (a).

**8.18** The following data (stored in the file [bankcost1.xls](#)) represent the bounced check fee, in dollars, charged by a sample of 23 banks for direct-deposit customers who maintain a \$100 balance:

26	28	20	20	21	22	25	25	18	25	15	20
18	20	25	25	22	30	30	30	15	20	29	

Source: Extracted from "The New Face of Banking," Consumer Reports, June 2000.

- Construct a 95% confidence interval for the population mean bounced check fee.
- Interpret the interval constructed in (a)

**8.19** The data in the file `chicken.xls` represent the total fat, in grams per serving, for a sample of 20 chicken sandwiches from fast-food chains. The data are as follows:

```
7 8 4 5 16 20 20 24 19 30
23 30 25 19 29 29 30 30 40 56
```

Source: Extracted from "Fast Food: Adding Health to the Menu," Consumer Reports, September 2004, pp. 28–31.

- Construct a 95% confidence interval for the population mean total fat, in grams per serving.
- Interpret the interval constructed in (a).

**8.20** One of the major measures of the quality of service provided by any organization is the speed with which it responds to customer complaints. A large family-held department store selling furniture and flooring, including carpet, had undergone a major expansion in the past several years. In particular, the flooring department had expanded from 2 installation crews to an installation supervisor, a measurer, and 15 installation crews. Last year, there were 50 complaints concerning carpet installation. The following data, in the file `furniture.xls`, represent the number of days between the receipt of a complaint and the resolution of the complaint:

```
54 5 35 137 31 27 152 2 123 81 74 27
11 19 126 110 110 29 61 35 94 31 26 5
12 4 165 32 29 28 29 26 25 1 14 13
13 10 5 27 4 52 30 22 36 26 20 23
33 68
```

- Construct a 95% confidence interval estimate of the mean number of days between the receipt of a complaint and the resolution of the complaint.
- What assumption must you make about the population distribution in (a)?
- Do you think that the assumption made in (b) is seriously violated? Explain.
- What effect might your conclusion in (c) have on the validity of the results in (a)?

**8.21** In New York State, savings banks are permitted to sell a form of life insurance called savings bank life insurance (SBLI). The approval process consists of underwriting, which includes a review of the application, a medical information bureau check, possible requests for additional medical information and medical exams, and a policy com-

pilation stage in which the policy pages are generated and sent to the bank for delivery. The ability to deliver approved policies to customers in a timely manner is critical to the profitability of this service to the bank. During a period of one month, a random sample of 27 approved policies was selected, and the total processing time, in days, was as shown below and stored in the file `insurance.xls`:

```
73 19 16 64 28 28 31 90 60 56 31 56 22 18
45 48 17 17 17 91 92 63 50 51 69 16 17
```

- Construct a 95% confidence interval estimate of the mean processing time.
- What assumption must you make about the population distribution in (a)?
- Do you think that the assumption made in (b) is seriously violated? Explain.

**8.22** The data in the file `batterylife.xls` represent the battery life (in shots) for three-pixel digital cameras:

```
300 180 85 170 380 460 260 35 380 120 110 240
```

Source: Extracted from "Cameras: More Features in the Mix," Consumer Reports, July 2005, pp. 14–18.

- Construct a 95% confidence interval for the population mean battery life (in shots).
- What assumption do you need to make about the population of interest to construct the interval in (a)?
- Given the data presented, do you think the assumption needed in (a) is valid? Explain.

**8.23** One operation of a mill is to cut pieces of steel into parts that are used later in the frame for front seats in an automobile. The steel is cut with a diamond saw and requires the resulting parts to be within  $\pm 0.005$  inch of the length specified by the automobile company. The measurement reported from a sample of 100 steel parts (and stored in the file `steel.xls`) is the difference, in inches, between the actual length of the steel part, as measured by a laser measurement device, and the specified length of the steel part. For example, the first observation,  $-0.002$ , represents a steel part that is 0.002 inch shorter than the specified length.

- Construct a 95% confidence interval estimate of the mean difference between the actual length of the steel part and the specified length of the steel part.
- What assumption must you make about the population distribution in (a)?
- Do you think that the assumption made in (b) is seriously violated? Explain.
- Compare the conclusions reached in (a) with those of Problem 2.23 on page 53.

### 8.3 CONFIDENCE INTERVAL ESTIMATION FOR THE PROPORTION

This section extends the concept of the confidence interval to categorical data. Here you are concerned with estimating the proportion of items in a population having a certain characteristic of interest. The unknown population proportion is represented by the Greek letter  $\pi$ . The



point estimate for  $\pi$  is the sample proportion,  $p = X/n$ , where  $n$  is the sample size and  $X$  is the number of items in the sample having the characteristic of interest. Equation (8.3) defines the confidence interval estimate for the population proportion.

**CONFIDENCE INTERVAL ESTIMATE FOR THE PROPORTION**

$$p \pm Z \sqrt{\frac{p(1 - p)}{n}}$$

or

$$p - Z \sqrt{\frac{p(1 - p)}{n}} \leq \pi \leq p + Z \sqrt{\frac{p(1 - p)}{n}} \tag{8.3}$$

where

$$p = \text{Sample proportion} = \frac{X}{n} = \frac{\text{Number of items having the characteristic}}{\text{Sample size}}$$

$\pi$  = population proportion

$Z$  = critical value from the standardized normal distribution

$n$  = sample size

assuming that both  $X$  and  $n - X$  are greater than 5

You can use the confidence interval estimate of the proportion defined in Equation (8.3) to estimate the proportion of sales invoices that contain errors (see the Using Statistics scenario on page 284). Suppose that in a sample of 100 sales invoices, 10 contain errors. Thus, for these data,  $p = X/n = 10/100 = 0.10$ . Using Equation (8.3) and  $Z = 1.96$  for 95% confidence,

$$\begin{aligned} & p \pm Z \sqrt{\frac{p(1 - p)}{n}} \\ &= 0.10 \pm (1.96) \sqrt{\frac{(0.10)(0.90)}{100}} \\ &= 0.10 \pm (1.96)(0.03) \\ &= 0.10 \pm 0.0588 \\ &0.0412 \leq \pi \leq 0.1588 \end{aligned}$$

Therefore, you have 95% confidence that between 4.12% and 15.88% of all the sales invoices contain errors. Figure 8.10 shows a Microsoft Excel worksheet for these data.

**FIGURE 8.10**

Microsoft Excel worksheet to construct a confidence interval estimate for the proportion of sales invoices that contain errors



See Section E8.3 to create this.

	A	B	
1	<b>Proportion of In-Error Sales Invoices</b>		
2			
3	<b>Data</b>		
4	<b>Sample Size</b>	<b>100</b>	
5	<b>Number of Successes</b>	<b>10</b>	
6	<b>Confidence Level</b>	<b>95%</b>	
7			
8	<b>Intermediate Calculations</b>		
9	Sample Proportion	0.1	=B5/B4
10	Z Value	-1.9600	=NORMSINV((1 - B6)/2)
11	Standard Error of the Proportion	0.03	=SQRT(B9 * (1 - B9)/B4)
12	Interval Half Width	0.0588	=ABS(B10 * B11)
13			
14	<b>Confidence Interval</b>		
15	<b>Interval Lower Limit</b>	<b>0.0412</b>	=B9 - B12
16	<b>Interval Upper Limit</b>	<b>0.1588</b>	=B9 + B12

Example 8.4 illustrates another application of a confidence interval estimate for the proportion.

**EXAMPLE 8.4****ESTIMATING THE PROPORTION OF NONCONFORMING NEWSPAPERS PRINTED**

A large newspaper wants to estimate the proportion of newspapers printed that have a nonconforming attribute, such as excessive ruboff, improper page setup, missing pages, or duplicate pages. A random sample of 200 newspapers is selected from all the newspapers printed during a single day. For this sample of 200, 35 contain some type of nonconformance. Construct and interpret a 90% confidence interval for the proportion of newspapers printed during the day that have a nonconforming attribute.

**SOLUTION** Using Equation (8.3),

$$p = \frac{35}{200} = 0.175, \text{ and with a 90\% level of confidence } Z = 1.645$$

$$\begin{aligned} p \pm Z \sqrt{\frac{p(1-p)}{n}} \\ &= 0.175 \pm (1.645) \sqrt{\frac{(0.175)(0.825)}{200}} \\ &= 0.175 \pm (1.645)(0.0269) \\ &= 0.175 \pm 0.0442 \\ &0.1308 \leq \pi \leq 0.2192 \end{aligned}$$

You conclude with 90% confidence that between 13.08% and 21.92% of the newspapers printed on that day have some type of nonconformance.

Equation (8.3) contains a  $Z$  statistic because you can use the normal distribution to approximate the binomial distribution when the sample size is sufficiently large. In Example 8.4, the confidence interval using  $Z$  provides an excellent approximation for the population proportion because both  $X$  and  $n - X$  are greater than 5. However, if you do not have a sufficiently large sample size, you should use the binomial distribution rather than Equation (8.3) (see references 1, 2, and 6). The exact confidence intervals for various sample sizes and proportions of successes have been tabulated by Fisher and Yates (reference 2).

## PROBLEMS FOR SECTION 8.3

### Learning the Basics

**PH Grade ASSIST** **8.24** If  $n = 200$  and  $X = 50$ , construct a 95% confidence interval estimate of the population proportion.

**PH Grade ASSIST** **8.25** If  $n = 400$  and  $X = 25$ , construct a 99% confidence interval estimate of the population proportion.

### Applying the Concepts

**PH Grade ASSIST** **8.26** The telephone company wants to estimate the proportion of households that would purchase an additional telephone line if it were made available at a substantially reduced installation cost. A random sample of 500 households is selected. The results indicate that 135 of the households would purchase the additional telephone line at a reduced installation cost.

- a. Construct a 99% confidence interval estimate of the population proportion of households that would purchase the additional telephone line.
- b. How would the manager in charge of promotional programs concerning residential customers use the results in (a)?

**8.27** According to the Center for Work-Life Policy, a survey of 500 highly educated women who left careers for family reasons found that 66% wanted to return to work (Extracted from A. M. Chaker and H. Stout, “After Years Off, Women Struggle to Revive Careers,” *The Wall Street Journal*, May 6, 2004, p. A1).

- a. Construct a 95% confidence interval for the population proportion of highly educated women who left careers for family reasons who want to return to work.
- b. Interpret the interval in (a).

**8.28** In a survey conducted for American Express, 27% of small business owners indicated that they never check in with the office when on vacation (“Snapshots,” *usatoday.com*, April 18, 2006.). The article did not disclose the sample size used in the study.

- a. Suppose that the survey was based on 500 small business owners. Construct a 95% confidence interval estimate for the population proportion of small business owners who never check in with the office when on vacation.
- b. Suppose that the survey was based on 1,000 small business owners. Construct a 95% confidence interval estimate for the population proportion of small business owners who never check in with the office when on vacation.
- c. Discuss the effect of sample size on the confidence interval estimate.

**8.29** The number of older consumers in the United States is growing, and they are becoming an even bigger economic force. Many feel overwhelmed when confronted with the task of selecting investments, banking services, health care providers, or phone service providers. A telephone survey of 1,900 older consumers found that 27% said they didn’t have enough time to be good money managers (Extracted from “Seniors Confused by Financial Choices—Study,” *msnbc.com*, May 6, 2004).

- a. Construct a 95% confidence interval for the population proportion of older consumers who don’t think they have enough time to be good money managers.
- b. Interpret the interval in (a).

**8.30** A survey of 705 workers (*USA Today Snapshots*, March 21, 2006, p. 1B) were asked how much they used

the Internet at work. 423 said they used it within limits, and 183 said that they did not use the Internet at work.

- a. Construct a 95% confidence interval for the proportion of all workers who used the Internet within limits.
- b. Construct a 95% confidence interval for the proportion of all workers who did not use the Internet at work.

**8.31** When do Americans decide what to make for dinner? An online survey (N. Hellmich, “Americans Go for the Quick Fix for Dinner,” *USA Today*, February 14, 2005, p. 1B) indicated that 74% of Americans decided either at the last minute or that day. Suppose that the survey was based on 500 respondents.

- a. Construct a 95% confidence interval for the proportion of Americans who decided what to make for dinner either at the last minute or that day.
- b. Construct a 99% confidence interval for the proportion of Americans who decided what to make for dinner either at the last minute or that day.
- c. Which interval is wider? Explain why this is true.

**8.32** In a survey of 894 respondents with salaries below \$100,000 per year, 367 indicated that the primary reason for staying on their job was interesting job responsibilities (“What Is the Primary Reason for Staying on Your Job?” *USA Today Snapshots*, October 5, 2005, p. 1B).

- a. Construct a 95% confidence interval for the proportion of all workers whose primary reason for staying on their job was interesting job responsibilities.
- b. Interpret the interval constructed in (a).

**8.33** A large number of companies are trying to reduce the cost of prescription drug benefits by requiring employees to purchase drugs through a mandatory mail-order program. In a survey of 600 employers, 126 indicated that they either had a mandatory mail-order program in place or were adopting one by the end of 2004 (B. Martinez, “Forcing Employees to Buy Drugs via Mail,” *The Wall Street Journal*, February 18, 2004, p. 1B).

- a. Construct a 95% confidence interval for the population proportion of employers who had a mandatory mail-order program in place or were adopting one by the end of 2004.
- b. Construct a 99% confidence interval for the population proportion of employers who had a mandatory mail-order program in place or were adopting one by the end of 2004.
- c. Interpret the intervals in (a) and (b).
- d. Discuss the effect on the confidence interval estimate when you change the level of confidence.

## 8.4 DETERMINING SAMPLE SIZE

In each example of confidence interval estimation so far in this chapter, the sample size was reported along with the results with little discussion with regard to the width of the resulting confidence interval. In the business world, sample sizes are determined prior to data

collection to ensure that the confidence interval is narrow enough to be useful in making decisions. Determining the proper sample size is a complicated procedure, subject to the constraints of budget, time, and the amount of acceptable sampling error. In the Saxon Home Improvement example, you want to estimate the mean dollar amount of the sales invoices, you must determine in advance how large a sampling error to allow in estimating the population mean. You must also determine in advance the level of confidence (that is, 90%, 95%, or 99%) to use in estimating the population parameter.

### Sample Size Determination for the Mean

To develop an equation for determining the appropriate sample size needed when constructing a confidence interval estimate of the mean, recall Equation (8.1) on page 287:

$$\bar{X} \pm Z \frac{\sigma}{\sqrt{n}}$$

The amount added to or subtracted from  $\bar{X}$  is equal to half the width of the interval. This quantity represents the amount of imprecision in the estimate that results from sampling error. The **sampling error**,<sup>1</sup>  $e$ , is defined as

<sup>1</sup>In this context, some statisticians refer to  $e$  as the “margin of error.”

$$e = Z \frac{\sigma}{\sqrt{n}}$$

Solving for  $n$  gives the sample size needed to construct the appropriate confidence interval estimate for the mean. “Appropriate” means that the resulting interval will have an acceptable amount of sampling error.

#### SAMPLE SIZE DETERMINATION FOR THE MEAN

The sample size,  $n$ , is equal to the product of the  $Z$  value squared and the variance,  $\sigma$ , squared, divided by the square of the sampling error,  $e$ ,

$$n = \frac{Z^2 \sigma^2}{e^2} \quad (8.4)$$

<sup>2</sup>You use  $Z$  instead of  $t$  because, to determine the critical value of  $t$ , you need to know the sample size, but you do not know it yet. For most studies, the sample size needed is large enough that the standardized normal distribution is a good approximation of the  $t$  distribution.

To determine the sample size, you must know three factors:

1. The desired confidence level, which determines the value of  $Z$ , the critical value from the standardized normal distribution<sup>2</sup>
2. The acceptable sampling error,  $e$
3. The standard deviation,  $\sigma$

In some business-to-business relationships that require estimation of important parameters, legal contracts specify acceptable levels of sampling error and the confidence level required. For companies in the food or drug sectors, government regulations often specify sampling errors and confidence levels. In general, however, it is usually not easy to specify

the two factors needed to determine the sample size. How can you determine the level of confidence and sampling error? Typically, these questions are answered only by the subject matter expert (that is, the individual most familiar with the variables under study). Although 95% is the most common confidence level used, if more confidence is desired, then 99% might be more appropriate; if less confidence is deemed acceptable, then 90% might be used. For the sampling error, you should think not of how much sampling error you would like to have (you really do not want any error) but of how much you can tolerate when drawing conclusions from the data.

In addition to specifying the confidence level and the sampling error, you need an estimate of the standard deviation. Unfortunately, you rarely know the population standard deviation,  $\sigma$ . In some instances, you can estimate the standard deviation from past data. In other situations, you can make an educated guess by taking into account the range and distribution of the variable. For example, if you assume a normal distribution, the range is approximately equal to  $6\sigma$  (that is,  $\pm 3\sigma$  around the mean) so that you estimate  $\sigma$  as the range divided by 6. If you cannot estimate  $\sigma$  in this way, you can conduct a small-scale study and estimate the standard deviation from the resulting data.

To explore how to determine the sample size needed for estimating the population mean, consider again the audit at Saxon Home Improvement. In Section 8.2, you selected a sample of 100 sales invoices and constructed a 95% confidence interval estimate of the population mean sales invoice amount. How was this sample size determined? Should you have selected a different sample size?

Suppose that, after consultation with company officials, you determine that a sampling error of no more than  $\pm \$5$  is desired, along with 95% confidence. Past data indicate that the standard deviation of the sales amount is approximately \$25. Thus,  $e = \$5$ ,  $\sigma = \$25$ , and  $Z = 1.96$  (for 95% confidence). Using Equation (8.4),

$$n = \frac{Z^2 \sigma^2}{e^2} = \frac{(1.96)^2 (25)^2}{(5)^2} = 96.04$$

Because the general rule is to slightly oversatisfy the criteria by rounding the sample size up to the next whole integer, you should select a sample of size 97. Thus, the sample of 100 used on page 292 is close to what is necessary to satisfy the needs of the company, based on the estimated standard deviation, desired confidence level, and sampling error. Because the calculated sample standard deviation is slightly higher than expected, \$28.95 compared to \$25.00, the confidence interval is slightly wider than desired. Figure 8.11 illustrates a Microsoft Excel worksheet to determine the sample size.

**FIGURE 8.11**

Microsoft Excel worksheet for determining sample size for estimating the mean sales invoice amount for the Saxon Home Improvement Company



See Section E8.4 to create this.

	A	B	
1	<b>For the Mean Sales Invoice Amount</b>		
2			
3	<b>Data</b>		
4	<b>Population Standard Deviation</b>	25	
5	<b>Sampling Error</b>	5	
6	<b>Confidence Level</b>	95%	
7			
8	<b>Intermediate Calculations</b>		
9	Z Value	-1.9600	=NORMSINV((1 - B6)/2)
10	Calculated Sample Size	96.0365	=(B9 * B4)/B5^2
11			
12	<b>Result</b>		
13	<b>Sample Size Needed</b>	97	=ROUNDUP(B10, 0)

Example 8.5 illustrates another application of determining the sample size needed to develop a confidence interval estimate for the mean.

**EXAMPLE 8.5****DETERMINING THE SAMPLE SIZE FOR THE MEAN**

Returning to Example 8.3 on page 293, suppose you want to estimate the population mean force required to break the insulator to within  $\pm 25$  pounds with 95% confidence. On the basis of a study taken the previous year, you believe that the standard deviation is 100 pounds. Find the sample size needed.

**SOLUTION** Using Equation (8.4) on page 300 and  $e = 25$ ,  $\sigma = 100$ , and  $Z = 1.96$  for 95% confidence,

$$\begin{aligned} n &= \frac{Z^2 \sigma^2}{e^2} = \frac{(1.96)^2 (100)^2}{(25)^2} \\ &= 61.47 \end{aligned}$$

Therefore, you should select a sample size of 62 insulators because the general rule for determining sample size is to always round up to the next integer value in order to slightly oversatisfy the criteria desired.

An actual sampling error slightly larger than 25 will result if the sample standard deviation calculated in this sample of 62 is greater than 100 and slightly smaller if the sample standard deviation is less than 100.

**Sample Size Determination for the Proportion**

So far in this section, you have learned how to determine the sample size needed for estimating the population mean. Now suppose that you want to determine the sample size necessary for estimating the proportion of sales invoices at Saxon Home Improvement that contain errors.

To determine the sample size needed to estimate a population proportion,  $\pi$ , you use a method similar to the method for a population mean. Recall that in developing the sample size for a confidence interval for the mean, the sampling error is defined by

$$e = Z \frac{\sigma}{\sqrt{n}}$$

When estimating a proportion, you replace  $\sigma$  with  $\sqrt{\pi(1 - \pi)}$ . Thus, the sampling error is

$$e = Z \sqrt{\frac{\pi(1 - \pi)}{n}}$$

Solving for  $n$ , you have the sample size necessary to develop a confidence interval estimate for a proportion.

**SAMPLE SIZE DETERMINATION FOR THE PROPORTION**

The sample size  $n$  is equal to the  $Z$  value squared times the population proportion,  $\pi$ , times 1 minus the population proportion,  $\pi$ , divided by the square of the sampling error,  $e$ ,

$$n = \frac{Z^2 \pi(1 - \pi)}{e^2} \quad (8.5)$$

To determine the sample size, you must know three factors:

1. The desired confidence level, which determines the value of  $Z$ , the critical value from the standardized normal distribution
2. The acceptable sampling error,  $e$
3. The population proportion,  $\pi$

In practice, selecting these quantities requires some planning. Once you determine the desired level of confidence, you can find the appropriate  $Z$  value from the standardized normal distribution. The sampling error,  $e$ , indicates the amount of error that you are willing to tolerate in estimating the population proportion. The third quantity,  $\pi$ , is actually the population parameter that you want to estimate! Thus, how do you state a value for what you are taking a sample in order to determine?

Here you have two alternatives. In many situations, you may have past information or relevant experience that provide an educated estimate of  $\pi$ . Or, if you do not have past information or relevant experience, you can try to provide a value for  $\pi$  that would never *underestimate* the sample size needed. Referring to Equation (8.5), you can see that the quantity  $\pi(1 - \pi)$  appears in the numerator. Thus, you need to determine the value of  $\pi$  that will make the quantity  $\pi(1 - \pi)$  as large as possible. When  $\pi = 0.5$ , the product  $\pi(1 - \pi)$  achieves its maximum result. To show this, several values of  $\pi$ , along with the accompanying products of  $\pi(1 - \pi)$ , are as follows

When  $\pi = 0.9$ , then  $\pi(1 - \pi) = (0.9)(0.1) = 0.09$

When  $\pi = 0.7$ , then  $\pi(1 - \pi) = (0.7)(0.3) = 0.21$

When  $\pi = 0.5$ , then  $\pi(1 - \pi) = (0.5)(0.5) = 0.25$

When  $\pi = 0.3$ , then  $\pi(1 - \pi) = (0.3)(0.7) = 0.21$

When  $\pi = 0.1$ , then  $\pi(1 - \pi) = (0.1)(0.9) = 0.09$

Therefore, when you have no prior knowledge or estimate of the population proportion,  $\pi$ , you should use  $\pi = 0.5$  for determining the sample size. This produces the largest possible sample size and results in the highest possible cost of sampling. Using  $\pi = 0.5$  may overestimate the sample size needed because you use the actual sample proportion in developing the confidence interval. You will get a confidence interval narrower than originally intended if the actual sample proportion is different from 0.5. The increased precision comes at the cost of spending more time and money for an increased sample size.

Returning to the Saxon Home Improvement Using Statistics scenario, suppose that the auditing procedures require you to have 95% confidence in estimating the population proportion of sales invoices with errors to within  $\pm 0.07$ . The results from past months indicate that the largest proportion has been no more than 0.15. Thus, using Equation (8.5) on page 302 and  $e = 0.07$ ,  $\pi = 0.15$ , and  $Z = 1.96$  for 95% confidence,

$$\begin{aligned} n &= \frac{Z^2 \pi(1 - \pi)}{e^2} \\ &= \frac{(1.96)^2(0.15)(0.85)}{(0.07)^2} \\ &= 99.96 \end{aligned}$$

Because the general rule is to round the sample size up to the next whole integer to slightly oversatisfy the criteria, a sample size of 100 is needed. Thus, the sample size needed to satisfy the requirements of the company, based on the estimated proportion, desired confidence level, and sampling error, is equal to the sample size taken on page 297. The actual confidence interval is narrower than required because the sample proportion is 0.10, while 0.15 was used for  $\pi$  in Equation (8.5). Figure 8.12 shows a Microsoft Excel worksheet for determining sample size.

**FIGURE 8.12**

Microsoft Excel worksheet for determining sample size for estimating the proportion of sales invoices with errors for the Saxon Home Improvement Company



See Section E8.5 to create this.

	A	B
1	<b>For the Proportion of In-Error Sales Invoices</b>	
2		
3	<b>Data</b>	
4	<b>Estimate of True Proportion</b>	<b>0.15</b>
5	<b>Sampling Error</b>	<b>0.07</b>
6	<b>Confidence Level</b>	<b>95%</b>
7		
8	<b>Intermediate Calculations</b>	
9	Z Value	-1.9600 =NORMSINV((1 - B6)/2)
10	Calculated Sample Size	99.9563 =(B9^2 * B4 * (1 - B4))/B5^2
11		
12	<b>Result</b>	
13	<b>Sample Size Needed</b>	<b>100</b> =ROUNDUP(B10, 0)

Example 8.6 provides a second application of determining the sample size for estimating the population proportion.

**EXAMPLE 8.6****DETERMINING THE SAMPLE SIZE FOR THE POPULATION PROPORTION**

You want to have 90% confidence of estimating the proportion of office workers who respond to email within an hour to within  $\pm 0.05$ . Because you have not previously undertaken such a study, there is no information available from past data. Determine the sample size needed.

**SOLUTION** Because no information is available from past data, assume that  $\pi = 0.50$ . Using Equation (8.5) on page 302 and  $e = 0.05$ ,  $\pi = 0.50$ , and  $Z = 1.645$  for 90% confidence,

$$n = \frac{(1.645)^2(0.50)(0.50)}{(0.05)^2}$$

$$= 270.6$$

Therefore, you need a sample of 271 office workers to estimate the population proportion to within  $\pm 0.05$  with 90% confidence.

**PROBLEMS FOR SECTION 8.4****Learning the Basics**

**PH Grade ASSIST** **8.34** If you want to be 95% confident of estimating the population mean to within a sampling error of  $\pm 5$  and the standard deviation is assumed to be 15, what sample size is required?

**PH Grade ASSIST** **8.35** If you want to be 99% confident of estimating the population mean to within a sampling error of  $\pm 20$  and the standard deviation is assumed to be 100, what sample size is required?

**PH Grade ASSIST** **8.36** If you want to be 99% confident of estimating the population proportion to within an error of  $\pm 0.04$ , what sample size is needed?

**PH Grade ASSIST** **8.37** If you want to be 95% confident of estimating the population proportion to within an error of  $\pm 0.02$  and there is historical evidence

that the population proportion is approximately 0.40, what sample size is needed?

**Applying the Concepts**

**SELF Test** **8.38** A survey is planned to determine the mean annual family medical expenses of employees of a large company. The management of the company wishes to be 95% confident that the sample mean is correct to within  $\pm \$50$  of the population mean annual family medical expenses. A previous study indicates that the standard deviation is approximately \$400.

- How large a sample size is necessary?
- If management wants to be correct to within  $\pm \$25$ , what sample size is necessary?

**8.39** If the manager of a paint supply store wants to estimate the mean amount of paint in a 1-gallon can to within



$\pm 0.004$  gallon with 95% confidence and also assumes that the standard deviation is 0.02 gallon, what sample size is needed?

**8.40** If a quality control manager wants to estimate the mean life of light bulbs to within  $\pm 20$  hours with 95% confidence and also assumes that the population standard deviation is 100 hours, what sample size is needed?

**8.41** If the inspection division of a county weights and measures department wants to estimate the mean amount of soft-drink fill in 2-liter bottles to within  $\pm 0.01$  liter with 95% confidence and also assumes that the standard deviation is 0.05 liter, what sample size is needed?

**PH Grade ASSIST** **8.42** A consumer group wants to estimate the mean electric bill for the month of July for single-family homes in a large city. Based on studies conducted in other cities, the standard deviation is assumed to be \$25. The group wants to estimate the mean bill for July to within  $\pm 5$  with 99% confidence.

- What sample size is needed?
- If 95% confidence is desired, what sample size is necessary?

**8.43** An advertising agency that serves a major radio station wants to estimate the mean amount of time that the station's audience spends listening to the radio daily. From past studies, the standard deviation is estimated as 45 minutes.

- What sample size is needed if the agency wants to be 90% confident of being correct to within  $\pm 5$  minutes?
- If 99% confidence is desired, what sample size is necessary?

**8.44** A growing niche in the restaurant business is gourmet-casual breakfast, lunch, and brunch. Chains in this group include Le Peep, Good Egg, Eggs & I, First Watch, and Eggs Up Grill. The mean per-person check for First Watch is approximately \$7, and the mean per-person check for Eggs Up Grill is \$6.50. (Extracted from J. Hayes, "Competition Heats Up as Breakfast Concepts Eye Growth," *Nation's Restaurant News*, April 24, 2006, pp. 8, 66.)

- Assuming a standard deviation of \$2.00, what sample size is needed to estimate the mean per-person check for Good Egg to within \$0.25 with 95% confidence?
- Assuming a standard deviation of \$2.50, what sample size is needed to estimate the mean per-person check for Good Egg to within \$0.25 with 95% confidence?
- Assuming a standard deviation of \$3.00, what sample size is needed to estimate the mean per-person check for Good Egg to within \$0.25 with 95% confidence?
- Discuss the effect of variation on selecting the sample size needed.

**8.45** The U.S. Department of Transportation defines an airline flight as being "on time" if it lands less than 15 minutes after the scheduled time shown in the carrier's comput-

erized reservation system. Cancelled and diverted flights are counted as late. A study of the 10 largest U.S. domestic airlines found Southwest Airlines to have the lowest proportion of late arrivals, at 0.1577 (Extracted from N. Tsirikrisis and J. Heineke, "The Impact of Process Variation on Customer Dissatisfaction: Evidence from the U.S. Domestic Airline Industry," *Decision Sciences*, Winter 2004, 35(1), pp. 129–142). Suppose you were asked to perform a follow-up study for Southwest Airlines in order to update the estimated proportion of late arrivals. What sample size would you use in order to estimate the population proportion to within an error of

- $\pm 0.06$  with 95% confidence?
- $\pm 0.04$  with 95% confidence?
- $\pm 0.02$  with 95% confidence?

**8.46** In 2005, 34% of workers reported that their jobs were more difficult, with more stress, and 37% reported that they worry about retiring comfortably. (Extracted from S. Armour, "Money Worries Hinder Job Performance," *USA Today*, October 5, 2005, p. D1). Consider a follow-up study to be conducted in the near future.

- What sample size is needed to estimate the population proportion of workers who reported that their jobs were more difficult, with more stress, to within  $\pm 0.02$  with 95% confidence?
- What sample size is needed to estimate the population proportion of workers who worried about retiring comfortably to within  $\pm 0.02$  with 95% confidence?
- Compare the results of (a) and (b). Explain why these results differ.
- If you were to design the follow-up study, would you use one sample and ask the respondents both questions, or would you select two separate samples? Explain the rationale behind your decision.

**8.47** What proportion of people hit snags with online transactions? According to a poll conducted by Harris Interactive, 89% hit snags with online transactions ("Top Online Transaction Trouble," *USA Today Snapshots*, April 4, 2006, p. 1D).

- To conduct a follow-up study that would provide 95% confidence that the point estimate is correct to within  $\pm 0.04$  of the population proportion, how large a sample size is required?
- To conduct a follow-up study that would provide 99% confidence that the point estimate is correct to within  $\pm 0.04$  of the population proportion, how large a sample size is required?
- To conduct a follow-up study that would provide 95% confidence that the point estimate is correct to within  $\pm 0.02$  of the population proportion, how large a sample size is required?
- To conduct a follow-up study that would provide 99% confidence that the point estimate is correct to within

$\pm 0.02$  of the population proportion, how large a sample size is required?

- e. Discuss the effects of changing the desired confidence level and the acceptable sampling error on sample size requirements.

**8.48** A poll of 1,286 young adult cell phone users was conducted in March 2006. These cell phone users, aged 18–29, were actively engaged in multiple uses of their cell phones. The data suggest that 707 took still pictures with their phones, 604 played games, and 360 used the Internet (Extracted from “Poll: Cellphones Are Annoying but Invaluable,” *usatoday.com*, April 3, 2006). Construct a 95% confidence interval estimate of the population proportion of young adults that used their cell phone to

- take still pictures.
- play games.
- use the Internet.

- d. You have been asked to update the results of this study. Determine the sample size necessary to estimate the population proportions in (a) through (c) to within  $\pm 0.02$  with 95% confidence.

**8.49** A study of 658 CEOs conducted by the Conference Board reported that 250 stated that their company’s greatest concern was sustained and steady top-line growth (“CEOs’ Greatest Concerns,” *USA Today Snapshots*, May 8, 2006, p. 1 D).

- Construct a 95% confidence interval for the proportion of CEOs whose greatest concern was sustained and steady top-line growth.
- Interpret the interval constructed in (a).
- To conduct a follow-up study to estimate the population proportion of CEOs whose greatest concern was sustained and steady top-line growth to within  $\pm 0.01$  with 95% confidence, how many CEOs would you survey?

## 8.5 APPLICATIONS OF CONFIDENCE INTERVAL ESTIMATION IN AUDITING

This chapter has focused on estimating either the population mean or the population proportion. In previous chapters, you have studied application to different business scenarios. Auditing is one of the areas in business that makes widespread use of probability sampling methods in order to construct confidence interval estimates.

### AUDITING

**Auditing** is the collection and evaluation of evidence about information relating to an economic entity, such as a sole business proprietor, a partnership, a corporation, or a government agency, in order to determine and report on how well the information corresponds to established criteria.

Auditors rarely examine a complete population of information. Instead, they rely on estimation techniques based on the probability sampling methods you have studied in this text. The following list contains some of the reasons sampling is advantageous to examining the entire population.

- Sampling is less time consuming.
- Sampling is less costly.
- Sampling provides results that are objective and defensible. Because the sample size is based on demonstrable statistical principles, the audit is defensible before one’s superiors and in a court of law.
- Sampling provides an objective way of estimating the sample size in advance.
- Sampling provides an estimate of the sampling error.
- Sampling is often more accurate for drawing conclusions about large populations than other methods. Examining large populations is time-consuming and therefore often subject to more nonsampling error than statistical sampling.
- Sampling allows auditors to combine, and then evaluate collectively, samples collected by different individuals.
- Sampling allows auditors to generalize their findings to the population with a known sampling error.

## Estimating the Population Total Amount

In auditing applications, you are often more interested in developing estimates of the population **total amount** than the population mean. Equation (8.6) shows how to estimate a population total amount.

### ESTIMATING THE POPULATION TOTAL

The point estimate for the population total is equal to the population size,  $N$ , times the sample mean.

$$\text{Total} = N\bar{X} \quad (8.6)$$

Equation (8.7) defines the confidence interval estimate for the population total.

### CONFIDENCE INTERVAL ESTIMATE FOR THE TOTAL

$$N\bar{X} \pm N(t_{n-1}) \frac{S}{\sqrt{n}} \sqrt{\frac{N-n}{N-1}} \quad (8.7)$$

To demonstrate the application of the confidence interval estimate for the population total amount, return to the Saxon Home Improvement Using Statistics scenario on page 284. One of the auditing tasks is to estimate the total dollar amount of all sales invoices for the month. If there are 5,000 invoices for that month and  $\bar{X} = \$110.27$ , then using Equation (8.6),

$$N\bar{X} = (5,000)(\$110.27) = \$551,350$$

If  $n = 100$  and  $S = \$28.95$ , then using Equation (8.7) with  $t_{99} = 1.9842$  for 95% confidence,

$$\begin{aligned} N\bar{X} \pm N(t_{n-1}) \frac{S}{\sqrt{n}} \sqrt{\frac{N-n}{N-1}} &= 551,350 \pm (5,000)(1.9842) \frac{28.95}{\sqrt{100}} \sqrt{\frac{5,000-100}{5,000-1}} \\ &= 551,350 \pm 28,721.295(0.99005) \\ &= 551,350 \pm 28,436 \end{aligned}$$

$$\$522,914 \leq \text{Population total} \leq \$579,786$$

Therefore, with 95% confidence, you estimate that the total amount of sales invoices is between \$522,914 and \$579,786. Figure 8.13 shows a Microsoft Excel worksheet for these data.

**FIGURE 8.13**

Microsoft Excel worksheet for the confidence interval estimate of the total amount of all invoices for the Saxon Home Improvement Company



See Section E8.6 to create this.

	A	B
1	<b>Total Amount of All Sales Invoices</b>	
2		
3	<b>Data</b>	
4	Population Size	5000
5	Sample Mean	110.27
6	Sample Size	100
7	Sample Standard Deviation	28.95
8	Confidence Level	95%
9		
10	<b>Intermediate Calculations</b>	
11	Population Total	551350.00 =B4 * B5
12	FPC Factor	0.9900 =SQRT((B4 - B6)/(B4 - 1))
13	Standard Error of the Total	14330.9521 =(B4 * B7 * B12)/SQRT(B6)
14	Degrees of Freedom	99 =B6 - 1
15	t Value	1.9842 =TINV(1 - B8, B14)
16	Interval Half Width	28435.72 =B15 * B13
17		
18	<b>Confidence Interval</b>	
19	Interval Lower Limit	522914.28 =B11 - B16
20	Interval Upper Limit	579785.72 =B11 + B16

Example 8.7 further illustrates the population total.

### EXAMPLE 8.7 DEVELOPING A CONFIDENCE INTERVAL ESTIMATE FOR THE POPULATION TOTAL

An auditor is faced with a population of 1,000 vouchers and wants to estimate the total value of the population of vouchers. A sample of 50 vouchers is selected, with the following results:

$$\text{Mean voucher amount } (\bar{X}) = \$1,076.39$$

$$\text{Standard deviation } (S) = \$273.62$$

Construct a 95% confidence interval estimate of the total amount for the population of vouchers.

**SOLUTION** Using Equation (8.6) on page 307, the point estimate of the population total is

$$N\bar{X} = (1,000)(1,076.39) = \$1,076,390$$

From Equation (8.7) on page 307, a 95% confidence interval estimate of the population total amount is

$$\begin{aligned} (1,000)(1,076.39) \pm (1,000)(2.0096) \frac{273.62}{\sqrt{50}} \sqrt{\frac{1,000 - 50}{1,000 - 1}} \\ = 1,076,390 \pm 77,762.878 (0.97517) \\ = 1,076,390 \pm 75,832 \\ \$1,000,558 \leq \text{Population total} \leq \$1,152,222 \end{aligned}$$

Therefore, with 95% confidence, you estimate that the total amount of the vouchers is between \$1,000,558 and \$1,152,222.

### Difference Estimation

An auditor uses **difference estimation** when he or she believes that errors exist in a set of items and he or she wants to estimate the magnitude of the errors based only on a sample. The following steps are used in difference estimation:

1. Determine the sample size required.
2. Calculate the differences between the values reached during the audit and the original values recorded. The difference in value  $i$ , denoted  $D_i$ , is equal to 0 if the auditor finds that the original value is correct, is a positive value when the audited value is larger than the original value, and is negative when the audited value is smaller than the original value.
3. Compute the mean difference in the sample,  $\bar{D}$ , by dividing the total difference by the sample size, as shown in Equation (8.8).

#### MEAN DIFFERENCE

$$\bar{D} = \frac{\sum_{i=1}^n D_i}{n} \quad (8.8)$$

where  $D_i = \text{Audited value} - \text{Original value}$

4. Compute the standard deviation of the differences,  $S_D$ , as shown in Equation (8.9). Remember that any item that is not in error has a difference value of 0.

### STANDARD DEVIATION OF THE DIFFERENCE

$$S_D = \sqrt{\frac{\sum_{i=1}^n (D_i - \bar{D})^2}{n-1}} \quad (8.9)$$

5. Use Equation (8.10) to construct a confidence interval estimate of the total difference in the population.

### CONFIDENCE INTERVAL ESTIMATE FOR THE TOTAL DIFFERENCE

$$N\bar{D} \pm N(t_{n-1}) \frac{S_D}{\sqrt{n}} \sqrt{\frac{N-n}{N-1}} \quad (8.10)$$

The auditing procedures for Saxon Home Improvement require a 95% confidence interval estimate of the difference between the actual dollar amounts on the sales invoices and the amounts entered into the integrated inventory and sales information system. Suppose that in a sample of 100 sales invoices, you have 12 invoices in which the actual amount on the sales invoice and the amount entered into the integrated inventory management and sales information system is different. These 12 differences (stored in the file [plumbinv.xls](#)) are

\$9.03 \$7.47 \$17.32 \$8.30 \$5.21 \$10.80 \$6.22 \$5.63 \$4.97 \$7.43 \$2.99 \$4.63

The other 88 invoices are not in error. Their *differences* are each 0. Thus,

$$\bar{D} = \frac{\sum_{i=1}^n D_i}{n} = \frac{90}{100} = 0.90$$

<sup>3</sup>In the numerator, there are 100 differences. Each of the last 88 are equal to  $(0 - 0.9)^2$

and<sup>3</sup>

$$\begin{aligned} S_D &= \sqrt{\frac{\sum_{i=1}^n (D_i - \bar{D})^2}{n-1}} \\ &= \sqrt{\frac{(9.03 - 0.9)^2 + (7.47 - 0.9)^2 + \dots + (0 - 0.9)^2}{100-1}} \\ S_D &= 2.752 \end{aligned}$$

Using Equation (8.10), construct the 95% confidence interval estimate for the total difference in the population of 5,000 sales invoices as follows:

$$\begin{aligned} (5,000)(0.90) \pm (5,000)(1.9842) \frac{2.752}{\sqrt{100}} \sqrt{\frac{5,000-100}{5,000-1}} \\ = 4,500 \pm 2,702.91 \\ \$1,797.09 \leq \text{Total difference} \leq \$7,202.91 \end{aligned}$$

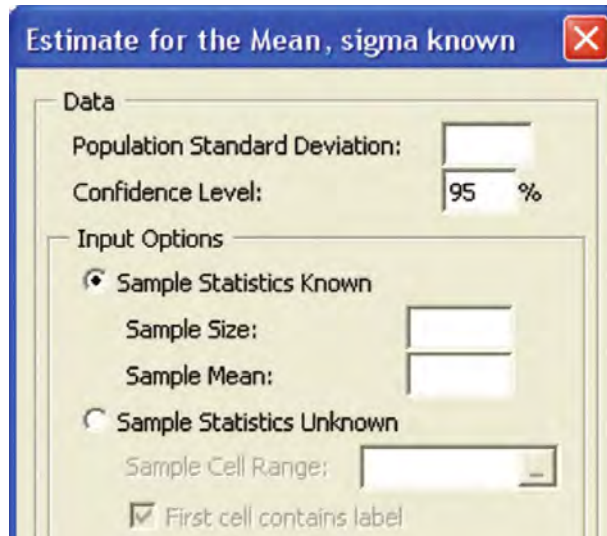
Thus, the auditor estimates with 95% confidence that the total difference between the sales invoices, as determined during the audit and the amount originally entered into the accounting system is between \$1,797.09 and \$7,202.91. Figure 8.14 shows an Excel worksheet for these data.

**FIGURE 8.14**

Microsoft Excel worksheet for the total difference between the invoice amounts found during audit and the amounts entered into the accounting system for the Saxon Home Improvement Company



See Section E8.7 to create this.



In the previous example, all 12 differences are positive because the actual amount on the sales invoice is more than the amount entered into the accounting system. In some circumstances, you could have negative errors. Example 8.8 illustrates such a situation.

**EXAMPLE 8.8****DIFFERENCE ESTIMATION**

Returning to Example 8.7 on page 308, suppose that 14 vouchers in the sample of 50 vouchers contain errors. The values of the 14 errors are listed below and stored in the file [difftest.xls](#). Observe that two differences are negative:

\$75.41	\$38.97	\$108.54	-\$37.18	\$62.75	\$118.32	-\$88.84
\$127.74	\$55.42	\$39.03	\$29.41	\$47.99	\$28.73	\$84.05

Construct a 95% confidence interval estimate of the total difference in the population of 1,000 vouchers.

**SOLUTION** For these data,

$$\bar{D} = \frac{\sum_{i=1}^n D_i}{n} = \frac{690.34}{50} = 13.8068$$

and

$$\begin{aligned} S_D &= \sqrt{\frac{\sum_{i=1}^n (D_i - \bar{D})^2}{n-1}} \\ &= \sqrt{\frac{(75.41 - 13.8068)^2 + (38.97 - 13.8068)^2 + \dots + (0 - 13.8068)^2}{50-1}} \\ &= 37.427 \end{aligned}$$

Using Equation (8.10) on page 309, construct the confidence interval estimate for the total difference in the population as follows:

$$\begin{aligned} (1,000)(13.8068) \pm (1,000)(2.0096) \frac{37.427}{\sqrt{50}} \sqrt{\frac{1,000 - 50}{1,000 - 1}} \\ = 13,806.8 \pm 10,372.4 \\ \$3,434.40 \leq \text{Total difference} \leq \$24,179.20 \end{aligned}$$

Therefore, with 95% confidence, you estimate that the total difference in the population of vouchers is between \$3,434.40 and \$24,179.20.

### One-Sided Confidence Interval Estimation of the Rate of Noncompliance with Internal Controls

Organizations use internal control mechanisms to ensure that individuals act in accordance with company guidelines. For example, Saxon Home Improvement requires that an authorized warehouse-removal slip be completed before goods are removed from the warehouse. During the monthly audit of the company, the auditing team is charged with the task of estimating the proportion of times goods were removed without proper authorization. This is referred to as the *rate of noncompliance with the internal control*. To estimate the rate of noncompliance, auditors take a random sample of sales invoices and determine how often merchandise was shipped without an authorized warehouse-removal slip. The auditors then compare their results with a previously established tolerable exception rate, which is the maximum allowable proportion of items in the population not in compliance. When estimating the rate of noncompliance, it is commonplace to use a **one-sided confidence interval**. That is, the auditors estimate an upper bound on the rate of noncompliance. Equation (8.11) defines a one-sided confidence interval for a proportion.

#### ONE-SIDED CONFIDENCE INTERVAL FOR A PROPORTION

$$\text{Upper bound} = p + Z \sqrt{\frac{p(1-p)}{n}} \sqrt{\frac{N-n}{N-1}} \quad (8.11)$$

where  $Z$  = the value corresponding to a cumulative area of  $(1 - \alpha)$  from the standardized normal distribution (that is, a right-hand tail probability of  $\alpha$ ).

If the tolerable exception rate is higher than the upper bound, the auditor concludes that the company is in compliance with the internal control. If the upper bound is higher than the tolerable exception rate, the auditor concludes that the control noncompliance rate is too high. The auditor may then request a larger sample.

Suppose that in the monthly audit, you select 400 sales invoices from a population of 10,000 invoices. In the sample of 400 sales invoices, 20 are in violation of the internal control. If the tolerable exception rate for this internal control is 6%, what should you conclude? Use a 95% level of confidence.

The one-sided confidence interval is computed using  $p = 20/400 = 0.05$  and  $Z = 1.645$ . Using Equation (8.11),

$$\begin{aligned} \text{Upper bound} &= p + Z \sqrt{\frac{p(1-p)}{n}} \sqrt{\frac{N-n}{N-1}} = 0.05 + 1.645 \sqrt{\frac{0.05(1-0.05)}{400}} \sqrt{\frac{10,000-400}{10,000-1}} \\ &= 0.05 + 1.645(0.0109)(0.98) = 0.05 + 0.0176 = 0.0676 \end{aligned}$$

Thus, you have 95% confidence that the rate of noncompliance is less than 6.76%. Because the tolerable exception rate is 6%, the rate of noncompliance may be too high for this internal control. In other words, it is possible that the noncompliance rate for the population is higher than the rate deemed tolerable. Therefore, you should request a larger sample.

In many cases, the auditor is able to conclude that the rate of noncompliance with the company's internal controls is acceptable. Example 8.9 illustrates such an occurrence.

### EXAMPLE 8.9 ESTIMATING THE RATE OF NONCOMPLIANCE

A large electronics firm writes 1 million checks a year. An internal control policy for the company is that the authorization to sign each check is granted only after an invoice has been initialed by an accounts payable supervisor. The company's tolerable exception rate for this control is 4%. If control deviations are found in 8 of the 400 invoices sampled, what should the auditor do? To solve this, use a 95% level of confidence.

**SOLUTION** The auditor constructs a 95% one-sided confidence interval for the proportion of invoices in noncompliance and compares this to the tolerable exception rate. Using Equation (8.11) on page 311,  $p = 8/400 = 0.02$ , and  $Z = 1.645$  for 95% confidence,

$$\begin{aligned} \text{Upper bound} &= p + Z \sqrt{\frac{p(1-p)}{n}} \sqrt{\frac{N-n}{N-1}} = 0.02 + 1.645 \sqrt{\frac{0.02(1-0.02)}{400}} \sqrt{\frac{1,000,000-400}{1,000,000-1}} \\ &= 0.02 + 1.645(0.007)(0.9998) = 0.02 + 0.0115 = 0.0315 \end{aligned}$$

The auditor concludes with 95% confidence that the rate of noncompliance is less than 3.15%. Because this is less than the tolerable exception rate, the auditor concludes that the internal control compliance is adequate. In other words, the auditor is more than 95% confident that the rate of noncompliance is less than 4%.

## PROBLEMS FOR SECTION 8.5

### Learning the Basics

**8.50** A sample of 25 is selected from a population of 500 items. The sample mean is 25.7, and the sample standard deviation is 7.8. Construct a 99% confidence interval estimate of the population total.

**8.51** Suppose that a sample of 200 (see the file [itemerr.xls](#)) is selected from a population of 10,000 items. Of these, 10 items are found to have errors of the following amounts:

13.76 42.87 34.65 11.09 14.54  
22.87 25.52 9.81 10.03 15.49

Construct a 95% confidence interval estimate of the total difference in the population.

**8.52** If  $p = 0.04$ ,  $n = 300$ , and  $N = 5,000$ , calculate the upper bound for a one-sided confidence interval estimate of the population proportion,  $\pi$ , using the following levels of confidence:

a. 90%   b. 95%   c. 99%

### Applying the Concepts

**8.53** A stationery store wants to estimate the total retail value of the 1,000 greeting cards it has in its inventory. Construct a 95% confidence interval estimate of the population total value of all greeting cards that are in inventory if a random sample of 100 greeting cards indicates a mean value of \$2.55 and a standard deviation of \$0.44.



**8.54** The personnel department of a large corporation employing 3,000 workers wants to estimate the family dental expenses of its employees to determine the feasibility of providing a dental insurance plan. A random sample of 10 employees reveals the following family dental expenses (in dollars) for the preceding year (see the [dental.xls](#) file):

110 362 246 85 510 208 173 425 316 179

Construct a 90% confidence interval estimate of the total family dental expenses for all employees in the preceding year.



**8.55** A branch of a chain of large electronics stores is conducting an end-of-month inventory of the merchandise in stock. There were 1,546 items in inventory at that time. A sample of 50 items was randomly selected, and an audit was conducted, with the following results:

Value of Merchandise

$$\bar{X} = \$252.28 \quad S = \$93.67$$

Construct a 95% confidence interval estimate of the total value of the merchandise in inventory at the end of the month.

**8.56** A customer in the wholesale garment trade is often entitled to a discount for a cash payment for goods. The amount of discount varies by vendor. A sample of 150 items selected from a population of 4,000 invoices at the end of a period of time (see the [discount.xls](#) file) revealed that in 13 cases, the customer failed to take the discount to which he or she was entitled. The amounts (in dollars) of the 13 discounts that were not taken were as follows:

6.45 15.32 97.36 230.63 104.18 84.92 132.76  
66.12 26.55 129.43 88.32 47.81 89.01

Construct a 99% confidence interval estimate of the population total amount of discounts not taken.

**8.57** Econe Dresses is a small company that manufactures women's dresses for sale to specialty stores. It has 1,200 inventory items, and the historical cost is recorded on a first in, first out (FIFO) basis. In the past, approximately 15% of the inventory items were incorrectly priced. However, any misstatements were usually not significant. A sample of 120 items was selected (see the [fifo.xls](#) file), and the historical cost of each item was compared with the audited value. The results indicated that 15 items differed in their historical costs and audited values. These values were as follows:

Sample Number	Historical Cost (\$)	Audited Value (\$)	Sample Number	Historical Cost (\$)	Audited Value (\$)
5	261	240	60	21	210
9	87	105	73	140	152
17	201	276	86	129	112
18	121	110	95	340	216
28	315	298	96	341	402
35	411	356	107	135	97
43	249	211	119	228	220
51	216	305			

Construct a 95% confidence interval estimate of the total population difference in the historical cost and audited value.

**8.58** Tom and Brent's Alpine Outfitters conduct an annual audit of its financial records. An internal control policy for the company is that a check can be issued only after the accounts payable manager initials the invoice. The tolerable exception rate for this internal control is 0.04. During an audit, a sample of 300 invoices is examined from a population of 10,000 invoices, and 11 invoices are found to violate the internal control.

- Calculate the upper bound for a 95% one-sided confidence interval estimate for the rate of noncompliance.
- Based on (a), what should the auditor conclude?

**8.59** An internal control policy for Rhonda's Online Fashion Accessories requires a quality assurance check before a shipment is made. The tolerable exception rate for this internal control is 0.05. During an audit, 500 shipping records were sampled from a population of 5,000 shipping records, and 12 were found that violated the internal control.

- Calculate the upper bound for a 95% one-sided confidence interval estimate for the rate of noncompliance.
- Based on (a), what should the auditor conclude?

## 8.6 CONFIDENCE INTERVAL ESTIMATION AND ETHICAL ISSUES

Ethical issues relating to the selection of samples and the inferences that accompany them can occur in several ways. The major ethical issue relates to whether confidence interval estimates are provided along with the sample statistics. To provide a sample statistic without also including the confidence interval limits (typically set at 95%), the sample size used, and an interpretation of the meaning of the confidence interval in terms that a layperson can understand raises ethical issues. Failure to include a confidence interval estimate might mislead the user of the results into thinking that the point estimate is all that is needed to predict the population characteristic with certainty. Thus, it is important that you indicate the interval estimate in a prominent place in any written communication, along with a simple explanation of the meaning of the confidence interval. In addition, you should highlight the sample size.

One of the most common areas where ethical issues concerning estimation occurs is in the publication of the results of political polls. Often, the results of the polls are highlighted on the

front page of the newspaper, and the sampling error involved along with the methodology used is printed on the page where the article is typically continued, often in the middle of the newspaper. To ensure an ethical presentation of statistical results, the confidence levels, sample size, and confidence limits should be made available for all surveys and other statistical studies.

## 8.7 (CD-ROM Topic) ESTIMATION AND SAMPLE SIZE DETERMINATION FOR FINITE POPULATIONS

In this section, confidence intervals are developed and the sample size is determined for situations in which sampling is done without replacement from a finite population. For further discussion, see [section 8.7.pdf](#) on the Student CD-ROM that accompanies this book.

## SUMMARY

This chapter discusses confidence intervals for estimating the characteristics of a population, along with how you can determine the necessary sample size. You learned how an accountant at Saxon Home Improvement can use the sample

data from an audit to estimate important population parameters such as the total dollar amount on invoices and the proportion of shipments made without the proper authorization. Table 8.3 provides a list of topics covered in this chapter.

**TABLE 8.3**

Summary of Topics in Chapter 8

Type of Analysis	Type of Data	
	Numerical	Categorical
Confidence interval for a population parameter	Confidence interval estimate for the mean (Sections 8.1 and 8.2)	Confidence interval estimate for the proportion (Section 8.3)
	Confidence interval estimate for the total and the mean difference (Section 8.5)	One-sided confidence interval estimate for the proportion (Section 8.5)

To determine what equation to use for a particular situation, you need to ask several questions:

- Are you developing a confidence interval or are you determining sample size?
- Do you have a numerical variable or do you have a categorical variable?

- If you have a numerical variable, do you know the population standard deviation? If you do, use the normal distribution. If you do not, use the  $t$  distribution.

The next four chapters develop a hypothesis-testing approach to making decisions about population parameters.

## KEY EQUATIONS

**Confidence Interval for the Mean ( $\sigma$  Known)**

$$\bar{X} \pm Z \frac{\sigma}{\sqrt{n}}$$

or

$$\bar{X} - Z \frac{\sigma}{\sqrt{n}} \leq \mu \leq \bar{X} + Z \frac{\sigma}{\sqrt{n}} \quad (8.1)$$

**Confidence Interval for the Mean ( $\sigma$  Unknown)**

$$\bar{X} \pm t_{n-1} \frac{S}{\sqrt{n}}$$

or

$$\bar{X} - t_{n-1} \frac{S}{\sqrt{n}} \leq \mu \leq \bar{X} + t_{n-1} \frac{S}{\sqrt{n}} \quad (8.2)$$

**Confidence Interval Estimate for the Proportion**

$$p \pm Z \sqrt{\frac{p(1-p)}{n}}$$

or

$$p - Z \sqrt{\frac{p(1-p)}{n}} \leq \pi \leq p + Z \sqrt{\frac{p(1-p)}{n}} \quad (8.3)$$

**Sample Size Determination for the Mean**

$$n = \frac{Z^2 \sigma^2}{e^2} \quad (8.4)$$

**Sample Size Determination for the Proportion**

$$n = \frac{Z^2 \pi(1-\pi)}{e^2} \quad (8.5)$$

**KEY TERMS**

auditing 306

confidence interval estimate 284

critical value 287

degrees of freedom 290

difference estimation 308

level of confidence 287

one-sided confidence interval 311

point estimate 284

sampling error 300

Student's *t* distribution 290

total amount 307

**CHAPTER REVIEW PROBLEMS****Checking Your Understanding**

**8.60** Why can you never really have 100% confidence of correctly estimating the population characteristic of interest?

**8.61** When do you use the *t* distribution to develop the confidence interval estimate for the mean?

**8.62** Why is it true that for a given sample size, *n*, an increase in confidence is achieved by widening (and making less precise) the confidence interval?

**8.63** Under what circumstances do you use a one-sided confidence interval instead of a two-sided confidence interval?

**8.64** When would you want to estimate the population total instead of the population mean?

**8.65** How does difference estimation differ from estimation of the mean?

**Applying the Concepts**

**8.66** You work in the corporate office for a nationwide convenience store franchise that operates nearly 10,000 stores. The per-store daily customer count has been steady at 900 for some time (that is, the mean number of customers in a store in one day is 900). To increase the customer count, the franchise is considering cutting coffee prices by approximately half. The 12-ounce size will now be \$.59 instead of \$.99, and the 16-ounce size will be \$.69 instead of \$1.19. Even with this reduction in price, the franchise will have a 40% gross margin on coffee. To test the new initiative, the franchise has reduced coffee prices in a sample of 34 stores, where customer counts have been running almost exactly at

the national average of 900. After four weeks, the sample stores stabilize at a mean customer count of 974 and a standard deviation of 96. This increase seems like a substantial amount to you, but it also seems like a pretty small sample. Is there some way to get a feel for what the mean per-store count in all the stores will be if you cut coffee prices nationwide? Do you think reducing coffee prices is a good strategy for increasing the mean customer count?

**8.67** Companies are spending more time screening applicants than in the past. A study of 102 recruiters conducted by ExecuNet found that 77 did Internet research on candidates. (Extracted from P. Kitchen, "Don't Let Any 'Digital Dirt' Bury Your Job Prospects," *Newsday*, August 21, 2005, p. A59).

a. Construct a 95% confidence interval estimate of the population proportion of recruiters who do Internet research on candidates.

b. Based on (a), is it correct to conclude that more than 70% of recruiters do Internet research on candidates?

c. Suppose that the study uses a sample size of 400 recruiters and 302 did Internet research on candidates. Construct a 95% confidence interval estimate of the population proportion of recruiters who do Internet research on candidates.

d. Based on (c), is it correct to conclude that more than 70% of recruiters do Internet research on candidates?

e. Discuss the effect of sample size on your answers to (a) through (d).

**8.68** High-fructose corn syrup (HFCS) was created in the 1970s and is used today in a wide variety of foods and

beverages. HFCS is cheaper than sugar and is about 75% sweeter than sucrose. Some researchers think that HFCS is linked to the growing obesity problem in the United States (Extracted from P. Lempert, “War of the Sugars,” *Progressive Grocer*, April 15, 2006, p. 20). The following consumer views are from a nationwide survey of 1,114 responses:

Views on HFCS

	Yes	No
Are you concerned about consuming HFCS?	80%	20%
Do you think HFCS should be banned in food sold to schools?	88%	12%
Do you think HFCS should be banned in all foods?	56%	44%

Construct a 95% confidence interval estimate of the population proportion of people who

- are concerned about consuming HFCS.
- think HFCS should be banned in food sold to schools.
- think HFCS should be banned in all foods.
- You are in charge of a follow-up survey. Determine the sample size necessary to estimate the proportions in (a) through (c) to within  $\pm 0.02$  with 95% confidence.

**8.69** Starwood Hotels conducted a survey of 401 top executives who play golf (Extracted from D. Jones, “Many CEOs Bend the Rules (of Golf),” *USA Today*, June 26, 2002). Among the results were the following:

- 329 cheat at golf.
- 329 hate others who cheat at golf.
- 289 believe business and golf behavior parallel.
- 80 would let a client win to get business.
- 40 would call in sick to play golf.

Construct a 95% confidence interval estimate for each of these questions. Based on these results, what conclusions can you reach about CEOs’ attitudes toward golf?

**8.70** A market researcher for a consumer electronics company wants to study the television viewing habits of residents of a particular area. A random sample of 40 respondents is selected, and each respondent is instructed to keep a detailed record of all television viewing in a particular week. The results are as follows:

- Viewing time per week:  $\bar{X} = 15.3$  hours,  $S = 3.8$  hours.
  - 27 respondents watch the evening news on at least 3 weeknights.
- Construct a 95% confidence interval estimate for the mean amount of television watched per week in this city.
  - Construct a 95% confidence interval estimate for the population proportion who watch the evening news on at least 3 weeknights per week.

Suppose that the market researcher wants to take another survey in a different city. Answer these questions:

- What sample size is required to be 95% confident of estimating the population mean to within  $\pm 2$  hours and assumes that the population standard deviation is equal to 5 hours?
- What sample size is needed to be 95% confident of being within  $\pm 0.035$  of the population proportion who watch the evening news on at least 3 weeknights if no previous estimate is available?
- Based on (c) and (d), what sample size should the market researcher select if a single survey is being conducted?

**8.71** The real estate assessor for a county government wants to study various characteristics of single-family houses in the county. A random sample of 70 houses reveals the following:

- Heated area of the houses (in square feet):  $\bar{X} = 1,759$ ,  $S = 380$ .
  - 42 houses have central air-conditioning.
- Construct a 99% confidence interval estimate of the population mean heated area of the houses.
  - Construct a 95% confidence interval estimate of the population proportion of houses that have central air-conditioning.

**8.72** The personnel director of a large corporation wishes to study absenteeism among clerical workers at the corporation’s central office during the year. A random sample of 25 clerical workers reveals the following:

- Absenteeism:  $\bar{X} = 9.7$  days,  $S = 4.0$  days.
  - 12 clerical workers were absent more than 10 days.
- Construct a 95% confidence interval estimate of the mean number of absences for clerical workers during the year.
  - Construct a 95% confidence interval estimate of the population proportion of clerical workers absent more than 10 days during the year.

Suppose that the personnel director also wishes to take a survey in a branch office. Answer these questions:

- What sample size is needed to have 95% confidence in estimating the population mean to within  $\pm 1.5$  days if the population standard deviation is 4.5 days?
- What sample size is needed to have 90% confidence in estimating the population proportion to within  $\pm 0.075$  if no previous estimate is available?
- Based on (c) and (d), what sample size is needed if a single survey is being conducted?

**8.73** The market research director for Dotty’s Department Store wants to study women’s spending on cosmetics. A survey of the store’s credit card holders is designed in order to estimate the proportion of women who purchase their

cosmetics primarily from Dotty's Department Store and the mean yearly amount that women spend on cosmetics. A previous survey found that the standard deviation of the amount women spend on cosmetics in a year is approximately \$18.

- What sample size is needed to have 99% confidence of estimating the population mean to within  $\pm\$5$ ?
- What sample size is needed to have 90% confidence of estimating the population proportion to within  $\pm 0.045$ ?
- Based on the results in (a) and (b), how many of the store's credit card holders should be sampled? Explain.

**8.74** The branch manager of a nationwide bookstore chain wants to study characteristics of her store's customers. She decides to focus on two variables: the amount of money spent by customers and whether the customers would consider purchasing educational DVDs relating to graduate preparation exams, such as the GMAT, GRE, or LSAT. The results from a sample of 70 customers are as follows:

- Amount spent:  $\bar{X} = \$28.52$ ,  $S = \$11.39$ .
- 28 customers stated that they would consider purchasing the educational DVDs.

- Construct a 95% confidence interval estimate of the population mean amount spent in the bookstore.
- Construct a 90% confidence interval estimate of the population proportion of customers who would consider purchasing educational DVDs.

Assume that the branch manager of another store in the chain wants to conduct a similar survey in his store. Answer the following questions:

- What sample size is needed to have 95% confidence of estimating the population mean amount spent in his store to within  $\pm\$2$  if the standard deviation is assumed to be \$10?
- What sample size is needed to have 90% confidence of estimating the population proportion who would consider purchasing the educational DVDs to within  $\pm 0.04$ ?
- Based on your answers to (c) and (d), how large a sample should the manager take?

**8.75** The branch manager of an outlet (Store 1) of a nationwide chain of pet supply stores wants to study characteristics of her customers. In particular, she decides to focus on two variables: the amount of money spent by customers and whether the customers own only one dog, only one cat, or more than one dog and/or cat. The results from a sample of 70 customers are as follows:

- Amount of money spent:  $\bar{X} = \$21.34$ ,  $S = \$9.22$ .
- 37 customers own only a dog.
- 26 customers own only a cat.
- 7 customers own more than one dog and/or cat.

- Construct a 95% confidence interval estimate of the population mean amount spent in the pet supply store.
- Construct a 90% confidence interval estimate of the population proportion of customers who own only a cat.

The branch manager of another outlet (Store 2) wishes to conduct a similar survey in his store. The manager does not have access to the information generated by the manager of Store 1. Answer the following questions:

- What sample size is needed to have 95% confidence of estimating the population mean amount spent in his store to within  $\pm\$1.50$  if the standard deviation is \$10?
- What sample size is needed to have 90% confidence of estimating the population proportion of customers who own only a cat to within  $\pm 0.045$ ?
- Based on your answers to (c) and (d), how large a sample should the manager take?

**8.76** The owner of a restaurant that serves continental food wants to study characteristics of his customers. He decides to focus on two variables: the amount of money spent by customers and whether customers order dessert. The results from a sample of 60 customers are as follows:

- Amount spent:  $\bar{X} = \$38.54$ ,  $S = \$7.26$ .
- 18 customers purchased dessert.

- Construct a 95% confidence interval estimate of the population mean amount spent per customer in the restaurant.
- Construct a 90% confidence interval estimate of the population proportion of customers who purchase dessert.

The owner of a competing restaurant wants to conduct a similar survey in her restaurant. This owner does not have access to the information of the owner of the first restaurant. Answer the following questions:

- What sample size is needed to have 95% confidence of estimating the population mean amount spent in her restaurant to within  $\pm\$1.50$ , assuming that the standard deviation is \$8?
- What sample size is needed to have 90% confidence of estimating the population proportion of customers who purchase dessert to within  $\pm 0.04$ ?
- Based on your answers to (c) and (d), how large a sample should the owner take?

**8.77** The manufacturer of "Ice Melt" claims its product will melt snow and ice at temperatures as low as  $0^\circ$  Fahrenheit. A representative for a large chain of hardware stores is interested in testing this claim. The chain purchases a large shipment of 5-pound bags for distribution. The representative wants to know with 95% confidence, within  $\pm 0.05$ , what proportion of bags of Ice Melt perform the job as claimed by the manufacturer.

- a. How many bags does the representative need to test? What assumption should be made concerning the population proportion? (This is called *destructive testing*; that is, the product being tested is destroyed by the test and is then unavailable to be sold.)
- b. The representative tests 50 bags, and 42 of them do the job as claimed. Construct a 95% confidence interval estimate for the population proportion that will do the job as claimed.
- c. How can the representative use the results of (b) to determine whether to sell the Ice Melt product?

**8.78** An auditor needs to estimate the percentage of times a company fails to follow an internal control procedure. A sample of 50 from a population of 1,000 items is selected, and in 7 instances, the internal control procedure was not followed.

- a. Construct a 90% one-sided confidence interval estimate of the population proportion of items in which the internal control procedure was not followed.
- b. If the tolerable exception rate is 0.15, what should the auditor conclude?

**8.79** An auditor for a government agency needs to evaluate payments for doctors' office visits paid by Medicare in a particular zip code during the month of June. A total of 25,056 visits occurred during June in this area. The auditor wants to estimate the total amount paid by Medicare to within ±\$5 with 95% confidence. On the basis of past experience, she believes that the standard deviation is approximately \$30.

- a. What sample size should she select? Using the sample size selected in (a), an audit is conducted with the following results.

*Amount of Reimbursement*

$$\bar{X} = \$93.70 \quad S = \$34.55$$

In 12 of the office visits, an incorrect amount of reimbursement was provided. For the 12 office visits in which there was an incorrect reimbursement, the differences between the amount reimbursed and the amount that the auditor determined should have been reimbursed were as follows (and are stored in the file [medicare.xls](#))

\$17 \$25 \$14 -\$10 \$20 \$40 \$35 \$30 \$28 \$22 \$15 \$5

- b. Construct a 90% confidence interval estimate of the population proportion of reimbursements that contain errors.
- c. Construct a 95% confidence interval estimate of the population mean reimbursement per office visit.
- d. Construct a 95% confidence interval estimate of the population total amount of reimbursements for this geographic area in June.
- e. Construct a 95% confidence interval estimate of the total difference between the amount reimbursed and the

amount that the auditor determined should have been reimbursed.

**8.80** A home furnishings store that sells bedroom furniture is conducting an end-of-month inventory of the beds (mattress, bed spring, and frame) in stock. An auditor for the store wants to estimate the mean value of the beds in stock at that time. She wants to have 99% confidence that her estimate of the mean value is correct to within ±\$100. On the basis of past experience, she estimates that the standard deviation of the value of a bed is \$200.

- a. What sample size should she select?
- b. Using the sample size selected in (a), an audit was conducted, with the following results:

$$\bar{X} = \$1,654.27 \quad S = \$184.62$$

Construct a 99% confidence interval estimate of the total value of the beds in stock at the end of the month if there were 258 beds in stock.

**8.81** A quality characteristic of interest for a tea-bag-filling process is the weight of the tea in the individual bags. In this example, the label weight on the package indicates that the mean amount is 5.5 grams of tea in a bag. If the bags are underfilled, two problems arise. First, customers may not be able to brew the tea to be as strong as they wish. Second, the company may be in violation of the truth-in-labeling laws. On the other hand, if the mean amount of tea in a bag exceeds the label weight, the company is giving away product. Getting an exact amount of tea in a bag is problematic because of variation in the temperature and humidity inside the factory, differences in the density of the tea, and the extremely fast filling operation of the machine (approximately 170 bags per minute). The following data are the weights, in grams, of a sample of 50 tea bags produced in one hour by a single machine (the data are stored in the file [teabags.xls](#)):

*Weight of Tea Bags, in Grams*

5.65 5.44 5.42 5.40 5.53 5.34 5.54 5.45 5.52 5.41  
 5.57 5.40 5.53 5.54 5.55 5.62 5.56 5.46 5.44 5.51  
 5.47 5.40 5.47 5.61 5.53 5.32 5.67 5.29 5.49 5.55  
 5.77 5.57 5.42 5.58 5.58 5.50 5.32 5.50 5.53 5.58  
 5.61 5.45 5.44 5.25 5.56 5.63 5.50 5.57 5.67 5.36

- a. Construct a 99% confidence interval estimate of the population mean weight of the tea bags.
- b. Is the company meeting the requirement set forth on the label that the mean amount of tea in a bag is 5.5 grams?

**8.82** A manufacturing company produces steel housings for electrical equipment. The main component part of the housing is a steel trough that is made out of a 14-gauge steel

coil. It is produced using a 250-ton progressive punch press with a wipe-down operation that puts two 90-degree forms in the flat steel to make the trough. The distance from one side of the form to the other is critical because of weather-proofing in outdoor applications. The data (stored in the file [trough.xls](#)) from a sample of 49 troughs follows:

*Width of Trough, in Inches*

8.312 8.343 8.317 8.383 8.348 8.410 8.351 8.373 8.481 8.422  
 8.476 8.382 8.484 8.403 8.414 8.419 8.385 8.465 8.498 8.447  
 8.436 8.413 8.489 8.414 8.481 8.415 8.479 8.429 8.458 8.462  
 8.460 8.444 8.429 8.460 8.412 8.420 8.410 8.405 8.323 8.420  
 8.396 8.447 8.405 8.439 8.411 8.427 8.420 8.498 8.409

- Construct a 95% confidence interval estimate of the mean width of the troughs.
- Interpret the interval developed in (a).

**8.83** The manufacturer of Boston and Vermont asphalt shingles know that product weight is a major factor in the customer's perception of quality. The last stage of the assembly line packages the shingles before they are placed on wooden pallets. Once a pallet is full (a pallet for most brands holds 16 squares of shingles), it is weighed, and the measurement is recorded. The data file [pallet.xls](#) contains the weight (in pounds) from a sample of 368 pallets of Boston shingles and 330 pallets of Vermont shingles.

- For the Boston shingles, construct a 95% confidence interval estimate of the mean weight.
- For the Vermont shingles, construct a 95% confidence interval estimate of the mean weight.
- Evaluate whether the assumption needed for (a) and (b) has been seriously violated.
- Based on the results of (a) and (b), what conclusions can you reach concerning the mean weight of the Boston and Vermont shingles?

**8.84** The manufacturer of Boston and Vermont asphalt shingles provides its customers with a 20-year warranty on most of its products. To determine whether a shingle will last as long as the warranty period, accelerated-life testing is conducted at the manufacturing plant. Accelerated-life testing exposes the shingle to the stresses it would be subject to in a lifetime of normal use via a laboratory experiment that takes only a few minutes to conduct. In this test, a shingle is repeatedly scraped with a brush for a short period of time, and the shingle granules removed by the brushing are weighed (in grams). Shingles that experience low amounts of granule loss are expected to last longer in normal use than shingles that experience high amounts of granule loss. In this situation, a shingle should experience no more than 0.8 grams of granule loss if it is expected to last the length of

the warranty period. The data file [granule.xls](#) contains a sample of 170 measurements made on the company's Boston shingles and 140 measurements made on Vermont shingles.

- For the Boston shingles, construct a 95% confidence interval estimate of the mean granule loss.
- For the Vermont shingles, construct a 95% confidence interval estimate of the mean granule loss.
- Evaluate whether the assumption needed for (a) and (b) has been seriously violated.
- Based on the results of (a) and (b), what conclusions can you reach concerning the mean granule loss of the Boston and Vermont shingles?

## Report Writing Exercises

**8.85** Referring to the results in Problem 8.82 on page 318 concerning the width of a steel trough, write a report that summarizes your conclusions.

## Team Project

**8.86** Refer to the team project on page 73 (see the [mutual funds.xls](#) file). Construct all appropriate confidence interval estimates of the population characteristics of low-risk, average-risk, and high-risk mutual funds. Include these estimates in a report to the vice president for research at the financial investment service.

## Student Survey Database

**8.87** Problem 1.27 on page 15 describes a survey of 50 undergraduate students (see the file [undergradsurvey.xls](#)).

- For these data, for each variable, construct a 95% confidence interval estimate of the population characteristic.
- Write a report that summarizes your conclusions.

**8.88** Problem 1.27 on page 15 describes a survey of 50 undergraduate students (see the file [undergradsurvey.xls](#)).

- Select a sample of 50 undergraduate students at your school and conduct a similar survey for those students.
- For the data collected in (a), repeat (a) and (b) of Problem 8.87.
- Compare the results of (b) to those of Problem 8.87.

**8.89** Problem 1.28 on page 15 describes a survey of 50 MBA students (see the file [gradsurvey.xls](#)).

- For these data, for each variable, construct a 95% confidence interval estimate of the population characteristic.
- Write a report that summarizes your conclusions.

**8.90** Problem 1.28 on page 15 describes a survey of 50 MBA students (see the file [gradsurvey.xls](#)).

- Select a sample of 50 graduate students in your MBA program and conduct a similar survey for those students.
- For the data collected in (a), repeat (a) and (b) of Problem 8.89.
- Compare the results of (b) to those of Problem 8.89.

## Managing the *Springville Herald*

The marketing department has been considering ways to increase the number of new subscriptions and increase the rate of retention among customers who agreed to a trial subscription. Following the suggestion of Assistant Manager Lauren Alfonso, the department staff designed a survey to help determine various characteristics of readers of the newspaper who were not home-delivery subscribers. The survey consists of the following 10 questions:

- Do you or a member of your household ever purchase the *Springville Herald*?  
(1) Yes (2) No  
[If the respondent answers no, the interview is terminated.]
- Do you receive the *Springville Herald* via home delivery?  
(1) Yes (2) No  
[If no, skip to question 4.]
- Do you receive the *Springville Herald*:  
(1) Monday–Saturday (2) Sunday only (3) Every day  
[If every day, skip to question 9.]
- How often during the Monday–Saturday period do you purchase the *Springville Herald*?  
(1) Every day (2) Most days (3) Occasionally or never
- How often do you purchase the *Springville Herald* on Sundays?  
(1) Every Sunday (2) 2–3 Sundays per month (3) No more than once a month
- Where are you most likely to purchase the *Springville Herald*?  
(1) Convenience store (2) Newsstand/candy store (3) Vending machine (4) Supermarket (5) Other
- Would you consider subscribing to the *Springville Herald* for a trial period if a discount were offered?  
(1) Yes (2) No  
[If no, skip to question 9.]
- The *Springville Herald* currently costs \$0.50 Monday–Saturday and \$1.50 on Sunday, for a total of \$4.50 per week. How much would you be willing to pay per week to get home delivery for a 90-day trial period?
- Do you read a daily newspaper other than the *Springville Herald*?  
(1) Yes (2) No
- As an incentive for long-term subscribers, the newspaper is considering the possibility of offering a card that would provide discounts at certain restaurants in the Springville area to all subscribers who pay in advance for six months of home delivery. Would you want to get such a card under the terms of this offer?  
(1) Yes (2) No

The group agreed to use a random-digit dialing method to poll 500 local households by telephone. Using this approach, the last four digits of a telephone number are randomly selected to go with an area code and exchange (the first 6 digits of a 10-digit telephone number). Only those pairs of area codes and exchanges that were for the Springville city area were used for this survey.

Of the 500 households selected, 94 households either refused to participate, could not be contacted after repeated attempts, or represented telephone numbers that were not in service. The summary results are as follows:

Households That Purchase the <i>Springville Herald</i>	Frequency
Yes	352
No	54

Households with Home Delivery	Frequency
Yes	136
No	216

Type of Home Delivery Subscription	Frequency
Monday–Saturday	18
Sunday only	25
7 days a week	93

Purchase Behavior of Nonsubscribers for Monday–Saturday Editions	Frequency
Every day	78
Most days	95
Occasionally or never	43

Purchase Behavior of Nonsubscribers for Sunday Editions	Frequency
Every Sunday	138
2–3 Sundays a month	54
No more than once a month	24

Nonsubscribers' Purchase Location	Frequency
Convenience store	74
Newsstand/candy store	95
Vending machine	21
Supermarket	13
Other locations	13

Would Consider Trial Subscription if Offered a Discount	Frequency
Yes	46
No	170



**Rate Willing to Pay per Week (in Dollars)** Data file [sh8.xls](#)  
for a 90-Day Home-Delivery Trial Subscription

4.15 3.60 4.10 3.60 3.60 3.60 4.40 3.15 4.00 3.75 4.00  
3.25 3.75 3.30 3.75 3.65 4.00 4.10 3.90 3.50 3.75 3.00  
3.40 4.00 3.80 3.50 4.10 4.25 3.50 3.90 3.95 4.30 4.20  
3.50 3.75 3.30 3.85 3.20 4.40 3.80 3.40 3.50 2.85 3.75  
3.80 3.90

**Read a Daily Newspaper Other Than  
the *Springville Herald***

	Frequency
Yes	138
No	214

**Would Prepay Six Months to Receive  
a Restaurant Discount Card**

	Frequency
Yes	66
No	286

## Web Case

Apply your knowledge about confidence interval estimation in this Web Case, which extends the *OurCampus! Web Case* from Chapter 6.

Among its other features, the OurCampus! Web site allows customers to purchase OurCampus! LifeStyles merchandise online. To handle payment processing, the management of OurCampus! has contracted with the following firms:

- PayAFriend (PAF): an online payment system with which customers and businesses such as OurCampus! register in order to exchange payments in a secure and convenient manner without the need for a credit card.
- Continental Banking Company (Conbanco): a processing services provider that allows OurCampus! customers to pay for merchandise using nationally recognized credit cards issued by a financial institution.

To reduce costs, the management is considering eliminating one of these two payment systems. However, Virginia Duffy of the sales department suspects that customers use the two forms of payment in unequal numbers

## EXERCISES

- SH8.1 Some members of the marketing department are concerned about the random-digit dialing method used to collect survey responses. Prepare a memorandum that examines the following issues:
- The advantages and disadvantages of using the random-digit dialing method.
  - Possible alternative approaches for conducting the survey and their advantages and disadvantages.
- SH8.2 Analyze the results of the survey of Springville households. Write a report that discusses the marketing implications of the survey results for the *Springville Herald*.

and that customers display different buying behaviors when using the two forms of payment. Therefore, she would like to first determine

- the proportion of customers using PAF and the proportion of customers using a credit card to pay for their purchases.
- the mean purchase amount when using PAF and the mean purchase amount when using a credit card.

Assist Ms. Duffy by preparing an appropriate analysis based on a random sample of 50 transactions that she has prepared and placed in an internal file on the OurCampus! Web site, [www.prenhall.com/Springville/OurCampus\\_PymtSample.htm](http://www.prenhall.com/Springville/OurCampus_PymtSample.htm). Summarize your findings and determine whether Ms. Duffy's conjectures about OurCampus! customer purchasing behaviors are correct. If you want the sampling error to be no more than \$3 when estimating the mean purchase amount, is Ms. Duffy's sample large enough to perform a valid analysis?

## REFERENCES

1. Cochran, W. G., *Sampling Techniques*, 3rd ed. (New York: Wiley, 1977).
2. Fisher, R. A., and F. Yates, *Statistical Tables for Biological, Agricultural and Medical Research*, 5th ed. (Edinburgh: Oliver & Boyd, 1957).
3. Kirk, R. E., ed., *Statistical Issues: A Reader for the Behavioral Sciences* (Belmont, CA: Wadsworth, 1972).
4. Larsen, R. L., and M. L. Marx, *An Introduction to Mathematical Statistics and Its Applications*, 4th ed. (Upper Saddle River, NJ: Prentice Hall, 2006).
5. *Microsoft Excel 2007* (Redmond, WA: Microsoft Corp., 2007).
6. Snedecor, G. W., and W. G. Cochran, *Statistical Methods*, 7th ed. (Ames, IA: Iowa State University Press, 1980).

# Excel Companion

## to Chapter 8

### E8.1 COMPUTING THE CONFIDENCE INTERVAL ESTIMATE FOR THE MEAN ( $\sigma$ KNOWN)

You compute the confidence interval estimate for the mean ( $\sigma$  known) either by using the PHStat2 Estimate for the Mean, sigma known procedure or by making entries in the [CIE sigma known.xls](#) workbook.

#### Using PHStat2 Estimate for the Mean, Sigma Known

Select **PHStat** → **Confidence Intervals** → **Estimate for the Mean, sigma known**. In the Estimate for the Mean, sigma known dialog box (shown below), enter values for the **Population Standard Deviation** and the **Confidence Level**. Click one of the input options and make the required entries. Enter a title as the **Title** and click **OK**.

If you know the sample size and sample mean of your sample, click **Sample Statistics Known** and enter those values. Otherwise, click **Sample Statistics Unknown** and enter the cell range of your sample as the **Sample Cell Range**.

#### Using the CIE\_SK Worksheet

Open to the **CIE\_SK** worksheet of the workbook. This worksheet uses the **NORMSINV( $P < X$ )** and **CONFIDENCE(1-confidence level, population standard deviation, sample size)** functions to compute the *Z* value and interval half-width for the Example 8.1 mean paper length problem on page 288. To adapt this worksheet to other problems, enter the appropriate population standard deviation, sample mean, sample size, and confidence level values in the tinted cells B4 through B7 and enter a new title in cell A1.

### E8.2 COMPUTING THE CONFIDENCE INTERVAL ESTIMATE FOR THE MEAN ( $\sigma$ UNKNOWN)

You compute the confidence interval estimate for the mean ( $\sigma$  unknown) either by using the PHStat2 Estimate for the Mean, sigma unknown procedure or by making entries in the [CIE sigma unknown.xls](#) workbook.

#### Using PHStat2 Estimate for the Mean, Sigma Unknown

Select **PHStat** → **Confidence Intervals** → **Estimate for the Mean, sigma unknown**. In the Estimate for the Mean, sigma unknown dialog box (shown on page 323), enter a **Confidence Level** value, click one of the input options, and make the required entries. Enter a title as the **Title** and click **OK**.

If you know the sample size, sample mean, and sample standard deviation of your sample, click **Sample Statistics Known** and enter those values. Otherwise, click **Sample Statistics Unknown** and enter the cell range of your sample as the **Sample Cell Range**.

### Using the CIE\_SU Worksheet

Open to the CIE\_SU worksheet of the [CIE sigma unknown.xls](#) workbook. The worksheet (see Figure 8.6 on page 293) uses the  $TINV(1-\text{confidence level}, \text{degrees of freedom})$  function to determine the critical value from the  $t$  distribution and compute the interval half-width for the Section 8.2 Saxon Home Improvement Company example. To adapt this worksheet to other problems, change the sample statistics and confidence level values in the tinted cells B4 through B7 and enter a new title in cell A1.

## E8.3 COMPUTING THE CONFIDENCE INTERVAL ESTIMATE FOR THE PROPORTION

You compute the confidence interval estimate for the proportion either by using the PHStat2 Estimate for the Proportion procedure or by making entries in the [CIE Proportion.xls](#) workbook.

### Using PHStat2 Estimate for the Proportion

Select **PHStat** → **Confidence Intervals** → **Estimate for the Proportion**. In the Estimate for the Proportion dialog

box (shown below), enter values for the **Sample Size**, the **Number of Successes**, and the **Confidence Level**. Enter a title as the **Title** and click **OK**.

### Using the CIE\_P worksheet

Open to the CIE\_P worksheet of the [CIE Proportion.xls](#) workbook. The worksheet (see Figure 8.10 on page 297) uses the  $NORMSINV(P<X)$  function to determine the  $Z$  value and uses the square root function to compute the standard error of the proportion for the Section 8.3 Saxon Home Improvement Company example. To adapt this worksheet to other problems, enter the appropriate sample size, number of successes, and confidence level values in the tinted cells B4, B5, and B6 and enter a new title in cell A1.

## E8.4 COMPUTING THE SAMPLE SIZE NEEDED FOR ESTIMATING THE MEAN

You compute the sample size needed for estimating the mean either by using the PHStat2 Determination for the Mean procedure or by making entries in the [Sample Size Mean.xls](#) workbook.

### Using PHStat2 Determination for the Mean

Select **PHStat** → **Sample Size** → **Determination for the Mean**. In the Sample Size Determination for the Mean dialog box (shown on page 324), enter values for the **Population Standard Deviation**, the **Sampling Error**, and the **Confidence Level**. Enter a title as the **Title** and click **OK**.

### Using the SampleSize\_M Worksheet

Open to the **SampleSize\_M** worksheet of the **Sample Size Mean.xls** workbook. The worksheet (see Figure 8.11 on page 301) uses the **NORMSINV( $P < X$ )** function to compute the Z value and uses the **ROUNDUP(value)** function to round up the sample size needed to the next higher integer for the Section 8.4 Saxon Home Improvement Company example. To adapt this worksheet to other problems, enter the appropriate population standard deviation, sampling error, and confidence level values in the tinted cells B4, B5, and B6 and enter a new title in cell A1.

## E8.5 COMPUTING THE SAMPLE SIZE NEEDED FOR ESTIMATING THE PROPORTION

You compute the sample size needed for estimating the proportion either by using the PHStat2 Determination for the Proportion procedure or by making entries in the **Sample Size Proportion.xls** workbook.

### Using PHStat2 Determination for the Proportion

Select **PHStat** → **Sample Size** → **Determination for the Proportion**. In the Sample Size Determination for the Proportion dialog box (shown at right), enter values for the **Estimate of True Proportion**, the **Sampling Error**, and the **Confidence Level**. Enter a title as the **Title** and click **OK**.

### Using the SampleSize\_P Worksheet

Open to the **SampleSize\_P** worksheet of the **Sample Size Proportion.xls** file. The worksheet (see Figure 8.12 on page 304) uses the **NORMSINV** and **ROUNDUP** functions, discussed in Section E8.3, for the Saxon Home Improvement Company example in Section 8.3. To adapt this worksheet to other problems, enter the appropriate estimate of true proportion, sampling error, and confidence level values in the tinted cells B4 through B6 and enter a new title in cell A1.

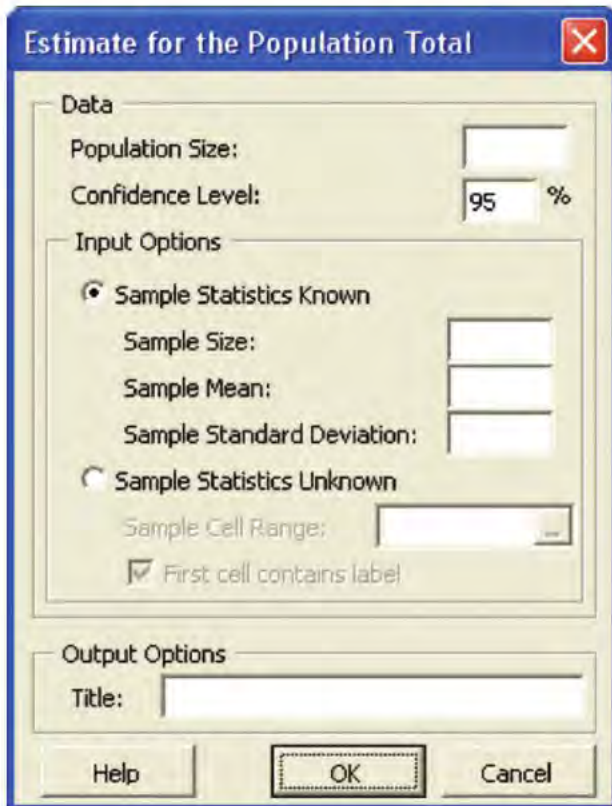
## E8.6 COMPUTING THE CONFIDENCE INTERVAL ESTIMATE FOR THE POPULATION TOTAL

You compute the confidence interval estimate for the population total either by using the PHStat2 Estimate for the Population Total procedure or by making entries in the **CIE Total.xls** workbook.

### Using PHStat2 Estimate for the Population Total

Select **PHStat** → **Confidence Intervals** → **Estimate for the Population Total**. In the Estimate for the Population Total dialog box (shown on page 325), enter values for the **Population Size** and the **Confidence Level**.

Click one of the input options and make the required entries.



If you know the sample size, sample mean, and sample standard deviation of your sample, click **Sample Statistics Known** and enter those values. Otherwise, click **Sample Statistics Unknown** and enter the cell range of your sample as the **Sample Cell Range**. Enter a title as the **Title** and click **OK**.

### Using the CIE\_T Worksheet

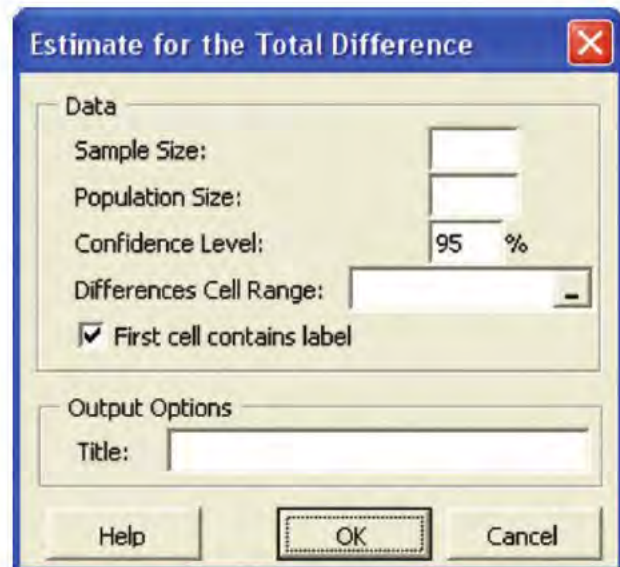
Open to the **CIE\_T** worksheet of the **CIE Total.xls** workbook. This worksheet (see Figure 8.13 on page 307) uses the **TINV(1-confidence level, degrees of freedom)** function to determine the critical value from the *t* distribution and the interval half-width for the Section 8.5 Saxon Home Improvement Company population total example. To adapt this worksheet to other problems, enter the appropriate population size, sample mean, sample size, sample standard deviation, and confidence level values in the tinted cells B4 through B8 and enter a new title in cell A1.

## E8.7 COMPUTING THE CONFIDENCE INTERVAL ESTIMATE FOR THE TOTAL DIFFERENCE

You compute the confidence interval estimate for the total difference either by using the PHStat2 Estimate for the Total Difference procedure or by making entries in the **CIE Total Difference.xls** workbook.

### Using PHStat2 Estimate for the Total Difference

Select **PHStat** → **Confidence Intervals** → **Estimate for the Total Difference**. In the Estimate for the Total Difference dialog box (shown below), enter values for the **Sample Size**, the **Population Size**, and the **Confidence Level**. Enter the cell range of the differences as the **Differences Cell Range**. If the first cell in the column of differences contains a label, click **First cell contains label**, enter a title as the **Title**, and click **OK**.



### Using the CIE\_TD Worksheet

Open to the **CIE\_TD** worksheet of the **CIE Total Difference.xls** file. This worksheet (see Figure 8.14 on page 310) uses the **TINV(1-confidence level, degrees of freedom)** function to determine the critical value from the *t* distribution and the interval half-width for the Section 8.5 Saxon Home Improvement Company total difference example. The worksheet also contains a calculation area in cell range D9:E16, as shown in Figure E8.1, that counts and sums the differences listed on a **DifferencesData** worksheet. Figure E8.2 illustrates the first 6 of the 13 rows in the Difference Data worksheet).

	D	E	
9	Calculations Area		
10	For standard deviation of differences:		
11	Number of Differences Not = 0	12	=COUNT(DifferencesData!A:A)
12	Number of Differences = 0	88	=B5 - E11
13	SS for Differences Not = 0	678.3864	=SUM(DifferencesData!B:B)
14	SS for Differences = 0	71.28	=E12 * (-B10)^2
15	Sum of Squares	749.6664	=E13 + E14
16	Variance of Differences	7.572388	=E15/B15

**FIGURE E8.1**

Calculations area in the Difference Data worksheet

	A	B	
1	<b>Differences (D-DBar)^2</b>		
2	9.03	66.0969	=(A2 - CIE_TD!\$B\$10)^2
3	7.47	43.1649	=(A3 - CIE_TD!\$B\$10)^2
4	17.32	269.6164	=(A4 - CIE_TD!\$B\$10)^2
5	8.3	54.76	=(A5 - CIE_TD!\$B\$10)^2
6	5.21	18.5761	=(A6 - CIE_TD!\$B\$10)^2

**FIGURE E8.2**

DifferencesData worksheet (partial)

To adapt this worksheet to other problems, you need to change both the CIE\_TD and DifferencesData worksheet. In the CIE\_TD worksheet, enter the appropriate population size, sample size, and confidence level values in the tinted cells B4 through B6 and enter a new title in cell A1. In the DifferencesData worksheet, enter the differences in column A. Then adjust column B by either copying down the

formula in cell B13 to all rows with difference data, if you have more than 12 differences; or by deleting unnecessary column B formulas, if you have fewer than 12 differences.

## E8.8 COMPUTING FINITE POPULATION CORRECTION FACTORS

The workbooks for confidence interval estimations of the mean and proportion and for computing the sample size needed for estimating the mean or proportion include a **FPC** worksheet that calculates the confidence interval estimate or sample size, using a finite population correction factor (see Section 8.7 on the Student CD-ROM). (Open to those worksheets for further information.) If you use PHStat2, you can include these computations by clicking the **Finite Population Correction** output option and entering the **Population Size** before clicking **OK** in the appropriate dialog boxes.

# CHAPTER 9

## Fundamentals of Hypothesis Testing: One-Sample Tests

USING STATISTICS @ Oxford Cereals, Part II

### 9.1 HYPOTHESIS-TESTING METHODOLOGY

The Null and Alternative Hypotheses  
The Critical Value of the Test Statistic  
Regions of Rejection and Nonrejection  
Risks in Decision Making Using Hypothesis-Testing Methodology

### 9.2 Z TEST OF HYPOTHESIS FOR THE MEAN ( $\sigma$ KNOWN)

The Critical Value Approach to Hypothesis Testing  
The  $p$ -Value Approach to Hypothesis Testing  
A Connection Between Confidence Interval Estimation and Hypothesis Testing

### 9.3 ONE-TAIL TESTS

The Critical Value Approach  
The  $p$ -Value Approach

### 9.4 $t$ TEST OF HYPOTHESIS FOR THE MEAN ( $\sigma$ UNKNOWN)

The Critical Value Approach  
The  $p$ -Value Approach  
Checking Assumptions

### 9.5 Z TEST OF HYPOTHESIS FOR THE PROPORTION

The Critical Value Approach  
The  $p$ -Value Approach

### 9.6 POTENTIAL HYPOTHESIS-TESTING PITFALLS AND ETHICAL ISSUES

### 9.7 (CD-ROM TOPIC) THE POWER OF A TEST

#### EXCEL COMPANION TO CHAPTER 9

- E9.1 Using the  $Z$  Test for the Mean ( $\sigma$  Known)
- E9.2 Using the  $t$  Test for the Mean ( $\sigma$  Unknown)
- E9.3 Using the  $Z$  Test for the Proportion

## LEARNING OBJECTIVES

In this chapter, you learn:

- The basic principles of hypothesis testing
- How to use hypothesis testing to test a mean or proportion
- The assumptions of each hypothesis-testing procedure, how to evaluate them, and the consequences if they are seriously violated
- How to avoid the pitfalls involved in hypothesis testing
- Ethical issues involved in hypothesis testing

## Using Statistics @ Oxford Cereals, Part II



As in Chapter 7, you again find yourself as plant operations manager for Oxford Cereals. You are responsible for monitoring the amount in each cereal box filled. Company specifications require a mean weight of 368 grams per box. It is your responsibility to adjust the process when the mean fill weight in the population of boxes deviates from 368 grams. How can you rationally make the decision whether or not to adjust the process when it is impossible to weigh every single box as it is being filled? You begin by selecting and weighing a random sample of 25 cereal boxes. After computing a sample mean, how do you proceed?

In Chapter 7, you learned methods to determine whether a sample mean is consistent with a known population mean. In this Oxford Cereals scenario, you seek to use a sample mean to validate a claim about the population mean, a somewhat different problem. For this type of problem, you use an inferential method called **hypothesis testing**. Hypothesis testing requires that you state a claim unambiguously. In this scenario, the claim is that the population mean is 368 grams. You examine a sample statistic to see if it better supports the stated claim, called the *null hypothesis*, or the mutually exclusive alternative (for this scenario, that the population mean is not 368 grams.)

In this chapter, you will learn several applications of hypothesis testing. You will learn how to make inferences about a population parameter by *analyzing differences* between the results observed, the sample statistic, and the results you would expect to get if some underlying hypothesis were actually true. For the Oxford Cereals scenario, hypothesis testing would allow you to infer one of the following:

- The mean weight of the cereal boxes in the sample is a value consistent with what you would expect if the mean of the entire population of cereal boxes is 368 grams.
- The population mean is not equal to 368 grams because the sample mean is significantly different from 368 grams.

### 9.1 HYPOTHESIS-TESTING METHODOLOGY

Hypothesis testing typically begins with some theory, claim, or assertion about a particular parameter of a population. For example, your initial hypothesis about the cereal example is that the process is working properly, so the mean fill is 368 grams, and no corrective action is needed.

#### The Null and Alternative Hypotheses

The hypothesis that the population parameter is equal to the company specification is referred to as the null hypothesis. A **null hypothesis** is always one of status quo and is identified by the symbol  $H_0$ . Here the null hypothesis is that the filling process is working properly, and therefore the mean fill is the 368-gram specification. This is stated as

$$H_0: \mu = 368$$



Even though information is available only from the sample, the null hypothesis is written in terms of the population. Remember, your focus is on the population of all cereal boxes. The sample statistic is used to make inferences about the entire filling process. One inference may be that the results observed from the sample data indicate that the null hypothesis is false. If the null hypothesis is considered false, something else must be true.

Whenever a null hypothesis is specified, an alternative hypothesis is also specified, and it must be true if the null hypothesis is false. The **alternative hypothesis**,  $H_1$ , is the opposite of the null hypothesis,  $H_0$ . This is stated in the cereal example as

$$H_1: \mu \neq 368$$

The alternative hypothesis represents the conclusion reached by rejecting the null hypothesis. The null hypothesis is rejected when there is sufficient evidence from the sample information that the null hypothesis is false. In the cereal example, if the weights of the sampled boxes are sufficiently above or below the expected 368-gram mean specified by the company, you reject the null hypothesis in favor of the alternative hypothesis that the mean fill is different from 368 grams. You stop production and take whatever action is necessary to correct the problem. If the null hypothesis is not rejected, you should continue to believe in the status quo, that the process is working correctly and therefore no corrective action is necessary. In this second circumstance, you have not proven that the process is working correctly. Rather, you have failed to prove that it is working incorrectly, and therefore you continue your belief (although unproven) in the null hypothesis.

In the hypothesis-testing methodology, the null hypothesis is rejected when the sample evidence suggests that it is far more likely that the alternative hypothesis is true. However, failure to reject the null hypothesis is not proof that it is true. You can never prove that the null hypothesis is correct because the decision is based only on the sample information, not on the entire population. Therefore, if you fail to reject the null hypothesis, you can only conclude that there is insufficient evidence to warrant its rejection. The following key points summarize the null and alternative hypotheses:

- The null hypothesis,  $H_0$ , represents the status quo or the current belief in a situation.
- The alternative hypothesis,  $H_1$ , is the opposite of the null hypothesis and represents a research claim or specific inference you would like to prove.
- If you reject the null hypothesis, you have statistical proof that the alternative hypothesis is correct.
- If you do not reject the null hypothesis, you have failed to prove the alternative hypothesis. The failure to prove the alternative hypothesis, however, does not mean that you have proven the null hypothesis.
- The null hypothesis,  $H_0$ , always refers to a specified value of the population parameter (such as  $\mu$ ), not a sample statistic (such as  $\bar{X}$ ).
- The statement of the null hypothesis always contains an equal sign regarding the specified value of the population parameter (for example,  $H_0: \mu = 368$  grams).
- The statement of the alternative hypothesis never contains an equal sign regarding the specified value of the population parameter (for example,  $H_1: \mu \neq 368$  grams).

### EXAMPLE 9.1

#### THE NULL AND ALTERNATIVE HYPOTHESES

You are the manager of a fast-food restaurant. You want to determine whether the waiting time to place an order has changed in the past month from its previous population mean value of 4.5 minutes. State the null and alternative hypotheses.

**SOLUTION** The null hypothesis is that the population mean has not changed from its previous value of 4.5 minutes. This is stated as

$$H_0: \mu = 4.5$$

The alternative hypothesis is the opposite of the null hypothesis. Because the null hypothesis is that the population mean is 4.5 minutes, the alternative hypothesis is that the population mean is not 4.5 minutes. This is stated as

$$H_1: \mu \neq 4.5$$

## The Critical Value of the Test Statistic

The logic behind the hypothesis-testing methodology is to determine how likely the null hypothesis is to be true by considering the information gathered in a sample. In the Oxford Cereal Company scenario, the null hypothesis is that the mean amount of cereal per box in the entire filling process is 368 grams (that is, the population parameter specified by the company). You select a sample of boxes from the filling process, weigh each box, and compute the sample mean. This statistic is an estimate of the corresponding parameter (the population mean,  $\mu$ ). Even if the null hypothesis is true, the statistic (the sample mean,  $\bar{X}$ ) is likely to differ from the value of the parameter (the population mean,  $\mu$ ) because of variation due to sampling. However, you expect the sample statistic to be close to the population parameter if the null hypothesis is true. If the sample statistic is close to the population parameter, you have insufficient evidence to reject the null hypothesis. For example, if the sample mean is 367.9, you conclude that the population mean has not changed (that is,  $\mu = 368$ ) because a sample mean of 367.9 is very close to the hypothesized value of 368. Intuitively, you think that it is likely that you could get a sample mean of 367.9 from a population whose mean is 368.

However, if there is a large difference between the value of the statistic and the hypothesized value of the population parameter, you conclude that the null hypothesis is false. For example, if the sample mean is 320, you conclude that the population mean is not 368 (that is,  $\mu \neq 368$ ), because the sample mean is very far from the hypothesized value of 368. In such a case, you conclude that it is very unlikely to get a sample mean of 320 if the population mean is really 368. Therefore, it is more logical to conclude that the population mean is not equal to 368. Here you reject the null hypothesis.

Unfortunately, the decision-making process is not always so clear-cut. Determining what is “very close” and what is “very different” is arbitrary without clear definitions. Hypothesis-testing methodology provides clear definitions for evaluating differences. Furthermore, it enables you to quantify the decision-making process by computing the probability of getting a given sample result if the null hypothesis is true. You calculate this probability by determining the sampling distribution for the sample statistic of interest (for example, the sample mean) and then computing the particular **test statistic** based on the given sample result. Because the sampling distribution for the test statistic often follows a well-known statistical distribution, such as the standardized normal distribution or  $t$  distribution, you can use these distributions to help determine whether the null hypothesis is true.

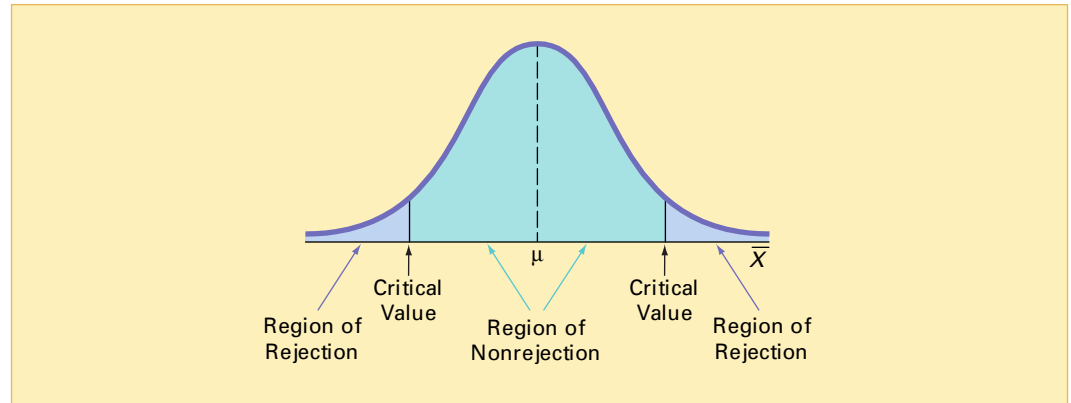
## Regions of Rejection and Nonrejection

The sampling distribution of the test statistic is divided into two regions, a **region of rejection** (sometimes called the critical region) and a **region of nonrejection** (see Figure 9.1).

If the test statistic falls into the region of nonrejection, you do not reject the null hypothesis. In the Oxford Cereals scenario, you conclude that there is insufficient evidence that the population mean fill is different from 368 grams. If the test statistic falls into the rejection region, you reject the null hypothesis. In this case, you conclude that the population mean is not 368 grams.

**FIGURE 9.1**

Regions of rejection and nonrejection in hypothesis testing



The region of rejection consists of the values of the test statistic that are unlikely to occur if the null hypothesis is true. These values are more likely to occur if the null hypothesis is false. Therefore, if a value of the test statistic falls into this *rejection region*, you reject the null hypothesis because that value is unlikely if the null hypothesis is true.

To make a decision concerning the null hypothesis, you first determine the **critical value** of the test statistic. The critical value divides the nonrejection region from the rejection region. Determining this critical value depends on the size of the rejection region. The size of the rejection region is directly related to the risks involved in using only sample evidence to make decisions about a population parameter.

### Risks in Decision Making Using Hypothesis-Testing Methodology

When using a sample statistic to make decisions about a population parameter, there is a risk that you will reach an incorrect conclusion. You can make two different types of errors when applying hypothesis-testing methodology, Type I and Type II errors.

A **Type I error** occurs if you reject the null hypothesis,  $H_0$ , when it is true and should not be rejected. The probability of a Type I error occurring is  $\alpha$ .

A **Type II error** occurs if you do not reject the null hypothesis,  $H_0$ , when it is false and should be rejected. The probability of a Type II error occurring is  $\beta$ .

In the Oxford Cereals scenario, you make a Type I error if you conclude that the population mean fill is *not* 368 when it *is* 368. This error causes you to adjust the filling process even though the process is working properly. You make a Type II error if you conclude that the population mean fill *is* 368 when it is *not* 368. Here, you would allow the process to continue without adjustment even though adjustments are needed.

**The Level of Significance ( $\alpha$ )** The probability of committing a Type I error, denoted by  $\alpha$  (the lowercase Greek letter *alpha*), is referred to as the **level of significance** of the statistical test. Traditionally, you control the Type I error by deciding the risk level,  $\alpha$ , that you are willing to have in rejecting the null hypothesis when it is true. Because you specify the level of significance before the hypothesis test is performed, the risk of committing a Type I error,  $\alpha$ , is directly under your control. Traditionally, you select levels of 0.01, 0.05, or 0.10. The choice of a particular risk level for making a Type I error depends on the cost of making a Type I error. After you specify the value for  $\alpha$ , you can then determine the critical values that divide the rejection and nonrejection regions. You know the size of the rejection region because  $\alpha$  is the probability of rejection when the null hypothesis is true. From this, you can then determine the critical value or values that divide the rejection and nonrejection regions.

**The Confidence Coefficient** The complement of the probability of a Type I error,  $(1 - \alpha)$ , is called the confidence coefficient. When multiplied by 100%, the confidence coefficient yields the confidence level that was studied when constructing confidence intervals (see Section 8.1).

The **confidence coefficient**,  $(1 - \alpha)$ , is the probability that you will not reject the null hypothesis,  $H_0$ , when it is true and should not be rejected. The **confidence level** of a hypothesis test is  $(1 - \alpha) \times 100\%$ .

In terms of hypothesis-testing methodology, the confidence coefficient represents the probability of concluding that the value of the parameter as specified in the null hypothesis is plausible when it is true. In the Oxford Cereals scenario, the confidence coefficient measures the probability of concluding that the population mean fill is 368 grams when it is actually 368 grams.

**The  $\beta$  Risk** The probability of committing a Type II error is denoted by  $\beta$  (the lowercase Greek letter *beta*). Unlike a Type I error, which you control by the selection of  $\alpha$ , the probability of making a Type II error depends on the difference between the hypothesized and actual values of the population parameter. Because large differences are easier to find than small ones, if the difference between the hypothesized and actual values of the population parameter is large,  $\beta$  is small. For example, if the population mean is 330 grams, there is a small chance ( $\beta$ ) that you will conclude that the mean has not changed from 368. However, if the difference between the hypothesized and actual values of the parameter is small,  $\beta$  is large. For example, if the population mean is actually 367 grams, there is a large chance ( $\beta$ ) that you will conclude that the mean is still 368 grams.

**The Power of a Test** The complement of the probability of a Type II error,  $(1 - \beta)$ , is called the power of a statistical test.

The **power of a statistical test**,  $(1 - \beta)$ , is the probability that you will reject the null hypothesis when it is false and should be rejected.

In the Oxford Cereals scenario, the power of the test is the probability that you will correctly conclude that the mean fill amount is not 368 grams when it actually is not 368 grams. For a detailed discussion of the power of the test, see Section 9.7 on the Student CD-ROM.

**Risks in Decision Making: A Delicate Balance** Table 9.1 illustrates the results of the two possible decisions (do not reject  $H_0$  or reject  $H_0$ ) that you can make in any hypothesis test. You can make a correct decision or make one of two types of errors.

**TABLE 9.1**  
Hypothesis Testing  
and Decision Making

Statistical Decision	Actual Situation	
	$H_0$ True	$H_0$ False
Do not reject $H_0$	Correct decision Confidence = $(1 - \alpha)$	Type II error $P(\text{Type II error}) = \beta$
Reject $H_0$	Type I error $P(\text{Type I error}) = \alpha$	Correct decision Power = $(1 - \beta)$

One way to reduce the probability of making a Type II error is by increasing the sample size. Large samples generally permit you to detect even very small differences between the hypothesized values and the population parameters. For a given level of  $\alpha$ , increasing the sample size decreases  $\beta$  and therefore increases the power of the test to detect that the null hypoth-

esis,  $H_0$ , is false. However, there is always a limit to your resources, and this affects the decision as to how large a sample you can take. Thus, for a given sample size, you must consider the trade-offs between the two possible types of errors. Because you can directly control the risk of Type I error, you can reduce this risk by selecting a smaller value for  $\alpha$ . For example, if the negative consequences associated with making a Type I error are substantial, you could select  $\alpha = 0.01$  instead of 0.05. However, when you decrease  $\alpha$ , you increase  $\beta$ , so reducing the risk of a Type I error results in an increased risk of a Type II error. However, if you wish to reduce  $\beta$ , you could select a larger value for  $\alpha$ . Therefore, if it is important to try to avoid a Type II error, you can select  $\alpha$  of 0.05 or 0.10 instead of 0.01.

In the Oxford Cereals scenario, the risk of a Type I error involves concluding that the mean fill amount has changed from the hypothesized 368 grams when it actually has not changed. The risk of a Type II error involves concluding that the mean fill amount has not changed from the hypothesized 368 grams when it actually has changed. The choice of reasonable values for  $\alpha$  and  $\beta$  depends on the costs inherent in each type of error. For example, if it is very costly to change the cereal-fill process, you would want to be very confident that a change is needed before making any changes. In this case, the risk of a Type I error is more important, and you would choose a small  $\alpha$ . However, if you want to be very certain of detecting changes from a mean of 368 grams, the risk of a Type II error is more important, and you would choose a higher level of  $\alpha$ .

## PROBLEMS FOR SECTION 9.1

### Learning the Basics

**PH Grade ASSIST** 9.1 You use the symbol  $H_0$  for which hypothesis?

**PH Grade ASSIST** 9.2 You use the symbol  $H_1$  for which hypothesis?

**PH Grade ASSIST** 9.3 What symbol do you use for the chance of committing a Type I error?

**PH Grade ASSIST** 9.4 What symbol do you use for the chance of committing a Type II error?

**PH Grade ASSIST** 9.5 What does  $1 - \alpha$  represent?

9.6 What is the relationship of  $\alpha$  to a Type I error?

9.7 What is the relationship of  $\beta$  to a Type II error?

9.8 How is power related to the probability of making a Type II error?

**PH Grade ASSIST** 9.9 Why is it possible to reject the null hypothesis when it is true?

**PH Grade ASSIST** 9.10 Why is it possible to not reject the null hypothesis when it is false?

9.11 For a given sample size, if  $\alpha$  is reduced from 0.05 to 0.01, what happens to  $\beta$ ?

9.12 For  $H_0: \mu = 100$ ,  $H_1: \mu \neq 100$ , and for a sample of size  $n$ , why is  $\beta$  larger if the actual value of  $\mu$  is 90 than if the actual value of  $\mu$  is 75?

### Applying the Concepts

**PH Grade ASSIST** 9.13 In the U.S. legal system, a defendant is presumed innocent until proven guilty. Consider a null hypothesis,  $H_0$ , that the defendant is innocent, and an alternative hypothesis,  $H_1$ , that the defendant is guilty. A jury has two possible decisions: Convict the defendant (that is, reject the null hypothesis) or do not convict the defendant (that is, do not reject the null hypothesis). Explain the meaning of the risks of committing either a Type I or Type II error in this example.

9.14 Suppose the defendant in Problem 9.13 is presumed guilty until proven innocent, as in some other judicial systems. How do the null and alternative hypotheses differ from those in Problem 9.13? What are the meanings of the risks of committing either a Type I or Type II error here?

9.15 The U.S. Food and Drug Administration (FDA) is responsible for approving new drugs. Many consumer groups feel that the approval process is too easy and, therefore, too many drugs are approved that are later found to be unsafe. On the other hand, a number of industry lobbyists are pushing for a more lenient approval process so that pharmaceutical companies can get new drugs approved more easily and quickly (R. Sharpe, "FDA Tries to Find Right Balance on Drug Approvals," *The Wall Street Journal*, April 20, 1999, p. A24). Consider a null hypothesis that a new, unapproved drug is unsafe and an alternative hypothesis that a new, unapproved drug is safe.

- Explain the risks of committing a Type I or Type II error.
- Which type of error are the consumer groups trying to avoid? Explain.
- Which type of error are the industry lobbyists trying to avoid? Explain.
- How would it be possible to lower the chances of both Type I and Type II errors?



**9.16** As a result of complaints from both students and faculty about lateness, the registrar at a large university wants to adjust the scheduled class times to allow for adequate travel time between classes and is ready to undertake a study. Until now, the registrar has believed that there should be 20 minutes between scheduled classes. State the null hypothesis,  $H_0$ , and the alternative hypothesis,  $H_1$ .

**9.17** The operations manager at a clothing factory needs to determine whether a new machine is producing a particular type of cloth according to the manufacturer's specifications, which indicate that the cloth should have a mean breaking strength of 70 pounds. State the null and alternative hypotheses.

**9.18** The manager of a paint supply store wants to determine whether the amount of paint contained in 1-gallon cans purchased from a nationally known manufacturer actually averages 1 gallon. State the null and alternative hypotheses.

**9.19** The quality control manager at a light bulb factory needs to determine whether the mean life of a large shipment of light bulbs is equal to the specified value of 375 hours. State the null and alternative hypotheses.

## 9.2 Z TEST OF HYPOTHESIS FOR THE MEAN ( $\sigma$ KNOWN)

Now that you have been introduced to hypothesis-testing methodology, recall that in the Using Statistics scenario on page 328, Oxford Cereals wants to determine whether the cereal-fill process is working properly (that is, whether the mean fill throughout the entire packaging process remains at the specified 368 grams, and no corrective action is needed). To evaluate the 368-gram requirement, you take a random sample of 25 boxes, weigh each box, and then evaluate the difference between the sample statistic and the hypothesized population parameter by comparing the mean weight (in grams) from the sample to the expected mean of 368 grams specified by the company. The null and alternative hypotheses are

$$H_0: \mu = 368$$

$$H_1: \mu \neq 368$$

When the standard deviation,  $\sigma$ , is known, you use the  $Z$  test if the population is normally distributed. If the population is not normally distributed, you can still use the  $Z$  test if the sample size is large enough for the Central Limit Theorem to take effect (see Section 7.4). Equation (9.1) defines the  **$Z$ -test statistic** for determining the difference between the sample mean,  $\bar{X}$ , and the population mean,  $\mu$ , when the standard deviation,  $\sigma$ , is known.

### Z TEST OF HYPOTHESIS FOR THE MEAN ( $\sigma$ KNOWN)

$$Z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} \quad (9.1)$$

In Equation (9.1) the numerator measures how far (in an absolute sense) the observed sample mean,  $\bar{X}$ , is from the hypothesized mean,  $\mu$ . The denominator is the standard error of the mean, so  $Z$  represents the difference between  $\bar{X}$  and  $\mu$  in standard error units.

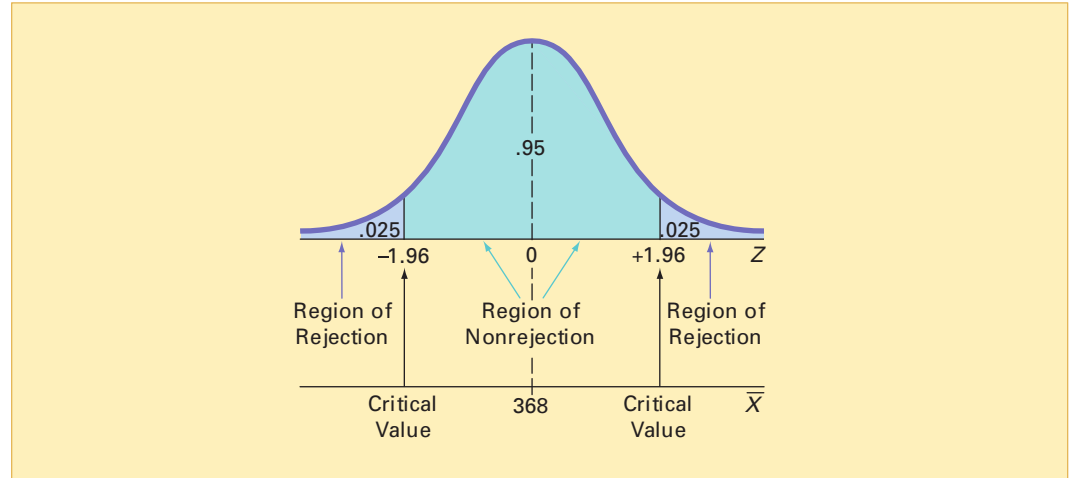
### The Critical Value Approach to Hypothesis Testing

The observed value of the  $Z$  test statistic, Equation (9.1), is compared to critical values. The **critical values** are expressed as standardized  $Z$  values (that is, in standard deviation units). For example, if you use a level of significance of 0.05, the size of the rejection region is 0.05. Because the rejection region is divided into the two tails of the distribution (this is called a **two-tail test**), you divide the 0.05 into two equal parts of 0.025 each. A rejection region of 0.025 in

each tail of the normal distribution results in a cumulative area of 0.025 below the lower critical value and a cumulative area of 0.975 below the upper critical value. According to the cumulative standardized normal distribution table (Table E.2), the critical values that divide the rejection and nonrejection regions are  $-1.96$  and  $+1.96$ . Figure 9.2 illustrates that if the mean is actually 368 grams, as  $H_0$  claims, the values of the test statistic  $Z$  have a standardized normal distribution centered at  $Z = 0$  (which corresponds to an  $\bar{X}$  value of 368 grams). Values of  $Z$  greater than  $+1.96$  or less than  $-1.96$  indicate that  $\bar{X}$  is so far from the hypothesized  $\mu = 368$  that it is unlikely that such a value would occur if  $H_0$  were true.

**FIGURE 9.2**

Testing a hypothesis about the mean ( $\sigma$  known) at the 0.05 level of significance



Therefore, the decision rule is

Reject  $H_0$  if  $Z > +1.96$   
or if  $Z < -1.96$ ;  
otherwise, do not reject  $H_0$ .

Suppose that the sample of 25 cereal boxes indicates a sample mean,  $\bar{X}$ , of 372.5 grams, and the population standard deviation,  $\sigma$ , is assumed to be 15 grams. Using Equation (9.1) on page 334,

$$Z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{372.5 - 368}{\frac{15}{\sqrt{25}}} = +1.50$$

Because the test statistic  $Z = +1.50$  is between  $-1.96$  and  $+1.96$ , you do not reject  $H_0$  (see Figure 9.3). You continue to believe that the mean fill amount is 368 grams. To take into account the possibility of a Type II error, you state the conclusion as “there is insufficient evidence that the mean fill is different from 368 grams.”

**FIGURE 9.3**

Testing a hypothesis about the mean cereal weight ( $\sigma$  known) at 0.05 level of significance

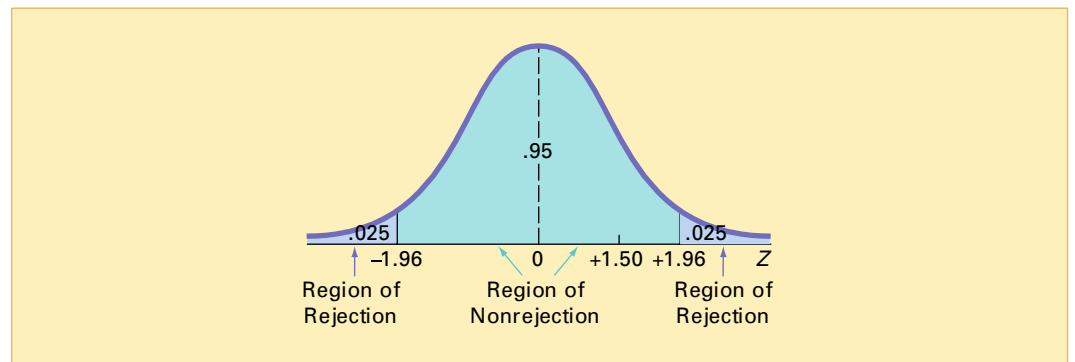


Exhibit 9.1 provides a summary of the critical value approach to hypothesis testing.

### EXHIBIT 9.1 THE SIX-STEP METHOD OF HYPOTHESIS TESTING

1. State the null hypothesis,  $H_0$ , and the alternative hypothesis,  $H_1$ .
2. Choose the level of significance,  $\alpha$ , and the sample size,  $n$ . The level of significance is based on the relative importance of the risks of committing Type I and Type II errors in the problem.
3. Determine the appropriate test statistic and sampling distribution.
4. Determine the critical values that divide the rejection and nonrejection regions.
5. Collect the sample data and compute the value of the test statistic.
6. Make the statistical decision and state the managerial conclusion. If the test statistic falls into the nonrejection region, you do not reject the null hypothesis,  $H_0$ . If the test statistic falls into the rejection region, you reject the null hypothesis. The managerial conclusion is written in the context of the real-world problem.

### EXAMPLE 9.2

#### APPLYING THE SIX-STEP METHOD OF HYPOTHESIS TESTING AT OXFORD CEREALS

State the six-step method of hypothesis testing at Oxford Cereals.

#### SOLUTION

- Step 1** State the null and alternative hypotheses. The null hypothesis,  $H_0$ , is always stated in statistical terms, using population parameters. In testing whether the mean fill is 368 grams, the null hypothesis states that  $\mu$  equals 368. The alternative hypothesis,  $H_1$ , is also stated in statistical terms, using population parameters. Therefore, the alternative hypothesis states that  $\mu$  is not equal to 368 grams.
- Step 2** Choose the level of significance and the sample size. You choose the level of significance,  $\alpha$ , according to the relative importance of the risks of committing Type I and Type II errors in the problem. The smaller the value of  $\alpha$ , the less risk there is of making a Type I error. In this example, a Type I error is to conclude that the population mean is not 368 grams when it is 368 grams. Thus, you will take corrective action on the filling process even though the process is working properly. Here,  $\alpha = 0.05$  is selected. The sample,  $n$ , is 25.
- Step 3** Select the appropriate test statistic. Because  $\sigma$  is known from information about the filling process, you use the normal distribution and the  $Z$  test statistic.
- Step 4** Determine the rejection region. Critical values for the appropriate test statistic are selected so that the rejection region contains a total area of  $\alpha$  when  $H_0$  is true and the nonrejection region contains a total area of  $1 - \alpha$  when  $H_0$  is true. Because  $\alpha = 0.05$  in the cereal example, the critical values of the  $Z$ -test statistic are  $-1.96$  and  $+1.96$ . The rejection region is therefore  $Z < -1.96$  or  $Z > +1.96$ . The nonrejection region is  $-1.96 < Z < +1.96$ .
- Step 5** Collect the sample data and compute the value of the test statistic. In the cereal example,  $\bar{X} = 372.5$ , and the value of the test statistic is  $Z = +1.50$ .
- Step 6** State the statistical decision and the managerial conclusion. First, determine whether the test statistic has fallen into the rejection region or the nonrejection region. For the cereal example,  $Z = +1.50$  is in the region of nonrejection because  $-1.96 < Z = +1.50 < +1.96$ . Because the test statistic falls into the nonrejection region, the statistical decision is to not reject the null hypothesis,  $H_0$ . The managerial conclusion is that insufficient evidence exists to prove that the mean fill is different from 368 grams. No corrective action on the filling process is needed.



**EXAMPLE 9.3****REJECTING A NULL HYPOTHESIS**

You are the manager of a fast-food restaurant. You want to determine whether the population mean waiting time to place an order has changed in the past month from its previous population mean value of 4.5 minutes. From past experience, you can assume that the population is normally distributed with a population standard deviation of 1.2 minutes. You select a sample of 25 orders during a one-hour period. The sample mean is 5.1 minutes. Use the six-step approach listed in Exhibit 9.1 to determine whether there is evidence at the 0.05 level of significance that the population mean waiting time to place an order has changed in the past month from its previous population mean value of 4.5 minutes.

**SOLUTION**

**Step 1** The null hypothesis is that the population mean has not changed from its previous value of 4.5 minutes:

$$H_0: \mu = 4.5$$

The alternative hypothesis is the opposite of the null hypothesis. Because the null hypothesis is that the population mean is 4.5 minutes, the alternative hypothesis is that the population mean is not 4.5 minutes:

$$H_1: \mu \neq 4.5$$

**Step 2** You have selected a sample of  $n = 25$ . The level of significance is 0.05 (that is,  $\alpha = 0.05$ ).

**Step 3** Because  $\sigma$  is known, you use the normal distribution and the  $Z$  test statistic.

**Step 4** Because  $\alpha = 0.05$ , the critical values of the  $Z$  test statistic are  $-1.96$  and  $+1.96$ . The rejection region is  $Z < -1.96$  or  $Z > +1.96$ . The nonrejection region is  $-1.96 < Z < +1.96$ .

**Step 5** You collect the sample data and compute  $\bar{X} = 5.1$ . Using Equation (9.1) on page 334, you compute the test statistic:

$$Z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{5.1 - 4.5}{\frac{1.2}{\sqrt{25}}} = 2.50$$

**Step 6** Because  $Z = 2.50 > 1.96$ , you reject the null hypothesis. You conclude that there is evidence that the population mean waiting time to place an order has changed from its previous value of 4.5 minutes. The mean waiting time for customers is longer now than it was last month.

**The  $p$ -Value Approach to Hypothesis Testing**

Most modern software, including Microsoft Excel, computes the  $p$ -value when performing a test of hypothesis.

The  **$p$ -value** is the probability of getting a test statistic equal to or more extreme than the sample result, given that the null hypothesis,  $H_0$ , is true.

The  $p$ -value, often referred to as the *observed level of significance*, is the smallest level at which  $H_0$  can be rejected.

The decision rules for rejecting  $H_0$  in the  $p$ -value approach are

- If the  $p$ -value is greater than or equal to  $\alpha$ , do not reject the null hypothesis.
- If the  $p$ -value is less than  $\alpha$ , reject the null hypothesis.

Many people confuse these rules, mistakenly believing that a high  $p$ -value is grounds for rejection. You can avoid this confusion by remembering the following mantra:

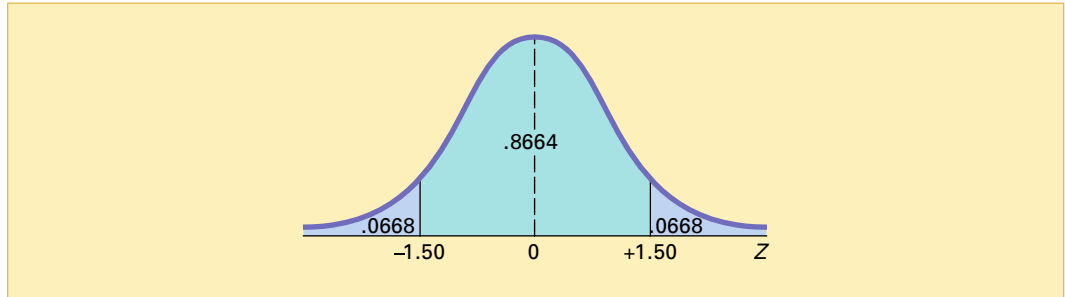
If the  $p$ -value is low, then  $H_0$  must go.

To understand the  $p$ -value approach, consider the Oxford Cereals scenario. You tested whether the mean fill was equal to 368 grams. The test statistic resulted in a  $Z$  value of +1.50, and you did not reject the null hypothesis because +1.50 was less than the upper critical value of +1.96 and more than the lower critical value of -1.96.

To use the  $p$ -value approach for the *two-tail test*, you find the probability of getting a test statistic  $Z$  that is equal to or *more extreme than* 1.50 standard deviation units from the center of a standardized normal distribution. In other words, you need to compute the probability of a  $Z$  value greater than +1.50, along with the probability of a  $Z$  value less than -1.50. Table E.2 shows that the probability of a  $Z$  value below -1.50 is 0.0668. The probability of a value below +1.50 is 0.9332, and the probability of a value above +1.50 is  $1 - 0.9332 = 0.0668$ . Therefore, the  $p$ -value for this two-tail test is  $0.0668 + 0.0668 = 0.1336$  (see Figure 9.4). Thus, the probability of a result equal to or more extreme than the one observed is 0.1336. Because 0.1336 is greater than  $\alpha = 0.05$ , you do not reject the null hypothesis.

**FIGURE 9.4**

Finding a  $p$ -value for a two-tail test



In this example, the observed sample mean is 372.5 grams, 4.5 grams above the hypothesized value, and the  $p$ -value is 0.1336. Thus, if the population mean is 368 grams, there is a 13.36% chance that the sample mean differs from 368 grams by more than 4.5 grams (that is, is  $\geq 372.5$  grams or  $\leq 363.5$  grams). Therefore, while 372.5 is above the hypothesized value of 368, a result as extreme as or more extreme than 372.5 is not highly unlikely when the population mean is 368.

Unless you are dealing with a test statistic that follows the normal distribution, computing the  $p$ -value can be very difficult. However, Microsoft Excel routinely computes the  $p$ -value in its hypothesis-testing procedures. Figure 9.5 displays a Microsoft Excel worksheet for the cereal-filling example discussed in this section.

**FIGURE 9.5**

Microsoft Excel  $Z$ -test results for the cereal-fill example

	A	B
1	<b>Cereal-Filling Process Hypothesis Test</b>	
2		
3	Data	
4	Null Hypothesis	$\mu = 368$
5	Level of Significance	0.05
6	Population Standard Deviation	15
7	Sample Size	25
8	Sample Mean	372.5
9		
10	Intermediate Calculations	
11	Standard Error of the Mean	3 =B6/SORT(B7)
12	Z Test Statistic	1.5 =(B8 - B4)/B11
13		
14	Two-Tail Test	
15	Lower Critical Value	-1.9600 =NORMSINV(B5/2)
16	Upper Critical Value	1.9600 =NORMSINV(1 - B5/2)
17	$p$ -Value	0.1336 =2 * (1 - NORMSDIST(ABS(B12)))
18	Do not reject the null hypothesis	=IF(B17 < B5, "Reject the null hypothesis", "Do not reject the null hypothesis")

Exhibit 9.2 provides a summary of the  $p$ -value approach for hypothesis testing.

### EXHIBIT 9.2 THE FIVE-STEP $p$ -VALUE APPROACH TO HYPOTHESIS TESTING

1. State the null hypothesis,  $H_0$ , and the alternative hypothesis,  $H_1$ .
2. Choose the level of significance,  $\alpha$ , and the sample size,  $n$ . The level of significance is based on the relative importance of the risks of committing Type I and Type II errors in the problem.
3. Determine the appropriate test statistic and sampling distribution.
4. Collect the sample data, compute the value of the test statistic, and compute the  $p$ -value.
5. Make the statistical decision and state the managerial conclusion. If the  $p$ -value is greater than or equal to  $\alpha$ , you do not reject the null hypothesis,  $H_0$ . If the  $p$ -value is less than  $\alpha$ , you reject the null hypothesis. Remember the mantra: If the  $p$ -value is low, then  $H_0$  must go. The managerial conclusion is written in the context of the real-world problem.

#### EXAMPLE 9.4

#### REJECTING A NULL HYPOTHESIS, USING THE $p$ -VALUE APPROACH

You are the manager of a fast-food restaurant. You want to determine whether the population mean waiting time to place an order has changed in the past month from its previous value of 4.5 minutes. From past experience, you can assume that the population standard deviation is 1.2 minutes. You select a sample of 25 orders during a one-hour period. The sample mean is 5.1 minutes. Use the five-step approach of Exhibit 9.2 above to determine whether there is evidence that the population mean waiting time to place an order has changed in the past month from its previous population mean value of 4.5 minutes.

#### SOLUTION

**Step 1** The null hypothesis is that the population mean has not changed from its previous value of 4.5 minutes:

$$H_0: \mu = 4.5$$

The alternative hypothesis is the opposite of the null hypothesis. Because the null hypothesis is that the population mean is 4.5 minutes, the alternative hypothesis is that the population mean is not 4.5 minutes:

$$H_1: \mu \neq 4.5$$

**Step 2** You have selected a sample of  $n = 25$ . You choose a 0.05 level of significance (that is,  $\alpha = 0.05$ ).

**Step 3** Select the appropriate test statistic. Because  $\sigma$  is known, you use the normal distribution and the  $Z$  test statistic.

**Step 4** You collect the data and compute  $\bar{X} = 5.1$ . Using Equation (9.1) on page 334, you compute the test statistic as follows:

$$Z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{5.1 - 4.5}{\frac{1.2}{\sqrt{25}}} = 2.50$$

To find the probability of getting a test statistic  $Z$  that is equal to or more extreme than 2.50 standard deviation units from the center of a standardized normal distribution, you compute the probability of a  $Z$  value greater than 2.50 along with the probability

of a  $Z$  value less than  $-2.50$ . From Table E.2, the probability of a  $Z$  value below  $-2.50$  is  $0.0062$ . The probability of a value below  $+2.50$  is  $0.9938$ . Therefore, the probability of a value above  $+2.50$  is  $1 - 0.9938 = 0.0062$ . Thus, the  $p$ -value for this two-tail test is  $0.0062 + 0.0062 = 0.0124$ .

**Step 5** Because the  $p$ -value  $= 0.0124 < \alpha = 0.05$ , you reject the null hypothesis. You conclude that there is evidence that the population mean waiting time to place an order has changed from its previous population mean value of 4.5 minutes. The mean waiting time for customers is longer now than it was last month.

## A Connection Between Confidence Interval Estimation and Hypothesis Testing

This chapter and Chapter 8 discuss the two major components of statistical inference: confidence interval estimation and hypothesis testing. Although they are based on the same set of concepts, they are used for different purposes. In Chapter 8, confidence intervals were used to estimate parameters. In this chapter, hypothesis testing is used for making decisions about specified values of population parameters. Hypothesis tests are used when trying to prove that a parameter is less than, more than, or not equal to a specified value. Proper interpretation of a confidence interval, however, can also indicate whether a parameter is less than, more than, or not equal to a specified value. For example, in this section, you tested whether the population mean fill amount was different from 368 grams by using Equation (9.1) on page 334:

$$Z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

Instead of testing the null hypothesis that  $\mu = 368$  grams, you can reach the same conclusion by constructing a confidence interval estimate of  $\mu$ . If the hypothesized value of  $\mu = 368$  is contained within the interval, you do not reject the null hypothesis because 368 would not be considered an unusual value. However, if the hypothesized value does not fall into the interval, you reject the null hypothesis because “ $\mu = 368$  grams” is then considered an unusual value. Using Equation (8.1) on page 287 and the following data:

$$n = 25, \bar{X} = 372.5 \text{ grams}, \sigma = 15 \text{ grams}$$

for a confidence level of 95% (corresponding to a 0.05 level of significance—that is,  $\alpha = 0.05$ ),

$$\begin{aligned} & \bar{X} \pm Z \frac{\sigma}{\sqrt{n}} \\ & 372.5 \pm (1.96) \frac{15}{\sqrt{25}} \\ & 372.5 \pm 5.88 \end{aligned}$$

so that

$$366.62 \leq \mu \leq 378.38$$

Because the interval includes the hypothesized value of 368 grams, you do not reject the null hypothesis. There is insufficient evidence that the mean fill amount over the entire filling process is not 368 grams. You reached the same decision by using two-tail hypothesis testing.

## PROBLEMS FOR SECTION 9.2

### Learning the Basics

**PH Grade ASSIST** **9.20** If you use a 0.05 level of significance in a (two-tail) hypothesis test, what will you decide if the computed value of the test statistic  $Z$  is  $+2.21$ ?

**PH Grade ASSIST** **9.21** If you use a 0.10 level of significance in a (two-tail) hypothesis test, what is your decision rule for rejecting a null hypothesis that the population mean is 500 if you use the  $Z$  test?

**PH Grade ASSIST** **9.22** If you use a 0.01 level of significance in a (two-tail) hypothesis test, what is your decision rule for rejecting  $H_0: \mu = 12.5$  if you use the  $Z$  test?

**PH Grade ASSIST** **9.23** What is your decision in Problem 9.22 if the computed value of the test statistic  $Z$  is  $-2.61$ ?

**PH Grade ASSIST** **9.24** Suppose that in a two-tail hypothesis test, you compute the value of the test statistic  $Z$  as  $+2.00$ . What is the  $p$ -value?

**PH Grade ASSIST** **9.25** In Problem 9.24, what is your statistical decision if you test the null hypothesis at the 0.10 level of significance?

**PH Grade ASSIST** **9.26** Suppose that in a two-tail hypothesis test, you compute the value of the test statistic  $Z$  as  $-1.38$ . What is the  $p$ -value?

**PH Grade ASSIST** **9.27** In Problem 9.26, what is your statistical decision if you test the null hypothesis at the 0.01 level of significance?

### Applying the Concepts

**SELF Test** **9.28** The operations manager at a clothing factory needs to determine whether a new machine is producing a particular type of cloth according to the manufacturer's specifications, which indicate that the cloth should have a mean breaking strength of 70 pounds and a standard deviation of 3.5 pounds. A sample of 49 pieces of cloth reveals a sample mean breaking strength of 69.1 pounds.

- Is there evidence that the machine is not meeting the manufacturer's specifications for mean breaking strength? (Use a 0.05 level of significance.)
- Compute the  $p$ -value and interpret its meaning.
- What is your answer in (a) if the standard deviation is 1.75 pounds?
- What is your answer in (a) if the sample mean is 69 pounds and the standard deviation is 3.5 pounds?

**PH Grade ASSIST** **9.29** The manager of a paint supply store wants to determine whether the mean amount of paint

contained in 1-gallon cans purchased from a nationally known manufacturer is actually 1 gallon. You know from the manufacturer's specifications that the standard deviation of the amount of paint is 0.02 gallon. You select a random sample of 50 cans, and the mean amount of paint per 1-gallon can is 0.995 gallon.

- Is there evidence that the population mean amount is different from 1.0 gallon (use  $\alpha = 0.01$ )?
- Compute the  $p$ -value and interpret its meaning.
- Construct a 99% confidence interval estimate of the population mean amount of paint.
- Compare the results of (a) and (c). What conclusions do you reach?

**9.30** The quality control manager at a light bulb factory needs to determine whether the mean life of a large shipment of light bulbs is equal to 375 hours. The population standard deviation is 100 hours. A random sample of 64 light bulbs indicates a sample mean life of 350 hours.

- At the 0.05 level of significance, is there evidence that the population mean life is different from 375 hours?
- Compute the  $p$ -value and interpret its meaning.
- Construct a 95% confidence interval estimate of the population mean life of the light bulbs.
- Compare the results of (a) and (c). What conclusions do you reach?

**9.31** The inspection division of the Lee County Weights and Measures Department is interested in determining whether the proper amount of soft drink has been placed in 2-liter bottles at the local bottling plant of a large nationally known soft-drink company. The bottling plant has informed the inspection division that the standard deviation for 2-liter bottles is 0.05 liter. A random sample of 100 2-liter bottles selected from this bottling plant indicates a sample mean of 1.99 liters.

- At the 0.05 level of significance, is there evidence that the population mean amount in the bottles is different from 2.0 liters?
- Compute the  $p$ -value and interpret its meaning.
- Construct a 95% confidence interval estimate of the population mean amount in the bottles.
- Compare the results of (a) and (c). What conclusions do you reach?

**9.32** A manufacturer of salad dressings uses machines to dispense liquid ingredients into bottles that move along a filling line. The machine that dispenses dressings is working properly when the mean amount dispensed is 8 ounces. The population standard deviation of the amount dispensed is 0.15 ounce. A sample of 50 bottles is selected periodically, and the filling line is stopped if there is evidence that

the mean amount dispensed is different from 8 ounces. Suppose that the mean amount dispensed in a particular sample of 50 bottles is 7.983 ounces.

- Is there evidence that the population mean amount is different from 8 ounces? (Use a 0.05 level of significance.)
- Compute the  $p$ -value and interpret its meaning.
- What is your answer in (a) if the standard deviation is 0.05 ounce?
- What is your answer in (a) if the sample mean is 7.952 ounces and the standard deviation is 0.15 ounce?

**9.33** ATMs must be stocked with enough cash to satisfy customers making withdrawals over an entire weekend. But if too much cash is unnecessarily kept in the ATMs, the bank is forgoing the opportunity of investing the money

and earning interest. Suppose that at a particular branch, the population mean amount of money withdrawn from ATMs per customer transaction over the weekend is \$160, with a population standard deviation of \$30.

- If a random sample of 36 customer transactions indicates that the sample mean withdrawal amount is \$172, is there evidence to believe that the population mean withdrawal amount is no longer \$160? (Use a 0.05 level of significance.)
- Compute the  $p$ -value and interpret its meaning.
- What is your answer in (b) if you use a 0.01 level of significance?
- What is your answer in (b) if the standard deviation is \$24 (use  $\alpha = 0.05$ )?

### 9.3 ONE-TAIL TESTS

In Section 9.2, hypothesis-testing methodology has been used to examine the question of whether the population mean amount of cereal filled is 368 grams. The alternative hypothesis ( $H_1: \mu \neq 368$ ) contains two possibilities: Either the mean is less than 368 grams, or the mean is more than 368 grams. For this reason, the rejection region is divided into the two tails of the sampling distribution of the mean.

In many situations, however, the alternative hypothesis focuses on a *particular direction*. One such situation occurs in the following application: A company that makes processed cheese is interested in determining whether some suppliers that provide milk for the processing operation are adding water to their milk to increase the amount supplied to the processing operation. It is known that excess water reduces the freezing point of the milk. The freezing point of natural milk is normally distributed, with a mean of  $-0.545^\circ$  Celsius (C). The standard deviation of the freezing temperature of natural milk is known to be  $0.008^\circ$ C. Because the cheese company is only interested in determining whether the freezing point of the milk is less than what would be expected from natural milk, the entire rejection region is located in the lower tail of the distribution.

#### The Critical Value Approach

Suppose you wish to determine whether the mean freezing point of milk is less than  $-0.545^\circ$ . To perform this one-tail hypothesis test, you use the six-step method listed in Exhibit 9.1 on page 336.

**Step 1**  $H_0: \mu \geq -0.545^\circ\text{C}$ .

$$H_1: \mu < -0.545^\circ\text{C}.$$

The alternative hypothesis contains the statement you are trying to prove. If the conclusion of the test is “reject  $H_0$ ,” there is statistical proof that the mean freezing point of the milk is less than the natural freezing point of  $-0.545^\circ\text{C}$ . If the conclusion of the test is “do not reject  $H_0$ ,” then there is insufficient evidence to prove that the mean freezing point is less than the natural freezing point of  $-0.545^\circ\text{C}$ .

**Step 2** You have selected a sample size of  $n = 25$ . You decide to use  $\alpha = 0.05$ .

**Step 3** Because  $\sigma$  is known, you use the normal distribution and the  $Z$ -test statistic.

**Step 4** The rejection region is entirely contained in the lower tail of the sampling distribution of the mean because you want to reject  $H_0$  only when the sample mean is significantly

less than  $-0.545^{\circ}\text{C}$ . When the entire rejection region is contained in one tail of the sampling distribution of the test statistic, the test is called a **one-tail** or **directional test**. When the alternative hypothesis includes the *less than* sign, the critical value of  $Z$  must be negative. As shown in Table 9.2 and Figure 9.6, because the entire rejection region is in the lower tail of the standardized normal distribution and contains an area of 0.05, the critical value of the  $Z$  test statistic is  $-1.645$ , halfway between  $-1.64$  and  $-1.65$ .

The decision rule is

Reject  $H_0$  if  $Z < -1.645$ ;  
otherwise, do not reject  $H_0$ .

**TABLE 9.2**

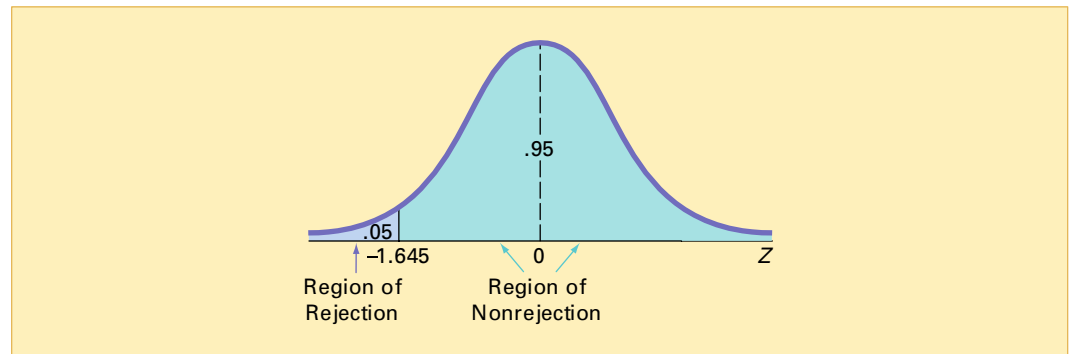
Finding the Critical Value of the  $Z$  Test Statistic from the Standardized Normal Distribution for a One-Tail Test with  $\alpha = 0.05$

$Z$	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455

Source: Extracted from Table E.2.

**FIGURE 9.6**

One-tail test of hypothesis for a mean ( $\sigma$  known) at the 0.05 level of significance



**Step 5** You select a sample of 25 containers of milk and find that the sample mean freezing point equals  $-0.550^{\circ}\text{C}$ . Using  $n = 25$ ,  $\bar{X} = -0.550^{\circ}\text{C}$ ,  $\sigma = 0.008^{\circ}\text{C}$ , and Equation (9.1) on page 334,

$$Z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{-0.550 - (-0.545)}{\frac{0.008}{\sqrt{25}}} = -3.125$$

**Step 6** Because  $Z = -3.125 < -1.645$ , you reject the null hypothesis (see Figure 9.6). You conclude that the mean freezing point of the milk provided is below  $-0.545^{\circ}\text{C}$ . The company should pursue an investigation of the milk supplier because the mean freezing point is significantly below what is expected to occur by chance.

### The $p$ -Value Approach

Use the five steps listed in Exhibit 9.2 on page 339 to illustrate the test from the preceding page using the  $p$ -value approach.

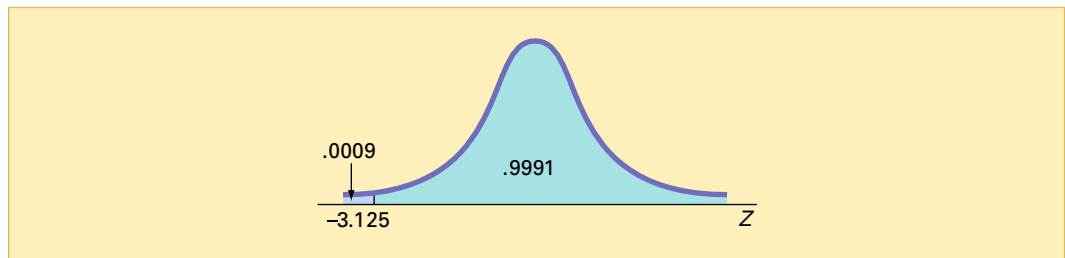
**Steps 1–3** These steps are the same as in the critical value approach.

**Step 4**  $Z = -3.125$  (see step 5 of the critical value approach). Because the alternative hypothesis indicates a rejection region entirely in the *lower* tail of the sampling distribution of

the  $Z$  test statistic, to compute the  $p$ -value, you need to find the probability that the  $Z$  value will be *less than* the test statistic of  $-3.125$ . From Table E.2, the probability that the  $Z$  value will be less than  $-3.125$  is 0.0009 (see Figures 9.7 and 9.8).

**FIGURE 9.7**

Determining the  $p$ -value for a one-tail test

**FIGURE 9.8**

Microsoft Excel  $Z$  test results for the milk production example



See Section E9.1 to create this.

	A	B
1	<b>Milk Production Hypothesis Test</b>	
2		
3	Data	
4	Null Hypothesis $\mu =$	-0.545
5	Level of Significance	0.05
6	Population Standard Deviation	0.008
7	Sample Size	25
8	Sample Mean	-0.55
9		
10	Intermediate Calculations	
11	Standard Error of the Mean	0.0016 =B6/SQRT(B7)
12	Z Test Statistic	-3.125 =(B8 - B4)/B11
13		
14	Lower-Tail Test	
15	Lower Critical Value	-1.6449 =NORMSINV(B5)
16	$p$ -Value	0.0009 =NORMSDIST(B12)
17	Reject the null hypothesis =IF(B16 < B5, "Reject the null hypothesis", "Do not reject the null hypothesis")	

**Step 5** The  $p$ -value of 0.0009 is less than  $\alpha = 0.05$ . You reject  $H_0$ . You conclude that the mean freezing point of the milk provided is less than  $-0.545^\circ\text{C}$ . The company should pursue an investigation of the milk supplier because the mean freezing point is significantly less than what is expected to occur by chance.

**EXAMPLE 9.5****A ONE-TAIL TEST FOR THE MEAN**

A company that manufactures chocolate bars is particularly concerned that the mean weight of a chocolate bar not be greater than 6.03 ounces. Past experience allows you to assume that the standard deviation is 0.02 ounces. A sample of 50 chocolate bars is selected, and the sample mean is 6.034 ounces. Using the  $\alpha = 0.01$  level of significance, is there evidence that the population mean weight of the chocolate bars is greater than 6.03 ounces?

**SOLUTION** Using the critical value approach,

**Step 1**  $H_0: \mu \leq 6.03$

$H_1: \mu > 6.03$

**Step 2** You have selected a sample size of  $n = 50$ . You decide to use  $\alpha = 0.01$ .

**Step 3** Because  $\sigma$  is known, you use the normal distribution and the  $Z$  test statistic.

**Step 4** The rejection region is entirely contained in the upper tail of the sampling distribution of the mean because you want to reject  $H_0$  only when the sample mean is significantly greater than 6.03 ounces. Because the entire rejection region is in the upper tail of the standardized normal distribution and contains an area of 0.01, the critical value of the  $Z$  test statistic is 2.33.



The decision rule is

Reject  $H_0$  if  $Z > 2.33$ ;  
otherwise, do not reject  $H_0$ .

**Step 5** You select a sample of 50 chocolate bars, and the sample mean weight is 6.034 ounces. Using  $n = 50$ ,  $\bar{X} = 6.034$ ,  $\sigma = 0.02$ , and Equation (9.1) on page 334,

$$Z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{6.034 - 6.03}{\frac{0.02}{\sqrt{50}}} = 1.414$$

**Step 6** Because  $Z = 1.414 < 2.33$ , you do not reject the null hypothesis. There is insufficient evidence to conclude that the population mean weight is greater than 6.03 ounces.

To perform one-tail tests of hypotheses, you must properly formulate  $H_0$  and  $H_1$ . A summary of the null and alternative hypotheses for one-tail tests is as follows:

1. The null hypothesis,  $H_0$ , represents the status quo or the current belief in a situation.
2. The alternative hypothesis,  $H_1$ , is the opposite of the null hypothesis and represents a research claim or specific inference you would like to prove.
3. If you reject the null hypothesis, you have statistical proof that the alternative hypothesis is correct.
4. If you do not reject the null hypothesis, then you have failed to prove the alternative hypothesis. The failure to prove the alternative hypothesis, however, does not mean that you have proven the null hypothesis.
5. The null hypothesis ( $H_0$ ) always refers to a specified value of the *population parameter* (such as  $\mu$ ), not to a *sample statistic* (such as  $\bar{X}$ ).
6. The statement of the null hypothesis *always* contains an equal sign regarding the specified value of the parameter (for example,  $H_0: \mu \geq -0.545^\circ\text{C}$ ).
7. The statement of the alternative hypothesis *never* contains an equal sign regarding the specified value of the parameter (for example,  $H_1: \mu < -0.545^\circ\text{C}$ ).

## PROBLEMS FOR SECTION 9.3

### Learning the Basics

**PH Grade ASSIST** **9.34** In a one-tail hypothesis test where you reject  $H_0$  only in the *upper* tail, what is the critical value of the  $Z$  test statistic at the 0.01 level of significance?

**PH Grade ASSIST** **9.35** In Problem 9.34, what is your statistical decision if the computed value of the  $Z$  test statistic is 2.39?

**PH Grade ASSIST** **9.36** In a one-tail hypothesis test where you reject  $H_0$  only in the *lower* tail, what is the critical value of the  $Z$  test statistic at the 0.01 level of significance?

**PH Grade ASSIST** **9.37** In Problem 9.36, what is your statistical decision if the computed value of the  $Z$  test statistic is  $-1.15$ ?

**PH Grade ASSIST** **9.38** Suppose that in a one-tail hypothesis test where you reject  $H_0$  only in the *upper* tail, you compute the value of the test statistic  $Z$  to be  $+2.00$ . What is the  $p$ -value?

**PH Grade ASSIST** **9.39** In Problem 9.38, what is your statistical decision if you test the null hypothesis at the 0.05 level of significance?


**9.40** Suppose that in a one-tail hypothesis test where you reject  $H_0$  only in the *lower* tail, you compute the value of the test statistic  $Z$  as  $-1.38$ . What is the  $p$ -value?

**9.41** In Problem 9.40, what is your statistical decision if you test the null hypothesis at the 0.01 level of significance?

**9.42** In a one-tail hypothesis test where you reject  $H_0$  only in the *lower* tail, you compute the value of the test statistic  $Z$  as  $+1.38$ . What is the  $p$ -value?

**9.43** In Problem 9.42, what is the statistical decision if you test the null hypothesis at the 0.01 level of significance?

### Applying the Concepts

 **9.44** The Glen Valley Steel Company manufactures steel bars. If the production process is working properly, it turns out steel bars that are normally distributed with mean length of *at least* 2.8 feet, with a standard deviation of 0.20 foot (as determined from engineering specifications on the production equipment involved). Longer steel bars can be used or altered, but shorter bars must be scrapped. You select a sample of 25 bars, and the mean length is 2.73 feet. Do you need to adjust the production equipment?

- If you test the null hypothesis at the 0.05 level of significance, what decision do you make using the critical value approach to hypothesis testing?
- If you test the null hypothesis at the 0.05 level of significance, what decision do you make using the  $p$ -value approach to hypothesis testing?
- Interpret the meaning of the  $p$ -value in this problem.
- Compare your conclusions in (a) and (b).

**9.45** You are the manager of a restaurant that delivers pizza to college dormitory rooms. You have just changed your delivery process in an effort to reduce the mean time between the order and completion of delivery from the current 25 minutes. From past experience, you can assume that the population standard deviation is 6 minutes. A sample of 36 orders using the new delivery process yields a sample mean of 22.4 minutes.

- Using the six-step critical value approach, at the 0.05 level of significance, is there evidence that the population mean delivery time has been reduced below the previous population mean value of 25 minutes?
- At the 0.05 level of significance, use the five-step  $p$ -value approach.
- Interpret the meaning of the  $p$ -value in (b).
- Compare your conclusions in (a) and (b).

**9.46** Children in the United States account directly for \$36 billion in sales annually. When their indirect influence over product decisions from stereos to vacations is consid-

ered, the total economic spending affected by children in the United States is \$290 billion. It is estimated that by age 10, a child makes an average of more than five trips a week to a store (M. E. Goldberg, G. J. Gorn, L. A. Peracchio, and G. Bamossy, “Understanding Materialism Among Youth,” *Journal of Consumer Psychology*, 2003, 13(3) pp. 278–288). Suppose that you want to prove that children in your city average more than five trips a week to a store. Let  $\mu$  represent the population mean number of times children in your city make trips to a store.

- State the null and alternative hypotheses.
- Explain the meaning of the Type I and Type II errors in the context of this scenario.
- Suppose that you carry out a similar study in the city in which you live. Based on past studies, you assume that the standard deviation of the number of trips to the store is 1.6. You take a sample of 100 children and find that the mean number of trips to the store is 5.47. At the 0.01 level of significance, is there evidence that the population mean number of trips to the store is greater than 5 per week?
- Interpret the meaning of the  $p$ -value in (c).

**9.47** The waiting time to place an order at a branch of a fast-food chain during the lunch hour has had a population mean of 3.55 minutes, with a population standard deviation of 1.1 minutes. Recently, in an effort to reduce the waiting time, the branch has experimented with a system in which there is a single waiting line. A sample of 100 customers during a recent lunch hour was selected, and their mean waiting time to place an order was 3.18 minutes. Assume that the population standard deviation of the waiting time has not changed from 1.1 minutes.

- At the 0.05 level of significance, using the critical value approach to hypothesis testing, is there evidence that the population mean waiting time to place an order is less than 3.55 minutes?
- At the 0.05 level of significance, using the  $p$ -value approach to hypothesis testing, is there evidence that the population mean waiting time to place an order is less than 3.55 minutes?
- Interpret the meaning of the  $p$ -value in this problem.
- Compare your conclusions in (a) and (b).

## 9.4 $t$ TEST OF HYPOTHESIS FOR THE MEAN ( $\sigma$ UNKNOWN)

In most hypothesis-testing situations concerning the population mean  $\mu$ , you do not know the population standard deviation,  $\sigma$ . Instead, you use the sample standard deviation,  $S$ . If you assume that the population is normally distributed, the sampling distribution of the mean follows a  $t$  distribution with  $n - 1$  degrees of freedom. If the population is not normally distributed, you can still use the  $t$  test if the sample size is large enough for the Central Limit Theorem to take effect (see Section 7.4). Equation (9.2) defines the test statistic  $t$  for determining the difference between the sample mean,  $\bar{X}$ , and the population mean,  $\mu$ , when the sample standard deviation,  $S$ , is used.

**t TEST OF HYPOTHESIS FOR THE MEAN ( $\sigma$  UNKNOWN)**

$$t = \frac{\bar{X} - \mu}{\frac{S}{\sqrt{n}}} \tag{9.2}$$

where the test statistic *t* follows a *t* distribution having *n* – 1 degrees of freedom.

To illustrate the use of this *t* test, return to the Using Statistics scenario concerning the Saxon Home Improvement Company on page 284. Over the past five years, the mean amount per sales invoice is \$120. As an accountant for the company, you need to inform the finance department if this amount changes. In other words, the hypothesis test is used to try to prove that the mean amount per sales invoice is increasing or decreasing.

**The Critical Value Approach**

To perform this two-tail hypothesis test, you use the six-step method listed in Exhibit 9.1 on page 336.

**Step 1**  $H_0: \mu = \$120$

$H_1: \mu \neq \$120$

The alternative hypothesis contains the statement you are trying to prove. If the null hypothesis is rejected, then there is statistical proof that the population mean amount per sales invoice is no longer \$120. If the statistical conclusion is “do not reject  $H_0$ ,” then you will conclude that there is insufficient evidence to prove that the mean amount differs from the long-term mean of \$120.

**Step 2** You have selected a sample of  $n = 12$ . You decide to use  $\alpha = 0.05$ .

**Step 3** Because  $\sigma$  is unknown, you use the *t* distribution and the *t* test statistic for this example. You must assume that the population of sales invoices is normally distributed. This assumption is discussed on page 346.

**Step 4** For a given sample size, *n*, the test statistic *t* follows a *t* distribution with *n* – 1 degrees of freedom. The critical values of the *t* distribution with  $12 - 1 = 11$  degrees of freedom are found in Table E.3, as illustrated in Table 9.3 and Figure 9.9. The alternative hypothesis,  $H_1, \mu \neq \$120$  is two-tailed. Thus, the area in the rejection region of the *t* distribution’s left (lower) tail is 0.025, and the area in the rejection region of the *t* distribution’s right (upper) tail is also 0.025.

**TABLE 9.3**

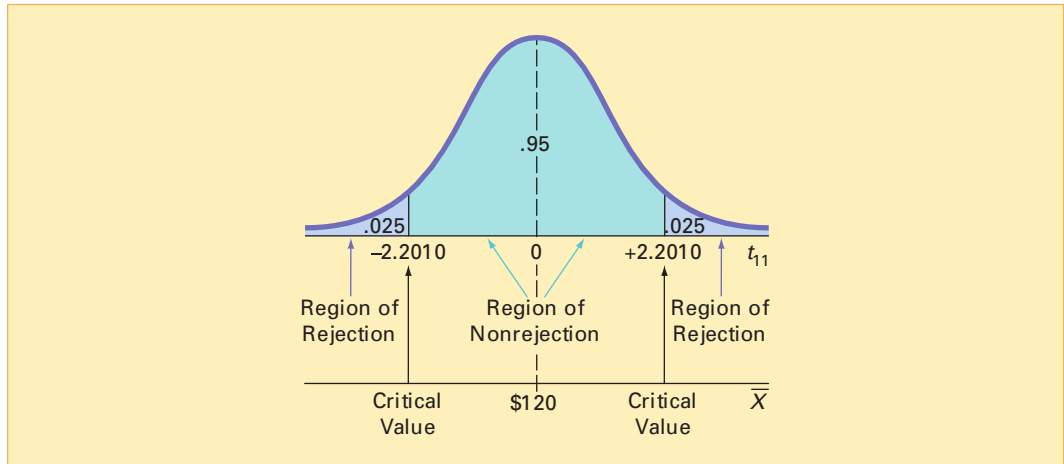
Determining the Critical Value from the *t* Table for an Area of 0.025 in Each Tail with 11 Degrees of Freedom

Degrees of Freedom	Upper-Tail Areas					
	.25	.10	.05	.025	.01	.005
1	1.0000	3.0777	6.3138	12.7062	31.8207	63.6574
2	0.8165	1.8856	2.9200	4.3027	6.9646	9.9248
3	0.7649	1.6377	2.3534	3.1824	4.5407	5.8409
4	0.7407	1.5332	2.1318	2.7764	3.7469	4.6041
5	0.7267	1.4759	2.0150	2.5706	3.3649	4.0322
6	0.7176	1.4398	1.9432	2.4469	3.1427	3.7074
7	0.7111	1.4149	1.8946	2.3646	2.9980	3.4995
8	0.7064	1.3968	1.8595	2.3060	2.8965	3.3554
9	0.7027	1.3830	1.8331	2.2622	2.8214	3.2498
10	0.6998	1.3722	1.8125	2.2281	2.7638	3.1693
11	0.6974	1.3634	1.7959	2.2010	2.7181	3.1058

Source: Extracted from Table E.3.

**FIGURE 9.9**

Testing a hypothesis about the mean ( $\sigma$  unknown) at the 0.05 level of significance with 11 degrees of freedom



From the  $t$  table as given in Table E.3, a portion of which is shown in Table 9.3, the critical values are  $\pm 2.2010$ . The decision rule is

Reject  $H_0$  if  $t < -t_{11} = -2.2010$   
 or if  $t > t_{11} = +2.2010$ ;  
 otherwise, do not reject  $H_0$ .

**Step 5** The data in the file **invoices.xls** are the amounts (in dollars) in a random sample of 12 sales invoices.

108.98 152.22 111.45 110.59 127.46 107.26  
 93.32 91.97 111.56 75.71 128.58 135.11

Using Equations (3.1) and (3.10) on pages 97 and 107, or Microsoft Excel, as shown in Figure 9.10,

$$\bar{X} = \frac{\sum_{i=1}^n X_i}{n} = \$112.85 \quad \text{and} \quad S = \sqrt{\frac{\sum_{i=1}^n (X_i - \bar{X})^2}{n - 1}} = \$20.80$$

From Equation (9.2) on page 347,

$$t = \frac{\bar{X} - \mu}{\frac{S}{\sqrt{n}}} = \frac{112.85 - 120}{\frac{20.80}{\sqrt{12}}} = -1.1908$$

**FIGURE 9.10**

Microsoft Excel results for the one-sample  $t$  test of sales invoices



See Section E9.2 to create this.

	A	B
1	<b>Mean Amount Per Invoice Hypothesis Test</b>	
2		
3	Data	
4	Null Hypothesis	$\mu = 120$
5	Level of Significance	0.05
6	Sample Size	12
7	Sample Mean	112.85
8	Sample Standard Deviation	20.8
9		
10	Intermediate Calculations	
11	Standard Error of the Mean	6.0044 =B8/SQRT(B6)
12	Degrees of Freedom	11 =B6 - 1
13	t Test Statistic	-1.1908 =(B7 - B4)/B11
14		
15	Two-Tail Test	
16	Lower Critical Value	-2.2010 =(TINV(B5, B12))
17	Upper Critical Value	2.2010 =TINV(B5, B12)
18	p-Value	0.2588 =TDIST(ABS(B13), B12, 2)
19	Do not reject the null hypothesis	=IF(B18 < B5, "Reject the null hypothesis", "Do not reject the null hypothesis")

**Step 6** Because  $-2.2010 < t = -1.1908 < 2.2010$ , you do not reject  $H_0$ . You have insufficient evidence to conclude that the mean amount per sales invoice differs from \$120. You should inform the finance department that the audit suggests that the mean amount per invoice has not changed.

## The *p*-Value Approach

**Steps 1–3** These steps are the same as in the critical value approach.

**Step 4** From the Microsoft Excel worksheet of Figure 9.10,  $t = -1.1908$  and the *p*-value = 0.2588 (see step 5 of the critical value approach).

**Step 5** The Microsoft Excel results in Figure 9.10 give the *p*-value for this two-tail test as 0.2588. Because the *p*-value of 0.2588 is greater than  $\alpha = 0.05$ , you do not reject  $H_0$ . The data provide insufficient evidence to conclude that the mean amount per sales invoice differs from \$120. You should inform the finance department that the audit suggests that the mean amount per invoice has not changed. The *p*-value indicates that if the null hypothesis is true, the probability that a sample of 12 invoices could have a monthly mean that differs by \$7.15 or more from the stated \$120 is 0.2588. In other words, if the mean amount per sales invoice is truly \$120, then there is a 25.88% chance of observing a sample mean below \$112.85 or above \$127.15.

In the preceding example, it is incorrect to state that there is a 25.88% chance that the null hypothesis is true. This misinterpretation of the *p*-value is sometimes used by those not properly trained in statistics. Remember that the *p*-value is a conditional probability, calculated by *assuming* that the null hypothesis is true. In general, it is proper to state the following:

If the null hypothesis is true, there is a (*p*-value)\*100% chance of observing a sample result at least as contradictory to the null hypothesis as the result observed.

## Checking Assumptions

You use the one-sample *t* test when the population standard deviation,  $\sigma$ , is not known and is estimated using the sample standard deviation,<sup>1</sup>  $S$ . To use the one-sample *t* test, the data are assumed to represent a random sample from a population that is normally distributed. In practice, as long as the sample size is not very small and the population is not very skewed, the *t* distribution provides a good approximation to the sampling distribution of the mean when  $\sigma$  is unknown.

There are several ways to evaluate the normality assumption necessary for using the *t* test. You can observe how closely the sample statistics match the normal distribution's theoretical properties. You can also use a histogram, stem-and-leaf display, box-and-whisker plot, or normal probability plot. For details on evaluating normality, see Section 6.3 on page 234.

Figure 9.11 presents descriptive statistics generated by Microsoft Excel. Figure 9.12 is a Microsoft Excel box-and-whisker plot. Figure 9.13 is a Microsoft Excel normal probability plot.

<sup>1</sup>When a large sample size is available,  $S$  estimates  $\sigma$  precisely enough that there is little difference between the *t* and *Z* distributions.

**FIGURE 9.11**

Microsoft Excel descriptive statistics for the sales invoice data



See Section E3.1 to create this.

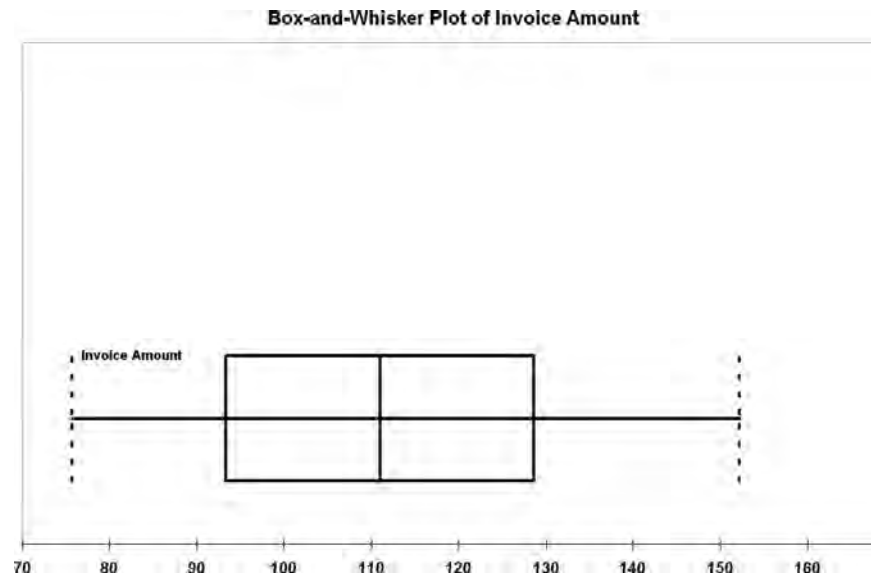
	A	B
1	<b>Invoice Amount</b>	
2		
3	<b>Mean</b>	112.8508
4	<b>Standard Error</b>	6.003863
5	<b>Median</b>	111.02
6	<b>Mode</b>	#N/A
7	<b>Standard Deviation</b>	20.79799
8	<b>Sample Variance</b>	432.5565
9	<b>Kurtosis</b>	0.172708
10	<b>Skewness</b>	0.133638
11	<b>Range</b>	76.51
12	<b>Minimum</b>	75.71
13	<b>Maximum</b>	152.22
14	<b>Sum</b>	1354.21
15	<b>Count</b>	12
16	<b>Largest(1)</b>	152.22
17	<b>Smallest(1)</b>	75.71

**FIGURE 9.12**

Microsoft Excel box-and-whisker plot for the sales invoice data



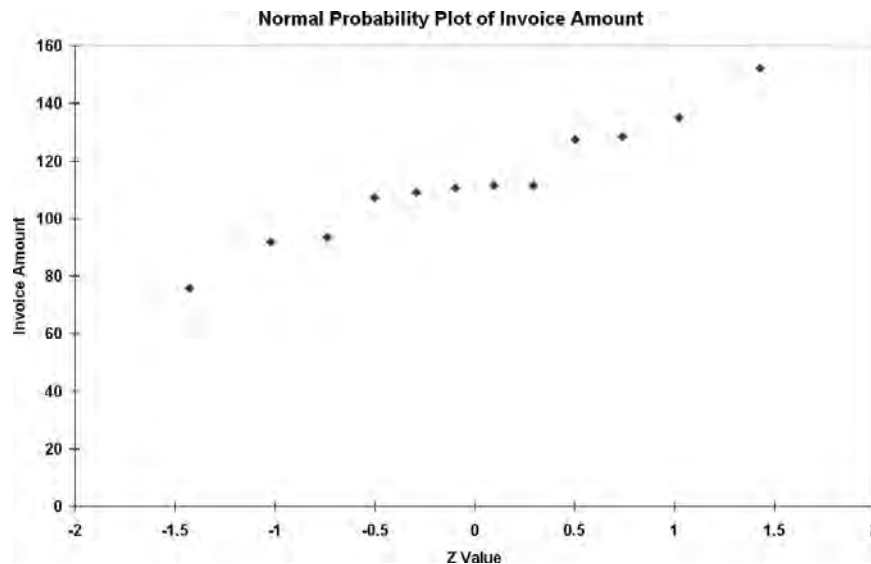
See Section E3.4 to create this.

**FIGURE 9.13**

Microsoft Excel normal probability plot for the sales invoice data



See Section E6.2 to create this.



Because the mean is very close to the median, the points on the normal probability plot appear to be increasing approximately in a straight line, and the box-and-whisker plot appears approximately symmetrical. You can assume that the population of sales invoices is approximately normally distributed. The normality assumption is valid, and therefore the auditor's results are valid.

The  $t$  test is a **robust** test. It does not lose power if the shape of the population departs somewhat from a normal distribution, particularly when the sample size is large enough to enable the test statistic  $t$  to be influenced by the Central Limit Theorem (see Section 7.4). However, you can make erroneous conclusions and can lose statistical power if you use the  $t$  test incorrectly. If the sample size,  $n$ , is small (that is, less than 30) and you cannot easily make the assumption that the underlying population is at least approximately normally distributed, other *nonparametric* testing procedures are more appropriate (see references 1 and 2).

## PROBLEMS FOR SECTION 9.4

### Learning the Basics

PH Grade  
ASSIST

**9.48** If, in a sample of  $n = 16$  selected from a normal population,  $\bar{X} = 56$  and  $S = 12$ , what is the value of the *t*-test statistic if you are testing the null hypothesis  $H_0: \mu = 50$ ?

**9.49** In Problem 9.48, how many degrees of freedom are there in the one-sample *t* test?

**9.50** In Problems 9.48 and 9.49, what are the critical values of *t* if the level of significance,  $\alpha$ , is 0.05 and the alternative hypothesis,  $H_1$ , is

- $\mu \neq 50$ ?
- $\mu > 50$ ?

**9.51** In Problems 9.48, 9.49, and 9.50, what is your statistical decision if the alternative hypothesis,  $H_1$ , is

- $\mu \neq 50$ ?
- $\mu > 50$ ?

**9.52** If, in a sample of  $n = 16$  selected from a left-skewed population,  $\bar{X} = 65$  and  $S = 21$ , would you use the *t* test to test the null hypothesis  $H_0: \mu = 60$ ? Discuss.

**9.53** If, in a sample of  $n = 160$  selected from a left-skewed population,  $\bar{X} = 65$  and  $S = 21$ , would you use the *t* test to test the null hypothesis  $H_0: \mu = 60$ ? Discuss.

### Applying the Concepts

**9.54** Late payment of medical claims can add to the cost of health care. An article (M. Freudenheim, “The Check Is Not in the Mail,” *The New York Times*, May 25, 2006, pp. C1, C6) reported that the mean time from the date of service to the date of payment for one insurance company was 41.4 days during a recent period. Suppose that a sample of 100 medical claims is selected during the latest time period. The sample mean time from the date of service to the date of payment was 39.6 days, and the sample standard deviation was 7.4 days.

- Using the 0.05 level of significance, is there evidence that the population mean has changed from 41.4 days?
- What is your answer in (a) if you use the 0.01 level of significance?
- What is your answer in (a) if the sample mean is 38.2 days and the sample standard deviation is 10.7 days?

**9.55** An article (N. Hellmich, “‘Supermarket Guru’ Has a Simple Mantra,” *USA Today*, June 19, 2002, p. 70) claimed that the typical supermarket trip takes a mean of 22 minutes. Suppose that in an effort to test this claim, you select a sample of 50 shoppers at a local supermarket. The mean shopping time for the sample of 50 shoppers is 25.36 minutes, with a standard deviation of 7.24 minutes. Using the

0.10 level of significance, is there evidence that the mean shopping time at the local supermarket is different from the claimed value of 22 minutes?

SELF  
TEST

**9.56** You are the manager of a restaurant for a fast-food franchise. Last month, the mean waiting time at the drive-through window, as measured from the time a customer places an order until the time the customer receives the order, was 3.7 minutes. The franchise helped you institute a new process intended to reduce waiting time. You select a random sample of 64 orders. The sample mean waiting time is 3.57 minutes, with a sample standard deviation of 0.8 minute. At the 0.05 level of significance, is there evidence that the population mean waiting time is now less than 3.7 minutes?

**9.57** A manufacturer of chocolate candies uses machines to package candies as they move along a filling line. Although the packages are labeled as 8 ounces, the company wants the packages to contain 8.17 ounces so that virtually none of the packages contain less than 8 ounces. A sample of 50 packages is selected periodically, and the packaging process is stopped if there is evidence that the mean amount packaged is different from 8.17 ounces. Suppose that in a particular sample of 50 packages, the mean amount dispensed is 8.159 ounces, with a sample standard deviation of 0.051 ounce.

- Is there evidence that the population mean amount is different from 8.17 ounces? (Use a 0.05 level of significance.)
- Determine the *p*-value and interpret its meaning.

**9.58** The data in the file [movieprices.xls](#) contains prices (in dollars) for two tickets, with online service charges, large popcorn, and two medium soft drinks at a sample of six theatre chains:

36.15 31.00 35.05 40.25 33.75 43.00

Source: Extracted from K. Kelly, “The Multiplex Under Siege,” *The Wall Street Journal*, December 24–25, 2005, pp. P1, P5.

- At the 0.05 level of significance, is there evidence that the mean price for two tickets, with online service charges, large popcorn, and two medium soft drinks, is different from \$35?
- Determine the *p*-value in (a) and interpret its meaning.
- What assumption about the population distribution is needed in (a) and (b)?
- Do you think that the assumption stated in (c) is seriously violated?

**9.59** In New York State, savings banks are permitted to sell a form of life insurance called savings bank life insurance (SBLI). The approval process consists of underwriting,

which includes a review of the application, a medical information bureau check, possible requests for additional medical information and medical exams, and a policy compilation stage in which the policy pages are generated and sent to the bank for delivery. The ability to deliver approved policies to customers in a timely manner is critical to the profitability of this service. During a period of one month, a random sample of 27 approved policies is selected, and the total processing time, in days, is recorded (as stored in the [insurance.xls](#) file):

73 19 16 64 28 28 31 90 60 56 31 56 22 18  
45 48 17 17 17 91 92 63 50 51 69 16 17

- In the past, the mean processing time was 45 days. At the 0.05 level of significance, is there evidence that the mean processing time has changed from 45 days?
- What assumption about the population distribution is needed in (a)?
- Do you think that the assumption stated in (b) is seriously violated? Explain.

**9.60** The following data (see the [drink.xls](#) file) represent the amount of soft-drink filled in a sample of 50 consecutive 2-liter bottles. The results, listed horizontally in the order of being filled, were:

2.109 2.086 2.066 2.075 2.065 2.057 2.052 2.044 2.036 2.038  
2.031 2.029 2.025 2.029 2.023 2.020 2.015 2.014 2.013 2.014  
2.012 2.012 2.012 2.010 2.005 2.003 1.999 1.996 1.997 1.992  
1.994 1.986 1.984 1.981 1.973 1.975 1.971 1.969 1.966 1.967  
1.963 1.957 1.951 1.951 1.947 1.941 1.941 1.938 1.908 1.894

- At the 0.05 level of significance, is there evidence that the mean amount of soft drink filled is different from 2.0 liters?
- Determine the  $p$ -value in (a) and interpret its meaning.
- Evaluate the population distribution assumption you made in (a) graphically. Are the results of (a) valid? Why?
- Examine the values of the 50 bottles in their sequential order, as given in the problem. Is there a pattern to the results? If so, what impact might this pattern have on the validity of the results in (a)?

**9.61** One of the major measures of the quality of service provided by any organization is the speed with which it responds to customer complaints. A large family-held department store selling furniture and flooring, including carpet, had undergone a major expansion in the past several years. In particular, the flooring department had expanded from 2 installation crews to an installation supervisor, a measurer, and 15 installation crews. Last year there were 50 complaints concerning carpet installation. The following data (stored in the [furniture.xls](#) file) represent the number of days between the receipt of a complaint and the resolution of the complaint:

54 5 35 137 31 27 152 2 123 81 74 27  
11 19 126 110 110 29 61 35 94 31 26 5  
12 4 165 32 29 28 29 26 25 1 14 13  
13 10 5 27 4 52 30 22 36 26 20 23  
33 68

- The installation supervisor claims that the mean number of days between the receipt of a complaint and the resolution of the complaint is 20 days or less. At the 0.05 level of significance, is there evidence that the claim is not true (that is, that the mean number of days is greater than 20)?
- What assumption about the population distribution must you make in (a)?
- Do you think that the assumption made in (b) is seriously violated? Explain.
- What effect might your conclusion in (c) have on the validity of the results in (a)?

**9.62** A manufacturing company produces steel housings for electrical equipment. The main component part of the housing is a steel trough that is made out of a 14-gauge steel coil. It is produced using a 250-ton progressive punch press with a wipe-down operation that puts two 90-degree forms in the flat steel to make the trough. The distance from one side of the form to the other is critical because of weatherproofing in outdoor applications. The company requires that the width of the trough be between 8.31 inches and 8.61 inches. The data file [trough.xls](#) contains the widths of the troughs, in inches, for a sample of  $n = 49$ .

8.312 8.343 8.317 8.383 8.348 8.410 8.351 8.373 8.481 8.422  
8.476 8.382 8.484 8.403 8.414 8.419 8.385 8.465 8.498 8.447  
8.436 8.413 8.489 8.414 8.481 8.415 8.479 8.429 8.458 8.462  
8.460 8.444 8.429 8.460 8.412 8.420 8.410 8.405 8.323 8.420  
8.396 8.447 8.405 8.439 8.411 8.427 8.420 8.498 8.409

- At the 0.05 level of significance, is there evidence that the mean widths of the troughs is different from 8.46 inches?
- What assumption about the population distribution do you need to make in (a)?
- Do you think that the assumption made in (a) has been seriously violated? Explain.

**9.63** One operation of a steel mill is to cut pieces of steel into parts that are used in the frame for front seats in an automobile. The steel is cut with a diamond saw and requires the resulting parts to be within  $\pm 0.005$  inch of the length specified by the automobile company. The data in the file [steel.xls](#) come from a sample of 100 steel parts. The measurement reported is the difference, in inches, between the actual length of the steel part, as measured by a laser measurement device, and the specified length of the steel part. For example, a value of  $-0.002$  represents a steel part that is 0.002 inch shorter than the specified length.



- a. At the 0.05 level of significance, is there evidence that the mean difference is not equal to 0.0 inches?
- b. Determine the  $p$ -value in (a) and interpret its meaning.
- c. What assumption about the differences between the actual length of the steel part and the specified length of the steel part must you make in (a)?
- d. Evaluate the assumption in (c) graphically. Are the results of (a) valid? Why?

**9.64** In Problem 3.63 on page 138, you were introduced to a tea-bag-filling operation. An important quality characteristic of interest for this process is the weight of the tea in the individual bags. The data in the file [teabags.xls](#) is an ordered array of the weight, in grams, of a sample of 50 tea bags produced during an eight-hour shift.

- a. Is there evidence that the mean amount of tea per bag is different from 5.5 grams (use  $\alpha = 0.01$ )?
- b. Construct a 99% confidence interval estimate of the population mean amount of tea per bag. Interpret this interval.
- c. Compare the conclusions reached in (a) and (b).

**9.65** Although many people think they can put a meal on the table in a short period of time, a recent article reported that they end up spending about 40 minutes doing so (N. Hellmich, "Americans Go for the Quick Fix for Dinner," *USA Today*, February 14, 2006). Suppose another study is conducted to test the validity of this statement. A sample of 25 people is selected, and the length of time to prepare and cook dinner (in minutes) is recorded, with the following results (stored in the file [dinner.xls](#)).

44.0 51.9 49.7 40.0 55.5 33.0 43.4 41.3 45.2 40.7 41.1 49.1 30.9  
45.2 55.3 52.1 55.1 38.8 43.1 39.2 58.6 49.8 43.2 47.9 46.6

- a. Is there evidence that the population mean time to prepare and cook dinner is greater than 40 minutes? Use a level of significance of 0.05.
- b. What assumptions are made to perform the test in (a)?
- c. Do you think the assumption needed in (a) is seriously violated? Explain.
- d. Determine the  $p$ -value and interpret its meaning.

## 9.5 Z TEST OF HYPOTHESIS FOR THE PROPORTION

In some situations, you want to test a hypothesis about the proportion of successes in the population,  $\pi$ , rather than testing the population mean. To begin, you select a random sample and compute the **sample proportion**,  $p = X/n$ . You then compare the value of this statistic to the hypothesized value of the parameter,  $\pi$ , in order to decide whether to reject the null hypothesis.

If the number of successes ( $X$ ) and the number of failures ( $n - X$ ) are each at least five, the sampling distribution of a proportion approximately follows a normal distribution. You use the **Z test for the proportion** given in Equation (9.3) to perform the hypothesis test for the difference between the sample proportion,  $p$ , and the hypothesized population proportion,  $\pi$ .

### ONE SAMPLE Z TEST FOR THE PROPORTION

$$Z = \frac{p - \pi}{\sqrt{\frac{\pi(1 - \pi)}{n}}} \tag{9.3}$$

where

$$p = \frac{X}{n} = \frac{\text{Number of successes in the sample}}{\text{Sample size}}$$

$p$  = sample proportion of successes

$\pi$  = hypothesized proportion of successes in the population

and the test statistic  $Z$  approximately follows a standardized normal distribution.

Alternatively, by multiplying the numerator and denominator by  $n$ , you can write the  $Z$  test statistic in terms of the number of successes,  $X$ , as shown in Equation (9.4).

### Z TEST FOR THE PROPORTION IN TERMS OF THE NUMBER OF SUCCESSES

$$Z = \frac{X - n\pi}{\sqrt{n\pi(1 - \pi)}} \tag{9.4}$$

To illustrate the one-sample  $Z$  test for a proportion, consider a survey of independent grocery owners. In the survey, the owners were asked to consider what they believed to be their biggest competitive threat. Some noted large-chain supermarkets, wholesale clubs, and other independent grocers. The most common response, given by 78 of the 151 owners, was that they viewed Wal-Mart as their biggest competitive threat (Extracted from J. Tarnowski and D. Chanil, "To Serve and Connect," *The Progressive Grocer*, January 1, 2006, pp. 60–65). For this survey, the null and alternative hypotheses are stated as follows:

$H_0: \pi = 0.50$  (that is, half of all independent grocery owners view Wal-Mart as their biggest competitive threat)

$H_1: \pi \neq 0.50$  (that is, either less than half or more than half of all independent grocery owners view Wal-Mart as their biggest competitive threat)

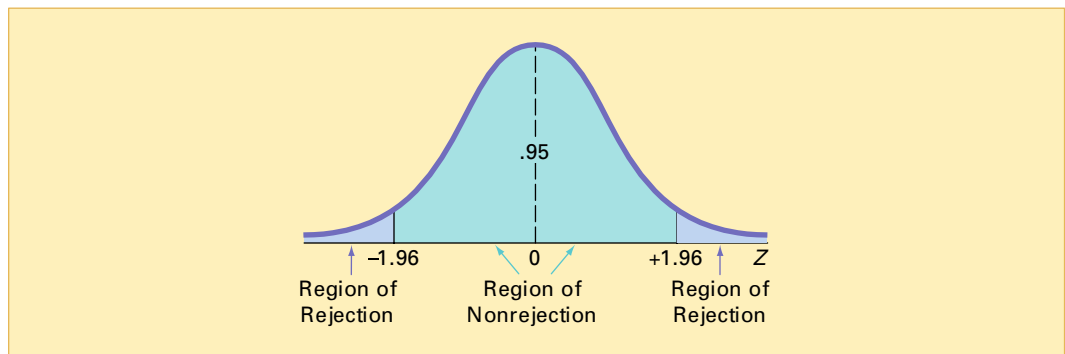
### The Critical Value Approach

Because you are interested in whether the proportion of independent grocery owners who view Wal-Mart as their biggest competitive threat is 0.50, you use a two-tail test. If you select the  $\alpha = 0.05$  level of significance, the rejection and nonrejection regions are set up as in Figure 9.14, and the decision rule is

Reject  $H_0$  if  $Z < -1.96$  or if  $Z > +1.96$ ;  
otherwise, do not reject  $H_0$ .

**FIGURE 9.14**

Two-tail test of hypothesis for the proportion at the 0.05 level of significance



Because 78 of the 151 of independent grocery owners view Wal-Mart as their biggest competitive threat,

$$p = \frac{78}{151} = 0.5166$$

Using Equation (9.3),

$$Z = \frac{p - \pi}{\sqrt{\frac{\pi(1 - \pi)}{n}}} = \frac{0.5166 - 0.50}{\sqrt{\frac{0.50(1 - 0.50)}{151}}} = \frac{0.0166}{0.0407} = 0.4069$$

or, using Equation (9.4),

$$Z = \frac{X - n\pi}{\sqrt{n\pi(1 - \pi)}} = \frac{78 - (151)(0.50)}{\sqrt{151(0.50)(0.50)}} = \frac{2.5}{6.1441} = 0.4069$$

Because  $-1.96 < Z = 0.4069 < 1.96$ , you do not reject  $H_0$ . There is insufficient evidence that the proportion of independent grocery owners who view Wal-Mart as their biggest competitive threat is not 0.50. Figure 9.15 presents Microsoft Excel results for these data. In other words, the current belief that half of all independent grocers view Wal-Mart as their biggest competitive threat is not contradicted by the Z test.

**FIGURE 9.15**

Microsoft Excel results for the survey of independent grocery owners and Wal-Mart as biggest competitive threat



See Section E9.3 to create this.

	A	B	
1	<b>Wal-Mart As Biggest Threat Hypothesis Test</b>		
2			
3	Data		
4	Null Hypothesis $p =$	0.5	
5	Level of Significance	0.05	
6	Number of Successes	78	
7	Sample Size	151	
8			
9	Intermediate Calculations		
10	Sample Proportion	0.5166	=B6/B7
11	Standard Error	0.0407	=SQRT(B4*(1 - B4)/B7)
12	Z Test Statistic	0.4069	=(B10 - B4)/B11
13			
14	Two-Tail Test		
15	Lower Critical Value	-1.9600	=NORMSINV(B5/2)
16	Upper Critical Value	1.9600	=NORMSINV(1 - B5/2)
17	p-Value	0.6841	=2 * (1 - NORMSDIST(ABS(B12)))
18	Do not reject the null hypothesis		=IF(B17 < B5, "Reject the null hypothesis", "Do not reject the null hypothesis")

## The $p$ -Value Approach

As an alternative to the critical value approach, you can compute the  $p$ -value. For this two-tail test in which the rejection region is located in the lower tail and the upper tail, you need to find the area below a Z value of  $-0.4069$  and above a Z value of  $+0.4069$ . Figure 9.15 reports a  $p$ -value of 0.6841. Because this value is greater than the selected level of significance ( $\alpha = 0.05$ ), you do not reject the null hypothesis.

### EXAMPLE 9.6

#### TESTING A HYPOTHESIS FOR A PROPORTION

A fast-food chain has developed a new process to ensure that orders at the drive-through are filled correctly. The previous process filled orders correctly 85% of the time. Based on a sample of 100 orders using the new process, 94 were filled correctly. At the 0.01 level of significance, can you conclude that the new process has increased the proportion of orders filled correctly?

**SOLUTION** The null and alternative hypotheses are

$$H_0: \pi \leq 0.85 \text{ (that is, the proportion of orders filled correctly is less than or equal to 0.85)}$$

$$H_1: \pi > 0.85 \text{ (that is, the proportion of orders filled correctly is greater than 0.85)}$$

Using Equation (9.3) on page 353,

$$p = \frac{X}{n} = \frac{94}{100} = 0.94$$

$$Z = \frac{p - \pi}{\sqrt{\frac{\pi(1 - \pi)}{n}}} = \frac{0.94 - 0.85}{\sqrt{\frac{0.85(1 - 0.85)}{100}}} = \frac{0.09}{0.0357} = 2.52$$

The  $p$ -value for  $Z > 2.52$  is 0.0059.

Using the critical value approach, you reject  $H_0$  if  $Z > 2.33$ . Using the  $p$ -value approach, you reject  $H_0$  if the  $p$ -value  $< 0.01$ . Because  $Z = 2.52 > 2.33$  or the  $p$ -value  $= 0.0059 < 0.01$ , you reject  $H_0$ . You have evidence that the new process has increased the proportion of correct orders above 0.85.

## PROBLEMS FOR SECTION 9.5

### Learning the Basics

**PH Grade ASSIST** **9.66** If, in a random sample of 400 items, 88 are defective, what is the sample proportion of defective items?

**PH Grade ASSIST** **9.67** In Problem 9.66, if the null hypothesis is that 20% of the items in the population are defective, what is the value of the  $Z$ -test statistic?

**PH Grade ASSIST** **9.68** In Problems 9.66 and 9.67, suppose you are testing the null hypothesis  $H_0: \pi = 0.20$  against the two-tail alternative hypothesis  $H_1: \pi \neq 0.20$  and you choose the level of significance  $\alpha = 0.05$ . What is your statistical decision?

### Applying the Concepts

**9.69** Late payment of medical claims can add to the cost of health care. An article (M. Freudenheim, “The Check Is Not in the Mail,” *The New York Times*, May 25, 2006, pp. C1, C6) reported that for one insurance company, 85.1% of the claims were paid in full when first submitted. Suppose that the insurance company developed a new payment system in an effort to increase this percentage. A sample of 200 claims processed under this system revealed that 180 of the claims were paid in full when first submitted.

- At the 0.05 level of significance, is there evidence that the proportion of claims processed under this new system is higher than the article reported for the previous system?
- Compute the  $p$ -value and interpret its meaning.

**SELF Test** **9.70** The high cost of gasoline in the spring of 2006 had many people reconsidering their summer vacation plans. A survey of 464 Cincinnati-area adults found that 255 said that they were planning on modifying or canceling their summer travel plans because of high gas prices (“Will You Travel?” *The Cincinnati Enquirer*, May 3, 2006, p. A10.)

- Use the six-step critical value approach to hypothesis testing and a 0.05 level of significance to try to prove that the majority of Cincinnati-area adults were planning on modifying or canceling their summer travel plans because of high gas prices.
- Use the five-step  $p$ -value approach to hypothesis testing and a 0.05 level of significance to try to prove that the majority of Cincinnati-area adults were planning on modifying or canceling their summer travel plans because of high gas prices.
- Compare the results of (a) and (b).

**9.71** A *Wall Street Journal* article suggested that age bias was a growing problem in the corporate world (C. Hymowitz, “Top Executives Chase Youthful Appearance, but Miss Real Issue,” *The Wall Street Journal*, February 17, 2004, p. B1). In

2001, an estimated 78% of executives believed that age bias was a serious problem. In a 2004 study by ExecuNet, 82% of the executives surveyed considered age bias a serious problem. The sample size for the 2004 study was not disclosed. Suppose 50 executives were surveyed.

- At the 0.05 level of significance, use the six-step critical value approach to hypothesis testing to try to prove that the 2004 proportion of executives who believed that age bias was a serious problem was higher than the 2001 value of 0.78.
- Use the five-step  $p$ -value approach. Interpret the meaning of the  $p$ -value.
- Suppose that the sample size used was 1,000. Redo (a) and (b).
- Discuss the effect that sample size had on the outcome of this analysis and, in general, the effect that sample size plays in hypothesis testing.

**9.72** A *Wall Street Journal* poll (“What’s News Online,” *The Wall Street Journal*, March 30, 2004, p. D7) asked respondents if they trusted energy-efficiency ratings on cars and appliances; 552 responded yes, and 531 responded no.

- At the 0.05 level of significance, use the six-step critical value approach to hypothesis testing to try to prove that the percentage of people who trust energy-efficiency ratings differs from 50%.
- Use the five-step  $p$ -value approach. Interpret the meaning of the  $p$ -value.

**9.73** One of the biggest issues facing e-retailers is the ability to reduce the proportion of customers who cancel their transactions after they have selected their products. It has been estimated that about half of prospective customers cancel their transactions after they have selected their products (B. Tedeschi, “E-Commerce, a Cure for Abandoned Shopping Carts: A Web Checkout System That Eliminates the Need for Multiple Screens,” *The New York Times*, February 14, 2005, p. C3). Suppose that a company changed its Web site so that customers could use a single-page checkout process rather than multiple pages. A sample of 500 customers who had selected their products were provided with the new checkout system. Of these 500 customers, 210 cancelled their transactions after they have selected their products.

- At the 0.01 level of significance, is there evidence that the proportion of customers who selected products and then cancelled their transaction was less than 0.50 with the new system?
- Suppose that a sample of 100 customers who had selected their products were provided with the new checkout system and that 42 of those customers cancelled their transactions after they had selected their products. At the 0.01 level of significance, is there evidence that the proportion of customers who selected

products and then cancelled their transaction was less than 0.50 with the new system?

- c. Compare the results of (a) and (b) and discuss the effect that sample size has on the outcome, and, in general, in hypothesis testing.

**9.74** More professional women than ever before are forgoing motherhood because of the time constraints of their careers. However, many women still manage to find time to climb the corporate ladder *and* set time aside to have children. A survey of 187 attendees at *Fortune Magazine's* Most Powerful Women in Business summit in March 2002 found that 133 had at least one child (C. Hymowitz, "Women

Plotting Mix of Work and Family Won't Find Perfect Plan," *The Wall Street Journal*, June 11, 2002, p. B1). Assume that the group of 187 women was a random sample from the population of all successful women executives.

- What was the sample proportion of successful women executives who had children?
- At the 0.05 level of significance, can you state that more than half of all successful women executives had children?
- At the 0.05 level of significance, can you state that more than two-thirds of all successful women executives had children?
- Do you think the random sample assumption is valid? Explain.

## 9.6 POTENTIAL HYPOTHESIS-TESTING PITFALLS AND ETHICAL ISSUES

To this point, you have studied the fundamental concepts of hypothesis testing. You have used hypothesis testing to analyze differences between sample estimates (that is, statistics) and hypothesized population characteristics (that is, parameters) in order to make business decisions concerning the underlying population characteristics. You have also learned how to evaluate the risks involved in making these decisions.

When planning to carry out a test of the hypothesis based on a survey, research study, or designed experiment, you must ask several questions to ensure that you use proper methodology. You need to raise and answer questions such as the following in the planning stage:

- What is the goal of the survey, study, or experiment? How can you translate the goal into a null hypothesis and an alternative hypothesis?
- Is the hypothesis test a two-tail test or one-tail test?
- Can you select a random sample from the underlying population of interest?
- What kinds of data will you collect from the sample? Are the variables numerical or categorical?
- At what significance level, or risk of committing a Type I error, should you conduct the hypothesis test?
- Is the intended sample size large enough to achieve the desired power of the test for the level of significance chosen?
- What statistical test procedure should you use and why?
- What conclusions and interpretations can you reach from the results of the hypothesis test?

Failing to consider these questions early in the planning process can lead to biased or incomplete results. Proper planning can help ensure that the statistical study will provide objective information needed to make good business decisions.

**Statistical Significance Versus Practical Significance** You need to make the distinction between the existence of a statistically significant result and its practical significance in the context within a field of application. Sometimes, due to a very large sample size, you may get a result that is statistically significant but has little practical significance. For example, suppose that prior to a national marketing campaign focusing on a series of expensive television commercials, you believe that the proportion of people who recognize your brand is 0.30. At the completion of the campaign, a survey of 20,000 people indicates that 6,168 recognized your brand. A one-tail test trying to prove that the proportion is now greater than 0.30 results in a  $p$ -value of 0.0047 and the correct statistical conclusion is that the proportion of consumers recognizing your brand name has now increased. Was the campaign successful? The result of the hypothesis test indicates a statistically significant increase in brand awareness, but is this increase practically important? The population proportion is now estimated at  $6,168/20,000 = 0.3084$ , or 30.84%. This increase is less than 1% more than the hypothesized value of 30%. Did the large expenses associated with the

marketing campaign produce a result with a meaningful increase in brand awareness? Because of the minimal real-world impact an increase of less than 1% has on the overall marketing strategy and the huge expenses associated with the marketing campaign, you should conclude that the campaign was not successful. On the other hand, if the campaign increased brand awareness by 20%, you could conclude that the campaign was successful.

**Ethical Issues** You also need to distinguish between poor research methodology and unethical behavior. Ethical considerations arise when the hypothesis-testing process is manipulated. Some of the areas where ethical issues can arise include the use of human subjects in experiments, data collection method, informed consent from human subjects being “treated,” the type of test (one-tail or two-tail test), the choice of the level of significance, data snooping, the cleansing and discarding of data, and the failure to report pertinent findings.

**Informed Consent from Human Respondents Being “Treated”** Ethical considerations require that any individual who is to be subjected to some “treatment” in an experiment be made aware of the research endeavor and any potential behavioral or physical side effects. The person should also provide informed consent with respect to participation.

**Data Collection Method—Randomization** To eliminate the possibility of potential biases in the results, you must use proper data collection methods. To draw meaningful conclusions, the data must be the outcome of a random sample from a population or from an experiment in which a **randomization** process was used. Potential respondents should not be permitted to self-select for a study, nor should they be purposely selected. Aside from the potential ethical issues that may arise, such a lack of randomization can result in serious coverage errors or selection biases that destroy the integrity of the study.

**Type of Test—Two-Tail or One-Tail Test** If prior information is available that leads you to test the null hypothesis against a specifically directed alternative, then a one-tail test is more powerful than a two-tail test. However, if you are interested only in *differences* from the null hypothesis, not in the *direction* of the difference, the two-tail test is the appropriate procedure to use. For example, if previous research and statistical testing have already established the difference in a particular direction, or if an established scientific theory states that it is possible for results to occur in only one direction, then a one-tail test is appropriate. It is never appropriate to change the direction of a test after the data are collected.

**Choice of Level of Significance,  $\alpha$**  In a well-designed study, you select the level of significance,  $\alpha$ , before data collection occurs. You cannot alter the level of significance after the fact to achieve a specific result. You should always report the *p*-value, not just the conclusions of the hypothesis test.

**Data Snooping** **Data snooping** is never permissible. It is unethical to perform a hypothesis test on a set of data, look at the results, and then specify the level of significance or decide whether to use a one-tail or two-tail test. You must make these decisions before the data are collected in order for the conclusions to be valid. In addition, you cannot arbitrarily change or discard extreme or unusual values in order to alter the results of the hypothesis tests.

**Cleansing and Discarding of Data** In the data preparation stage of editing, coding, and transcribing, you have an opportunity to review the data for any extreme or unusual values. After reviewing the data, you should construct a stem-and-leaf display and/or a box-and-whisker plot to determine whether there are possible outliers to double-check against the original data.

The process of data cleansing raises a major ethical question. Should you ever remove a value from a study? The answer is a qualified yes. If you can determine that a measurement is incomplete or grossly in error because of some equipment problem or unusual behavioral

occurrence unrelated to the study, you can discard the value. Sometimes you have no choice—an individual may decide to quit a particular study he or she has been participating in before a final measurement can be made. In a well-designed experiment or study, you should decide, in advance, on all rules regarding the possible discarding of data.

**Reporting of Findings** In conducting research, you should document both good and bad results. You should not just report the results of hypothesis tests that show statistical significance but omit those for which there is insufficient evidence in the findings. In instances in which there is insufficient evidence to reject  $H_0$ , you must make it clear that this does not prove that the null hypothesis is true. What the result does indicate is that with the sample size used, there is not enough information to *disprove* the null hypothesis.

**Summary** To summarize, in discussing ethical issues concerning hypothesis testing, the key is *intent*. You must distinguish between poor data analysis and unethical practice. Unethical practice occurs when researchers *intentionally* create a selection bias in data collection, manipulate the treatment of human subjects without informed consent, use data snooping to select the type of test (two-tail or one-tail) and/or level of significance, hide the facts by discarding values that do not support a stated hypothesis, or fail to report pertinent findings.

### 9.7 (CD-ROM Topic) THE POWER OF A TEST

Section 9.1 defines Type I and Type II errors and the power of the test. In [section 9.7.pdf](#) on the Student CD-ROM, the power of the test is examined in greater depth.

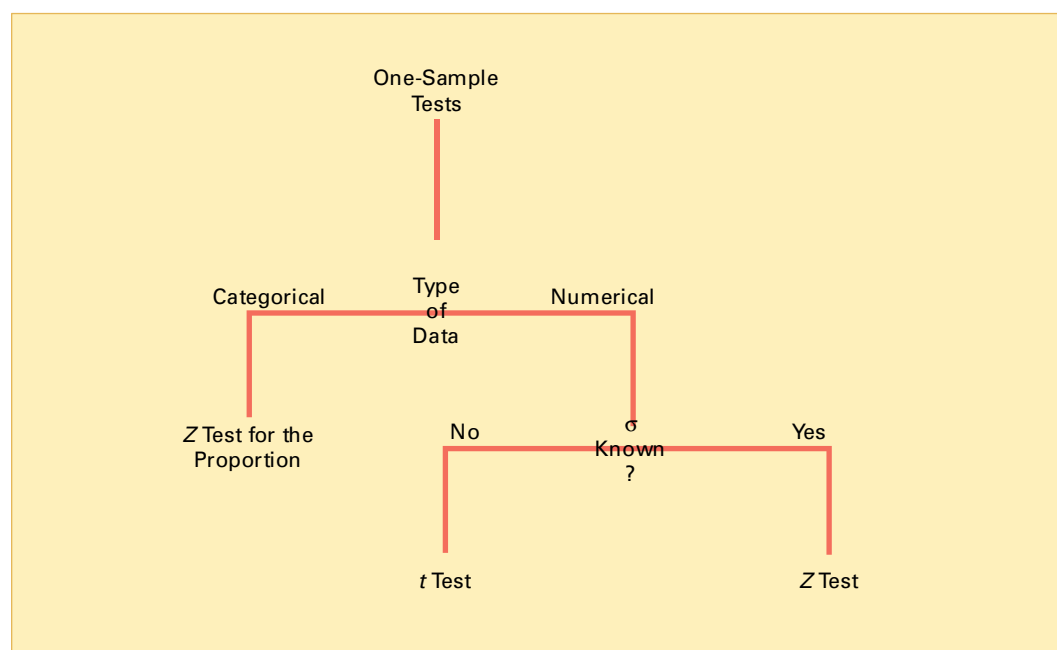
## SUMMARY

This chapter presented the foundation of hypothesis testing. You learned how to perform  $Z$  and  $t$  tests on the population mean and the  $Z$  test on the population proportion. You also learned how an operations manager of a production facility can use hypothesis testing to monitor and

improve a cereal-fill process. The three chapters that follow build on the foundation of hypothesis testing discussed here.

Figure 9.16 presents a roadmap for selecting the correct one-sample hypothesis test to use.

**FIGURE 9.16**  
Roadmap for selecting a one-sample test



In deciding which test to use, you should ask the following questions:

- Does the test involve a numerical variable or a categorical variable? If the test involves a categorical variable, use the  $Z$  test for the proportion.

- If the test involves a numerical variable, do you know the population standard deviation? If you know the population standard deviation, use the  $Z$  test for the mean. If you do not know the population standard deviation, use the  $t$  test for the mean.

Table 9.4 provides a list of topics covered in this chapter.

**TABLE 9.4**

Summary of Topics in Chapter 9

Type of Analysis	Type of Data	
	Numerical	Categorical
Hypothesis test concerning a single parameter	$Z$ test of hypothesis for the mean (Section 9.2)  $t$ test of hypothesis for the mean (Section 9.4)	$Z$ test of hypothesis for the proportion (Section 9.5)

## KEY EQUATIONS

### $Z$ Test of Hypothesis for the Mean ( $\sigma$ Known)

$$Z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} \quad (9.1)$$

### $t$ Test of Hypothesis for the Mean ( $\sigma$ Unknown)

$$t = \frac{\bar{X} - \mu}{\frac{S}{\sqrt{n}}} \quad (9.2)$$

### One Sample $Z$ Test for the Proportion

$$Z = \frac{p - \pi}{\sqrt{\frac{\pi(1 - \pi)}{n}}} \quad (9.3)$$

### $Z$ Test for the Proportion in Terms of the Number of Successes

$$Z = \frac{X - n\pi}{\sqrt{n\pi(1 - \pi)}} \quad (9.4)$$

## KEY TERMS

$\alpha$  (level of significance) 331  
 alternative hypothesis ( $H_1$ ) 329  
 $\beta$  risk 332  
 confidence coefficient ( $1 - \alpha$ ) 332  
 confidence level 332  
 critical value 331  
 data snooping 358  
 directional test 343  
 hypothesis testing 328

level of significance ( $\alpha$ ) 331  
 null hypothesis ( $H_0$ ) 328  
 one-tail test 343  
 $p$ -value 337  
 power of a statistical test 332  
 randomization 358  
 region of nonrejection 330  
 region of rejection 330  
 robust 350

sample proportion 353  
 $t$  test of hypothesis for a mean 346  
 test statistic 330  
 two-tail test 334  
 Type I error 331  
 Type II error 331  
 $Z$  test of hypothesis for the mean 334  
 $Z$  test for the proportion 353  
 $Z$  test statistic 334

## CHAPTER REVIEW PROBLEMS

### Checking Your Understanding

**9.75** What is the difference between a null hypothesis,  $H_0$ , and an alternative hypothesis,  $H_1$ ?

**9.76** What is the difference between a Type I error and a Type II error?

**9.77** What is meant by the power of a test?

**9.78** What is the difference between a one-tail test and a two-tail test?

**9.79** What is meant by a  $p$ -value?



**9.80** How can a confidence interval estimate for the population mean provide conclusions to the corresponding hypothesis test for the population mean?

**9.81** What is the six-step critical value approach to hypothesis testing?

**9.82** What are some of the ethical issues involved with performing a hypothesis test?

### Applying the Concepts

**9.83** An article in *Marketing News* (T. T. Semon, “Consider a Statistical Insignificance Test,” *Marketing News*, February 1, 1999) argued that the level of significance used when comparing two products is often too low—that is, sometimes you should be using an  $\alpha$  value greater than 0.05. Specifically, the article recounted testing the proportion of potential customers with a preference for product 1 over product 2. The null hypothesis was that the population proportion of potential customers preferring product 1 was 0.50, and the alternative hypothesis was that it was not equal to 0.50. The  $p$ -value for the test was 0.22. The article suggested that in some cases, this should be enough evidence to reject the null hypothesis.

- State the null and alternative hypotheses for this example in statistical terms.
- Explain the risks associated with Type I and Type II errors in this case.
- What would be the consequences if you rejected the null hypothesis for a  $p$ -value of 0.22?
- Why do you think the article suggested raising the value of  $\alpha$ ?
- What would you do in this situation?
- What is your answer in (e) if the  $p$ -value equals 0.12? What if it equals 0.06?

**9.84** La Quinta Motor Inns developed a computer model to help predict the profitability of sites that are being considered as locations for new hotels. If the computer model predicts large profits, La Quinta buys the proposed site and builds a new hotel. If the computer model predicts small or moderate profits, La Quinta chooses not to proceed with that site (Extracted from S. E. Kimes and J. A. Fitzsimmons, “Selecting Profitable Hotel Sites at La Quinta Motor Inns,” *Interfaces*, Vol. 20, March–April 1990, pp. 12–20). This decision-making procedure can be expressed in the hypothesis-testing framework. The null hypothesis is that the site is not a profitable location. The alternative hypothesis is that the site is a profitable location.

- Explain the risks associated with committing a Type I error in this case.
- Explain the risks associated with committing a Type II error in this case.
- Which type of error do you think the executives at La Quinta Motor Inns are trying hard to avoid? Explain.

**d.** How do changes in the rejection criterion affect the probabilities of committing Type I and Type II errors?

**9.85** In 2006, Visa wanted to move away from its long-running television advertising theme of “Visa, it’s everywhere you want to be.” During the Winter Olympics, Visa featured Olympians in commercials with a broader message, including security, check cards, and payment technologies such as contactless processing. One of the first commercials featured snowboarder Lindsey Jacobellis being coached to calm down before a big race by imagining that her Visa Check Card got stolen. A key metric for the success of television advertisements is the proportion of viewers who “like the ads a lot.” Harris Ad Research Service conducted a study of 903 adults who viewed the new Visa advertisement and reported that 54 indicated that they “like the ad a lot.” According to Harris, the proportion of a typical television advertisement receiving the “like the ad a lot” score is 0.21 (Extracted from T. Howard, “Visa to Change Strategies in Upcoming Ads,” *usatoday.com*, January 23, 2006.).

- Use the six-step critical value approach to hypothesis testing and a 0.05 level of significance to try to prove that the new Visa ad is less successful than a typical television advertisement.
- Use the five-step  $p$ -value approach to hypothesis testing and a 0.05 level of significance to try to prove that the new Visa ad is less successful than a typical television advertisement.
- Compare the results of (a) and (b).

**9.86** The owner of a gasoline station wants to study gasoline purchasing habits by motorists at his station. He selects a random sample of 60 motorists during a certain week, with the following results:

- The amount purchased was  $\bar{X} = 11.3$  gallons,  $S = 3.1$  gallons.
  - 11 motorists purchased premium-grade gasoline.
- At the 0.05 level of significance, is there evidence that the mean purchase was different from 10 gallons?
  - Determine the  $p$ -value in (a).
  - At the 0.05 level of significance, is there evidence that fewer than 20% of all the motorists at the station purchased premium-grade gasoline?
  - What is your answer to (a) if the sample mean equals 10.3 gallons?
  - What is your answer to (c) if 7 motorists purchased premium-grade gasoline?

**9.87** An auditor for a government agency is assigned the task of evaluating reimbursement for office visits to physicians paid by Medicare. The audit was conducted on a sample of 75 of the reimbursements, with the following results:

- In 12 of the office visits, an incorrect amount of reimbursement was provided.
- The amount of reimbursement was  $\bar{X} = \$93.70$ ,  $S = \$34.55$ .

- At the 0.05 level of significance, is there evidence that the mean reimbursement was less than \$100?
- At the 0.05 level of significance, is there evidence that the proportion of incorrect reimbursements in the population was greater than 0.10?
- Discuss the underlying assumptions of the test used in (a).
- What is your answer to (a) if the sample mean equals \$90?
- What is your answer to (b) if 15 office visits had incorrect reimbursements?

**9.88** A bank branch located in a commercial district of a city has developed an improved process for serving customers during the noon-to-1:00 p.m. lunch period. The waiting time (defined as the time the customer enters the line until he or she reaches the teller window) of all customers during this hour is recorded over a period of 1 week. A random sample of 15 customers (see the data file [bank1.xls](#)) is selected, and the results are as follows:

4.21 5.55 3.02 5.13 4.77 2.34 3.54 3.20  
4.50 6.10 0.38 5.12 6.46 6.19 3.79

- At the 0.05 level of significance, is there evidence that the mean waiting time is less than 5 minutes?
- What assumption must hold in order to perform the test in (a)?
- Evaluate the assumption in (b) through a graphical approach. Discuss.
- As a customer walks into the branch office during the lunch hour, she asks the branch manager how long she can expect to wait. The branch manager replies, “Almost certainly not longer than 5 minutes.” On the basis of the results of (a), evaluate this statement.

**9.89** A manufacturing company produces electrical insulators. If the insulators break when in use, a short circuit is likely to occur. To test the strength of the insulators, destructive testing is carried out to determine how much *force* is required to break the insulators. Force is measured by observing the number of pounds of force applied to the insulator before it breaks. The following data (stored in the [force.xls](#) file) are from 30 insulators subject to this testing:

*Force (in the Number of Pounds Required  
to Break the Insulator)*

1,870 1,728 1,656 1,610 1,634 1,784 1,522 1,696 1,592 1,662  
1,866 1,764 1,734 1,662 1,734 1,774 1,550 1,756 1,762 1,866  
1,820 1,744 1,788 1,688 1,810 1,752 1,680 1,810 1,652 1,736

- At the 0.05 level of significance, is there evidence that the mean force is greater than 1,500 pounds?
- What assumption must hold in order to perform the test in (a)?
- Evaluate the assumption in (b) through a graphical approach. Discuss.
- Based on (a), what can you conclude about the strength of the insulators?

**9.90** An important quality characteristic used by the manufacturer of Boston and Vermont asphalt shingles is the amount of moisture the shingles contain when they are packaged. Customers may feel that they have purchased a product lacking in quality if they find moisture and wet shingles inside the packaging. In some cases, excessive moisture can cause the granules attached to the shingle for texture and coloring purposes to fall off the shingle, resulting in appearance problems. To monitor the amount of moisture present, the company conducts moisture tests. A shingle is weighed and then dried. The shingle is then reweighed, and, based on the amount of moisture taken out of the product, the pounds of moisture per 100 square feet are calculated. The company would like to show that the mean moisture content is less than 0.35 pounds per 100 square feet. The data file [moisture.xls](#) includes 36 measurements (in pounds per 100 square feet) for Boston shingles and 31 for Vermont shingles.

- For the Boston shingles, is there evidence at the 0.05 level of significance that the mean moisture content is less than 0.35 pounds per 100 square feet?
- Interpret the meaning of the  $p$ -value in (a).
- For the Vermont shingles, is there evidence at the 0.05 level of significance that the mean moisture content is less than 0.35 pounds per 100 square feet?
- Interpret the meaning of the  $p$ -value in (c).
- What assumption must hold in order to perform the tests in (a) and (c)?
- Evaluate the assumption in (e) for Boston shingles and Vermont shingles by using a graphical approach. Discuss.

**9.91** Studies conducted by the manufacturer of Boston and Vermont asphalt shingles have shown product weight to be a major factor in the customer’s perception of quality. Moreover, the weight represents the amount of raw materials being used and is therefore very important to the company from a cost standpoint. The last stage of the assembly line packages the shingles before the packages are placed on wooden pallets. Once a pallet is full (a pallet for most brands holds 16 squares of shingles), it is weighed, and the measurement is recorded. The data file [pallet.xls](#) contains the weight (in pounds) from a sample of 368 pallets of Boston shingles and 330 pallets of Vermont shingles.

- For the Boston shingles, is there evidence that the mean weight is different from 3,150 pounds?
- Interpret the meaning of the  $p$ -value in (a).
- For the Vermont shingles, is there evidence that the mean weight is different from 3,700 pounds?
- Interpret the meaning of the  $p$ -value in (c).
- What assumption is needed for (a) and (c)?
- Evaluate this assumption through a graphical approach. Discuss.

**9.92** The manufacturer of Boston and Vermont asphalt shingles provides its customers with a 20-year warranty on most of its products. To determine whether a shingle will

last as long as the warranty period, accelerated-life testing is conducted at the manufacturing plant. Accelerated-life testing exposes the shingle to the stresses it would be subject to in a lifetime of normal use in a laboratory setting via an experiment that takes only a few minutes to conduct. In this test, a shingle is repeatedly scraped with a brush for a short period of time, and the shingle granules removed by the brushing are weighed (in grams). Shingles that experience low amounts of granule loss are expected to last longer in normal use than shingles that experience high amounts of granule loss. The data file [granule.xls](#) contains a sample of 170 measurements made on the company's Boston shingles and 140 measurements made on Vermont shingles.

- a. For the Boston shingles, is there evidence that the mean granule loss is different from 0.50 grams?
- b. Interpret the meaning of the  $p$ -value in (a).
- c. For the Vermont shingles, is there evidence that the mean granule loss is different from 0.50 grams?
- d. Interpret the meaning of the  $p$ -value in (c).
- e. What assumption is needed for (a) and (c)?
- f. Evaluate this assumption through a graphical approach. Discuss.

### Report Writing Exercises

**9.93** Referring to the results of Problems 9.90–9.92 concerning Boston and Vermont shingles, write a report that evaluates the moisture level, weight, and granule loss of the two types of shingles.

## Managing the Springville Herald

Continuing its monitoring of the blackness of the newspaper print, first described in the Chapter 6 Managing the *Springville Herald* case, the production department of the newspaper wants to ensure that the mean blackness of the print for all newspapers is at least 0.97 on a standard scale in which the target value is 1.0. A random sample of 50 newspapers has been selected, and the blackness of each paper has been measured (and stored in the [sh9.xls](#) file). Calculate the sample statistics and determine whether there

is evidence that the mean blackness is less than 0.97. Write a memo to management that summarizes your conclusions.

### *Blackness of 50 Newspapers*

0.854	1.023	1.005	1.030	1.219	0.977	1.044	0.778	1.122	1.114
1.091	1.086	1.141	0.931	0.723	0.934	1.060	1.047	0.800	0.889
1.012	0.695	0.869	0.734	1.131	0.993	0.762	0.814	1.108	0.805
1.223	1.024	0.884	0.799	0.870	0.898	0.621	0.818	1.113	1.286
1.052	0.678	1.162	0.808	1.012	0.859	0.951	1.112	1.003	0.972

## Web Case

Apply your knowledge about hypothesis testing in this Web Case, which continues the cereal-fill-packaging dispute Web Case from Chapter 7.

Concerned about statements of groups such as the Consumers Concerned About Cereal Cheaters (CCACC) (see the Web Case for Chapter 7). Oxford Cereals recently conducted a public experiment concerning cereal packaging. The company claims that the results of the experiments refute the CCACC allegations that Oxford Cereals has been cheating consumers by packaging cereals at less than labeled weights. Review the Oxford Cereals' press release and supporting documents that describe the experiment at the company's Web site,

[www.prenhall.com/Springville/OC\\_FullUp.htm](http://www.prenhall.com/Springville/OC_FullUp.htm), and then answer the following:

1. Are the results of the independent testing valid? Why or why not? If you were conducting the experiment, is there anything you would change?
2. Do the results support the claim that Oxford Cereals is not cheating its customers?
3. Is the claim of the Oxford Cereals CEO that many cereal boxes contain *more* than 368 grams surprising? Is it true?
4. Could there ever be a circumstance in which the results of the public experiment *and* the CCACC's results are both correct? Explain.

## REFERENCES

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2. Daniel, W., *Applied Nonparametric Statistics*, 2nd ed. (Boston: Houghton Mifflin, 1990).
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# Excel Companion

## to Chapter 9

### E9.1 USING THE Z TEST FOR THE MEAN ( $\sigma$ KNOWN)

You conduct a Z test for the mean ( $\sigma$  known) by either selecting the PHStat2 Z Test for the Mean, sigma known procedure or by making entries in the [Z Mean.xls](#) workbook.

#### Using PHStat2 Z Test for the Mean, Sigma Known

Select **PHStat** → **One-Sample Tests** → **Z Test for the Mean, sigma known**. In the Z Test for the Mean, sigma known, dialog box (shown below), enter values for the **Null Hypothesis**, the **Level of Significance**, and the **Population Standard Deviation**. If you know the sample size and sample mean of your sample, click **Sample**

**Statistics Known** and enter those values. Otherwise, click **Sample Statistics Unknown** and enter the cell range of your sample as the **Sample Cell Range**. Then click one of the test options, enter a title as the **Title**, and click **OK**.

#### Using Z Mean.xls

You open and use either the **ZMean TT** or **ZMean\_All** worksheets of the [Z Mean.xls](#) workbook to apply the Z test for the mean ( $\sigma$  known). These worksheets use the **NORMSINV( $P < X$ )** function to determine the lower and upper critical values and use the **NORMSDIST( $Z$  value)** function to compute the  $p$ -values from the  $Z$  value calculated in cell B12. The **ZMean\_TT** worksheet (see Figure 9.5 on page 338) applies the two-tail Z test to the Example 9.2 cereal-fill example. The **ZMean\_All** worksheet (see Figure E9.1) includes both the two-tail test and the upper- and lower-tail tests on one worksheet. To adapt these worksheets to other problems, change (if necessary) the null hypothesis, level of significance, population standard deviation, sample size, and sample mean values in the tinted cells B4 through B8.

If you want the ZMean\_All worksheet to show only one of the single-tail tests, first make a copy of that worksheet (see the Excel Companion to Chapter 1). For a lower-tail-test-only worksheet, select and delete rows 25 through 28 and then select and delete rows 14 through 19. For an upper-tail-test-only worksheet, select and delete rows 14 through 24.

#### About the IF Function

Both worksheets of the [Z Mean.xls](#) workbook and all other hypothesis-testing worksheets presented in the rest of the book use the **IF (comparison, what to do if comparison holds, what to do if comparison fails)** function to suggest whether you should reject the null hypothesis. (You should always verify the results of worksheets and not blindly accept results reported by formulas containing the IF function.)

You need to supply three things when using an IF function in a formula. The **comparison** is an algebraic or logical comparison between two things. In Figure E9.1, the  $p$ -values are compared to the level of significance.

FIGURE E9.1

ZMean\_All worksheet

	A	B
1	<b>Z Test of the Hypothesis for the Mean</b>	
2		
3	<b>Data</b>	
4	<b>Null Hypothesis</b>	$\mu =$ 368
5	<b>Level of Significance</b>	0.05
6	<b>Population Standard Deviation</b>	15
7	<b>Sample Size</b>	25
8	<b>Sample Mean</b>	372.5
9		
10	<b>Intermediate Calculations</b>	
11	Standard Error of the Mean	3 =B6/SQRT(B7)
12	<b>Z Test Statistic</b>	1.5 =(B8 - B4)/B11
13		
14	<b>Two-Tail Test</b>	
15	<b>Lower Critical Value</b>	-1.9600 =NORMSINV(B5/2)
16	<b>Upper Critical Value</b>	1.9600 =NORMSINV(1 - B5/2)
17	<b>p-Value</b>	0.1336 =2 * (1 - NORMSDIST(ABS(B12)))
18	<b>Do not reject the null hypothesis</b>	
19		=IF(B17 < B5, "Reject the null hypothesis", "Do not reject the null hypothesis")
20	<b>Lower-Tail Test</b>	
21	<b>Lower Critical Value</b>	-1.6449 =NORMSINV(B5)
22	<b>p-Value</b>	0.9332 =NORMSDIST(B12)
23	<b>Do not reject the null hypothesis</b>	
24		=IF(B22 < B5, "Reject the null hypothesis", "Do not reject the null hypothesis")
25	<b>Upper-Tail Test</b>	
26	<b>Upper Critical Value</b>	1.6449 =NORMSINV(1 - B5)
27	<b>p-Value</b>	0.0668 =1 - NORMSDIST(B12)
28	<b>Do not reject the null hypothesis</b>	
		=IF(B27 < B5, "Reject the null hypothesis", "Do not reject the null hypothesis")

*What to do if comparison holds/fails* are either text values to display (as in Figure E9.1) or numeric values or formulas to use as the cell contents. In Figure E9.1, “Do not reject the null hypothesis” is displayed in rows 18, 23, and 28 because all three  $p$ -values (in cells B17, B22, and B27) are greater than the level of significance in cell B5. (By the way, text values in IF functions must be enclosed by a pair of quotation marks as they are in Figure E9.1.)

You should always enter formulas containing the IF function as a single, continuous line, even though such formulas are generally shown in this text on two physical lines (as in Figure E9.1). For the hypothesis-testing worksheets, the formulas containing the IF function have been entered into column A cells so that their text values display across columns A and B.

## E9.2 USING THE $t$ TEST FOR THE MEAN ( $\sigma$ unknown)

You conduct a  $t$  test for the mean ( $\sigma$  unknown) by either selecting the PHStat2  $t$  Test for the Mean, sigma unknown procedure or by making entries in the [T Mean.xls](#) workbook.

### Using PHStat2 $t$ Test for the Mean, Sigma Unknown

Select **PHStat** → **One-Sample Tests** →  **$t$  Test for the Mean, sigma unknown**. In the  $t$  Test for the Mean, sigma unknown dialog box (shown at right), enter values for the **Null Hypothesis** and the **Level of Significance**. If you know the sample size, sample mean, and sample standard deviation, click **Sample Statistics Known** and enter those

values. Otherwise, click **Sample Statistics Unknown** and enter the cell range of your data as the **Sample Cell Range**. Then, click one of the test options, enter a title as the **Title**, and click **OK**.

FIGURE E9.2

TMean\_All worksheet

	A	B	
1	<b>t Test for the Hypothesis of the Mean</b>		
2			
3	<b>Data</b>		
4	Null Hypothesis $\mu =$	120	
5	Level of Significance	0.05	
6	Sample Size	12	
7	Sample Mean	112.85	
8	Sample Standard Deviation	20.8	
9			
10	<b>Intermediate Calculations</b>		
11	Standard Error of the Mean	6.0044	=B8/SQRT(B6)
12	Degrees of Freedom	11	=B6 - 1
13	t Test Statistic	-1.1908	=(B7 - B4)/B11
14			
15	<b>Two-Tail Test</b>		
16	Lower Critical Value	-2.2010	=(TINV(B5, B12))
17	Upper Critical Value	2.2010	=TINV(B5, B12)
18	p-Value	0.2588	=TDIST(ABS(B13), B12, 2)
19	Do not reject the null hypothesis		=IF(B18 < B5, "Reject the null hypothesis", "Do not reject the null hypothesis")
20			
21	<b>Lower-Tail Test</b>		
22	Lower Critical Value	-1.7959	=(TINV(2 * B5, B12))
23	p-Value	0.1294	=IF(B13 < 0, E22, E23)
24	Do not reject the null hypothesis		=IF(B23 < B5, "Reject the null hypothesis", "Do not reject the null hypothesis")
25			
26	<b>Upper-Tail Test</b>		
27	Upper Critical Value	1.7959	=(TINV(2 * B5, B12))
28	p-Value	0.8706	=IF(B13 < 0, E23, E22)
29	Do not reject the null hypothesis		=IF(B28 < B5, "Reject the null hypothesis", "Do not reject the null hypothesis")

**Not shown**  
 Cell E22: =TDIST(ABS(B13), B12, 1)  
 Cell E23: =1 - E22

## Using T Mean.xls

You open and use either the **TMean\_TT** or **TMean\_All** worksheets of the **T Mean.xls** workbook to apply the  $t$  test for the mean ( $\sigma$  unknown). These worksheets use the **TINV(1-confidence level, degrees of freedom)** function to determine the lower and upper critical values and to compute the  $p$ -values. They also use the **TDIST(ABS( $t$ ), degrees of freedom, tails)** function in which **ABS( $t$ )** is the absolute value of the  $t$  test statistic and **tails** is either 1 for a one-tail test or 2 for a two-tail test.

The **TMean\_TT** worksheet (see Figure 9.10 on page 348) applies the two-tail  $t$  test to the Section 9.4 sales invoices example. The **TMean\_All** worksheet (see Figure E9.2) includes both the two-tail test and the upper- and lower-tail tests on one worksheet. To adapt these worksheets to other problems, change (if necessary) the null hypothesis, level of significance, sample size, sample mean, and sample standard deviation values in the tinted cells B4 through B8. To better understand how a message gets displayed in these worksheets, read the "About the IF Function" part of Section E9.1 on page 364.

If you want the **TMean\_All** worksheet to show only one of the single-tail tests, first make a copy of that worksheet (see the Excel Companion to Chapter 1). For a lower-tail-test-only worksheet, select and delete rows 26 through 29 and then select and delete cell range A15:B20. For an upper-tail-test-only worksheet, select and delete cell range A15:B25. These instructions ask you to select and delete certain cell ranges in order to preserve the D20:E23 calculations area cell range (which is not shown in Figure E9.2).

## E9.3 USING THE Z TEST FOR THE PROPORTION

You conduct a  $Z$  test for the proportion by either selecting the **PHStat2 Z Test for the Proportion** procedure or by making entries in the **Z Proportion.xls** workbook.

### Using PHStat2 Z Test for the Proportion

Select **PHStat** → **One-Sample Tests** → **Z Test for the Proportion**. In the **Z Test for the Proportion** dialog box (shown below), enter values for the **Null Hypothesis**, the

**Level of Significance**, the **Number of Successes**, and the **Sample Size**. Click one of the test options, enter a title as the **Title**, and click **OK**.

### Using Z Proportion.xls

You open and use either the **ZProp\_TT** or **ZProp\_All** worksheets of the **Z Proportion.xls** workbook to apply the Z test for the proportion. These worksheets use the **NORMSINV( $P < X$ )** function to determine the lower and upper critical values and the **NORMSDIST( $Z$  value)** function to compute the  $p$ -values.

The **ZProp\_TT** worksheet (see Figure 9.15 on page 355) applies the two-tail Z test to the Section 9.5 competitive threat example. The **ZProp\_All** worksheet (see Figure E9.3)

includes both the two-tail test and the upper- and lower-tail tests on one worksheet. To adapt these worksheet to other problems, change (if necessary) the null hypothesis, level of significance, number of successes, and sample size values in the tinted cells B4 through B8. To better understand how messages get displayed in these worksheets, read the “About the IF Function” part of Section E9.1 on page 364.

If you want the ZProp\_All worksheet to show only one of the single-tail tests, first make a copy of that worksheet (see the Excel Companion to Chapter 1). For a lower-tail-test-only worksheet, select and delete rows 25 through 28 and then select and delete rows 14 through 19. For an upper-tail-test-only worksheet, select and delete rows 14 through 24.

**FIGURE E9.3**  
ZProp\_All worksheet

	A	B	
1	<b>Wal-Mart As Biggest Threat Hypothesis Test</b>		
2			
3	Data		
4	Null Hypothesis	$p =$	0.5
5	Level of Significance		0.05
6	Number of Successes		78
7	Sample Size		151
8			
9	Intermediate Calculations		
10	Sample Proportion		=B6/B7
11	Standard Error		=SQRT(B4*(1 - B4)/B7)
12	Z Test Statistic		=(B10 - B4)/B11
13			
14	Two-Tail Test		
15	Lower Critical Value		=NORMSINV(B5/2)
16	Upper Critical value		=NORMSINV(1 - B5/2)
17	p-Value		=2 * (1 - NORMSDIST(ABS(B12)))
18	Do not reject the null hypothesis		=IF(B17 < B5, "Reject the null hypothesis", "Do not reject the null hypothesis")
19			
20	Lower-Tail Test		
21	Lower Critical Value		=NORMSINV(B5)
22	p-Value		=NORMSDIST(B12)
23	Do not reject the null hypothesis		=IF(B22 < B5, "Reject the null hypothesis", "Do not reject the null hypothesis")
24			
25	Upper-Tail Test		
26	Upper Critical Value		=NORMSINV(1 - B5)
27	p-Value		=1 - NORMSDIST(B12)
28	Do not reject the null hypothesis		=IF(B27 < B5, "Reject the null hypothesis", "Do not reject the null hypothesis")





# CHAPTER 10

## Simple Linear Regression

### USING STATISTICS @ Sunflowers Apparel

#### 10.1 TYPES OF REGRESSION MODELS

#### 10.2 DETERMINING THE SIMPLE LINEAR REGRESSION EQUATION

- The Least-Squares Method
- Visual Explorations: Exploring Simple Linear Regression Coefficients
- Predictions in Regression Analysis: Interpolation Versus Extrapolation
- Computing the  $Y$  Intercept,  $b_0$ , and the Slope,  $b_1$

#### 10.3 MEASURES OF VARIATION

- Computing the Sum of Squares
- The Coefficient of Determination
- Standard Error of the Estimate

#### 10.4 ASSUMPTIONS

#### 10.5 RESIDUAL ANALYSIS

- Evaluating the Assumptions

#### 10.6 MEASURING AUTOCORRELATION: THE DURBIN-WATSON STATISTIC

- Residual Plots to Detect Autocorrelation
- The Durbin-Watson Statistic

#### 10.7 INFERENCES ABOUT THE SLOPE AND CORRELATION COEFFICIENT

- $t$  Test for the Slope
- $F$  Test for the Slope
- Confidence Interval Estimate of the Slope ( $\beta_1$ )
- $t$  Test for the Correlation Coefficient

#### 10.8 ESTIMATION OF MEAN VALUES AND PREDICTION OF INDIVIDUAL VALUES

- The Confidence Interval Estimate
- The Prediction Interval

#### 10.9 PITFALLS IN REGRESSION AND ETHICAL ISSUES

#### EXCEL COMPANION TO CHAPTER 10

- E10.1 Performing Simple Linear Regression Analyses
- E10.2 Creating Scatter Plots and Adding a Prediction Line
- E10.3 Performing Residual Analyses
- E10.4 Computing the Durbin-Watson Statistic
- E10.5 Estimating the Mean of  $Y$  and Predicting  $Y$  Values
- E10.6 Example: Sunflowers Apparel Data

### LEARNING OBJECTIVES

In this chapter, you learn:

- To use regression analysis to predict the value of a dependent variable based on an independent variable
- The meaning of the regression coefficients  $b_0$  and  $b_1$
- To evaluate the assumptions of regression analysis and know what to do if the assumptions are violated
- To make inferences about the slope and correlation coefficient
- To estimate mean values and predict individual values

## Using Statistics @ Sunflowers Apparel



The sales for Sunflowers Apparel, a chain of upscale clothing stores for women, have increased during the past 12 years as the chain has expanded the number of stores open. Until now, Sunflowers managers selected sites based on subjective factors, such as the availability of a good lease or the perception that a location seemed ideal for an apparel store. As the new director of planning, you need to develop a systematic approach that will lead to making better decisions during the site selection process. As a starting point, you believe that the size of the store significantly contributes to store sales, and you want to use this relationship in the decision-making process. How can you use statistics so that you can forecast the annual sales of a proposed store based on the size of that store?

In this chapter, you learn how **regression analysis** enables you to develop a model to predict the values of a numerical variable, based on the value of other variables.

In regression analysis, the variable you wish to predict is called the **dependent variable**. The variables used to make the prediction are called **independent variables**. In addition to predicting values of the dependent variable, regression analysis also allows you to identify the type of mathematical relationship that exists between a dependent and an independent variable, to quantify the effect that changes in the independent variable have on the dependent variable, and to identify unusual observations. For example, as the director of planning, you may wish to predict sales for a Sunflowers store, based on the size of the store. Other examples include predicting the monthly rent of an apartment, based on its size, and predicting the monthly sales of a product in a supermarket, based on the amount of shelf space devoted to the product.

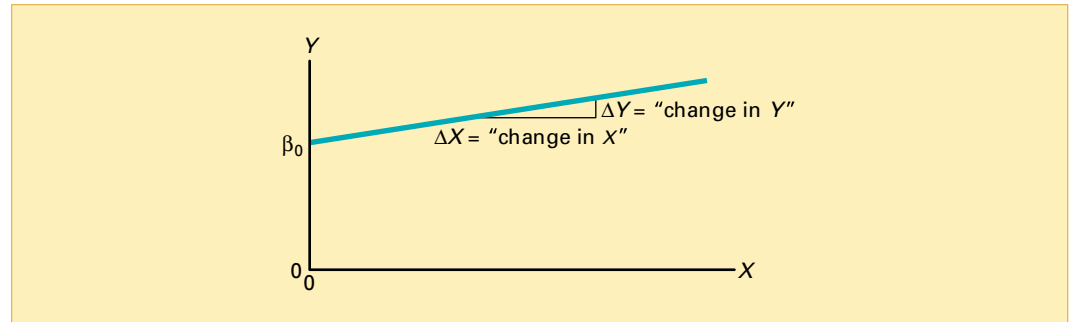
This chapter discusses **simple linear regression**, in which a *single* numerical independent variable,  $X$ , is used to predict the numerical dependent variable  $Y$ , such as using the size of a store to predict the annual sales of the store. Chapter 11 discusses *multiple regression models*, which use several independent variables to predict a numerical dependent variable,  $Y$ . For example, you could use the amount of advertising expenditures, price, and the amount of shelf space devoted to a product to predict its monthly sales.

### 10.1 TYPES OF REGRESSION MODELS

In Section 2.5, you used a **scatter plot** (also known as a **scatter diagram**) to examine the relationship between an  $X$  variable on the horizontal axis and a  $Y$  variable on the vertical axis. The nature of the relationship between two variables can take many forms, ranging from simple to extremely complicated mathematical functions. The simplest relationship consists of a straight-line, or **linear relationship**. An example of this relationship is shown in Figure 10.1.

**FIGURE 10.1**

A positive straight-line relationship



Equation (10.1) represents the straight-line (linear) model.

### SIMPLE LINEAR REGRESSION MODEL

$$Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i \quad (10.1)$$

where

$\beta_0$  =  $Y$  intercept for the population

$\beta_1$  = slope for the population

$\varepsilon_i$  = random error in  $Y$  for observation  $i$

$Y_i$  = dependent variable (sometimes referred to as the **response variable**) for observation  $i$

$X_i$  = independent variable (sometimes referred to as the **explanatory variable**) for observation  $i$

The portion  $Y_i = \beta_0 + \beta_1 X_i$  of the simple linear regression model expressed in Equation (10.1) is a straight line. The **slope** of the line,  $\beta_1$ , represents the expected change in  $Y$  per unit change in  $X$ . It represents the mean amount that  $Y$  changes (either positively or negatively) for a one-unit change in  $X$ . The  **$Y$  intercept**,  $\beta_0$ , represents the mean value of  $Y$  when  $X$  equals 0. The last component of the model,  $\varepsilon_i$ , represents the random error in  $Y$  for each observation,  $i$ . In other words,  $\varepsilon_i$  is the vertical distance of the actual value of  $Y_i$  above or below the predicted value of  $Y_i$  on the line.

The selection of the proper mathematical model depends on the distribution of the  $X$  and  $Y$  values on the scatter plot. In Panel A of Figure 10.2 on page 372, the values of  $Y$  are generally increasing linearly as  $X$  increases. This panel is similar to Figure 10.3 on page 373, which illustrates the positive relationship between the square footage of the store and the annual sales at branches of the Sunflowers Apparel women's clothing store chain.

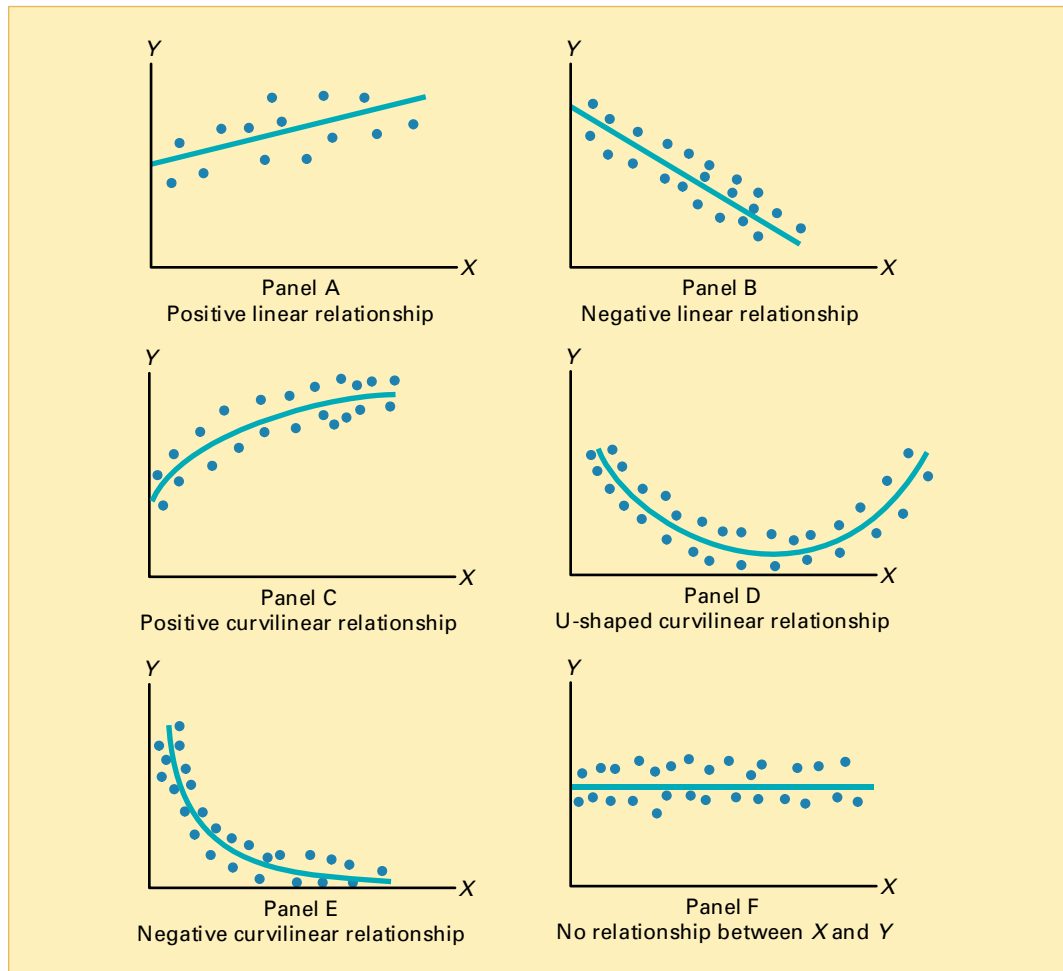
Panel B is an example of a negative linear relationship. As  $X$  increases, the values of  $Y$  are generally decreasing. An example of this type of relationship might be the price of a particular product and the amount of sales.

The data in Panel C show a positive curvilinear relationship between  $X$  and  $Y$ . The values of  $Y$  increase as  $X$  increases, but this increase tapers off beyond certain values of  $X$ . An example of a positive curvilinear relationship might be the age and maintenance cost of a machine. As a machine gets older, the maintenance cost may rise rapidly at first, but then level off beyond a certain number of years.

Panel D shows a U-shaped relationship between  $X$  and  $Y$ . As  $X$  increases, at first  $Y$  generally decreases; but as  $X$  continues to increase,  $Y$  not only stops decreasing but actually increases above its minimum value. An example of this type of relationship might be the number of errors per hour at a task and the number of hours worked. The number of errors per hour

**FIGURE 10.2**

Examples of types of relationships found in scatter plots



decreases as the individual becomes more proficient at the task, but then it increases beyond a certain point because of factors such as fatigue and boredom.

Panel E indicates an exponential relationship between  $X$  and  $Y$ . In this case,  $Y$  decreases very rapidly as  $X$  first increases, but then it decreases much less rapidly as  $X$  increases further. An example of an exponential relationship could be the resale value of an automobile and its age. In the first year, the resale value drops drastically from its original price; however, the resale value then decreases much less rapidly in subsequent years.

Finally, Panel F shows a set of data in which there is very little or no relationship between  $X$  and  $Y$ . High and low values of  $Y$  appear at each value of  $X$ .

In this section, a variety of different models that represent the relationship between two variables were briefly examined. Although scatter plots are useful in visually displaying the mathematical form of a relationship, more sophisticated statistical procedures are available to determine the most appropriate model for a set of variables. The rest of this chapter discusses the model used when there is a *linear* relationship between variables.

## 10.2 DETERMINING THE SIMPLE LINEAR REGRESSION EQUATION

In the Using Statistics scenario on page 370, the stated goal is to forecast annual sales for all new stores, based on store size. To examine the relationship between the store size in square feet and its annual sales, a sample of 14 stores was selected. Table 10.1 summarizes the results for these 14 stores, which are stored in the file [site.xls](#).

**TABLE 10.1**

Square Footage (in Thousands of Square Feet) and Annual Sales (in Millions of Dollars) for a Sample of 14 Branches of Sunflowers Apparel

Store	Square Feet (Thousands)	Annual Sales (in Millions of Dollars)	Store	Square Feet (Thousands)	Annual Sales (in Millions of Dollars)
1	1.7	3.7	8	1.1	2.7
2	1.6	3.9	9	3.2	5.5
3	2.8	6.7	10	1.5	2.9
4	5.6	9.5	11	5.2	10.7
5	1.3	3.4	12	4.6	7.6
6	2.2	5.6	13	5.8	11.8
7	1.3	3.7	14	3.0	4.1

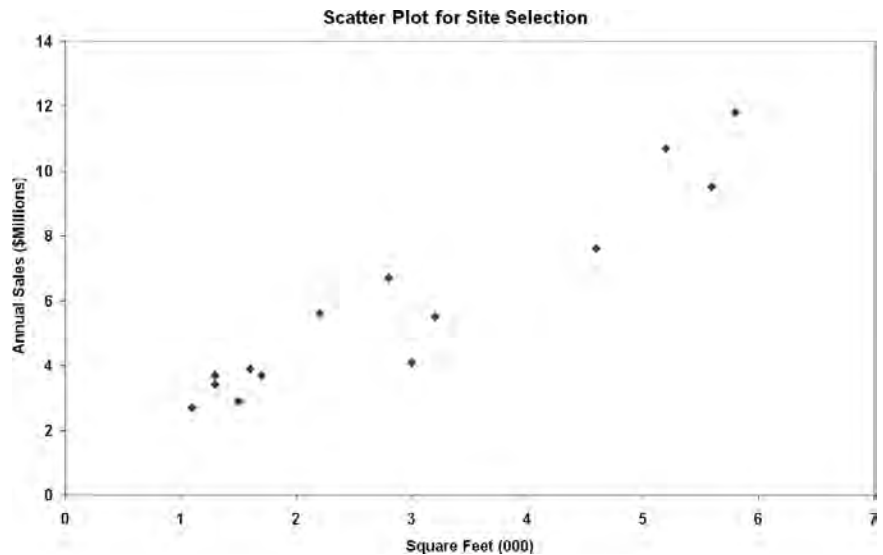
Figure 10.3 displays the scatter plot for the data in Table 10.1. Observe the increasing relationship between square feet ( $X$ ) and annual sales ( $Y$ ). As the size of the store increases, annual sales increase approximately as a straight line. Thus, you can assume that a straight line provides a useful mathematical model of this relationship. Now you need to determine the specific straight line that is the *best* fit to these data.

**FIGURE 10.3**

Microsoft Excel scatter plot for the Sunflowers Apparel data



See Section E2.12 to create this.



## The Least-Squares Method

In the preceding section, a statistical model is hypothesized to represent the relationship between two variables, square footage and sales, in the entire population of Sunflowers Apparel stores. However, as shown in Table 10.1, the data are from only a random sample of stores. If certain assumptions are valid (see Section 10.4), you can use the sample  $Y$  intercept,  $b_0$ , and the sample slope,  $b_1$ , as estimates of the respective population parameters,  $\beta_0$  and  $\beta_1$ . Equation (10.2) uses these estimates to form the **simple linear regression equation**. This straight line is often referred to as the **prediction line**.

### SIMPLE LINEAR REGRESSION EQUATION: THE PREDICTION LINE

The predicted value of  $Y$  equals the  $Y$  intercept plus the slope times the value of  $X$ .

$$\hat{Y}_i = b_0 + b_1 X_i \quad (10.2)$$

where

$\hat{Y}_i$  = predicted value of  $Y$  for observation  $i$

$X_i$  = value of  $X$  for observation  $i$

$b_0$  = sample  $Y$  intercept

$b_1$  = sample slope

Equation (10.2) requires the determination of two **regression coefficients**— $b_0$  (the sample  $Y$  intercept) and  $b_1$  (the sample slope). The most common approach to finding  $b_0$  and  $b_1$  is the method of least squares. This method minimizes the sum of the squared differences between the actual values ( $Y_i$ ) and the predicted values ( $\hat{Y}_i$ ) using the simple linear regression equation [that is, the prediction line; see Equation (10.2)]. This sum of squared differences is equal to

$$\sum_{i=1}^n (Y_i - \hat{Y}_i)^2$$

Because  $\hat{Y}_i = b_0 + b_1 X_i$ ,

$$\sum_{i=1}^n (Y_i - \hat{Y}_i)^2 = \sum_{i=1}^n [Y_i - (b_0 + b_1 X_i)]^2$$

Because this equation has two unknowns,  $b_0$  and  $b_1$ , the sum of squared differences depends on the sample  $Y$  intercept,  $b_0$ , and the sample slope,  $b_1$ . The **least-squares method** determines the values of  $b_0$  and  $b_1$  that minimize the sum of squared differences. Any values for  $b_0$  and  $b_1$  other than those determined by the least-squares method result in a greater sum of squared differences between the actual values ( $Y_i$ ) and the predicted values  $\hat{Y}_i$ . In this book, Microsoft Excel is used to perform the computations involved in the least-squares method. For the data of Table 10.1, Figure 10.4 presents results from Microsoft Excel.

**FIGURE 10.4**

Microsoft Excel results for the Sunflowers Apparel data



See Section E10.1 to create this.

	A	B	C	D	E	F	G
1	<b>Site Selection Analysis</b>						
2							
3	<b>Regression Statistics</b>						
4	Multiple R	0.9509					
5	R Square	0.9042					
6	Adjusted R Square	0.8962					
7	Standard Error	0.9664	$S_{YX}$				
8	Observations	14	$n$				
9							
10	<b>ANOVA</b>						
11		<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>Significance F</i>	
12	Regression	1	SSR—105.7476	105.7476	113.2335	0.0000	
13	Residual	12	SSE—11.2067	0.9339		<i>p-value</i>	
14	Total	13	SST—116.9543				
15							
16		<i>Coefficients</i>	<i>Standard Error</i>	<i>t Stat</i>	<i>P-value</i>	<i>Lower 95%</i>	<i>Upper 95%</i>
17	Intercept	$b_0$ —0.9645	0.5262	1.8329	0.0917	-0.1820	2.1110
18	Square Feet	$b_1$ —1.6699	0.1569	10.6411	0.0000	1.3280	2.0118

To understand how the results are computed, many of the computations involved are illustrated in Examples 10.3 and 10.4 on pages 378–379 and 384–385. In Figure 10.4, observe that  $b_0 = 0.9645$  and  $b_1 = 1.6699$ . Thus, the prediction line [see Equation (10.2) on page 373] for these data is

$$\hat{Y}_i = 0.9645 + 1.6699 X_i$$

The slope,  $b_1$ , is +1.6699. This means that for each increase of 1 unit in  $X$ , the mean value of  $Y$  is estimated to increase by 1.6699 units. In other words, for each increase of 1.0 thousand square feet in the size of the store, the mean annual sales are estimated to increase by 1.6699 millions of dollars. Thus, the slope represents the portion of the annual sales that are estimated to vary according to the size of the store.

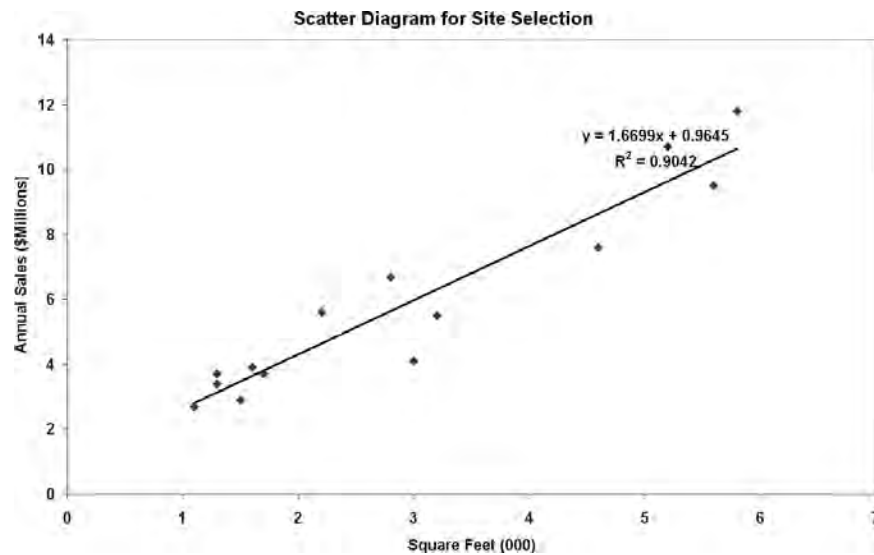
The  $Y$  intercept,  $b_0$ , is +0.9645. The  $Y$  intercept represents the mean value of  $Y$  when  $X$  equals 0. Because the square footage of the store cannot be 0, this  $Y$  intercept has no practical interpretation. Also, the  $Y$  intercept for this example is outside the range of the observed values of the  $X$  variable, and therefore interpretations of the value of  $b_0$  should be made cautiously. Figure 10.5 displays the actual observations and the prediction line. To illustrate a situation in which there is a direct interpretation for the  $Y$  intercept,  $b_0$ , see Example 10.1.

**FIGURE 10.5**

Microsoft Excel scatter plot and prediction line for Sunflowers Apparel data



See Section E10.2 to create this.



**EXAMPLE 10.1**

**INTERPRETING THE YINTERCEPT,  $b_0$ , AND THE SLOPE,  $b_1$**

A statistics professor wants to use the number of hours a student studies for a statistics final exam ( $X$ ) to predict the final exam score ( $Y$ ). A regression model was fit based on data collected for a class during the previous semester, with the following results:

$$\hat{Y}_i = 35.0 + 3 X_i$$

What is the interpretation of the  $Y$  intercept,  $b_0$ , and the slope,  $b_1$ ?

**SOLUTION** The  $Y$  intercept  $b_0 = 35.0$  indicates that when the student does not study for the final exam, the mean final exam score is 35.0. The slope  $b_1 = 3$  indicates that for each increase of one hour in studying time, the mean change in the final exam score is predicted to be +3.0. In other words, the final exam score is predicted to increase by 3 points for each one-hour increase in studying time.

## VISUAL EXPLORATIONS Exploring Simple Linear Regression Coefficients

Use the Visual Explorations Simple Linear Regression procedure to produce a prediction line that is as close as possible to the prediction line defined by the least-squares solution. Open the **Visual Explorations.xla** add-in workbook and select **VisualExplorations → Simple Linear Regression** (Excel 97-2003) or **Add-ins → Visual Explorations → Simple Linear Regression** (Excel 2007). (See Section E1.6 to learn about using add-ins.)

When a scatter plot of the Sunflowers Apparel data of Table 10.1 on page 373 with an initial prediction line appears (shown below), click the spinner buttons to change the values for  $b_1$ , the slope of the prediction line, and  $b_0$ , the  $Y$  intercept of the prediction line.

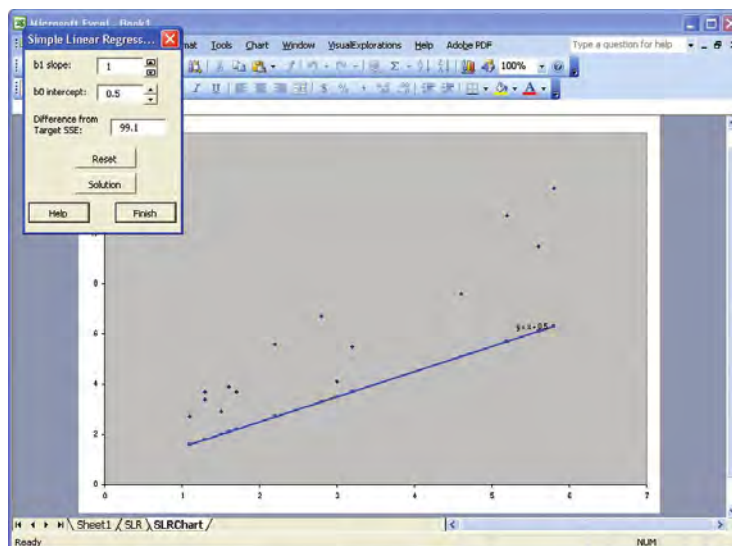
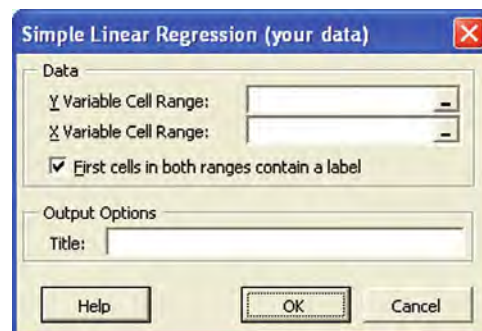
Try to produce a prediction line that is as close as possible to the prediction line defined by the least-squares estimates, using the chart display and the Difference from Target SSE value as feedback (see page 383 for an explanation of SSE). Click **Finish** when you are done with this exploration.

At any time, click **Reset** to reset the  $b_1$  and  $b_0$  values, **Help** for more information, or **Solution** to reveal the prediction line defined by the least-squares method.

### Using Your Own Regression Data

To use Visual Explorations to find a prediction line for your own data, open the **Visual Explorations.xla** add-in

workbook and select **VisualExplorations → Simple Linear Regression with your worksheet data** (97-2003) or **Add-ins → Visual Explorations → Simple Linear Regression with your worksheet data** (2007). In the procedure's dialog box (shown below), enter your  $Y$  variable cell range as the **Y Variable Cell Range** and your  $X$  variable cell range as the **X Variable Cell Range**. Click **First cells in both ranges contain a label**, enter a title as the **Title**, and click **OK**. When the scatter plot with an initial prediction line appears, use the instructions in the first part of this section to try to produce the prediction line defined by the least-squares method.





Return to the Using Statistics scenario concerning the Sunflowers Apparel stores. Example 10.2 illustrates how you use the prediction equation to predict the mean annual sales.

**EXAMPLE 10.2****PREDICTING MEAN ANNUAL SALES, BASED ON SQUARE FOOTAGE**

Use the prediction line to predict the mean annual sales for a store with 4,000 square feet.

**SOLUTION** You can determine the predicted value by substituting  $X = 4$  (thousands of square feet) into the simple linear regression equation:

$$\hat{Y}_i = 0.9645 + 1.6699 X_i$$

$$\hat{Y}_i = 0.9645 + 1.6699(4) = 7.644 \text{ or } \$7,644,000$$

Thus, the predicted mean annual sales of a store with 4,000 square feet is \$7,644,000.

**Predictions in Regression Analysis: Interpolation Versus Extrapolation**

When using a regression model for prediction purposes, you need to consider only the **relevant range** of the independent variable in making predictions. This relevant range includes all values from the smallest to the largest  $X$  used in developing the regression model. Hence, when predicting  $Y$  for a given value of  $X$ , you can interpolate within this relevant range of the  $X$  values, but you should not extrapolate beyond the range of  $X$  values. When you use the square footage to predict annual sales, the square footage (in thousands of square feet) varies from 1.1 to 5.8 (see Table 10.1 on page 373). Therefore, you should predict annual sales *only* for stores whose size is between 1.1 and 5.8 thousands of square feet. Any prediction of annual sales for stores outside this range assumes that the observed relationship between sales and store size for store sizes from 1.1 to 5.8 thousand square feet is the same as for stores outside this range. For example, you cannot extrapolate the linear relationship beyond 5,800 square feet in Example 10.2. It would be improper to use the prediction line to forecast the sales for a new store containing 8,000 square feet. It is quite possible that store size has a point of diminishing returns. If that is true, as square footage increases beyond 5,800 square feet, the effect on sales might become smaller and smaller.

**Computing the Y Intercept,  $b_0$ , and the Slope,  $b_1$** 

For small data sets, you can use a hand calculator to compute the least-squares regression coefficients. Equations (10.3) and (10.4) give the values of  $b_0$  and  $b_1$ , which minimize

$$\sum_{i=1}^n (Y_i - \hat{Y}_i)^2 = \sum_{i=1}^n [Y_i - (b_0 + b_1 X_i)]^2$$

**COMPUTATIONAL FORMULA FOR THE SLOPE,  $b_1$** 

$$b_1 = \frac{SSXY}{SSX} \quad (10.3)$$

where

$$SSXY = \sum_{i=1}^n (X_i - \bar{X})(Y_i - \bar{Y}) = \sum_{i=1}^n X_i Y_i - \frac{\left(\sum_{i=1}^n X_i\right)\left(\sum_{i=1}^n Y_i\right)}{n}$$

$$SSX = \sum_{i=1}^n (X_i - \bar{X})^2 = \sum_{i=1}^n X_i^2 - \frac{\left(\sum_{i=1}^n X_i\right)^2}{n}$$

COMPUTATIONAL FORMULA FOR THE Y INTERCEPT,  $b_0$ 

$$b_0 = \bar{Y} - b_1 \bar{X} \quad (10.4)$$

where

$$\bar{Y} = \frac{\sum_{i=1}^n Y_i}{n}$$

$$\bar{X} = \frac{\sum_{i=1}^n X_i}{n}$$

**EXAMPLE 10.3****COMPUTING THE Y INTERCEPT,  $b_0$ , AND THE SLOPE,  $b_1$** 

Compute the  $Y$  intercept,  $b_0$ , and the slope,  $b_1$ , for the Sunflowers Apparel data.

**SOLUTION** Examining Equations (10.3) and (10.4), you see that five quantities must be calculated to determine  $b_1$  and  $b_0$ . These are  $n$ , the sample size;  $\sum_{i=1}^n X_i$ , the sum of the  $X$  values;  $\sum_{i=1}^n Y_i$ , the sum of the  $Y$  values;  $\sum_{i=1}^n X_i^2$ , the sum of the squared  $X$  values; and  $\sum_{i=1}^n X_i Y_i$ , the sum of the product of  $X$  and  $Y$ . For the Sunflowers Apparel data, the number of square feet is used to predict the annual sales in a store. Table 10.2 presents the computations of the various sums needed for the site selection problem, plus  $\sum_{i=1}^n Y_i^2$ , the sum of the squared  $Y$  values that will be used to compute  $SST$  in Section 10.3.

**TABLE 10.2**

Computations for the Sunflowers Apparel Data

Store	Square Feet ( $X$ )	Annual Sales ( $Y$ )	$X^2$	$Y^2$	$XY$
1	1.7	3.7	2.89	13.69	6.29
2	1.6	3.9	2.56	15.21	6.24
3	2.8	6.7	7.84	44.89	18.76
4	5.6	9.5	31.36	90.25	53.20
5	1.3	3.4	1.69	11.56	4.42
6	2.2	5.6	4.84	31.36	12.32
7	1.3	3.7	1.69	13.69	4.81
8	1.1	2.7	1.21	7.29	2.97
9	3.2	5.5	10.24	30.25	17.60
10	1.5	2.9	2.25	8.41	4.35
11	5.2	10.7	27.04	114.49	55.64
12	4.6	7.6	21.16	57.76	34.96
13	5.8	11.8	33.64	139.24	68.44
14	3.0	4.1	9.00	16.81	12.30
Totals	40.9	81.8	157.41	594.90	302.30

Using Equations (10.3) and (10.4), you can compute the values of  $b_0$  and  $b_1$ :

$$b_1 = \frac{SSXY}{SSX}$$

$$SSXY = \sum_{i=1}^n (X_i - \bar{X})(Y_i - \bar{Y}) = \sum_{i=1}^n X_i Y_i - \frac{\left(\sum_{i=1}^n X_i\right)\left(\sum_{i=1}^n Y_i\right)}{n}$$

$$\begin{aligned} SSXY &= 302.3 - \frac{(40.9)(81.8)}{14} \\ &= 302.3 - 238.97285 \\ &= 63.32715 \end{aligned}$$

$$\begin{aligned} SSX &= \sum_{i=1}^n (X_i - \bar{X})^2 = \sum_{i=1}^n X_i^2 - \frac{\left(\sum_{i=1}^n X_i\right)^2}{n} \\ &= 157.41 - \frac{(40.9)^2}{14} \\ &= 157.41 - 119.48642 \\ &= 37.92358 \end{aligned}$$

so that

$$\begin{aligned} b_1 &= \frac{63.32715}{37.92358} \\ &= 1.6699 \end{aligned}$$

and

$$\begin{aligned} b_0 &= \bar{Y} - b_1 \bar{X} \\ \bar{Y} &= \frac{\sum_{i=1}^n Y_i}{n} = \frac{81.8}{14} = 5.842857 \\ \bar{X} &= \frac{\sum_{i=1}^n X_i}{n} = \frac{40.9}{14} = 2.92143 \\ b_0 &= 5.842857 - (1.6699)(2.92143) \\ &= 0.9645 \end{aligned}$$

## PROBLEMS FOR SECTION 10.2

### Learning the Basics

**PH Grade ASSIST** **10.1** Fitting a straight line to a set of data yields the following prediction line:

$$\hat{Y}_i = 2 + 5X_i$$

- Interpret the meaning of the  $Y$  intercept,  $b_0$ .
- Interpret the meaning of the slope,  $b_1$ .
- Predict the mean value of  $Y$  for  $X=3$ .

**10.2** If the values of  $X$  in Problem 10.1 range from 2 to 25, should you use this model to predict the mean value of  $Y$  when  $X$  equals

- 3?
- 3?
- 0?
- 24?

**PH Grade ASSIST** **10.3** Fitting a straight line to a set of data yields the following prediction line:

$$\hat{Y}_i = 16 - 0.5X_i$$

- Interpret the meaning of the  $Y$  intercept,  $b_0$ .
- Interpret the meaning of the slope,  $b_1$ .
- Predict the mean value of  $Y$  for  $X=6$ .

### Applying the Concepts

**PH Grade ASSIST** **10.4** The marketing manager of a large super-market chain would like to use shelf space to predict the sales of pet food. A random sample of 12 equal-sized stores is selected, with the following results (stored in the file [petfood.xls](#)):

Store	Shelf Space ( $X$ ) (Feet)	Weekly Sales ( $Y$ ) (\$)
1	5	160
2	5	220
3	5	140
4	10	190
5	10	240
6	10	260
7	15	230
8	15	270
9	15	280
10	20	260
11	20	290
12	20	310

- Construct a scatter plot.

For these data,  $b_0 = 145$  and  $b_1 = 7.4$ .

- Interpret the meaning of the slope,  $b_1$ , in this problem.
- Predict the mean weekly sales (in hundreds of dollars) of pet food for stores with 8 feet of shelf space for pet food.

**10.5** Circulation is the lifeblood of the publishing business. The larger the sales of a magazine, the more it can charge advertisers. Recently, a circulation gap has appeared between the publishers' reports of magazines' newsstand sales and subsequent audits by the Audit Bureau of Circulations. The data in the file [circulation.xls](#) represent the reported and audited newsstand sales (in thousands) in 2001 for the following 10 magazines:

Magazine	Reported ( $X$ )	Audited ( $Y$ )
<i>YM</i>	621.0	299.6
<i>CosmoGirl</i>	359.7	207.7
<i>Rosie</i>	530.0	325.0
<i>Playboy</i>	492.1	336.3
<i>Esquire</i>	70.5	48.6
<i>TeenPeople</i>	567.0	400.3
<i>More</i>	125.5	91.2
<i>Spin</i>	50.6	39.1
<i>Vogue</i>	353.3	268.6
<i>Elle</i>	263.6	214.3

Source: Extracted from M. Rose, "In Fight for Ads, Publishers Often Overstate Their Sales," The Wall Street Journal, August 6, 2003, pp. A1, A10.

- Construct a scatter plot.

For these data  $b_0 = 26.724$  and  $b_1 = 0.5719$ .

- Interpret the meaning of the slope,  $b_1$ , in this problem.
- Predict the mean audited newsstand sales for a magazine that reports newsstand sales of 400,000.

**10.6** The owner of a moving company typically has his most experienced manager predict the total number of labor hours that will be required to complete an upcoming move. This approach has proved useful in the past, but he would like to be able to develop a more accurate method of predicting labor hours by using the number of cubic feet moved. In a preliminary effort to provide a more accurate method, he has collected data for 36 moves in which the origin and destination were within the borough of Manhattan in New York City and in which the travel time was an insignificant portion of the hours worked. The data are stored in the file [moving.xls](#).

- Construct a scatter plot.
- Assuming a linear relationship, use the least-squares method to find the regression coefficients  $b_0$  and  $b_1$ .
- Interpret the meaning of the slope,  $b_1$ , in this problem.
- Predict the mean labor hours for moving 500 cubic feet.

**PH Grade ASSIST** **10.7** A large mail-order house believes that there is a linear relationship between the weight of the mail it receives and the number of orders to be filled. It would like to investigate the relationship in order to predict the number of orders, based on the weight of the mail. From an operational perspective, knowledge of the number of orders will help in the planning of the order-fulfillment process. A sample of 25 mail shipments is selected that range from 200 to 700 pounds. The results (stored in the file [mail.xls](#)) are as follows:

Weight of Mail (Pounds)	Orders (Thousands)	Weight of Mail (Pounds)	Orders (Thousands)
216	6.1	432	13.6
283	9.1	409	12.8
237	7.2	553	16.5
203	7.5	572	17.1
259	6.9	506	15.0
374	11.5	528	16.2
342	10.3	501	15.8
301	9.5	628	19.0
365	9.2	677	19.4
384	10.6	602	19.1
404	12.5	630	18.0
426	12.9	652	20.2
482	14.5		

- Construct a scatter plot.
- Assuming a linear relationship, use the least-squares method to find the regression coefficients  $b_0$  and  $b_1$ .
- Interpret the meaning of the slope,  $b_1$ , in this problem.
- Predict the mean number of orders when the weight of the mail is 500 pounds.

**10.8** The value of a sports franchise is directly related to the amount of revenue that a franchise can generate. The data in the file [bbrevenue.xls](#) represent the value in 2005 (in millions of dollars) and the annual revenue (in millions of dollars) for 30 baseball franchises. Suppose you want to develop a simple linear regression model to predict franchise value based on annual revenue generated.

- Construct a scatter plot.
- Use the least-squares method to find the regression coefficients  $b_0$  and  $b_1$ .
- Interpret the meaning of  $b_0$  and  $b_1$  in this problem.
- Predict the mean value of a baseball franchise that generates \$150 million of annual revenue.

**10.9** An agent for a residential real estate company in a large city would like to be able to predict the monthly rental cost for apartments, based on the size of the apartment, as defined by square footage. A sample of 25 apartments (stored in the file [rent.xls](#)) in a particular residential neighborhood was selected, and the information gathered revealed the following:

Apartment	Monthly Rent (\$)	Size (Square Feet)	Apartment	Monthly Rent (\$)	Size (Square Feet)
1	950	850	14	1,800	1,369
2	1,600	1,450	15	1,400	1,175
3	1,200	1,085	16	1,450	1,225
4	1,500	1,232	17	1,100	1,245
5	950	718	18	1,700	1,259
6	1,700	1,485	19	1,200	1,150
7	1,650	1,136	20	1,150	896
8	935	726	21	1,600	1,361
9	875	700	22	1,650	1,040
10	1,150	956	23	1,200	755
11	1,400	1,100	24	800	1,000
12	1,650	1,285	25	1,750	1,200
13	2,300	1,985			

- Construct a scatter plot.
- Use the least-squares method to find the regression coefficients  $b_0$  and  $b_1$ .
- Interpret the meaning of  $b_0$  and  $b_1$  in this problem.
- Predict the mean monthly rent for an apartment that has 1,000 square feet.
- Why would it not be appropriate to use the model to predict the monthly rent for apartments that have 500 square feet?
- Your friends Jim and Jennifer are considering signing a lease for an apartment in this residential neighborhood. They are trying to decide between two apartments, one with 1,000 square feet for a monthly rent of \$1,275 and the other with 1,200 square feet for a monthly rent of \$1,425. What would you recommend to them based on (a) through (d)?

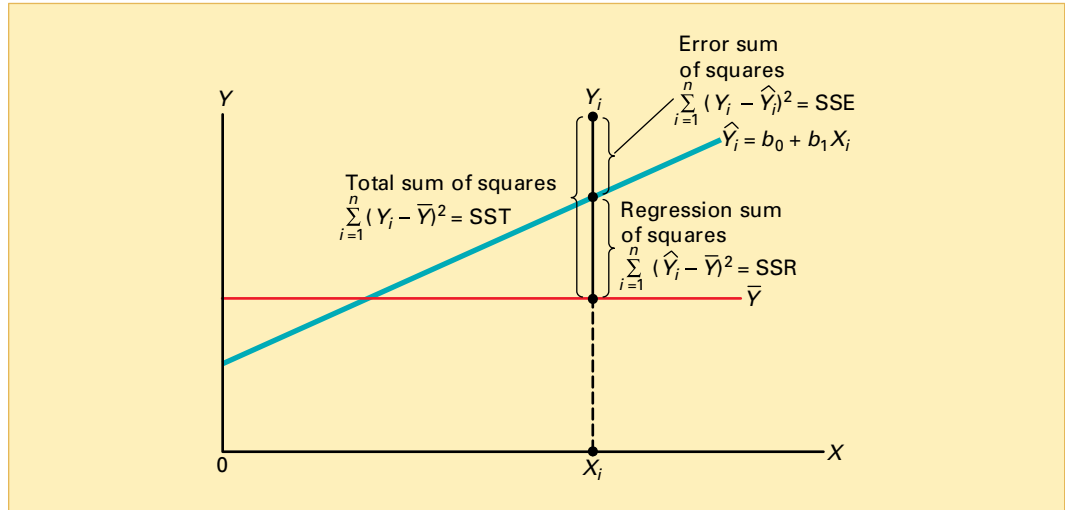
**10.10** The data in the file [hardness.xls](#) provide measurements on the hardness and tensile strength for 35 specimens of die-cast aluminum. It is believed that hardness (measured in Rockwell E units) can be used to predict tensile strength (measured in thousands of pounds per square inch).

- Construct a scatter plot.
- Assuming a linear relationship, use the least-squares method to find the regression coefficients  $b_0$  and  $b_1$ .
- Interpret the meaning of the slope,  $b_1$ , in this problem.
- Predict the mean tensile strength for die-cast aluminum that has a hardness of 30 Rockwell E units.

### 10.3 MEASURES OF VARIATION

When using the least-squares method to determine the regression coefficients for a set of data, you need to compute three important measures of variation. The first measure, the **total sum of squares (SST)**, is a measure of variation of the  $Y_i$  values around their mean,  $\bar{Y}$ . In a regression analysis, the **total variation** or total sum of squares is subdivided into **explained variation** and **unexplained variation**. The explained variation or **regression sum of squares (SSR)** is due to the relationship between  $X$  and  $Y$ , and the **unexplained variation**, or **error sum of squares (SSE)** is due to factors other than the relationship between  $X$  and  $Y$ . Figure 10.6 shows these different measures of variation.

**FIGURE 10.6**  
Measures of variation



#### Computing the Sum of Squares

The regression sum of squares ( $SSR$ ) is based on the difference between  $\hat{Y}_i$  (the predicted value of  $Y$  from the prediction line) and  $\bar{Y}$  (the mean value of  $Y$ ). The error sum of squares ( $SSE$ ) represents the part of the variation in  $Y$  that is not explained by the regression. It is based on the difference between  $Y_i$  and  $\hat{Y}_i$ . Equations (10.5), (10.6), (10.7), and (10.8) define these measures of variation.

#### MEASURES OF VARIATION IN REGRESSION

The total sum of squares is equal to the regression sum of squares plus the error sum of squares.

$$SST = SSR + SSE \quad (10.5)$$

#### TOTAL SUM OF SQUARES (SST)

The total sum of squares ( $SST$ ) is equal to the sum of the squared differences between each observed  $Y$  value and  $\bar{Y}$ , the mean value of  $Y$ .

$$SST = \text{Total sum of squares} \quad (10.6)$$

$$= \sum_{i=1}^n (Y_i - \bar{Y})^2$$

**REGRESSION SUM OF SQUARES (SSR)**

The regression sum of squares (*SSR*) is equal to the sum of the squared differences between the predicted value of *Y* and  $\bar{Y}$ , the mean value of *Y*.

$$SSR = \text{Explained variation or regression of squares} \quad (10.7)$$

$$= \sum_{i=1}^n (\hat{Y}_i - \bar{Y})^2$$

**ERROR SUM OF SQUARES (SSE)**

The error sum of squares (*SSE*) is equal to the sum of the squared differences between the observed value of *Y* and the predicted value of *Y*.

$$SSE = \text{Unexplained variation or error sum of squares} \quad (10.8)$$

$$= \sum_{i=1}^n (Y_i - \hat{Y}_i)^2$$

Figure 10.7 shows the sum of squares area of the worksheet containing the Microsoft Excel results for the Sunflowers Apparel data. The total variation, *SST*, is equal to 116.9543. This amount is subdivided into the sum of squares explained by the regression (*SSR*), equal to 105.7476, and the sum of squares unexplained by the regression (*SSE*), equal to 11.2067. From Equation (10.5) on page 382:

$$SST = SSR + SSE$$

$$116.9543 = 105.7476 + 11.2067$$

**FIGURE 10.7**  
Microsoft Excel sum of squares for the Sunflowers Apparel data



See Section E10.1 to create the worksheet that contains this area.

	A	B	C	D	E	F	G
10	<b>ANOVA</b>						
11		<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>Significance F</i>	
12	<b>Regression</b>	1	105.7476	105.7476	113.2335	0.0000	
13	<b>Residual</b>	12	11.2067	0.9339			
14	<b>Total</b>	13	116.9543				
15							
16		<i>Coefficients</i>	<i>Standard Error</i>	<i>t Stat</i>	<i>P-value</i>	<i>Lower 95%</i>	<i>Upper 95%</i>
17	<b>Intercept</b>	0.9645	0.5262	1.8329	0.0917	-0.1820	2.1110
18	<b>Square Feet</b>	1.6699	0.1569	10.6411	0.0000	1.3280	2.0118

In a data set that has a large number of significant digits, the results of a regression analysis are sometimes displayed using a numerical format known as *scientific notation*. This type of format is used to display very small or very large values. The number after the letter E represents the number of digits that the decimal point needs to be moved to the left (for a negative number) or to the right (for a positive number). For example, the number 3.7431 E+02 means that the decimal point should be moved two places to the right, producing the number 374.31. The number 3.7431 E-02 means that the decimal point should be moved two places to the left, producing the number 0.037431. When scientific notation is used, fewer significant digits are usually displayed, and the numbers may appear to be rounded.

## The Coefficient of Determination

By themselves,  $SSR$ ,  $SSE$ , and  $SST$  provide little information. However, the ratio of the regression sum of squares ( $SSR$ ) to the total sum of squares ( $SST$ ) measures the proportion of variation in  $Y$  that is explained by the independent variable  $X$  in the regression model. This ratio is called the **coefficient of determination**,  $r^2$ , and is defined in Equation (10.9).

### COEFFICIENT OF DETERMINATION

The coefficient of determination is equal to the regression sum of squares (that is, explained variation) divided by the total sum of squares (that is, total variation).

$$r^2 = \frac{\text{Regression sum of squares}}{\text{Total sum of squares}} = \frac{SSR}{SST} \quad (10.9)$$

The coefficient of determination measures the proportion of variation in  $Y$  that is explained by the independent variable  $X$  in the regression model. For the Sunflowers Apparel data, with  $SSR = 105.7476$ ,  $SSE = 11.2067$ , and  $SST = 116.9543$ ,

$$r^2 = \frac{105.7476}{116.9543} = 0.9042$$

Therefore, 90.42% of the variation in annual sales is explained by the variability in the size of the store, as measured by the square footage. This large  $r^2$  indicates a strong positive linear relationship between two variables because the use of a regression model has reduced the variability in predicting annual sales by 90.42%. Only 9.58% of the sample variability in annual sales is due to factors other than what is accounted for by the linear regression model that uses square footage.

Figure 10.8 presents the coefficient of determination portion of the Microsoft Excel results for the Sunflowers Apparel data.

**FIGURE 10.8**

Partial Microsoft Excel regression results for the Sunflowers Apparel data



See Section E10.1 to create the worksheet that contains this area.

	A	B
3	<b>Regression Statistics</b>	
4	<b>Multiple R</b>	<b>0.9509</b>
5	<b>R Square</b>	<b>0.9042</b>
6	<b>Adjusted R Square</b>	<b>0.8962</b>
7	<b>Standard Error</b>	$S_{YX}$ <b>0.9664</b>
8	<b>Observations</b>	<b>14</b>

### EXAMPLE 10.4

#### COMPUTING THE COEFFICIENT OF DETERMINATION

Compute the coefficient of determination,  $r^2$ , for the Sunflowers Apparel data.

**SOLUTION** You can compute  $SST$ ,  $SSR$ , and  $SSE$ , that are defined in Equations (10.6), (10.7), and (10.8) on pages 382–383, by using Equations (10.10), (10.11), and (10.12).

#### COMPUTATIONAL FORMULA FOR $SST$

$$SST = \sum_{i=1}^n (Y_i - \bar{Y})^2 = \sum_{i=1}^n Y_i^2 - \frac{\left( \sum_{i=1}^n Y_i \right)^2}{n} \quad (10.10)$$



**COMPUTATIONAL FORMULA FOR SSR**

$$SSR = \sum_{i=1}^n (\hat{Y}_i - \bar{Y})^2 = b_0 \sum_{i=1}^n Y_i + b_1 \sum_{i=1}^n X_i Y_i - \frac{\left( \sum_{i=1}^n Y_i \right)^2}{n} \quad (10.11)$$

**COMPUTATIONAL FORMULA FOR SSE**

$$SSE = \sum_{i=1}^n (Y_i - \hat{Y}_i)^2 = \sum_{i=1}^n Y_i^2 - b_0 \sum_{i=1}^n Y_i - b_1 \sum_{i=1}^n X_i Y_i \quad (10.12)$$

Using the summary results from Table 10.2 on page 378,

$$\begin{aligned} SST &= \sum_{i=1}^n (\hat{Y}_i - \bar{Y})^2 = \sum_{i=1}^n Y_i^2 - \frac{\left( \sum_{i=1}^n Y_i \right)^2}{n} \\ &= 594.9 - \frac{(81.8)^2}{14} \\ &= 594.9 - 477.94571 \\ &= 116.95429 \\ SSR &= \sum_{i=1}^n (\hat{Y}_i - \bar{Y})^2 \\ &= b_0 \sum_{i=1}^n Y_i + b_1 \sum_{i=1}^n X_i Y_i - \frac{\left( \sum_{i=1}^n Y_i \right)^2}{n} \\ &= (0.964478)(81.8) + (1.66986)(302.3) - \frac{(81.8)^2}{14} \\ &= 105.74726 \\ SSE &= \sum_{i=1}^n (Y_i - \hat{Y}_i)^2 \\ &= \sum_{i=1}^n Y_i^2 - b_0 \sum_{i=1}^n Y_i - b_1 \sum_{i=1}^n X_i Y_i \\ &= 594.9 - (0.964478)(81.8) - (1.66986)(302.3) \\ &= 11.2067 \end{aligned}$$

Therefore,

$$r^2 = \frac{105.74726}{116.95429} = 0.9042$$

## Standard Error of the Estimate

Although the least-squares method results in the line that fits the data with the minimum amount of error, unless all the observed data points fall on a straight line, the prediction line is not a perfect predictor. Just as all data values cannot be expected to be exactly equal to their mean, neither can they be expected to fall exactly on the prediction line. An important statistic, called the **standard error of the estimate**, measures the variability of the actual  $Y$  values from the predicted  $Y$  values in the same way that the standard deviation in Chapter 3 measures the variability of each value around the sample mean. In other words, the standard error of the estimate is the standard deviation *around* the prediction line, whereas the standard deviation in Chapter 3 is the standard deviation *around* the sample mean.

Figure 10.5 on page 375 illustrates the variability around the prediction line for the Sunflowers Apparel data. Observe that although many of the actual values of  $Y$  fall near the prediction line, none of the values are exactly on the line.

The standard error of the estimate, represented by the symbol  $S_{YX}$  is defined in Equation (10.13).

### STANDARD ERROR OF THE ESTIMATE

$$S_{YX} = \sqrt{\frac{SSE}{n-2}} = \sqrt{\frac{\sum_{i=1}^n (Y_i - \hat{Y}_i)^2}{n-2}} \quad (10.13)$$

where

$Y_i$  = actual value of  $Y$  for a given  $X_i$

$\hat{Y}_i$  = predicted value of  $Y$  for a given  $X_i$

$SSE$  = error sum of squares

From Equation (10.8) and Figure 10.4 on page 374,  $SSE = 11.2067$ . Thus,

$$S_{YX} = \sqrt{\frac{11.2067}{14-2}} = 0.9664$$

This standard error of the estimate, equal to 0.9664 millions of dollars (that is, \$966,400), is labeled Standard Error in the Microsoft Excel results shown in Figure 10.8 on page 384. The standard error of the estimate represents a measure of the variation around the prediction line. It is measured in the same units as the dependent variable  $Y$ . The interpretation of the standard error of the estimate is similar to that of the standard deviation. Just as the standard deviation measures variability around the mean, the standard error of the estimate measures variability around the prediction line. For Sunflowers Apparel, the typical difference between actual annual sales at a store and the predicted annual sales using the regression equation is approximately \$966,400.

## PROBLEMS FOR SECTION 10.3

### Learning the Basics

**PH Grade ASSIST** **10.11** How do you interpret a coefficient of determination,  $r^2$ , equal to 0.80?

**PH Grade ASSIST** **10.12** If  $SSR = 36$  and  $SSE = 4$ , determine  $SST$  and then compute the coefficient of determination,  $r^2$ , and interpret its meaning.

**PH Grade ASSIST** **10.13** If  $SSR = 66$  and  $SST = 88$ , compute the coefficient of determination,  $r^2$ , and interpret its meaning.

**PH Grade ASSIST** **10.14** If  $SSE = 10$  and  $SSR = 30$ , compute the coefficient of determination,  $r^2$ , and interpret its meaning.

**10.15** If  $SSR = 120$ , why is it impossible for  $SST$  to equal 110?

### Applying the Concepts

**PH Grade ASSIST** **10.16** In Problem 10.4 on page 380, the marketing manager used shelf space for pet food to predict weekly sales (stored in the file **petfood.xls**). For that data,  $SSR = 20,535$  and  $SST = 30,025$ .

- Determine the coefficient of determination,  $r^2$ , and interpret its meaning.
- Determine the standard error of the estimate.
- How useful do you think this regression model is for predicting sales?

**10.17** In Problem 10.5 on page 380, you used reported magazine newsstand sales to predict audited sales (stored in the file **circulation.xls**). For that data,  $SSR = 130,301.41$  and  $SST = 144,538.64$ .

- Determine the coefficient of determination,  $r^2$ , and interpret its meaning.
- Determine the standard error of the estimate.
- How useful do you think this regression model is for predicting audited sales?

**10.18** In Problem 10.6 on page 380, an owner of a moving company wanted to predict labor hours, based on the cubic feet moved (stored in the file **moving.xls**). Using the results of that problem,

- determine the coefficient of determination,  $r^2$ , and interpret its meaning.
- determine the standard error of the estimate.
- How useful do you think this regression model is for predicting labor hours?

**PH Grade ASSIST** **10.19** In Problem 10.7 on page 381, you used the weight of mail to predict the number of orders

received (stored in the file **mail.xls**). Using the results of that problem,

- determine the coefficient of determination,  $r^2$ , and interpret its meaning.
- find the standard error of the estimate.
- How useful do you think this regression model is for predicting the number of orders?

**10.20** In Problem 10.8 on page 381, you used annual revenues to predict the value of a baseball franchise (stored in the file **bbrevenue.xls**). Using the results of that problem,

- determine the coefficient of determination,  $r^2$ , and interpret its meaning.
- determine the standard error of the estimate.
- How useful do you think this regression model is for predicting the value of a baseball franchise?

**10.21** In Problem 10.9 on page 381, an agent for a real estate company wanted to predict the monthly rent for apartments, based on the size of the apartment (stored in the file **rent.xls**). Using the results of that problem,

- determine the coefficient of determination,  $r^2$ , and interpret its meaning.
- determine the standard error of the estimate.
- How useful do you think this regression model is for predicting the monthly rent?

**10.22** In Problem 10.10 on page 381, you used hardness to predict the tensile strength of die-cast aluminum (stored in the file **hardness.xls**). Using the results of that problem,

- determine the coefficient of determination,  $r^2$ , and interpret its meaning.
- find the standard error of the estimate.
- How useful do you think this regression model is for predicting the tensile strength of die-cast aluminum?

## 10.4 ASSUMPTIONS

The discussion of hypothesis testing and the analysis of variance emphasized the importance of the assumptions to the validity of any conclusions reached. The assumptions necessary for regression are similar to those of the analysis of variance because both topics fall in the general category of *linear models* (reference 4).

The four **assumptions of regression** (known by the acronym LINE) are as follows:

- **L**inearity
- **I**ndependence of errors
- **N**ormality of error
- **E**qual variance

The first assumption, linearity, states that the relationship between variables is linear.

The second assumption, **independence of errors**, requires that the errors ( $\epsilon_i$ ) are independent of one another. This assumption is particularly important when data are collected over a period of time. In such situations, the errors for a specific time period are sometimes correlated with those of the previous time period.

The third assumption, **normality**, requires that the errors ( $\varepsilon_i$ ) are normally distributed at each value of  $X$ . Like the  $t$  test and the ANOVA  $F$  test, regression analysis is fairly robust against departures from the normality assumption. As long as the distribution of the errors at each level of  $X$  is not extremely different from a normal distribution, inferences about  $\beta_0$  and  $\beta_1$  are not seriously affected.

The fourth assumption, **equal variance** or **homoscedasticity**, requires that the variance of the errors ( $\varepsilon_i$ ) are constant for all values of  $X$ . In other words, the variability of  $Y$  values is the same when  $X$  is a low value as when  $X$  is a high value. The equal variance assumption is important when making inferences about  $\beta_0$  and  $\beta_1$ . If there are serious departures from this assumption, you can use either data transformations or weighted least-squares methods (see reference 4).

## 10.5 RESIDUAL ANALYSIS

In Section 10.1, regression analysis was introduced. In Sections 10.2 and 10.3, a regression model was developed using the least-squares approach for the Sunflowers Apparel data. Is this the correct model for these data? Are the assumptions introduced in Section 10.4 valid? In this section, a graphical approach called **residual analysis** is used to evaluate the assumptions and determine whether the regression model selected is an appropriate model.

The **residual** or estimated error value,  $e_i$ , is the difference between the observed ( $Y_i$ ) and predicted ( $\hat{Y}_i$ ) values of the dependent variable for a given value of  $X_i$ . Graphically, a residual appears on a scatter plot as the vertical distance between an observed value of  $Y$  and the prediction line. Equation (10.14) defines the residual.

### RESIDUAL

The residual is equal to the difference between the observed value of  $Y$  and the predicted value of  $Y$ .

$$e_i = Y_i - \hat{Y}_i \quad (10.14)$$

### Evaluating the Assumptions

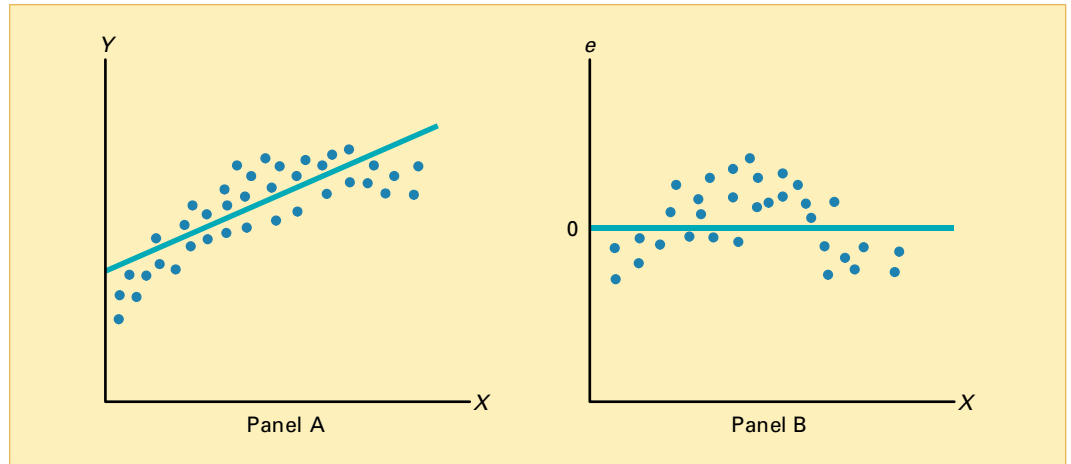
Recall from Section 10.4 that the four assumptions of regression (known by the acronym LINE) are linearity, independence, normality, and equal variance.

**Linearity** To evaluate linearity, you plot the residuals on the vertical axis against the corresponding  $X_i$  values of the independent variable on the horizontal axis. If the linear model is appropriate for the data, there is no apparent pattern in this plot. However, if the linear model is not appropriate, there is a relationship between the  $X_i$  values and the residuals,  $e_i$ . You can see such a pattern in Figure 10.9. Panel A shows a situation in which, although there is an increasing trend in  $Y$  as  $X$  increases, the relationship seems curvilinear because the upward trend decreases for increasing values of  $X$ . This quadratic effect is highlighted in Panel B, where there is a clear relationship between  $X_i$  and  $e_i$ . By plotting the residuals, the linear trend of  $X$  with  $Y$  has been removed, thereby exposing the lack of fit in the simple linear model. Thus, a quadratic model is a better fit and should be used in place of the simple linear model.

To determine whether the simple linear regression model is appropriate, return to the evaluation of the Sunflowers Apparel data. Figure 10.10 provides the predicted and residual values of the response variable (annual sales) computed by Microsoft Excel.

**FIGURE 10.9**

Studying the appropriateness of the simple linear regression model



**FIGURE 10.10**

Microsoft Excel residual statistics for the Sunflowers Apparel data



See Section E10.3 to create the worksheet that contains this area.

	A	B	C
22	<b>RESIDUAL OUTPUT</b>		
23			
24	<i>Observation</i>	<i>Predicted Annual Sales</i>	<i>Residuals</i>
25	1	3.803239598	-0.103239598
26	2	3.636253367	0.263746633
27	3	5.640088147	1.059911853
28	4	10.31570263	-0.815702635
29	5	3.135294672	0.264705328
30	6	4.638170757	0.961829243
31	7	3.135294672	0.564705328
32	8	2.801322208	-0.101322208
33	9	6.308033074	-0.808033074
34	10	3.469267135	-0.569267135
35	11	9.647757708	1.052242292
36	12	8.645840318	-1.045840318
37	13	10.6496751	1.150324902
38	14	5.974060611	-1.874060611

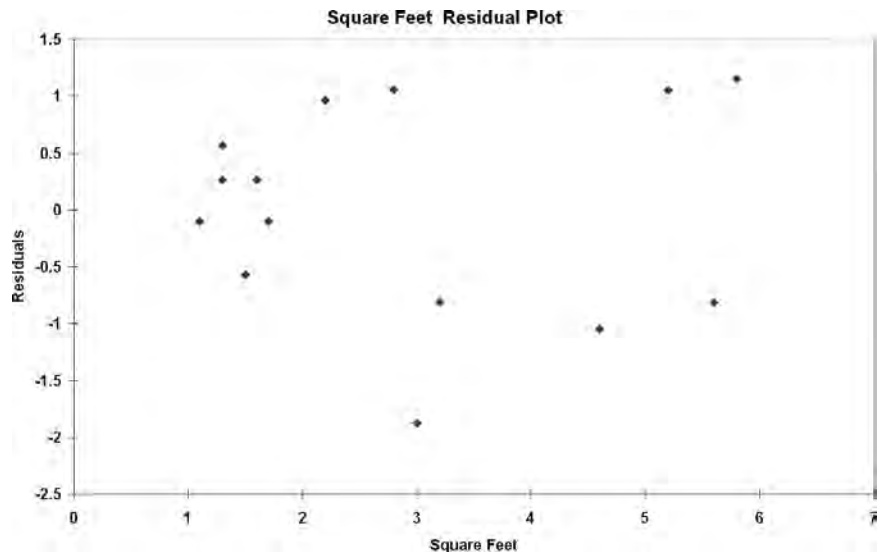
To assess linearity, the residuals are plotted against the independent variable (store size, in thousands of square feet) in Figure 10.11. Although there is widespread scatter in the residual plot, there is no apparent pattern or relationship between the residuals and  $X_i$ . The residuals appear to be evenly spread above and below 0 for the differing values of  $X$ . You can conclude that the linear model is appropriate for the Sunflowers Apparel data.

**FIGURE 10.11**

Microsoft Excel plot of residuals against the square footage of a store for the Sunflowers Apparel data



See Section E2.12 to create this.



**Independence** You can evaluate the assumption of independence of the errors by plotting the residuals in the order or sequence in which the data were collected. Data collected over periods of time sometimes exhibit an autocorrelation effect among successive observations. In these instances, there is a relationship between consecutive residuals. If this relationship exists (which violates the assumption of independence), it is apparent in the plot of the residuals versus the time in which the data were collected. You can also test for autocorrelation by using the Durbin-Watson statistic, which is the subject of Section 10.6. Because the Sunflowers Apparel data were collected during the same time period, you do not need to evaluate the independence assumption.

**Normality** You can evaluate the assumption of normality in the errors by tallying the residuals into a frequency distribution and displaying the results in a histogram (see Section 2.3). For the Sunflowers Apparel data, the residuals have been tallied into a frequency distribution in Table 10.3. (There are an insufficient number of values, however, to construct a histogram.) You can also evaluate the normality assumption by comparing the actual versus theoretical values of the residuals or by constructing a normal probability plot of the residuals (see Section 6.3). Figure 10.12 is a normal probability plot of the residuals for the Sunflower Apparel data.

**TABLE 10.3**

Frequency Distribution of 14 Residual Values for the Sunflowers Apparel Data

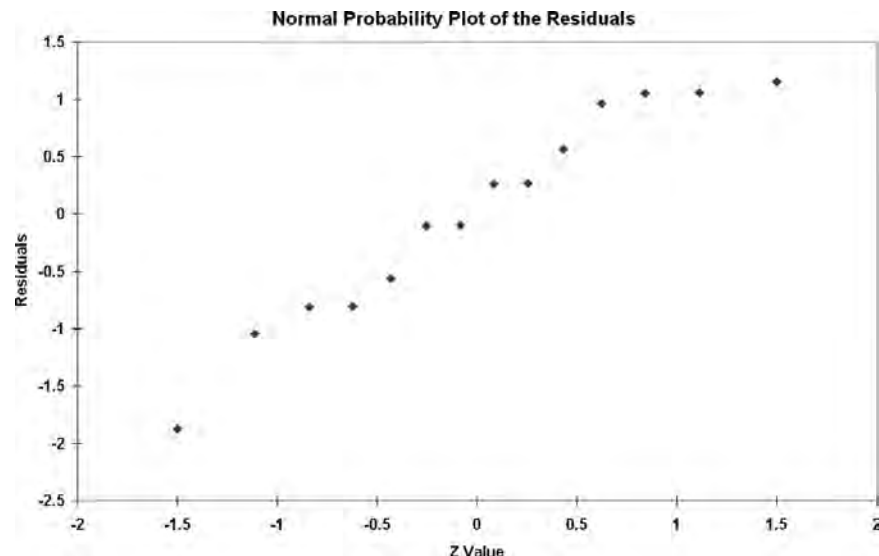
Residuals	Frequency
-2.25 but less than -1.75	1
-1.75 but less than -1.25	0
-1.25 but less than -0.75	3
-0.75 but less than -0.25	1
-0.25 but less than +0.25	2
+0.25 but less than +0.75	3
+0.75 but less than +1.25	4
	<u>14</u>

**FIGURE 10.12**

Microsoft Excel normal probability plot of the residuals for the Sunflowers Apparel data



See Section E6.2 to create this.

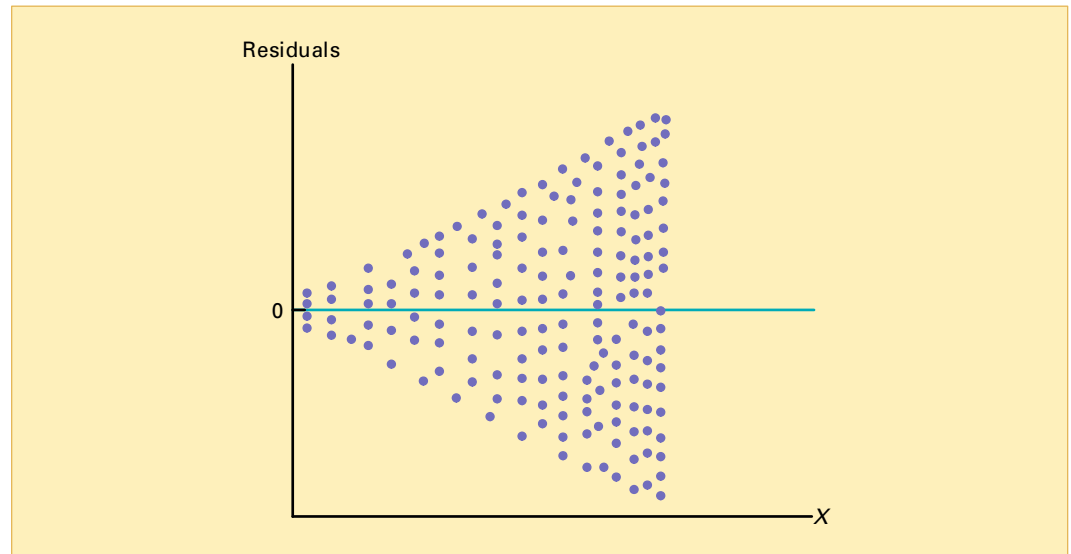


It is difficult to evaluate the normality assumption for a sample of only 14 values, regardless of whether you use a histogram, stem-and-leaf display, box-and-whisker plot, or normal probability plot. You can see from Figure 10.12 that the data do not appear to depart substantially from a normal distribution. The robustness of regression analysis with modest departures from normality enables you to conclude that you should not be overly concerned about departures from this normality assumption in the Sunflowers Apparel data.

**Equal Variance** You can evaluate the assumption of equal variance from a plot of the residuals with  $X_i$ . For the Sunflowers Apparel data of Figure 10.11 on page 389, there do not appear to be major differences in the variability of the residuals for different  $X_i$  values. Thus, you can conclude that there is no apparent violation in the assumption of equal variance at each level of  $X$ .

To examine a case in which the equal variance assumption is violated, observe Figure 10.13, which is a plot of the residuals with  $X_i$  for a hypothetical set of data. In this plot, the variability of the residuals increases dramatically as  $X$  increases, demonstrating the lack of homogeneity in the variances of  $Y_i$  at each level of  $X$ . For these data, the equal variance assumption is invalid.

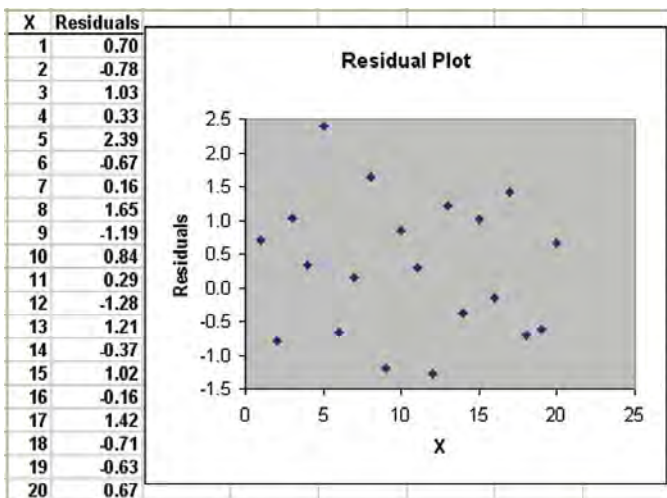
**FIGURE 10.13**  
Violation of equal variance



## PROBLEMS FOR SECTION 10.5

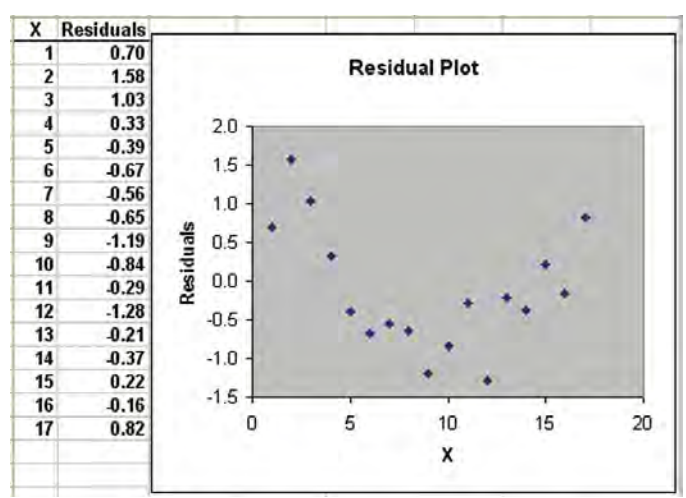
### Learning the Basics

**10.23** The results below provide the  $X$  values, residuals, and a residual plot from a regression analysis:



Is there any evidence of a pattern in the residuals? Explain.

**10.24** The results below show the  $X$  values, residuals, and a residual plot from a regression analysis:



Is there any evidence of a pattern in the residuals? Explain.

## Applying the Concepts

**10.25** In Problem 10.5 on page 380, you used reported magazine newsstand sales to predict audited sales. The data are stored in the file [circulation.xls](#). Perform a residual analysis for these data.

- Determine the adequacy of the fit of the model.
- Evaluate whether the assumptions of regression have been seriously violated.



**10.26** In Problem 10.4 on page 380, the marketing manager used shelf space for pet food to predict weekly sales. The data are stored in the file [petfood.xls](#). Perform a residual analysis for these data.

- Determine the adequacy of the fit of the model.
- Evaluate whether the assumptions of regression have been seriously violated.

**10.27** In Problem 10.7 on page 381, you used the weight of mail to predict the number of orders received. Perform a residual analysis for these data. The data are stored in the file [mail.xls](#). Based on these results,

- determine the adequacy of the fit of the model.
- evaluate whether the assumptions of regression have been seriously violated.

**10.28** In Problem 10.6 on page 380, the owner of a moving company wanted to predict labor hours based on the cubic feet moved. Perform a residual analysis for these data. The data are stored in the file [moving.xls](#). Based on these results,

- determine the adequacy of the fit of the model.
- evaluate whether the assumptions of regression have been seriously violated.

**10.29** In Problem 10.9 on page 381, an agent for a real estate company wanted to predict the monthly rent for apartments, based on the size of the apartments. Perform a residual analysis for these data. The data are stored in the file [rent.xls](#). Based on these results,

- determine the adequacy of the fit of the model.
- evaluate whether the assumptions of regression have been seriously violated.

**10.30** In Problem 10.8 on page 381, you used annual revenues to predict the value of a baseball franchise. The data are stored in the file [bbrevenue.xls](#). Perform a residual analysis for these data. Based on these results,

- determine the adequacy of the fit of the model.
- evaluate whether the assumptions of regression have been seriously violated.

**10.31** In Problem 10.10 on page 381, you used hardness to predict the tensile strength of die-cast aluminum. The data are stored in the file [hardness.xls](#). Perform a residual analysis for these data. Based on these results,

- determine the adequacy of the fit of the model.
- evaluate whether the assumptions of regression have been seriously violated.

## 10.6 MEASURING AUTOCORRELATION: THE DURBIN-WATSON STATISTIC

One of the basic assumptions of the regression model is the independence of the errors. This assumption is sometimes violated when data are collected over sequential time periods because a residual at any one time period may tend to be similar to residuals at adjacent time periods. This pattern in the residuals is called **autocorrelation**. When a set of data has substantial autocorrelation, the validity of a regression model can be in serious doubt.

### Residual Plots to Detect Autocorrelation

As mentioned in Section 10.5, one way to detect autocorrelation is to plot the residuals in time order. If a positive autocorrelation effect is present, there will be clusters of residuals with the same sign, and you will readily detect an apparent pattern. If negative autocorrelation exists, residuals will tend to jump back and forth from positive to negative to positive, and so on. This type of pattern is very rarely seen in regression analysis. Thus, the focus of this section is on positive autocorrelation. To illustrate positive autocorrelation, consider the following example.

The manager of a package delivery store wants to predict weekly sales, based on the number of customers making purchases for a period of 15 weeks. In this situation, because data are collected over a period of 15 consecutive weeks at the same store, you need to determine whether autocorrelation is present. Table 10.4 presents the data (stored in the file [custsale.xls](#)). Figure 10.14 illustrates Microsoft Excel results for these data.



**TABLE 10.4**  
Customers and Sales for a Period of 15 Consecutive Weeks

Week	Customers	Sales (Thousands of Dollars)	Week	Customers	Sales (Thousands of Dollars)
1	794	9.33	9	880	12.07
2	799	8.26	10	905	12.55
3	837	7.48	11	886	11.92
4	855	9.08	12	843	10.27
5	845	9.83	13	904	11.80
6	844	10.09	14	950	12.15
7	863	11.01	15	841	9.64
8	875	11.49			

**FIGURE 10.14**  
Microsoft Excel results for the package delivery store data of Table 10.4



See Section E10.1 to create this.

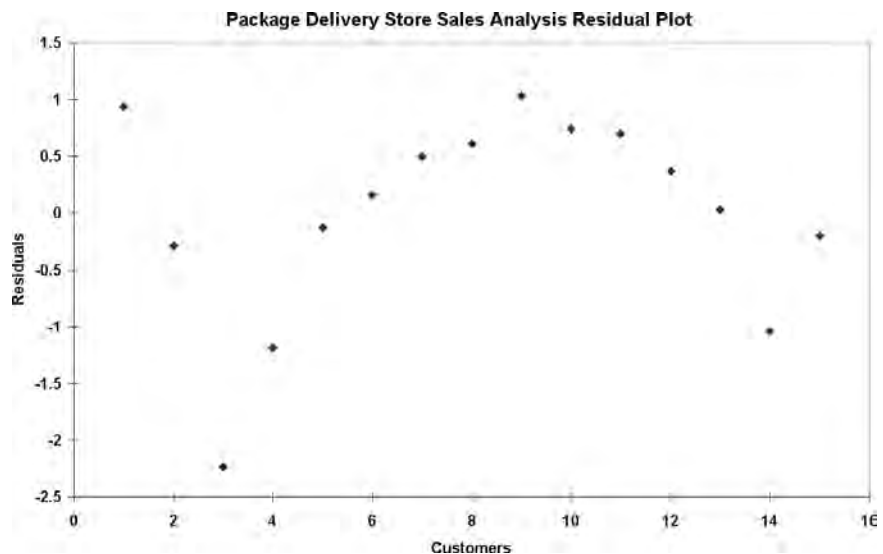
	A	B	C	D	E	F	G
1	Package Delivery Store Sales Analysis						
2							
3	Regression Statistics						
4	Multiple R	0.8108					
5	R Square	0.6574					
6	Adjusted R Square	0.6311					
7	Standard Error	0.9360					
8	Observations	15					
9							
10	ANOVA						
11		df	SS	MS	F	Significance F	
12	Regression	1	21.8604	21.8604	24.9501	0.0002	
13	Residual	13	11.3901	0.8762			
14	Total	14	33.2506				
15							
16		Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
17	Intercept	-16.0322	5.3102	-3.0192	0.0099	-27.5041	-4.5603
18	Customers	0.0308	0.0062	4.9950	0.0002	0.0175	0.0441

From Figure 10.14, observe that  $r^2$  is 0.6574, indicating that 65.74% of the variation in sales is explained by variation in the number of customers. In addition, the  $Y$  intercept,  $b_0$ , is  $-16.0322$ , and the slope,  $b_1$ , is 0.0308. However, before using this model for prediction, you must undertake proper analyses of the residuals. Because the data have been collected over a consecutive period of 15 weeks, in addition to checking the linearity, normality, and equal-variance assumptions, you must investigate the independence-of-errors assumption. You can plot the residuals versus time to help you see whether a pattern exists. In Figure 10.15, you can see that the residuals tend to fluctuate up and down in a cyclical pattern. This cyclical pattern provides strong cause for concern about the autocorrelation of the residuals and, hence, a violation of the independence-of-errors assumption.

**FIGURE 10.15**  
Microsoft Excel residual plot for the package delivery store data of Table 10.4



See Section E10.3 to create this.



### The Durbin-Watson Statistic

The **Durbin-Watson statistic** is used to measure autocorrelation. This statistic measures the correlation between each residual and the residual for the time period immediately preceding the one of interest. Equation (10.15) defines the Durbin-Watson statistic.

#### DURBIN-WATSON STATISTIC

$$D = \frac{\sum_{i=2}^n (e_i - e_{i-1})^2}{\sum_{i=1}^n e_i^2} \tag{10.15}$$

where

$e_i$  = residual at the time period  $i$

To better understand the Durbin-Watson statistic,  $D$ , you can examine Equation (10.15).

The numerator,  $\sum_{i=2}^n (e_i - e_{i-1})^2$ , represents the squared difference between two successive residuals, summed from the second value to the  $n$ th value. The denominator,  $\sum_{i=1}^n e_i^2$ , represents

the sum of the squared residuals. When successive residuals are positively autocorrelated, the value of  $D$  approaches 0. If the residuals are not correlated, the value of  $D$  will be close to 2. (If there is negative autocorrelation,  $D$  will be greater than 2 and could even approach its maximum value of 4.) For the package delivery store data, as shown in the Microsoft Excel results of Figure 10.16, the Durbin-Watson statistic,  $D$ , is 0.8830.

**FIGURE 10.16**

Microsoft Excel results of the Durbin-Watson statistic for the package delivery store data



See Section E10.4 to create this.

	A	B
1	<b>Durbin-Watson Calculations</b>	
2		
3	Sum of Squared Difference of Residuals	10.0575
4	Sum of Squared Residuals	11.3901
5		
6	<b>Durbin-Watson Statistic</b>	<b>0.8830</b> =B3/B4

You need to determine when the autocorrelation is large enough to make the Durbin-Watson statistic,  $D$ , fall sufficiently below 2 to conclude that there is significant positive autocorrelation. After computing  $D$ , you compare it to the critical values of the Durbin-Watson statistic found in Table E.10, a portion of which is presented in Table 10.5. The critical values depend on  $\alpha$ , the significance level chosen,  $n$ , the sample size, and  $k$ , the number of independent variables in the model (in simple linear regression,  $k = 1$ ).

**TABLE 10.5**

Finding Critical Values of the Durbin-Watson Statistic

		$\alpha = .05$									
		$k = 1$		$k = 2$		$k = 3$		$k = 4$		$k = 5$	
$n$		$d_L$	$D_U$	$d_L$	$d_U$	$d_L$	$d_U$	$d_L$	$d_U$	$d_L$	$d_U$
15	→	1.08	1.36	.95	1.54	.82	1.75	.69	1.97	.56	2.21
16		1.10	1.37	.98	1.54	.86	1.73	.74	1.93	.62	2.15
17		1.13	1.38	1.02	1.54	.90	1.71	.78	1.90	.67	2.10
18		1.16	1.39	1.05	1.53	.93	1.69	.82	1.87	.71	2.06

In Table 10.5, two values are shown for each combination of  $\alpha$  (level of significance),  $n$  (sample size), and  $k$  (number of independent variables in the model). The first value,  $d_L$ , represents the lower critical value. If  $D$  is below  $d_L$ , you conclude that there is evidence of positive autocorrelation among the residuals. If this occurs, the least-squares method used in this chapter is inappropriate, and you should use alternative methods (see reference 4). The second value,  $d_U$ , represents the upper critical value of  $D$ , above which you would conclude that there is no evidence of positive autocorrelation among the residuals. If  $D$  is between  $d_L$  and  $d_U$ , you are unable to arrive at a definite conclusion.

For the package delivery store data, with one independent variable ( $k = 1$ ) and 15 values ( $n = 15$ ),  $d_L = 1.08$  and  $d_U = 1.36$ . Because  $D = 0.8830 < 1.08$ , you conclude that there is positive autocorrelation among the residuals. The least-squares regression analysis of the data is inappropriate because of the presence of significant positive autocorrelation among the residuals. In other words, the independence-of-errors assumption is invalid. You need to use alternative approaches discussed in reference 4.

## PROBLEMS FOR SECTION 10.6

### Learning the Basics

**PH Grade ASSIST** **10.32** The residuals for 10 consecutive time periods are as follows:

Time Period	Residual	Time Period	Residual
1	-5	6	+1
2	-4	7	+2
3	-3	8	+3
4	-2	9	+4
5	-1	10	+5

- Plot the residuals over time. What conclusion can you reach about the pattern of the residuals over time?
- Based on (a), what conclusion can you reach about the autocorrelation of the residuals?

**PH Grade ASSIST** **10.33** The residuals for 15 consecutive time periods are as follows:

Time Period	Residual	Time Period	Residual
1	+4	9	+6
2	-6	10	-3
3	-1	11	+1
4	-5	12	+3
5	+2	13	0
6	+5	14	-4
7	-2	15	-7
8	+7		

- Plot the residuals over time. What conclusion can you reach about the pattern of the residuals over time?

- Compute the Durbin-Watson statistic. At the 0.05 level of significance, is there evidence of positive autocorrelation among the residuals?
- Based on (a) and (b), what conclusion can you reach about the autocorrelation of the residuals?

### Applying the Concepts

**PH Grade ASSIST** **10.34** In Problem 10.4 on page 380 concerning pet food sales, the marketing manager used shelf space for pet food to predict weekly sales.

- Is it necessary to compute the Durbin-Watson statistic in this case? Explain.
- Under what circumstances is it necessary to compute the Durbin-Watson statistic before proceeding with the least-squares method of regression analysis?

**10.35** The owner of a single-family home in a suburban county in the northeastern United States would like to develop a model to predict electricity consumption in his all-electric house (lights, fans, heat, appliances, and so on), based on average atmospheric temperature (in degrees Fahrenheit). Monthly kilowatt usage and temperature data are available for a period of 24 consecutive months in the file [elecuse.xls](#).

- Assuming a linear relationship, use the least-squares method to find the regression coefficients  $b_0$  and  $b_1$ .
- Predict the mean kilowatt usage when the average atmospheric temperature is  $50^\circ$  Fahrenheit.
- Plot the residuals versus the time period.
- Compute the Durbin-Watson statistic. At the 0.05 level of significance, is there evidence of positive autocorrelation among the residuals?
- Based on the results of (c) and (d), is there reason to question the validity of the model?

**10.36** A mail-order catalog business that sells personal computer supplies, software, and hardware maintains a centralized warehouse for the distribution of products ordered. Management is currently examining the process of distribution from the warehouse and is interested in studying the factors that affect warehouse distribution costs. Currently, a small handling fee is added to the order, regardless of the amount of the order. Data have been collected over the past 24 months, indicating the warehouse distribution costs and the number of orders received. They are stored in the file [warecost.xls](#). The results are as follows:

Months	Distribution Cost (Thousands of Dollars)	Number of Orders
1	52.95	4,015
2	71.66	3,806
3	85.58	5,309
4	63.69	4,262
5	72.81	4,296
6	68.44	4,097
7	52.46	3,213
8	70.77	4,809
9	82.03	5,237
10	74.39	4,732
11	70.84	4,413
12	54.08	2,921
13	62.98	3,977
14	72.30	4,428
15	58.99	3,964
16	79.38	4,582
17	94.44	5,582
18	59.74	3,450
19	90.50	5,079
20	93.24	5,735
21	69.33	4,269
22	53.71	3,708
23	89.18	5,387
24	66.80	4,161

- Assuming a linear relationship, use the least-squares method to find the regression coefficients  $b_0$  and  $b_1$ .
- Predict the monthly warehouse distribution costs when the number of orders is 4,500.
- Plot the residuals versus the time period.
- Compute the Durbin-Watson statistic. At the 0.05 level of significance, is there evidence of positive autocorrelation among the residuals?
- Based on the results of (c) and (d), is there reason to question the validity of the model?

**10.37** A freshly brewed shot of espresso has three distinct components: the heart, body, and crema. The separation of these three components typically lasts only 10 to 20 seconds.

To use the espresso shot in making a latte, cappuccino, or other drinks, the shot must be poured into the beverage during the separation of the heart, body, and crema. If the shot is used after the separation occurs, the drink becomes excessively bitter and acidic, ruining the final drink. Thus, a longer separation time allows the drink-maker more time to pour the shot and ensure that the beverage will meet expectations. An employee at a coffee shop hypothesized that the harder the espresso grounds were tamped down into the portafilter before brewing, the longer the separation time would be. An experiment using 24 observations was conducted to test this relationship. The independent variable Tamp measures the distance, in inches, between the espresso grounds and the top of the portafilter (that is, the harder the tamp, the larger the distance). The dependent variable Time is the number of seconds the heart, body, and crema are separated (that is, the amount of time after the shot is poured before it must be used for the customer's beverage). The data are stored in the file [espresso.xls](#):

Shot	Tamp	Time	Shot	Tamp	Time
1	0.20	14	13	0.50	18
2	0.50	14	14	0.50	13
3	0.50	18	15	0.35	19
4	0.20	16	16	0.35	19
5	0.20	16	17	0.20	17
6	0.50	13	18	0.20	18
7	0.20	12	19	0.20	15
8	0.35	15	20	0.20	16
9	0.50	9	21	0.35	18
10	0.35	15	22	0.35	16
11	0.50	11	23	0.35	14
12	0.50	16	24	0.35	16

- Determine the prediction line, using Time as the dependent variable and Tamp as the independent variable.
- Predict the mean separation time for a Tamp distance of 0.50 inch.
- Plot the residuals versus the time order of experimentation. Are there any noticeable patterns?
- Compute the Durbin-Watson statistic. At the 0.05 level of significance, is there evidence of positive autocorrelation among the residuals?
- Based on the results of (c) and (d), is there reason to question the validity of the model?

**10.38** The owner of a chain of ice cream stores would like to study the effect of atmospheric temperature on sales during the summer season. A sample of 21 consecutive days is selected, with the results stored in the data file [icecream.xls](#).

**(Hint:** Determine which are the independent and dependent variables.)

- a. Assuming a linear relationship, use the least-squares method to find the regression coefficients  $b_0$  and  $b_1$ .
- b. Predict the sales per store for a day in which the temperature is 83°F.
- c. Plot the residuals versus the time period.
- d. Compute the Durbin-Watson statistic. At the 0.05 level of significance, is there evidence of positive autocorrelation among the residuals?
- e. Based on the results of (c) and (d), is there reason to question the validity of the model?

## 10.7 INFERENCE ABOUT THE SLOPE AND CORRELATION COEFFICIENT

In Sections 10.1 through 10.3, regression was used solely for descriptive purposes. You learned how the least-squares method determines the regression coefficients and how to predict  $Y$  for a given value of  $X$ . In addition, you learned how to compute and interpret the standard error of the estimate and the coefficient of determination.

When residual analysis, as discussed in Section 10.5, indicates that the assumptions of a least-squares regression model are not seriously violated and that the straight-line model is appropriate, you can make inferences about the linear relationship between the variables in the population.

### t Test for the Slope

To determine the existence of a significant linear relationship between the  $X$  and  $Y$  variables, you test whether  $\beta_1$  (the population slope) is equal to 0. The null and alternative hypotheses are as follows:

$$H_0: \beta_1 = 0 \text{ (There is no linear relationship.)}$$

$$H_1: \beta_1 \neq 0 \text{ (There is a linear relationship.)}$$

If you reject the null hypothesis, you conclude that there is evidence of a linear relationship. Equation (10.16) defines the test statistic.

#### TESTING A HYPOTHESIS FOR A POPULATION SLOPE, $\beta_1$ , USING THE $t$ TEST

The  $t$  statistic equals the difference between the sample slope and hypothesized value of the population slope divided by the standard error of the slope.

$$t = \frac{b_1 - \beta_1}{S_{b_1}} \quad (10.16)$$

where

$$S_{b_1} = \frac{S_{YX}}{\sqrt{SSX}}$$

$$SSX = \sum_{i=1}^n (X_i - \bar{X})^2$$

The test statistic  $t$  follows a  $t$  distribution with  $n - 2$  degrees of freedom.

Return to the Using Statistics scenario concerning Sunflowers Apparel. To test whether there is a significant linear relationship between the size of the store and the annual sales at the 0.05 level of significance, refer to the Microsoft Excel worksheet for the  $t$  test presented in Figure 10.17.

**FIGURE 10.17**

Microsoft Excel  $t$  test for the slope for the Sunflowers Apparel data

	A	B	C	D	E	F	G
15							
16		<b>Coefficients</b>	<b>Standard Error</b>	<b><math>t</math> Stat</b>	<b><math>P</math>-value</b>	<b>Lower 95%</b>	<b>Upper 95%</b>
17	<b>Intercept</b>	0.9645	0.5262	1.8329	0.0917	-0.1820	2.1110
18	<b>Square Feet</b>	1.6699	0.1569	10.6411	0.0000	1.3280	2.0118



See Section E10.1 to create the worksheet that contains this area.

From Figure 10.17,

$$b_1 = +1.6699 \quad n = 14 \quad S_{b_1} = 0.1569$$

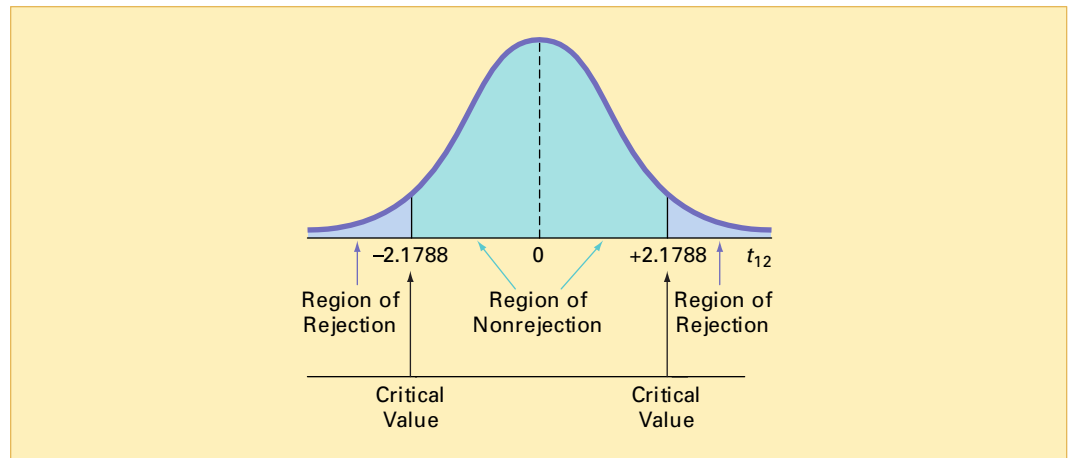
and

$$\begin{aligned} t &= \frac{b_1 - \beta_1}{S_{b_1}} \\ &= \frac{1.6699 - 0}{0.1569} = 10.6411 \end{aligned}$$

Microsoft Excel labels this  $t$  statistic  $t$  Stat (see Figure 10.17). Using the 0.05 level of significance, the critical value of  $t$  with  $n - 2 = 12$  degrees of freedom is 2.1788. Because  $t = 10.6411 > 2.1788$ , you reject  $H_0$  (see Figure 10.18). Using the  $p$ -value, you reject  $H_0$  because the  $p$ -value is approximately 0 which is less than  $\alpha = 0.05$ . Hence, you can conclude that there is a significant linear relationship between mean annual sales and the size of the store.

**FIGURE 10.18**

Testing a hypothesis about the population slope at the 0.05 level of significance, with 12 degrees of freedom



### F Test for the Slope

As an alternative to the  $t$  test, you can use an  $F$  test to determine whether the slope in simple linear regression is statistically significant. Equation (10.17) defines the  $F$  test for the slope as the ratio of the variance that is due to the regression ( $MSR$ ) divided by the error variance ( $MSE = S_{YX}^2$ ).

#### TESTING A HYPOTHESIS FOR A POPULATION SLOPE, $\beta_1$ , USING THE $F$ TEST

The  $F$  statistic is equal to the regression mean square ( $MSR$ ) divided by the error mean square ( $MSE$ ).

$$F = \frac{MSR}{MSE} \quad (10.17)$$

where

$$MSR = \frac{SSR}{k}$$

$$MSE = \frac{SSE}{n - k - 1}$$

$k$  = number of independent variables in the regression model

The test statistic  $F$  follows an  $F$  distribution with  $k$  and  $n - k - 1$  degrees of freedom.

Using a level of significance  $\alpha$ , the decision rule is

Reject  $H_0$  if  $F > F_U$ ;  
otherwise, do not reject  $H_0$ .

Table 10.6 organizes the complete set of results into an ANOVA table.

**TABLE 10.6**  
ANOVA Table  
for Testing the  
Significance of a  
Regression Coefficient

Source	$df$	Sum of Squares	Mean Square (Variance)	$F$
Regression	$k$	$SSR$	$MSR = \frac{SSR}{k}$	$F = \frac{MSR}{MSE}$
Error	$n - k - 1$	$SSE$	$MSE = \frac{SSE}{n - k - 1}$	
Total	$n - 1$	$SST$		

The completed ANOVA table is also part of the Microsoft Excel results shown in Figure 10.19. Figure 10.19 shows that the computed  $F$  statistic is 110.2335 and the  $p$ -value is approximately 0.

**FIGURE 10.19**  
Microsoft Excel  $F$  test  
for the Sunflowers  
Apparel data



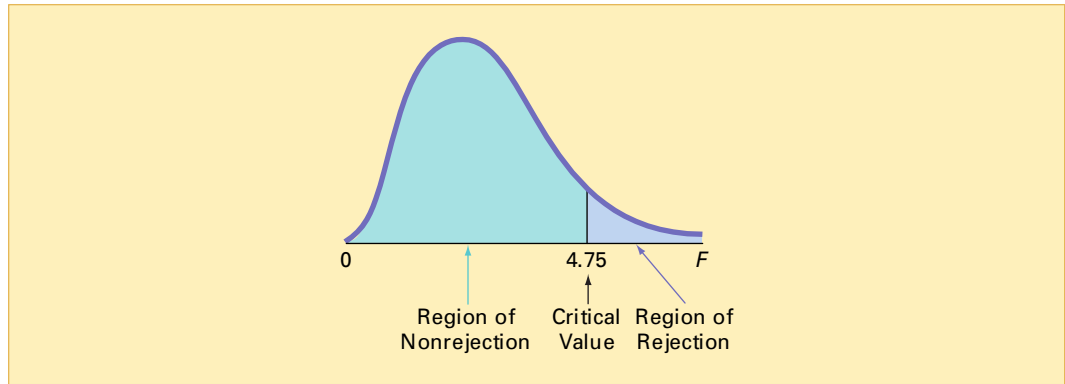
See Section E10.1 to create the worksheet that contains this area.

	A	B	C	D	E	F
10	<b>ANOVA</b>					
11		<b>df</b>	<b>SS</b>	<b>MS</b>	<b>F</b>	<b>Significance F</b>
12	<b>Regression</b>	1	105.7476	105.7476	113.2335	0.0000
13	<b>Residual</b>	12	11.2067	0.9339		
14	<b>Total</b>	13	116.9543			

Using a level of significance of 0.05, from Table E.5, the critical value of the  $F$  distribution, with 1 and 12 degrees of freedom, is 4.75 (see Figure 10.20). Because  $F = 113.2335 > 4.75$  or because the  $p$ -value = 0.0000 < 0.05, you reject  $H_0$  and conclude that the size of the store is significantly related to annual sales. Because the  $F$  test in Equation 10.17 on page 398 is equivalent to the  $t$  test on page 397, you reach the same conclusion.

**FIGURE 10.20**

Regions of rejection and nonrejection when testing for significance of slope at the 0.05 level of significance, with 1 and 12 degrees of freedom



### Confidence Interval Estimate of the Slope ( $\beta_1$ )

As an alternative to testing for the existence of a linear relationship between the variables, you can construct a confidence interval estimate of  $\beta_1$  and determine whether the hypothesized value ( $\beta_1 = 0$ ) is included in the interval. Equation (10.18) defines the confidence interval estimate of  $\beta_1$ .

#### CONFIDENCE INTERVAL ESTIMATE OF THE SLOPE, $\beta_1$

The confidence interval estimate for the slope can be constructed by taking the sample slope,  $b_1$ , and adding and subtracting the critical  $t$  value multiplied by the standard error of the slope.

$$b_1 \pm t_{n-2} S_{b_1} \quad (10.18)$$

From the Microsoft Excel results of Figure 10.17 on page 398,

$$b_1 = 1.6699 \quad n = 14 \quad S_{b_1} = 0.1569$$

To construct a 95% confidence interval estimate,  $\alpha/2 = 0.025$ , and from Table E.3,  $t_{12} = 2.1788$ . Thus,

$$\begin{aligned} b_1 \pm t_{n-2} S_{b_1} &= 1.6699 \pm (2.1788)(0.1569) \\ &= 1.6699 \pm 0.3419 \\ 1.3280 &\leq \beta_1 \leq 2.0118 \end{aligned}$$

Therefore, you estimate with 95% confidence that the population slope is between 1.3280 and 2.0118. Because these values are above 0, you conclude that there is a significant linear relationship between annual sales and the size of the store. Had the interval included 0, you would have concluded that no significant relationship exists between the variables. The confidence interval indicates that for each increase of 1,000 square feet, mean annual sales are estimated to increase by at least \$1,328,000 but no more than \$2,011,800.

### t Test for the Correlation Coefficient

In Section 3.5 on page 130, the strength of the relationship between two numerical variables was measured, using the **correlation coefficient**,  $r$ . You can use the correlation coefficient to determine whether there is a statistically significant linear relationship between  $X$  and  $Y$ . To do



so, you hypothesize that the population correlation coefficient,  $\rho$ , is 0. Thus, the null and alternative hypotheses are

$$\begin{aligned}H_0: \rho &= 0 \text{ (no correlation)} \\H_1: \rho &\neq 0 \text{ (correlation)}\end{aligned}$$

Equation (10.19) defines the test statistic for determining the existence of a significant correlation.

### TESTING FOR THE EXISTENCE OF CORRELATION

$$t = \frac{r - \rho}{\sqrt{\frac{1 - r^2}{n - 2}}} \quad (10.19)$$

where

$$\begin{aligned}r &= +\sqrt{r^2} \text{ if } b_1 > 0 \\r &= -\sqrt{r^2} \text{ if } b_1 < 0\end{aligned}$$

The test statistic  $t$  follows a  $t$  distribution with  $n - 2$  degrees of freedom.

In the Sunflowers Apparel problem,  $r^2 = 0.9042$  and  $b_1 = +1.6699$  (see Figure 10.4 on page 374). Because  $b_1 > 0$ , the correlation coefficient for annual sales and store size is the positive square root of  $r^2$ , that is,  $r = +\sqrt{0.9042} = +0.9509$ . Testing the null hypothesis that there is no correlation between these two variables results in the following observed  $t$  statistic:

$$\begin{aligned}t &= \frac{r - 0}{\sqrt{\frac{1 - r^2}{n - 2}}} \\&= \frac{0.9509 - 0}{\sqrt{\frac{1 - (0.9509)^2}{14 - 2}}} = 10.6411\end{aligned}$$

Using the 0.05 level of significance, because  $t = 10.6411 > 2.1788$ , you reject the null hypothesis. You conclude that there is evidence of an association between annual sales and store size. This  $t$  statistic is equivalent to the  $t$  statistic found when testing whether the population slope,  $\beta_1$ , is equal to zero (see Figure 10.17 on page 398).

When inferences concerning the population slope were discussed, confidence intervals and tests of hypothesis were used interchangeably. However, developing a confidence interval for the correlation coefficient is more complicated because the shape of the sampling distribution of the statistic  $r$  varies for different values of the population correlation coefficient. Methods for developing a confidence interval estimate for the correlation coefficient are presented in reference 4.

## PROBLEMS FOR SECTION 10.7

### Learning the Basics

**10.39** You are testing the null hypothesis that there is no linear relationship between two variables,  $X$  and  $Y$ . From your sample of  $n = 10$ , you determine that  $r = 0.80$ .

- What is the value of the  $t$  test statistic?
- At the  $\alpha = 0.05$  level of significance, what are the critical values?
- Based on your answers to (a) and (b), what statistical decision should you make?

**PH Grade ASSIST** **10.40** You are testing the null hypothesis that there is no relationship between two variables,  $X$  and  $Y$ . From your sample of  $n = 18$ , you determine that  $b_1 = +4.5$  and  $S_{b_1} = 1.5$ .

- What is the value of the  $t$  test statistic?
- At the  $\alpha = 0.05$  level of significance, what are the critical values?
- Based on your answers to (a) and (b), what statistical decision should you make?
- Construct a 95% confidence interval estimate of the population slope,  $\beta_1$ .

**PH Grade ASSIST** **10.41** You are testing the null hypothesis that there is no relationship between two variables,  $X$  and  $Y$ . From your sample of  $n = 20$ , you determine that  $SSR = 60$  and  $SSE = 40$ .

- What is the value of the  $F$  test statistic?
- At the  $\alpha = 0.05$  level of significance, what is the critical value?
- Based on your answers to (a) and (b), what statistical decision should you make?
- Compute the correlation coefficient by first computing  $r^2$  and assuming that  $b_1$  is negative.
- At the 0.05 level of significance, is there a significant correlation between  $X$  and  $Y$ ?

### Applying the Concepts

**SELF Test** **10.42** In Problem 10.4 on page 380, the marketing manager used shelf space for pet food to predict weekly sales. The data are stored in the file [petfood.xls](#). From the results of that problem,  $b_1 = 7.4$  and  $S_{b_1} = 1.59$ .

- At the 0.05 level of significance, is there evidence of a linear relationship between shelf space and sales?
- Construct a 95% confidence interval estimate of the population slope,  $\beta_1$ .

**10.43** In Problem 10.5 on page 380, you used reported magazine newsstand sales to predict audited sales. The data are stored in the file [circulation.xls](#). Using the results of that problem,  $b_1 = 0.5719$  and  $S_{b_1} = 0.0668$ .

- At the 0.05 level of significance, is there evidence of a linear relationship between reported sales and audited sales?
- Construct a 95% confidence interval estimate of the population slope,  $\beta_1$ .

**10.44** In Problem 10.6 on pages 380–381, the owner of a moving company wanted to predict labor hours, based on the number of cubic feet moved. The data are stored in the file [moving.xls](#). Using the results of that problem,

- at the 0.05 level of significance, is there evidence of a linear relationship between the number of cubic feet moved and labor hours?
- construct a 95% confidence interval estimate of the population slope,  $\beta_1$ .

**PH Grade ASSIST** **10.45** In Problem 10.7 on page 381, you used the weight of mail to predict the number of orders received. The data are stored in the file [mail.xls](#).

Using the results of that problem,

- at the 0.05 level of significance, is there evidence of a linear relationship between the weight of mail and the number of orders received?
- construct a 95% confidence interval estimate of the population slope,  $\beta_1$ .

**10.46** In Problem 10.8 on page 381, you used annual revenues to predict the value of a baseball franchise. The data are stored in the file [bbrevenue.xls](#). Using the results of that problem,

- at the 0.05 level of significance, is there evidence of a linear relationship between annual revenue and franchise value?
- construct a 95% confidence interval estimate of the population slope,  $\beta_1$ .

**10.47** In Problem 10.9 on page 381, an agent for a real estate company wanted to predict the monthly rent for apartments, based on the size of the apartment. The data are stored in the file [rent.xls](#). Using the results of that problem,

- at the 0.05 level of significance, is there evidence of a linear relationship between the size of the apartment and the monthly rent?
- construct a 95% confidence interval estimate of the population slope,  $\beta_1$ .

**10.48** In Problem 10.10 on page 381, you used hardness to predict the tensile strength of die-cast aluminum. The data are stored in the file [hardness.xls](#). Using the results of that problem,

- at the 0.05 level of significance, is there evidence of a linear relationship between hardness and tensile strength?
- construct a 95% confidence interval estimate of the population slope,  $\beta_1$ .

**10.49** The volatility of a stock is often measured by its beta value. You can estimate the beta value of a stock by developing a simple linear regression model, using the percentage weekly change in the stock as the dependent variable and the percentage weekly change in a market index as the independent variable. The S&P 500 Index is a common index to use. For example, if you wanted to estimate the beta for IBM, you could use the following model, which is sometimes referred to as a *market model*:

$$(\% \text{ weekly change in IBM}) = \beta_0 + \beta_1 (\% \text{ weekly change in S \& P 500 index}) + \epsilon$$

The least-squares regression estimate of the slope  $b_1$  is the estimate of the beta value for IBM. A stock with a beta value of 1.0 tends to move the same as the overall market. A stock with a beta value of 1.5 tends to move 50% more than the overall market, and a stock with a beta value of 0.6

tends to move only 60% as much as the overall market. Stocks with negative beta values tend to move in a direction opposite that of the overall market. The following table gives some beta values for some widely held stocks:

Company	Ticker Symbol	Beta
AT&T	T	0.80
IBM	IBM	1.20
Disney Company	DIS	1.40
Alcoa	AA	2.26
LSI Logic	LSI	3.61

Source: Extracted from [finance.yahoo.com](http://finance.yahoo.com), May 31, 2006.

- For each of the five companies, interpret the beta value.
- How can investors use the beta value as a guide for investing?

**10.50** Index funds are mutual funds that try to mimic the movement of leading indexes, such as the S&P 500 Index, the NASDAQ 100 Index, or the Russell 2000 Index. The beta values for these funds (as described in Problem 10.49) are therefore approximately 1.0. The estimated market models for these funds are approximately

$$(\% \text{ weekly change in index fund}) = 0.0 + 1.0 (\% \text{ weekly change in the index})$$

Leveraged index funds are designed to magnify the movement of major indexes. An article in *Mutual Funds* (L. O'Shaughnessy, "Reach for Higher Returns," *Mutual Funds*, July 1999, pp. 44–49) described some of the risks and rewards associated with these funds and gave details on some of the most popular leveraged funds, including those in the following table:

Name (Ticker Symbol)	Fund Description
Potomac Small Cap Plus (POSCX)	125% of Russell 2000 Index
Rydex "Inv" Nova (RYNVX)	150% of the S&P 500 Index
ProFund UltraOTC "Inv" (UOPIX)	Double (200%) the NASDAQ 100 Index

Thus, estimated market models for these funds are approximately

$$(\% \text{ weekly change in POSCX}) = 0.0 + 1.25 (\% \text{ weekly change in the Russell 2000 Index})$$

$$(\% \text{ weekly change in RYNVX}) = 0.0 + 1.50 (\% \text{ weekly change in the S\&P 500 Index})$$

$$(\% \text{ weekly change in UOPIX fund}) = 0.0 + 2.0 (\% \text{ weekly change in the NASDAQ 100 Index})$$

Thus, if the Russell 2000 Index gains 10% over a period of time, the leveraged mutual fund POSCX gains approxi-

mately 12.5%. On the downside, if the same index loses 20%, POSCX loses approximately 25%.

- Consider the leveraged mutual fund ProFund UltraOTC "Inv" (UOPIX), whose description is 200% of the performance of the S&P 500 Index. What is its approximate market model?
- If the NASDAQ gains 30% in a year, what return do you expect UOPIX to have?
- If the NASDAQ loses 35% in a year, what return do you expect UOPIX to have?
- What type of investors should be attracted to leveraged funds? What type of investors should stay away from these funds?

**10.51** The data in the file [coffeedrink.xls](#) represent the calories and fat (in grams) of 16-ounce iced coffee drinks at Dunkin' Donuts and Starbucks:

Product	Calories	Fat
Dunkin' Donuts Iced Mocha Swirl latte (whole milk)	240	8.0
Starbucks Coffee Frappuccino blended coffee	260	3.5
Dunkin' Donuts Coffee Coolatta (cream)	350	22.0
Starbucks Iced Coffee Mocha Espresso (whole milk and whipped cream)	350	20.0
Starbucks Mocha Frappuccino blended coffee (whipped cream)	420	16.0
Starbucks Chocolate Brownie Frappuccino blended coffee (whipped cream)	510	22.0
Starbucks Chocolate Frappuccino Blended Crème (whipped cream)	530	19.0

Source: Extracted from "Coffee as Candy at Dunkin' Donuts and Starbucks," *Consumer Reports*, June 2004, p. 9.

- Compute and interpret the coefficient of correlation,  $r$ .
- At the 0.05 level of significance, is there a significant linear relationship between the calories and fat?

**10.52** There are several methods for calculating fuel economy. The following table (contained in the file [mileage.xls](#)) indicates the mileage as calculated by owners and by current government standards:

Vehicle	Owner	Government Standards
2005 Ford F-150	14.3	16.8
2005 Chevrolet Silverado	15.0	17.8
2002 Honda Accord LX	27.8	26.2
2002 Honda Civic	27.9	34.2
2004 Honda Civic Hybrid	48.8	47.6
2002 Ford Explorer	16.8	18.3
2005 Toyota Camry	23.7	28.5
2003 Toyota Corolla	32.8	33.1
2005 Toyota Prius	37.3	56.0

- a. Compute and interpret the coefficient of correlation,  $r$ .
- b. At the 0.05 level of significance, is there a significant linear relationship between the mileage as calculated by owners and by current government standards?

**10.53** College basketball is big business, with coaches' salaries, revenues, and expenses in millions of dollars. The data in the file [colleges-basketball.xls](#) represent the coaches' salaries and revenues for college basketball at selected schools in a recent year (extracted from R. Adams, "Pay for Playoffs," *The Wall Street Journal*, March 11–12, 2006, pp. P1, P8).

- a. Compute and interpret the coefficient of correlation,  $r$ .
- b. At the 0.05 level of significance, is there a significant linear relationship between a coach's salary and revenue?

**10.54** College football players trying out for the NFL are given the Wonderlic standardized intelligence test. The data in the file [wonderlic.xls](#) represent the average Wonderlic scores of football players trying out for the NFL and the graduation rates for football players at selected schools (extracted from S. Walker, "The NFL's Smartest Team," *The Wall Street Journal*, September 30, 2005, pp. W1, W10).

- a. Compute and interpret the coefficient of correlation,  $r$ .
- b. At the 0.05 level of significance, is there a significant linear relationship between the average Wonderlic score of football players trying out for the NFL and the graduation rates for football players at selected schools?
- c. What conclusions can you reach about the relationship between the average Wonderlic score of football players trying out for the NFL and the graduation rates for football players at selected schools?

## 10.8 ESTIMATION OF MEAN VALUES AND PREDICTION OF INDIVIDUAL VALUES

This section presents methods of making inferences about the mean of  $Y$  and predicting individual values of  $Y$ .

### The Confidence Interval Estimate

In Example 10.2 on page 377, you used the prediction line to predict the value of  $Y$  for a given  $X$ . The mean annual sales for stores with 4,000 square feet was predicted to be 7.644 millions of dollars (\$7,644,000). This estimate, however, is a *point estimate* of the population mean. In Chapter 8, you studied the concept of the confidence interval as an estimate of the population mean. In a similar fashion, Equation (10.20) defines the **confidence interval estimate for the mean response** for a given  $X$ .

#### CONFIDENCE INTERVAL ESTIMATE FOR THE MEAN OF $Y$

$$\hat{Y}_i \pm t_{n-2} S_{YX} \sqrt{h_i}$$

$$\hat{Y}_i - t_{n-2} S_{YX} \sqrt{h_i} \leq \mu_{Y|X=X_i} \leq \hat{Y}_i + t_{n-2} S_{YX} \sqrt{h_i} \quad (10.20)$$

$$h_i = \frac{1}{n} + \frac{(X_i - \bar{X})^2}{SSX}$$

where

$\hat{Y}_i$  = predicted value of  $Y$ ;  $\hat{Y}_i = b_0 + b_1 X_i$

$S_{YX}$  = standard error of the estimate

$n$  = sample size

$X_i$  = given value of  $X$

$\mu_{Y|X=X_i}$  = mean value of  $Y$  when  $X = X_i$

$$SSX = \sum_{i=1}^n (X_i - \bar{X})^2$$

The width of the confidence interval in Equation (10.20) depends on several factors. For a given level of confidence, increased variation around the prediction line, as measured by the standard error of the estimate, results in a wider interval. However, as you would expect, increased sample size reduces the width of the interval. In addition, the width of the interval also varies at different values of  $X$ . When you predict  $Y$  for values of  $X$  close to  $\bar{X}$ , the interval is narrower than for predictions for  $X$  values more distant from  $\bar{X}$ .

In the Sunflowers Apparel example, suppose you want to construct a 95% confidence interval estimate of the mean annual sales for the entire population of stores that contain 4,000 square feet ( $X = 4$ ). Using the simple linear regression equation,

$$\begin{aligned}\hat{Y}_i &= 0.9645 + 1.6699 X_i \\ &= 0.9645 + 1.6699(4) = 7.6439 \text{ (millions of dollars)}\end{aligned}$$

Also, given the following:

$$\begin{aligned}\bar{X} &= 2.9214 \quad S_{YX} = 0.9664 \\ SSX &= \sum_{i=1}^n (X_i - \bar{X})^2 = 37.9236\end{aligned}$$

From Table E.3,  $t_{12} = 2.1788$ . Thus,

$$\hat{Y}_i \pm t_{n-2} S_{YX} \sqrt{h_i}$$

where

$$h_i = \frac{1}{n} + \frac{(X_i - \bar{X})^2}{SSX}$$

so that

$$\begin{aligned}\hat{Y}_i \pm t_{n-2} S_{YX} \sqrt{\frac{1}{n} + \frac{(X_i - \bar{X})^2}{SSX}} \\ &= 7.6439 \pm (2.1788)(0.9664) \sqrt{\frac{1}{14} + \frac{(4 - 2.9214)^2}{37.9236}} \\ &= 7.6439 \pm 0.6728\end{aligned}$$

so

$$6.9711 \leq \mu_{Y|X=4} \leq 8.3167$$

Therefore, the 95% confidence interval estimate is that the mean annual sales are between \$6,971,100 and \$8,316,700 for the population of stores with 4,000 square feet.

## The Prediction Interval

In addition to the need for a confidence interval estimate for the mean value, you often want to predict the response for an individual value. Although the form of the prediction interval is similar to that of the confidence interval estimate of Equation (10.20), the prediction interval is predicting an individual value, not estimating a parameter. Equation (10.21) defines the **prediction interval for an individual response,  $Y$** , at a particular value,  $X_i$ , denoted by  $Y_{X=X_i}$ .

**PREDICTION INTERVAL FOR AN INDIVIDUAL RESPONSE, Y**

$$\begin{aligned} \hat{Y}_i \pm t_{n-2} S_{YX} \sqrt{1 + h_i} \\ \hat{Y}_i - t_{n-2} S_{YX} \sqrt{1 + h_i} \leq Y_{X=X_i} \leq \hat{Y}_i + t_{n-2} S_{YX} \sqrt{1 + h_i} \end{aligned} \quad (10.21)$$

where  $h_i$ ,  $\hat{Y}_i$ ,  $S_{YX}$ ,  $n$ , and  $X_i$  are defined as in Equation (10.20) on page 404 and  $Y_{X=X_i}$  is a future value of  $Y$  when  $X = X_i$ .

To construct a 95% prediction interval of the annual sales for an individual store that contains 4,000 square feet ( $X=4$ ), you first compute  $\hat{Y}_i$ . Using the prediction line:

$$\begin{aligned} \hat{Y}_i &= 0.9645 + 1.6699 X_i \\ &= 0.9645 + 1.6699(4) \\ &= 7.6439 \text{ (millions of dollars)} \end{aligned}$$

Also, given the following:

$$\begin{aligned} \bar{X} &= 2.9214 \quad S_{YX} = 0.9664 \\ SSX &= \sum_{i=1}^n (X_i - \bar{X})^2 = 37.9236 \end{aligned}$$

From Table E.3,  $t_{12} = 2.1788$ . Thus,

$$\hat{Y}_i \pm t_{n-2} S_{YX} \sqrt{1 + h_i}$$

where

$$h_i = \frac{1}{n} + \frac{(X_i - \bar{X})^2}{\sum_{i=1}^n (X_i - \bar{X})^2}$$

so that

$$\begin{aligned} \hat{Y}_i \pm t_{n-2} S_{YX} \sqrt{1 + \frac{1}{n} + \frac{(X_i - \bar{X})^2}{SSX}} \\ = 7.6439 \pm (2.1788)(0.9664) \sqrt{1 + \frac{1}{14} + \frac{(4 - 2.9214)^2}{37.9236}} \\ = 7.6439 \pm 2.2104 \end{aligned}$$

so

$$5.4335 \leq Y_{X=4} \leq 9.8543$$

Therefore, with 95% confidence, you predict that the annual sales for an individual store with 4,000 square feet is between \$5,433,500 and \$9,854,300.

Figure 10.21 is a Microsoft Excel worksheet that illustrates the confidence interval estimate and the prediction interval for the Sunflowers Apparel problem. If you compare the results of the confidence interval estimate and the prediction interval, you see that the width of the prediction interval for an individual store is much wider than the confidence interval estimate for the mean. Remember that there is much more variation in predicting an individual value than in estimating a mean value.

**FIGURE 10.21**  
Microsoft Excel confidence interval estimate and prediction interval for the Sunflowers Apparel data



See Section E10.5 to create this.

A	B	
1	Site Selection Analysis	
2		
3	Data	
4	X Value	4
5	Confidence Level	95%
6		
7	Intermediate Calculations	
8	Sample Size	14 =DataCopy!F2
9	Degrees of Freedom	12 =B8 - 2
10	t Value	2.1788 =TINV(1 - B5, B9)
11	Sample Mean	2.9214 =DataCopy!F3
12	Sum of Squared Difference	37.9236 =DataCopy!F4
13	Standard Error of the Estimate	0.9664 from regression worksheet cell B7
14	h Statistic	0.1021 =1/B8 + (B4 - B11)^2/B12
15	Predicted Y (YHat)	7.6439 =DataCopy!F5
16		
17	For Average Y	
18	Interval Half Width	0.6728 =B10 * B13 * SQRT(B14)
19	Confidence Interval Lower Limit	6.9711 =B15 - B18
20	Confidence Interval Upper Limit	8.3167 =B15 + B18
21		
22	For Individual Response Y	
23	Interval Half Width	2.2104 =B10 * B13 * SQRT(1 + B14)
24	Prediction Interval Lower Limit	5.4335 =B15 - B23
25	Prediction Interval Upper Limit	9.8544 =B15 + B23

## PROBLEMS FOR SECTION 10.8

### Learning the Basics

**PH Grade ASSIST 10.55** Based on a sample of  $n = 20$ , the least-squares method was used to develop the following prediction line:  $\hat{Y}_i = 5 + 3X_i$ . In addition,

$$S_{YX} = 1.0 \quad \bar{X} = 2 \quad \sum_{i=1}^n (X_i - \bar{X})^2 = 20$$

- Construct a 95% confidence interval estimate of the population mean response for  $X = 2$ .
- Construct a 95% prediction interval of an individual response for  $X = 2$ .

**PH Grade ASSIST 10.56** Based on a sample of  $n = 20$ , the least-squares method was used to develop the following prediction line:  $\hat{Y}_i = 5 + 3X_i$ . In addition,

$$S_{YX} = 1.0 \quad \bar{X} = 2 \quad \sum_{i=1}^n (X_i - \bar{X})^2 = 20$$

- Construct a 95% confidence interval estimate of the population mean response for  $X = 4$ .
- Construct a 95% prediction interval of an individual response for  $X = 4$ .
- Compare the results of (a) and (b) with those of Problem 10.55 (a) and (b). Which interval is wider? Why?

### Applying the Concepts

**10.57** In Problem 10.5 on page 380, you used reported sales to predict audited sales of magazines. The data are stored in the file [circulation.xls](#). For these data  $S_{YX} = 42.186$  and  $h_i = 0.108$  when  $X = 400$ .

- Construct a 95% confidence interval estimate of the mean audited sales for magazines that report newsstand sales of 400,000.
- Construct a 95% prediction interval of the audited sales for an individual magazine that reports newsstand sales of 400,000.
- Explain the difference in the results in (a) and (b).

**PH Grade ASSIST 10.58** In Problem 10.4 on page 380, the marketing manager used shelf space for pet food to predict weekly sales. The data are stored in the file [petfood.xls](#). For these data  $S_{YX} = 30.81$  and  $h_i = 0.1373$  when  $X = 8$ .

- Construct a 95% confidence interval estimate of the mean weekly sales for all stores that have 8 feet of shelf space for pet food.
- Construct a 95% prediction interval of the weekly sales of an individual store that has 8 feet of shelf space for pet food.
- Explain the difference in the results in (a) and (b).

**10.59** In Problem 10.7 on page 381, you used the weight of mail to predict the number of orders received. The data are stored in the file [mail.xls](#).

- Construct a 95% confidence interval estimate of the mean number of orders received for all packages with a weight of 500 pounds.
- Construct a 95% prediction interval of the number of orders received for an individual package with a weight of 500 pounds.
- Explain the difference in the results in (a) and (b).

**10.60** In Problem 10.6 on page 380, the owner of a moving company wanted to predict labor hours based on the number of cubic feet moved. The data are stored in the file [moving.xls](#).

- Construct a 95% confidence interval estimate of the mean labor hours for all moves of 500 cubic feet.
- Construct a 95% prediction interval of the labor hours of an individual move that has 500 cubic feet.
- Explain the difference in the results in (a) and (b).

**10.61** In Problem 10.9 on page 381, an agent for a real estate company wanted to predict the monthly rent for apartments, based on the size of the apartment. The data are stored in the file [rent.xls](#).

- Construct a 95% confidence interval estimate of the mean monthly rental for all apartments that are 1,000 square feet in size.

- Construct a 95% prediction interval of the monthly rental of an individual apartment that is 1,000 square feet in size.
- Explain the difference in the results in (a) and (b).

**10.62** In Problem 10.8 on page 381, you predicted the value of a baseball franchise, based on current revenue. The data are stored in the file [bbrevenue.xls](#).

- Construct a 95% confidence interval estimate of the mean value of all baseball franchises that generate \$150 million of annual revenue.
- Construct a 95% prediction interval of the value of an individual baseball franchise that generates \$150 million of annual revenue.
- Explain the difference in the results in (a) and (b).

**10.63** In Problem 10.10 on page 381, you used hardness to predict the tensile strength of die-cast aluminum. The data are stored in the file [hardness.xls](#).

- Construct a 95% confidence interval estimate of the mean tensile strength for all specimens with a hardness of 30 Rockwell E units.
- Construct a 95% prediction interval of the tensile strength for an individual specimen that has a hardness of 30 Rockwell E units.
- Explain the difference in the results in (a) and (b).

## 10.9 PITFALLS IN REGRESSION AND ETHICAL ISSUES

Some of the pitfalls involved in using regression analysis are as follows:

- Lacking an awareness of the assumptions of least-squares regression
- Not knowing how to evaluate the assumptions of least-squares regression
- Not knowing what the alternatives to least-squares regression are if a particular assumption is violated
- Using a regression model without knowledge of the subject matter
- Extrapolating outside the relevant range
- Concluding that a significant relationship identified in an observational study is due to a cause-and-effect relationship

The widespread availability of spreadsheet and statistical software has made regression analysis much more feasible. However, for many users, this enhanced availability of software has not been accompanied by an understanding of how to use regression analysis properly. Someone who is not familiar with either the assumptions of regression or how to evaluate the assumptions cannot be expected to know what the alternatives to least-squares regression are if a particular assumption is violated.

The data in Table 10.7 (stored in the file [anscombe.xls](#)) illustrate the importance of using scatter plots and residual analysis to go beyond the basic number crunching of computing the  $Y$  intercept, the slope, and  $r^2$ .



**TABLE 10.7**  
Four Sets of Artificial  
Data

Data Set A		Data Set B		Data Set C		Data Set D	
$X_i$	$Y_i$	$X_i$	$Y_i$	$X_i$	$Y_i$	$X_i$	$Y_i$
10	8.04	10	9.14	10	7.46	8	6.58
14	9.96	14	8.10	14	8.84	8	5.76
5	5.68	5	4.74	5	5.73	8	7.71
8	6.95	8	8.14	8	6.77	8	8.84
9	8.81	9	8.77	9	7.11	8	8.47
12	10.84	12	9.13	12	8.15	8	7.04
4	4.26	4	3.10	4	5.39	8	5.25
7	4.82	7	7.26	7	6.42	19	12.50
11	8.33	11	9.26	11	7.81	8	5.56
13	7.58	13	8.74	13	12.74	8	7.91
6	7.24	6	6.13	6	6.08	8	6.89

Source: Extracted from F. J. Anscombe, "Graphs in Statistical Analysis," American Statistician, Vol. 27 (1973), pp. 17–21.

Anscombe (reference 1) showed that all four data sets given in Table 10.7 have the following identical results:

$$\hat{Y}_i = 3.0 + 0.5 X_i$$

$$S_{YX} = 1.237$$

$$S_{b_1} = 0.118$$

$$r^2 = 0.667$$

$$SSR = \text{Explained variation} = \sum_{i=1}^n (\hat{Y}_i - \bar{Y})^2 = 27.51$$

$$SSE = \text{Unexplained variation} = \sum_{i=1}^n (Y_i - \hat{Y}_i)^2 = 13.76$$

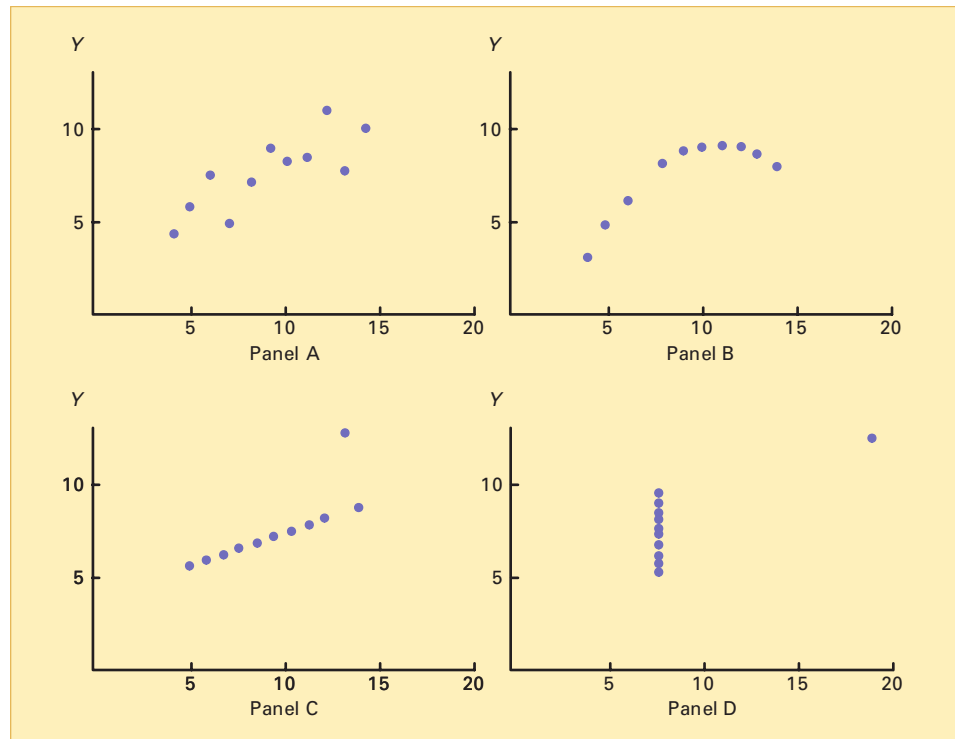
$$SST = \text{Total variation} = \sum_{i=1}^n (Y_i - \bar{Y})^2 = 41.27$$

Thus, with respect to these statistics associated with a simple linear regression analysis, the four data sets are identical. Were you to stop the analysis at this point, you would fail to observe the important differences among the four data sets. By examining the scatter plots for the four data sets in Figure 10.22 on page 410, and their residual plots in Figure 10.23 on page 410, you can clearly see that each of the four data sets has a different relationship between  $X$  and  $Y$ .

From the scatter plots of Figure 10.22 and the residual plots of Figure 10.23, you see how different the data sets are. The only data set that seems to follow an approximate straight line is data set A. The residual plot for data set A does not show any obvious patterns or outlying residuals. This is certainly not true for data sets B, C, and D. The scatter plot for data set B shows that a quadratic regression model is more appropriate. This conclusion is reinforced by the residual plot for data set B. The scatter plot and the residual plot for data set C clearly show an outlying observation. If this is the case, you may want to remove the outlier and reestimate the regression model (see reference 4). Similarly, the scatter plot for data set D represents the situation in which the model is heavily dependent on the outcome of a single response ( $X_8 = 19$  and  $Y_8 = 12.50$ ). You would have to cautiously evaluate any regression model because its regression coefficients are heavily dependent on a single observation.

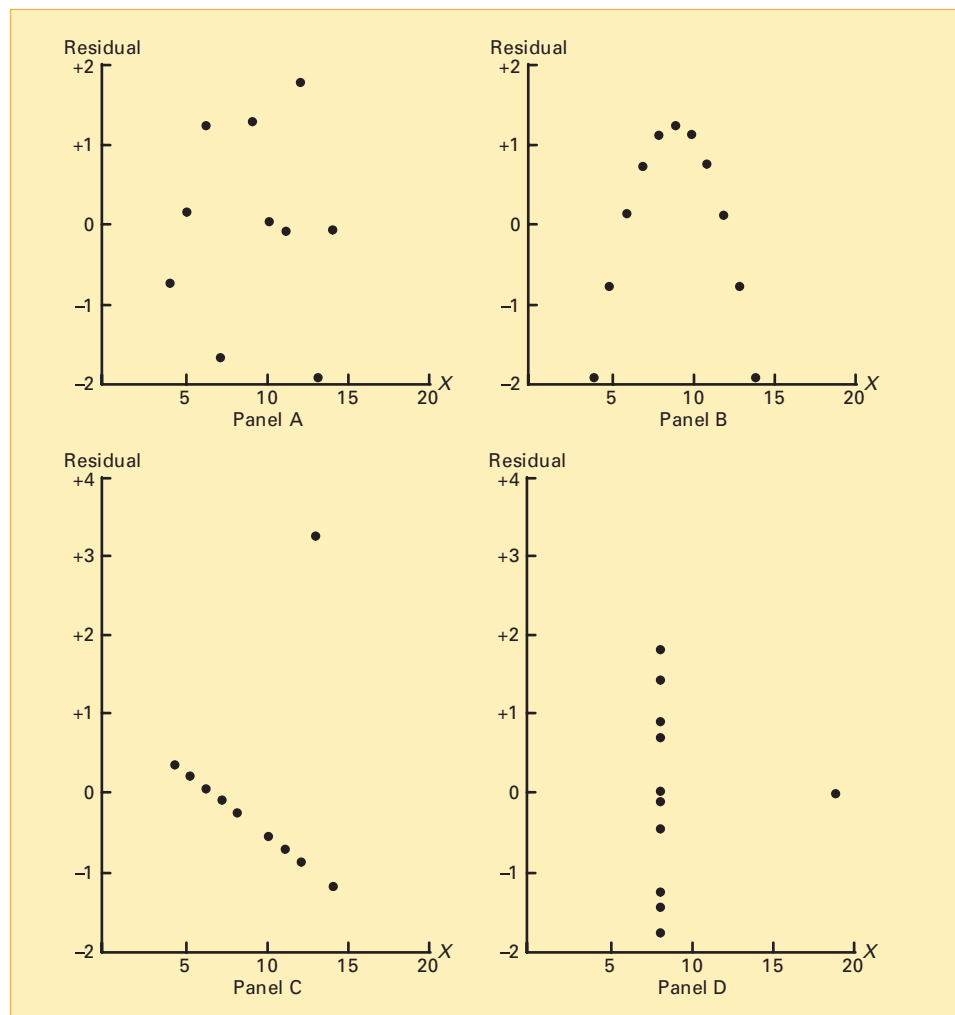
**FIGURE 10.22**

Scatter plots for four data sets



**FIGURE 10.23**

Residual plots for four data sets



In summary, scatter plots and residual plots are of vital importance to a complete regression analysis. The information they provide is so basic to a credible analysis that you should always include these graphical methods as part of a regression analysis. Thus, a strategy that you can use to help avoid the pitfalls of regression is as follows:

1. Start with a scatter plot to observe the possible relationship between  $X$  and  $Y$ .
2. Check the assumptions of regression before moving on to using the results of the model.
3. Plot the residuals versus the independent variable to determine whether the linear model is appropriate and to check the equal-variance assumption.
4. Use a histogram, stem-and-leaf display, box-and-whisker plot, or normal probability plot of the residuals to check the normality assumption.
5. If you collected the data over time, plot the residuals versus time and use the Durbin-Watson test to check the independence assumption.
6. If there are violations of the assumptions, use alternative methods to least-squares regression or alternative least-squares models.
7. If there are no violations of the assumptions, carry out tests for the significance of the regression coefficients and develop confidence and prediction intervals.
8. Avoid making predictions and forecasts outside the relevant range of the independent variable.
9. Keep in mind that the relationships identified in observational studies may or may not be due to cause-and-effect relationships. Remember that while causation implies correlation, correlation does not imply causation.

## America's Top Models

### From the Author's Desktop

Perhaps you are familiar with the TV competition organized by model Tyra Banks to find "America's top model." You may be less familiar with another set of top models that are emerging from the business world.

In a *Business Week* article from its January 23, 2006, edition (S. Baker, "Why Math Will Rock Your World: More Math Geeks Are Calling the Shots in Business. Is Your Industry Next?" *Business Week*, pp. 54–62), Stephen Baker talks about how "quants" turned finance upside down and is moving on to other business fields. The name *quants* derives from the fact that "math geeks" develop models and forecasts by using "quantitative methods." These methods are built on the principles of regression analysis discussed in this chapter, although the actual models are much more complicated than the simple linear models discussed in this chapter.

Regression-based models have become the top models for many types of business analyses. Some examples include

- **Advertising and marketing** Managers use econometric models (in other words,

regression models) to determine the effect of an advertisement on sales, based on a set of factors. Also, managers use data mining to predict patterns of behavior of what customers will buy in the future, based on historic information about the consumer.

- **Finance** Any time you read about a financial "model," you should understand that some type of regression model is being used. For example, a *New York Times* article on June 18, 2006, titled "An Old Formula That Points to New Worry" by Mark Hulbert (p. BU8) discusses a market timing model that predicts the return of stocks in the next three to five years, based on the dividend yield of the stock market and the interest rate of 90-day Treasury bills.
- **Food and beverage** Believe it or not, Enoloxig, a California consulting company, has developed a "formula" (a regression model) that predicts a wine's quality index, based on a set of chemical compounds found in the wine (see D. Darlington, "The Chemistry of a 90+ Wine," *The New York Times Magazine*, August 7, 2005, pp. 36–39).

- **Publishing** A study of the effect of price changes at Amazon.com and BN.com on sales (again, regression analysis) found that a 1% price change at BN.com pushed sales down 4%, but it pushed sales down only 0.5% at Amazon.com. (You can download the paper at <http://gsbadg.uchicago.edu/vitae.htm>.)

- **Transportation** Farecast.com uses data mining and predictive technologies to objectively predict airfare pricing (see D. Darlin, "Airmiles Made Easy (Or Easier)," *The New York Times*, July 1, 2006, pp. C1, C6).

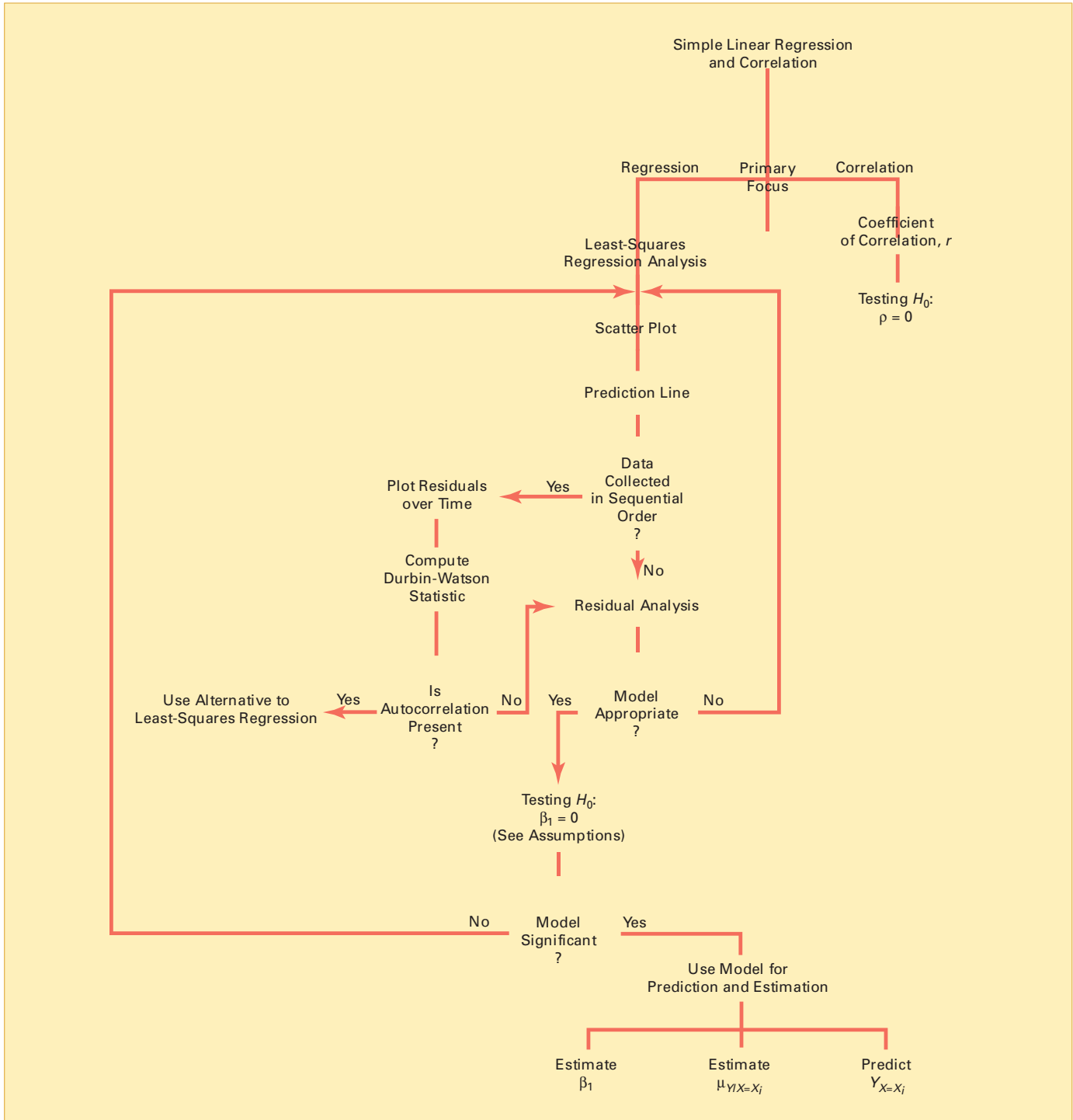
- **Real estate** Zillow.com uses information about the features contained in a home and its location to develop estimates about the market value of the home, using a "formula" built with a proprietary algorithm.

In the article, Baker stated that statistics and probability will become core skills for businesspeople and consumers. Those who are successful will know how to use statistics, whether they are building financial models or making marketing plans. He also strongly endorsed the need for everyone in business to have knowledge of Microsoft Excel to be able to produce statistical analysis and reports.

# SUMMARY

As you can see from the chapter roadmap in Figure 10.24, this chapter develops the simple linear regression model and discusses the assumptions and how to evaluate them.

Once you are assured that the model is appropriate, you can predict values by using the prediction line and test for the significance of the slope.



**FIGURE 10.24** Roadmap for simple linear regression

You have learned how the director of planning for a chain of clothing stores can use regression analysis to investigate the relationship between the size of a store and its annual sales. You have used this analysis to make better decisions

when selecting new sites for stores as well as to forecast sales for existing stores. In Chapter 11, regression analysis is extended to situations in which more than one independent variable is used to predict the value of a dependent variable.

## KEY EQUATIONS

### Simple Linear Regression Model

$$Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i \quad (10.1)$$

### Simple Linear Regression Equation: The Prediction Line

$$\hat{Y}_i = b_0 + b_1 X_i \quad (10.2)$$

### Computational Formula for the Slope, $b_1$

$$b_1 = \frac{SSXY}{SSX} \quad (10.3)$$

### Computational Formula for the $Y$ Intercept, $b_0$

$$b_0 = \bar{Y} - b_1 \bar{X} \quad (10.4)$$

### Measures of Variation in Regression

$$SST = SSR + SSE \quad (10.5)$$

### Total Sum of Squares ( $SST$ )

$$SST = \text{Total sum of squares} = \sum_{i=1}^n (Y_i - \bar{Y})^2 \quad (10.6)$$

### Regression Sum of Squares ( $SSR$ )

$SSR$  = Explained variation or regression of squares

$$= \sum_{i=1}^n (\hat{Y}_i - \bar{Y})^2 \quad (10.7)$$

### Error Sum of Squares ( $SSE$ )

$SSE$  = Unexplained variation or error sum of squares

$$= \sum_{i=1}^n (Y_i - \hat{Y}_i)^2 \quad (10.8)$$

### Coefficient of Determination

$$r^2 = \frac{\text{Regression sum of squares}}{\text{Total sum of squares}} = \frac{SSR}{SST} \quad (10.9)$$

### Computational Formula for $SST$

$$SST = \sum_{i=1}^n (Y_i - \bar{Y})^2 = \sum_{i=1}^n Y_i^2 - \frac{\left(\sum_{i=1}^n Y_i\right)^2}{n} \quad (10.10)$$

### Computational Formula for $SSR$

$$\begin{aligned} SSR &= \sum_{i=1}^n (\hat{Y}_i - \bar{Y})^2 \\ &= b_0 \sum_{i=1}^n Y_i + b_1 \sum_{i=1}^n X_i Y_i - \frac{\left(\sum_{i=1}^n Y_i\right)^2}{n} \end{aligned} \quad (10.11)$$

### Computational Formula for $SSE$

$$SSE = \sum_{i=1}^n (Y_i - \hat{Y}_i)^2 = \sum_{i=1}^n Y_i^2 - b_0 \sum_{i=1}^n Y_i - b_1 \sum_{i=1}^n X_i Y_i \quad (10.12)$$

### Standard Error of the Estimate

$$S_{YX} = \sqrt{\frac{SSE}{n-2}} = \sqrt{\frac{\sum_{i=1}^n (Y_i - \hat{Y}_i)^2}{n-2}} \quad (10.13)$$

### Residual

$$e_i = Y_i - \hat{Y}_i \quad (10.14)$$

### Durbin-Watson Statistic

$$D = \frac{\sum_{i=2}^n (e_i - e_{i-1})^2}{\sum_{i=1}^n e_i^2} \quad (10.15)$$

### Testing a Hypothesis for a Population Slope, $\beta_1$ , Using the $t$ Test

$$t = \frac{b_1 - \beta_1}{S_{b_1}} \quad (10.16)$$

### Testing a Hypothesis for a Population Slope, $\beta_1$ , Using the $F$ Test

$$F = \frac{MSR}{MSE} \quad (10.17)$$

### Confidence Interval Estimate of the Slope, $\beta_1$

$$\begin{aligned} &b_1 \pm t_{n-2} S_{b_1} \\ &b_1 - t_{n-2} S_{b_1} \leq \beta_1 \leq b_1 + t_{n-2} S_{b_1} \end{aligned} \quad (10.18)$$

**Testing for the Existence of Correlation**

$$t = \frac{r - \rho}{\sqrt{\frac{1 - r^2}{n - 2}}} \quad (10.19)$$

**Confidence Interval Estimate for the Mean of  $Y$** 

$$\begin{aligned} \hat{Y}_i \pm t_{n-2} S_{YX} \sqrt{h_i} \\ \hat{Y}_i - t_{n-2} S_{YX} \sqrt{h_i} \leq \mu_{Y|X=X_i} \leq \hat{Y}_i + t_{n-2} S_{YX} \sqrt{h_i} \end{aligned} \quad (10.20)$$

**Prediction Interval for an Individual Response,  $Y$** 

$$\begin{aligned} \hat{Y}_i \pm t_{n-2} S_{YX} \sqrt{1 + h_i} \\ \hat{Y}_i - t_{n-2} S_{YX} \sqrt{1 + h_i} \leq Y_{X=X_i} \leq \hat{Y}_i + t_{n-2} S_{YX} \sqrt{1 + h_i} \end{aligned} \quad (10.21)$$

**KEY TERMS**

assumptions of regression 387	independence of errors 387	residual analysis 388
autocorrelation 392	independent variable 370	response variable 371
coefficient of determination 384	least-squares method 374	scatter diagram 370
confidence interval estimate for the mean response 404	linear relationship 370	scatter plot 370
correlation coefficient 400	normality 388	simple linear regression 370
dependent variable 370	prediction interval for an individual response, $Y$ 405	simple linear regression equation 373
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error sum of squares ( $SSE$ ) 382	regression analysis 370	standard error of the estimate 386
equal variance 388	regression coefficient 374	total sum of squares ( $SST$ ) 382
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homoscedasticity 388	residual 388	$Y$ intercept 371

**CHAPTER REVIEW PROBLEMS****Checking Your Understanding**

- 10.64** What is the interpretation of the  $Y$  intercept and the slope in the simple linear regression equation?
- 10.65** What is the interpretation of the coefficient of determination?
- 10.66** When is the unexplained variation (that is, error sum of squares) equal to 0?
- 10.67** When is the explained variation (that is, regression sum of squares) equal to 0?
- 10.68** Why should you always carry out a residual analysis as part of a regression model?
- 10.69** What are the assumptions of regression analysis?
- 10.70** How do you evaluate the assumptions of regression analysis?
- 10.71** When and how do you use the Durbin-Watson statistic?

**10.72** What is the difference between a confidence interval estimate of the mean response,  $\mu_{Y|X=X_i}$ , and a prediction interval of  $Y_{X=X_i}$ ?

**Applying the Concepts**

**10.73** Researchers from the Lubin School of Business at Pace University in New York City conducted a study on Internet-supported courses. In one part of the study, four numerical variables were collected on 108 students in an introductory management course that met once a week for an entire semester. One variable collected was *hit consistency*. To measure hit consistency, the researchers did the following: If a student did not visit the Internet site between classes, the student was given a 0 for that time period. If a student visited the Internet site one or more times between classes, the student was given a 1 for that time period. Because there were 13 time periods, a student's score on hit consistency could range from 0 to 13.

The other three variables included the student's course average, the student's cumulative grade point average

(GPA), and the total number of hits the student had on the Internet site supporting the course. The following table gives the correlation coefficient for all pairs of variables. Note that correlations marked with an \* are statistically significant, using  $\alpha = 0.001$ :

Variable	Correlation
Course Average, Cumulative GPA	0.72*
Course Average, Total Hits	0.08
Course Average, Hit Consistency	0.37*
Cumulative GPA, Total Hits	0.12
Cumulative GPA, Hit Consistency	0.32*
Total Hits, Hit Consistency	0.64*

Source: Extracted from D. Baugher, A. Varanelli, and E. Weisbord, "Student Hits in an Internet-Supported Course: How Can Instructors Use Them and What Do They Mean?" *Decision Sciences Journal of Innovative Education*, Fall 2003, 1(2), pp. 159–179.

- What conclusions can you reach from this correlation analysis?
- Are you surprised by the results, or are they consistent with your own observations and experiences?

**10.74** Management of a soft-drink bottling company wants to develop a method for allocating delivery costs to customers. Although one cost clearly relates to travel time within a particular route, another variable cost reflects the time required to unload the cases of soft drink at the delivery point. A sample of 20 deliveries within a territory was selected. The delivery times and the numbers of cases delivered were recorded in the [delivery.xls](#) file:

Customer	Number of Cases	Delivery Time (Minutes)	Customer	Number of Cases	Delivery Time (Minutes)
1	52	32.1	11	161	43.0
2	64	34.8	12	184	49.4
3	73	36.2	13	202	57.2
4	85	37.8	14	218	56.8
5	95	37.8	15	243	60.6
6	103	39.7	16	254	61.2
7	116	38.5	17	267	58.2
8	121	41.9	18	275	63.1
9	143	44.2	19	287	65.6
10	157	47.1	20	298	67.3

Develop a regression model to predict delivery time, based on the number of cases delivered.

- Use the least-squares method to compute the regression coefficients  $b_0$  and  $b_1$ .
- Interpret the meaning of  $b_0$  and  $b_1$  in this problem.
- Predict the delivery time for 150 cases of soft drink.
- Should you use the model to predict the delivery time for a customer who is receiving 500 cases of soft drink? Why or why not?

- Determine the coefficient of determination,  $r^2$ , and explain its meaning in this problem.
- Perform a residual analysis. Is there any evidence of a pattern in the residuals? Explain.
- At the 0.05 level of significance, is there evidence of a linear relationship between delivery time and the number of cases delivered?
- Construct a 95% confidence interval estimate of the mean delivery time for 150 cases of soft drink.
- Construct a 95% prediction interval of the delivery time for a single delivery of 150 cases of soft drink.
- Construct a 95% confidence interval estimate of the population slope.
- Explain how the results in (a) through (j) can help allocate delivery costs to customers.

**10.75** A brokerage house wants to predict the number of trade executions per day, using the number of incoming phone calls as a predictor variable. Data were collected over a period of 35 days and are stored in the file [trades.xls](#).

- Use the least-squares method to compute the regression coefficients  $b_0$  and  $b_1$ .
- Interpret the meaning of  $b_0$  and  $b_1$  in this problem.
- Predict the number of trades executed for a day in which the number of incoming calls is 2,000.
- Should you use the model to predict the number of trades executed for a day in which the number of incoming calls is 5,000? Why or why not?
- Determine the coefficient of determination,  $r^2$ , and explain its meaning in this problem.
- Plot the residuals against the number of incoming calls and also against the days. Is there any evidence of a pattern in the residuals with either of these variables? Explain.
- Determine the Durbin-Watson statistic for these data.
- Based on the results of (f) and (g), is there reason to question the validity of the model? Explain.
- At the 0.05 level of significance, is there evidence of a linear relationship between the volume of trade executions and the number of incoming calls?
- Construct a 95% confidence interval estimate of the mean number of trades executed for days in which the number of incoming calls is 2,000.
- Construct a 95% prediction interval of the number of trades executed for a particular day in which the number of incoming calls is 2,000.
- Construct a 95% confidence interval estimate of the population slope.
- Based on the results of (a) through (l), do you think the brokerage house should focus on a strategy of increasing the total number of incoming calls or on a strategy that relies on trading by a small number of heavy traders? Explain.

**10.76** You want to develop a model to predict the selling price of homes based on assessed value. A sample of 30

recently sold single-family houses in a small city is selected to study the relationship between selling price (in thousands of dollars) and assessed value (in thousands of dollars). The houses in the city had been reassessed at full value one year prior to the study. The results are in the file [house1.xls](#).

**(Hint:** First, determine which are the independent and dependent variables.)

- Construct a scatter plot and, assuming a linear relationship, use the least-squares method to compute the regression coefficients  $b_0$  and  $b_1$ .
- Interpret the meaning of the  $Y$  intercept,  $b_0$ , and the slope,  $b_1$ , in this problem.
- Use the prediction line developed in (a) to predict the selling price for a house whose assessed value is \$170,000.
- Determine the coefficient of determination,  $r^2$ , and interpret its meaning in this problem.
- Perform a residual analysis on your results and determine the adequacy of the fit of the model.
- At the 0.05 level of significance, is there evidence of a linear relationship between selling price and assessed value?
- Construct a 95% confidence interval estimate of the mean selling price for houses with an assessed value of \$170,000.
- Construct a 95% prediction interval of the selling price of an individual house with an assessed value of \$170,000.
- Construct a 95% confidence interval estimate of the population slope.

**10.77** You want to develop a model to predict the assessed value of houses, based on heating area. A sample of 15 single-family houses is selected in a city. The assessed value (in thousands of dollars) and the heating area of the houses (in thousands of square feet) are recorded, with the following results, stored in the file [house2.xls](#):

House	Assessed Value (\$000)	Heating Area of Dwelling (Thousands of Square Feet)
1	184.4	2.00
2	177.4	1.71
3	175.7	1.45
4	185.9	1.76
5	179.1	1.93
6	170.4	1.20
7	175.8	1.55
8	185.9	1.93
9	178.5	1.59
10	179.2	1.50
11	186.7	1.90
12	179.3	1.39
13	174.5	1.54
14	183.8	1.89
15	176.8	1.59

**(Hint:** First, determine which are the independent and dependent variables.)

- Construct a scatter plot and, assuming a linear relationship, use the least-squares method to compute the regression coefficients  $b_0$  and  $b_1$ .
- Interpret the meaning of the  $Y$  intercept,  $b_0$ , and the slope,  $b_1$ , in this problem.
- Use the prediction line developed in (a) to predict the assessed value for a house whose heating area is 1,750 square feet.
- Determine the coefficient of determination,  $r^2$ , and interpret its meaning in this problem.
- Perform a residual analysis on your results and determine the adequacy of the fit of the model.
- At the 0.05 level of significance, is there evidence of a linear relationship between assessed value and heating area?
- Construct a 95% confidence interval estimate of the mean assessed value for houses with a heating area of 1,750 square feet.
- Construct a 95% prediction interval of the assessed value of an individual house with a heating area of 1,750 square feet.
- Construct a 95% confidence interval estimate of the population slope.

**10.78** The director of graduate studies at a large college of business would like to predict the grade point average (GPA) of students in an MBA program based on the Graduate Management Admission Test (GMAT) score. A sample of 20 students who had completed 2 years in the program is selected. The results are stored in the file [gpigmat.xls](#):

Observation	GMAT Score	GPA	Observation	GMAT Score	GPA
1	688	3.72	11	567	3.07
2	647	3.44	12	542	2.86
3	652	3.21	13	551	2.91
4	608	3.29	14	573	2.79
5	680	3.91	15	536	3.00
6	617	3.28	16	639	3.55
7	557	3.02	17	619	3.47
8	599	3.13	18	694	3.60
9	616	3.45	19	718	3.88
10	594	3.33	20	759	3.76

**(Hint:** First, determine which are the independent and dependent variables.)

- Construct a scatter plot and, assuming a linear relationship, use the least-squares method to compute the regression coefficients  $b_0$  and  $b_1$ .
- Interpret the meaning of the  $Y$  intercept,  $b_0$ , and the slope,  $b_1$ , in this problem.
- Use the prediction line developed in (a) to predict the GPA for a student with a GMAT score of 600.
- Determine the coefficient of determination,  $r^2$ , and interpret its meaning in this problem.
- Perform a residual analysis on your results and determine the adequacy of the fit of the model.



- f. At the 0.05 level of significance, is there evidence of a linear relationship between GMAT score and GPA?
- g. Construct a 95% confidence interval estimate of the mean GPA of students with a GMAT score of 600.
- h. Construct a 95% prediction interval of the GPA for a particular student with a GMAT score of 600.
- i. Construct a 95% confidence interval estimate of the population slope.

**10.79** The manager of the purchasing department of a large banking organization would like to develop a model to predict the amount of time it takes to process invoices. Data are collected from a sample of 30 days, and the number of invoices processed and completion time, in hours, is stored in the file [invoice.xls](#).

(**Hint:** First, determine which are the independent and dependent variables.)

- a. Assuming a linear relationship, use the least-squares method to compute the regression coefficients  $b_0$  and  $b_1$ .
- b. Interpret the meaning of the  $Y$  intercept,  $b_0$ , and the slope,  $b_1$ , in this problem.
- c. Use the prediction line developed in (a) to predict the amount of time it would take to process 150 invoices.
- d. Determine the coefficient of determination,  $r^2$ , and interpret its meaning.
- e. Plot the residuals against the number of invoices processed and also against time.
- f. Based on the plots in (e), does the model seem appropriate?
- g. Compute the Durbin-Watson statistic and, at the 0.05 level of significance, determine whether there is any autocorrelation in the residuals.
- h. Based on the results of (e) through (g), what conclusions can you reach concerning the validity of the model?
- i. At the 0.05 level of significance, is there evidence of a linear relationship between the amount of time and the number of invoices processed?
- j. Construct a 95% confidence interval estimate of the mean amount of time it would take to process 150 invoices.
- k. Construct a 95% prediction interval of the amount of time it would take to process 150 invoices on a particular day.

**10.80** On January 28, 1986, the space shuttle *Challenger* exploded, and seven astronauts were killed. Prior to the launch, the predicted atmospheric temperature was for freezing weather at the launch site. Engineers for Morton Thiokol (the manufacturer of the rocket motor) prepared charts to make the case that the launch should not take place due to the cold weather. These arguments were rejected, and the launch tragically took place. Upon investigation after the tragedy, experts agreed that the disaster occurred because of leaky rubber O-rings that did not seal properly due to the cold temperature. Data indicating the atmospheric temperature at the time of 23 previous launches and the O-ring damage index are stored in the file [o-ring.xls](#).

Flight Number	Temperature (°F)	O-Ring Damage Index
1	66	0
2	70	4
3	69	0
5	68	0
6	67	0
7	72	0
8	73	0
9	70	0
41-B	57	4
41-C	63	2
41-D	70	4
41-G	78	0
51-A	67	0
51-B	75	0
51-C	53	11
51-D	67	0
51-F	81	0
51-G	70	0
51-I	67	0
51-J	79	0
61-A	75	4
61-B	76	0
61-C	58	4

Note: Data from flight 4 is omitted due to unknown O-ring condition.

Source: Extracted from Report of the Presidential Commission on the Space Shuttle Challenger Accident, *Washington, DC, 1986, Vol. II (H1-H3) and Vol. IV (664)*, and Post Challenger Evaluation of Space Shuttle Risk Assessment and Management, *Washington, DC, 1988, pp. 135-136*.

- a. Construct a scatter plot for the seven flights in which there was O-ring damage (O-ring damage index  $\neq 0$ ). What conclusions, if any, can you draw about the relationship between atmospheric temperature and O-ring damage?
- b. Construct a scatter plot for all 23 flights.
- c. Explain any differences in the interpretation of the relationship between atmospheric temperature and O-ring damage in (a) and (b).
- d. Based on the scatter plot in (b), provide reasons why a prediction should not be made for an atmospheric temperature of 31 °F, the temperature on the morning of the launch of the *Challenger*.
- e. Although the assumption of a linear relationship may not be valid, fit a simple linear regression model to predict O-ring damage, based on atmospheric temperature.
- f. Include the prediction line found in (e) on the scatter plot developed in (b).
- g. Based on the results of (f), do you think a linear model is appropriate for these data? Explain.
- h. Perform a residual analysis. What conclusions do you reach?

**10.81** Crazy Dave, a well-known baseball analyst, would like to study various team statistics for the 2005 baseball season to determine which variables might be useful in predicting the number of wins achieved by teams during the season. He has decided to begin by using a team's earned run average (ERA), a measure of pitching performance, to predict the number of wins. The data for the 30 Major League Baseball teams are in the file [bb2005.xls](#).

(Hint: First, determine which are the independent and dependent variables.)

- Assuming a linear relationship, use the least-squares method to compute the regression coefficients  $b_0$  and  $b_1$ .
- Interpret the meaning of the  $Y$  intercept,  $b_0$ , and the slope,  $b_1$ , in this problem.
- Use the prediction line developed in (a) to predict the number of wins for a team with an ERA of 4.50.
- Compute the coefficient of determination,  $r^2$ , and interpret its meaning.
- Perform a residual analysis on your results and determine the adequacy of the fit of the model.
- At the 0.05 level of significance, is there evidence of a linear relationship between the number of wins and the ERA?
- Construct a 95% confidence interval estimate of the mean number of wins expected for teams with an ERA of 4.50.
- Construct a 95% prediction interval of the number of wins for an individual team that has an ERA of 4.50.
- Construct a 95% confidence interval estimate of the slope.
- The 30 teams constitute a population. In order to use statistical inference, as in (f) through (i), the data must be assumed to represent a random sample. What "population" would this sample be drawing conclusions about?
- What other independent variables might you consider for inclusion in the model?

**10.82** College football players trying out for the NFL are given the Wonderlic standardized intelligence test. The data in the file [wonderlic.xls](#) contains the average Wonderlic scores of football players trying out for the NFL and the graduation rates for football players at selected schools (extracted from S. Walker, "The NFL's Smartest Team," *The Wall Street Journal*, September 30, 2005, pp. W1, W10). You plan to develop a regression model to predict the Wonderlic scores for football players trying out for the NFL, based on the graduation rate of the school they attended.

- Assuming a linear relationship, use the least-squares method to compute the regression coefficients  $b_0$  and  $b_1$ .
- Interpret the meaning of the  $Y$  intercept,  $b_0$ , and the slope,  $b_1$ , in this problem.
- Use the prediction line developed in (a) to predict the Wonderlic score for football players trying out for the NFL from a school that has a graduation rate of 50%.

- Compute the coefficient of determination,  $r^2$ , and interpret its meaning.
- Perform a residual analysis on your results and determine the adequacy of the fit of the model.
- At the 0.05 level of significance, is there evidence of a linear relationship between the Wonderlic score for a football player trying out for the NFL from a school and the school's graduation rate?
- Construct a 95% confidence interval estimate of the mean Wonderlic score for football players trying out for the NFL from a school that has a graduation rate of 50%.
- Construct a 95% prediction interval of the Wonderlic score for a football player trying out for the NFL from a school that has a graduation rate of 50%.
- Construct a 95% confidence interval estimate of the slope.

**10.83** College basketball is big business, with coaches' salaries, revenues, and expenses in millions of dollars. The data in the file [colleges-basketball.xls](#) contains the coaches' salaries and revenues for college basketball at selected schools in a recent year (extracted from R. Adams, "Pay for Playoffs," *The Wall Street Journal*, March 11–12, 2006, pp. P1, P8). You plan to develop a regression model to predict a coach's salary based on revenue.

- Assuming a linear relationship, use the least-squares method to compute the regression coefficients  $b_0$  and  $b_1$ .
- Interpret the meaning of the  $Y$  intercept,  $b_0$ , and the slope,  $b_1$ , in this problem.
- Use the prediction line developed in (a) to predict the coach's salary for a school that has revenue of \$7 million.
- Compute the coefficient of determination,  $r^2$ , and interpret its meaning.
- Perform a residual analysis on your results and determine the adequacy of the fit of the model.
- At the 0.05 level of significance, is there evidence of a linear relationship between the coach's salary for a school and revenue?
- Construct a 95% confidence interval estimate of the mean salary of coaches at schools that have revenue of \$7 million.
- Construct a 95% prediction interval of the coach's salary for a school that has revenue of \$7 million.
- Construct a 95% confidence interval estimate of the slope.

**10.84** During the fall harvest season in the United States, pumpkins are sold in large quantities at farm stands. Often, instead of weighing the pumpkins prior to sale, the farm stand operator will just place the pumpkin in the appropriate circular cutout on the counter. When asked why this was done, one farmer replied, "I can tell the weight of the pumpkin from its circumference." To determine whether this was really true, a sample of 23 pumpkins were mea-

sured for circumference and weighed, with the following results, stored in the file [pumpkin.xls](#):

Circumference (cm)	Weight (Grams)	Circumference (cm)	Weight (Grams)
50	1,200	57	2,000
55	2,000	66	2,500
54	1,500	82	4,600
52	1,700	83	4,600
37	500	70	3,100
52	1,000	34	600
53	1,500	51	1,500
47	1,400	50	1,500
51	1,500	49	1,600
63	2,500	60	2,300
33	500	59	2,100
43	1,000		

- Assuming a linear relationship, use the least-squares method to compute the regression coefficients  $b_0$  and  $b_1$ .
- Interpret the meaning of the slope,  $b_1$ , in this problem.
- Predict the mean weight for a pumpkin that is 60 centimeters in circumference.
- Do you think it is a good idea for the farmer to sell pumpkins by circumference instead of weight? Explain.
- Determine the coefficient of determination,  $r^2$ , and interpret its meaning.
- Perform a residual analysis for these data and determine the adequacy of the fit of the model.
- At the 0.05 level of significance, is there evidence of a linear relationship between the circumference and the weight of a pumpkin?
- Construct a 95% confidence interval estimate of the population slope,  $\beta_1$ .
- Construct a 95% confidence interval estimate of the population mean weight for pumpkins that have a circumference of 60 centimeters.
- Construct a 95% prediction interval of the weight for an individual pumpkin that has a circumference of 60 centimeters.

**10.85** Can demographic information be helpful in predicting sales of sporting goods stores? The data stored in the file [sporting.xls](#) are the monthly sales totals from a random sample of 38 stores in a large chain of nationwide sporting goods stores. All stores in the franchise, and thus within the sample, are approximately the same size and carry the same merchandise. The county or, in some cases, counties in which the store draws the majority of its customers is referred to here as the customer base. For each of the 38 stores, demographic information about the customer base is provided. The data are real, but the name of the franchise is not used, at the request of the company. The variables in the data set are

Sales—Latest one-month sales total (dollars)  
 Age—Median age of customer base (years)  
 HS—Percentage of customer base with a high school diploma  
 College—Percentage of customer base with a college diploma  
 Growth—Annual population growth rate of customer base over the past 10 years  
 Income—Median family income of customer base (dollars)

- Construct a scatter plot, using sales as the dependent variable and median family income as the independent variable. Discuss the scatter diagram.
- Assuming a linear relationship, use the least-squares method to compute the regression coefficients  $b_0$  and  $b_1$ .
- Interpret the meaning of the  $Y$  intercept,  $b_0$ , and the slope,  $b_1$ , in this problem.
- Compute the coefficient of determination,  $r^2$ , and interpret its meaning.
- Perform a residual analysis on your results and determine the adequacy of the fit of the model.
- At the 0.05 level of significance, is there evidence of a linear relationship between the independent variable and the dependent variable?
- Construct a 95% confidence interval estimate of the slope and interpret its meaning.

**10.86** For the data of Problem 10.85, repeat (a) through (g), using median age as the independent variable.

**10.87** For the data of Problem 10.85, repeat (a) through (g), using high school graduation rate as the independent variable.

**10.88** For the data of Problem 10.85, repeat (a) through (g), using college graduation rate as the independent variable.

**10.89** For the data of Problem 10.85, repeat (a) through (g), using population growth as the independent variable.

**10.90** Zagat's publishes restaurant ratings for various locations in the United States. The data file [restaurants.xls](#) contains the Zagat rating for food, decor, service, and the price per person for a sample of 50 restaurants located in an urban area (New York City) and 50 restaurants located in a suburb of New York City. Develop a regression model to predict the price per person, based on a variable that represents the sum of the ratings for food, decor, and service.

Source: Extracted from Zagat Survey 2002 New York City Restaurants and Zagat Survey 2001–2002, Long Island Restaurants.

- Assuming a linear relationship, use the least-squares method to compute the regression coefficients  $b_0$  and  $b_1$ .
- Interpret the meaning of the  $Y$  intercept,  $b_0$ , and the slope,  $b_1$ , in this problem.
- Use the prediction line developed in (a) to predict the price per person for a restaurant with a summated rating of 50.
- Compute the coefficient of determination,  $r^2$ , and interpret its meaning.

- e. Perform a residual analysis on your results and determine the adequacy of the fit of the model.
- f. At the 0.05 level of significance, is there evidence of a linear relationship between the price per person and the summated rating?
- g. Construct a 95% confidence interval estimate of the mean price per person for all restaurants with a summated rating of 50.
- h. Construct a 95% prediction interval of the price per person for a restaurant with a summated rating of 50.
- i. Construct a 95% confidence interval estimate of the slope.
- j. How useful do you think the summated rating is as a predictor of price? Explain.

**10.91** Refer to the discussion of beta values and market models in Problem 10.49 on pages 402–403. One hundred weeks of data, ending the week of May 22, 2006, for the S&P 500 and three individual stocks are included in the data file [sp500.xls](#). Note that the *weekly percentage change* for both the S&P 500 and the individual stocks is measured as the percentage change from the previous week's closing value to the current week's closing value. The variables included are

- Week—Current week
- SP500—Weekly percentage change in the S&P 500 Index
- WALMART—Weekly percentage change in stock price of Wal-Mart Stores, Inc.
- TARGET—Weekly percentage change in stock price of the Target Corporation
- SARALEE—Weekly percentage change in stock price of the Sara Lee Corporation

Source: Extracted from [finance.yahoo.com](#), May 31, 2006.

- a. Estimate the market model for Wal-Mart Stores Inc. (*Hint*: Use the percentage change in the S&P 500 Index as the independent variable and the percentage change in Wal-Mart Stores, Inc.'s stock price as the dependent variable.)
- b. Interpret the beta value for Wal-Mart Stores, Inc.
- c. Repeat (a) and (b) for Target Corporation.
- d. Repeat (a) and (b) for Sara Lee Corporation.
- e. Write a brief summary of your findings.

**10.92** The data file [returns.xls](#) contains the stock prices of four companies, collected weekly for 53 consecutive weeks, ending May 22, 2006. The variables are

- Week—Closing date for stock prices
- MSFT—Stock price of Microsoft, Inc.
- Ford—Stock price of Ford Motor Company
- GM—Stock price of General Motors, Inc.
- IAL—Stock price of International Aluminum, Inc.

Source: Extracted from [finance.yahoo.com](#), May 31, 2006.

- a. Calculate the correlation coefficient,  $r$ , for each pair of stocks. (There are six of them.)
- b. Interpret the meaning of  $r$  for each pair.
- c. Is it a good idea to have all the stocks in an individual's portfolio be strongly positively correlated among each other? Explain.

**10.93** Is the daily performance of stocks and bonds correlated? The data file [stocks&bonds.xls](#) contains information concerning the closing value of the Dow Jones Industrial Average and the Vanguard Long-Term Bond Index Fund for 60 consecutive business days, ending May 30, 2006. The variables included are

- Date—Current day
- Bonds—Closing price of Vanguard Long-Term Bond Index Fund
- Stocks—Closing price of the Dow Jones Industrial Average

Source: Extracted from [finance.yahoo.com](#), May 31, 2006.

- a. Compute and interpret the correlation coefficient,  $r$ , for the variables Stocks and Bonds.
- b. At the 0.05 level of significance, is there a relationship between these two variables? Explain.

## Report Writing Exercises

**10.94** In Problems 10.85–10.89 on page 419, you developed regression models to predict monthly sales at a sporting goods store. Now, write a report based on the models you developed. Append to your report all appropriate charts and statistical information.

## Managing the Springville Herald

To ensure that as many trial subscriptions as possible are converted to regular subscriptions, the *Herald* marketing department works closely with the distribution department to accomplish a smooth initial delivery process for the trial subscription customers. To assist in this effort, the marketing department needs to accurately forecast the number of new regular subscriptions for the coming months.

A team consisting of managers from the marketing and distribution departments was convened to develop a better

method of forecasting new subscriptions. Previously, after examining new subscription data for the prior three months, a group of three managers would develop a subjective forecast of the number of new subscriptions. Lauren Hall, who was recently hired by the company to provide special skills in quantitative forecasting methods, suggested that the department look for factors that might help in predicting new subscriptions.

Members of the team found that the forecasts in the past year had been particularly inaccurate because in some

months, much more time was spent on telemarketing than in other months. In particular, in the past month, only 1,055 hours were completed because callers were busy during the first week of the month attending training sessions on the personal but formal greeting style and a new standard presentation guide. Lauren collected data (stored in the file [sh13.xls](#)) for the number of new subscriptions and hours spent on telemarketing for each month for the past two years.

## EXERCISES

SH10.1 What criticism can you make concerning the method of forecasting that involved taking the new subscriptions data for the prior three months as the basis for future projections?

SH10.2 What factors other than number of telemarketing hours spent might be useful in predicting the number of new subscriptions? Explain.

SH10.3 a. Analyze the data and develop a regression model to predict the mean number of new subscriptions for a month, based on the number of hours spent on telemarketing for new subscriptions.

b. If you expect to spend 1,200 hours on telemarketing per month, estimate the mean number of new subscriptions for the month. Indicate the assumptions on which this prediction is based. Do you think these assumptions are valid? Explain.

c. What would be the danger of predicting the number of new subscriptions for a month in which 2,000 hours were spent on telemarketing?

## Web Case

Apply your knowledge of simple linear regression in this Web Case, which extends the *Sunflowers Apparel Using Statistics* scenario from this chapter.

Leasing agents from the Triangle Mall Management Corporation have suggested that Sunflowers consider several locations in some of Triangle's newly renovated lifestyle malls that cater to shoppers with higher-than-mean disposable income. Although the locations are smaller than the typical Sunflowers location, the leasing agents argue that higher-than-mean disposable income in the surrounding community is a better predictor of higher sales than store size. The leasing agents maintain that sample data from 14 Sunflowers stores prove that this is true.

Review the leasing agents' proposal and supporting documents that describe the data at the company's Web site,

[www.prenhall.com/Springville/Triangle\\_Sunflower.htm](http://www.prenhall.com/Springville/Triangle_Sunflower.htm), (or open this Web case file from the Student CD-ROM's Web Case folder), and then answer the following:

1. Should mean disposable income be used to predict sales based on the sample of 14 Sunflowers stores?
2. Should the management of Sunflowers accept the claims of Triangle's leasing agents? Why or why not?
3. Is it possible that the mean disposable income of the surrounding area is not an important factor in leasing new locations? Explain.
4. Are there any other factors not mentioned by the leasing agents that might be relevant to the store leasing decision?

## REFERENCES

1. Anscombe, F. J., "Graphs in Statistical Analysis," *The American Statistician* 27 (1973): 17–21.
2. Hoaglin, D. C., and R. Welsch, "The Hat Matrix in Regression and ANOVA," *The American Statistician* 32 (1978): 17–22.
3. Hocking, R. R., "Developments in Linear Regression Methodology: 1959–1982," *Technometrics* 25 (1983): 219–250.
4. Kutner, M. H., C. J. Nachtsheim, J. Neter, and W. Li, *Applied Linear Statistical Models*, 5th ed. (New York: McGraw-Hill/Irwin, 2005).
5. *Microsoft Excel 2007* (Redmond, WA: Microsoft Corp., 2007).

# Excel Companion

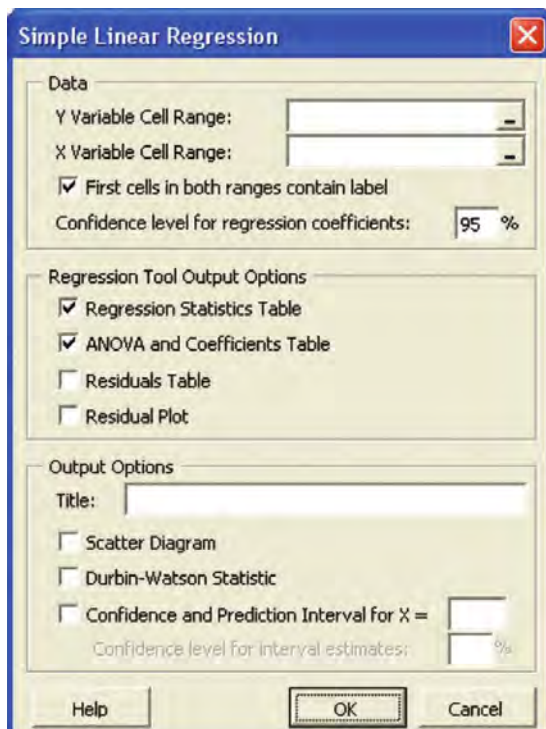
## to Chapter 10

### E10.1 PERFORMING SIMPLE LINEAR REGRESSION ANALYSES

You perform a simple linear regression analysis by either using the PHStat2 Simple Linear Regression procedure or by using the ToolPak Regression procedure.

#### Using PHStat2 Simple Linear Regression

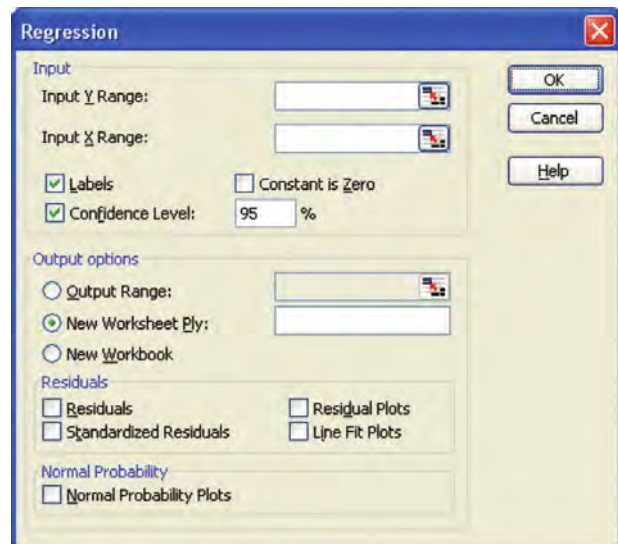
Open to the worksheet that contains the data for the regression analysis. Select **PHStat** → **Regression** → **Simple Linear Regression**. In the procedure's dialog box (shown below), enter the cell range of the *Y* variable as the **Y Variable Cell Range** and the cell range of the *X* variable as the **X Variable Cell Range**. Click **First cells in both ranges contain label** and enter a value for the **Confidence level for regression coefficients**. Click the **Regression Statistics Table** and the **ANOVA and Coefficients Table** Regression Tool Output Options, enter a title as the **Title**, and click **OK**.



PHStat2 performs the regression analysis, using the ToolPak Regression procedure. Therefore, the worksheet produced does *not* dynamically change if you change your data. (Rerun the procedure to create revised results.) The three Output Options available in the PHStat2 dialog box enhance the ToolPak procedure and are explained in Sections E1 0.2, E1 0.4, and E1 0.5.

#### Using ToolPak Regression

Open to the worksheet that contains the data for the regression analysis. Select **Tools** → **Data Analysis**, select **Regression** from the Data Analysis list, and click **OK**. In the procedure's dialog box (shown below), enter the cell range of the *Y* variable data as the **Input Y Range** and enter the cell range of the *X* variable data as the **Input X Range**. Click **Labels**, click **Confidence Level** and enter a value in its box, and then click **OK**. Results appear on a new worksheet.



### E10.2 CREATING SCATTER PLOTS AND ADDING A PREDICTION LINE

You use Excel charting features to create a scatter plot and add a prediction line to that plot. If you select the **Scatter Diagram** output option of the PHStat2 Simple Linear Regression procedure (see Section E1 0.1), you can skip to the “Adding a Prediction Line” section that applies to the Excel version you use.

## Creating a Scatter Plot

Use either the Section E2.12 instructions to create a scatter plot (see page 93) or use the Section E1 0.1 instructions in “Using PHStat2 Simple Linear Regression”, but clicking **Scatter Diagram** before you click **OK**.

## Adding a Prediction Line (97–2003)

Open to the chart sheet that contains your scatter plot and select **Chart** → **Add Trendline**. In the Add Trendline dialog box (see Figure E10.1), click the **Type** tab and then click **Linear**. Click the **Options** tab and select the **Automatic** option. Click **Display equation on chart** and **Display R-squared value on chart** and then click **OK**. If you have included a label as part of your data range, you will see that label displayed in place of **Series1** in this dialog box.

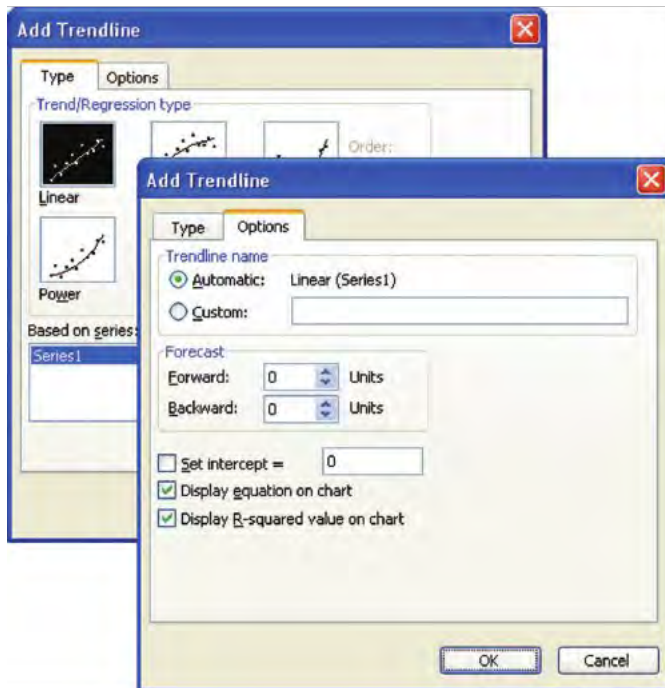


FIGURE E10.1 Add Trendline dialog box (97–2003)

## Adding a Prediction Line (2007)

Open to the chart sheet that contains your scatter plot and select **Layout** → **Trendline** and in the Trendline gallery, select **More Trendline Options**. In the Trendline Options panel of the Format Trendline dialog box (see Figure E10.2), select the **Linear** option, click **Display equation on chart** and **Display R-squared value on chart**, and click **Close**.

## Relocating an X Axis

If there are  $Y$  values on a residual plot or scatter plot that are less than zero, Microsoft Excel places the  $X$  axis at the point  $Y = 0$ , possibly obscuring some of the data points. To

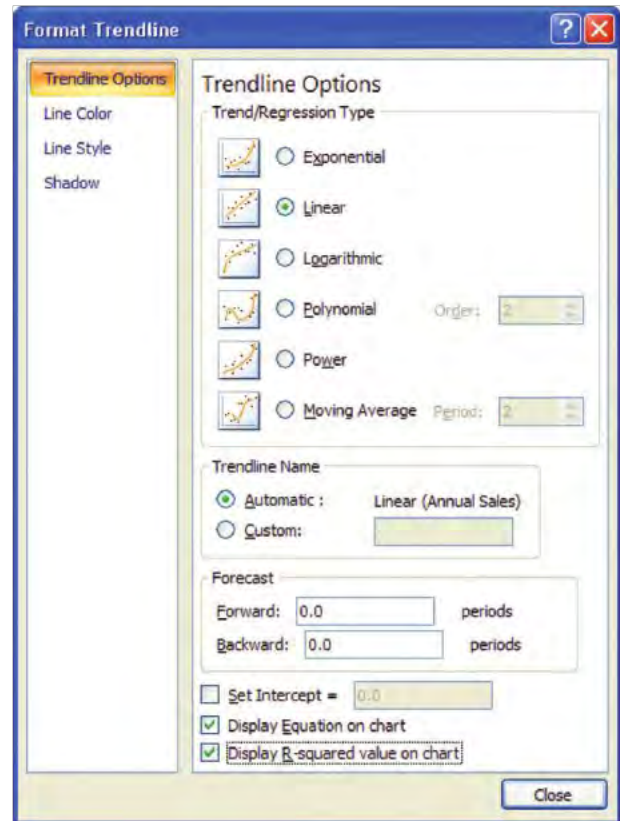


FIGURE E10.2 Format Trendline dialog box (2007)

relocate the  $X$  axis to the bottom of the chart, open to the chart, right-click the  $Y$  axis and select **Format Axis** from the shortcut menu.

If you use Excel 97–2003, select the **Scale** tab in the Format Axis dialog box (see Figure E10.3), and enter the value found in the **Minimum** box (-6 in Figure E10.3) as the **Value (X) axis Crosses at** value and click **OK**. (As you enter this value, the check box for this entry is cleared automatically.)

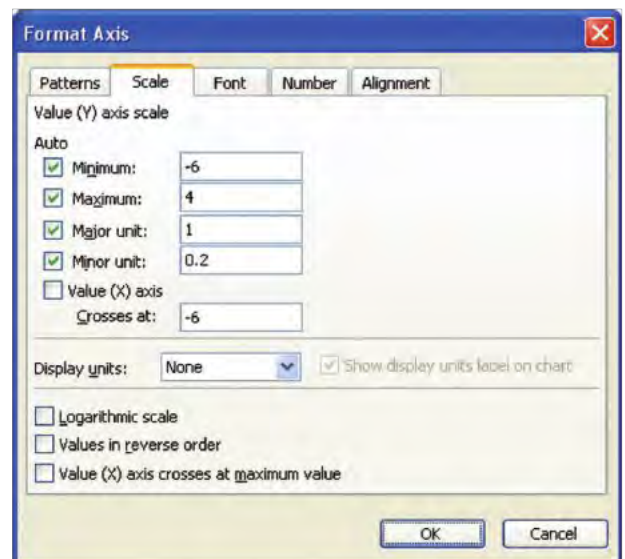


FIGURE E10.3 Format Axis dialog box (97–2003)

If you use Excel 2007, in the Axis Options panel of the Format Axis dialog box (see Figure E10.4), select the **Axis value** option, change its default value of 0.0 (shown in Figure E10.4) to a value less than the minimum *Y* value, and click **Close**.

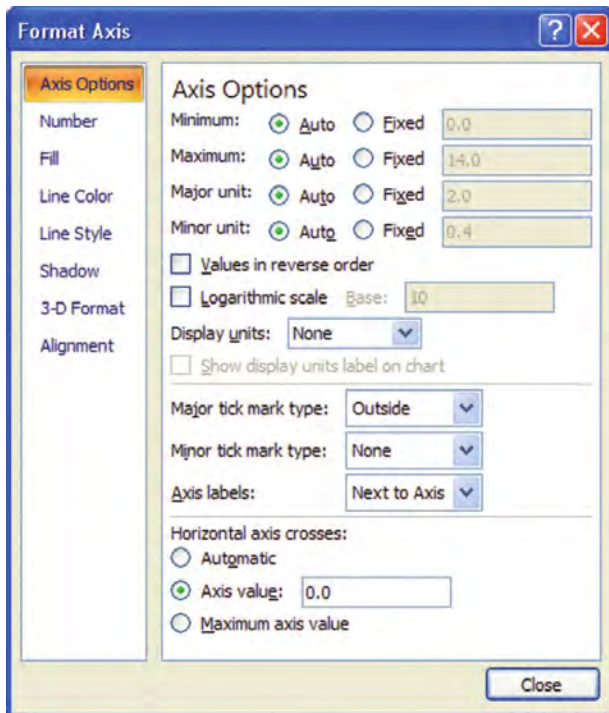


FIGURE E10.4 Format Axis dialog box (2007)

### E10.3 PERFORMING RESIDUAL ANALYSES

You modify the procedures of Section E10.1 to perform a residual analysis. If you use the PHStat2 Simple Linear Regression procedure, click all the Regression Tool output options (**Regression Statistics Table**, **ANOVA and Coefficients Table**, **Residuals Table**, and **Residual Plot**). If you use the ToolPak Regression procedure, click **Residuals** and **Residual Plots** before clicking **OK**. If you need to relocate an *X* axis to the bottom of a residual plot, review the “Relocating an *X* Axis” part of Section E10.2.

### E10.4 COMPUTING THE DURBIN-WATSON STATISTIC

You compute the Durbin-Watson Statistic by either using the PHStat2 Simple Linear Regression procedure or by using a several-step process that uses the [Durbin-Watson.xls](#) workbook.

#### Using PHStat2 Simple Linear Regression

Use the Section E10.1 instructions in “Using PHStat2 Simple Linear Regression,” but clicking **Durbin-Watson Statistic** before you click **OK**. Choosing the Durbin-

Watson Statistic causes PHStat2 to create a residuals table, even if you did not check the **Residuals Table** Regression Tool output option.

The Durbin-Watson Statistic output option creates a new **Durbin-Watson** worksheet similar to the one shown in Figure 10.16 on page 394. This worksheet references cells in the regression results worksheet that is also created by the procedure. If you delete the regression results worksheet, the Durbin-Watson worksheet displays an error message.

### Using Durbin-Watson.xls

Open to the **DurbinWatson** worksheet of the [Durbin-Watson.xls](#) workbook. This worksheet (see Figure 10.16 on page 394) uses the **SUMXMY2** (*cell range 1*, *cell range 2*) function in cell B3 to compute the sum of squared difference of the residuals, and the **SUMSQ** (*residuals cell range*) function in cell B4 to compute the sum of squared residuals for the Section 10.6 package delivery store example.

By setting *cell range 1* to the cell range of the first residual through the second-to-last residual and *cell range 2* to the cell range of the second residual through the last *residual*, you can get SUMXMY2 to compute the squared difference between two successive residuals, which is the numerator term of Equation (10.15). Because residuals appear in a regression results worksheet, cell references used in the SUMXMY2 function must refer to the regression results worksheet by name.

In the Durbin-Watson workbook, the **SLR** worksheet contains the simple linear regression analysis for the Section 10.6 package delivery example. The residuals appear in the cell range C25:C39. Therefore, cell range 1 is set to **SLR!C25:C38**, and cell range 2 is set to **SLR!C26:C39**. This makes the cell B3 formula =SUMXMY2(SLR!C26:C39, SLR!C25:C38). The cell B4 formula, which also must refer to the SLR worksheet, is =SUMSQ(SLR!C25:C39).

To adapt the Durbin-Watson workbook to other problems, first create a simple linear regression results worksheet that contains residual output and copy that worksheet to the Durbin-Watson workbook. Then open to the **Durbin-Watson** worksheet and edit the formulas in cells B3 and B4 so that they refer to the correct cell ranges on your regression worksheet. Finally, delete the no-longer-needed SLR worksheet.

### E10.5 ESTIMATING THE MEAN OF Y AND PREDICTING Y VALUES

You compute a confidence interval estimate for the mean response and the prediction interval for an individual response either by selecting the PHStat2 Simple Linear Regression procedure or by making entries in the [CIEandPIforSLR.xls](#) workbook.



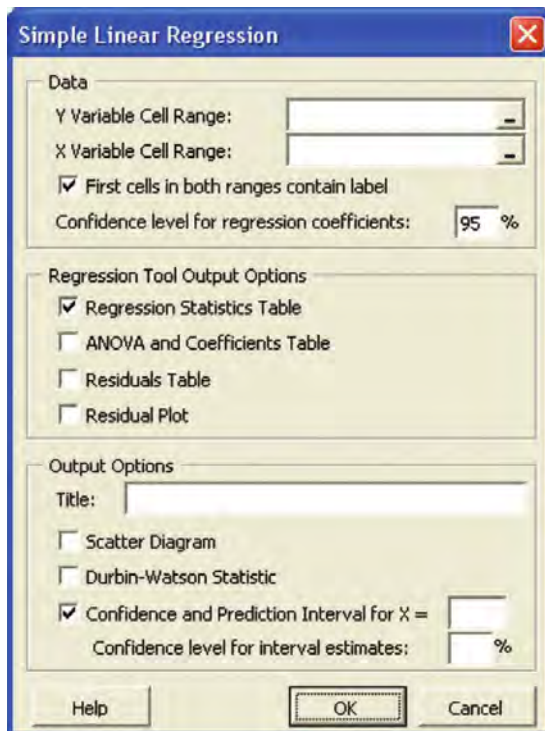
**FIGURE E10.5**DataCopy worksheet  
(first six rows)

	A	B	C	D	E	F
1	<b>Square Feet</b>	<b>Annual Sales</b>	$(X-X\bar{A})^2$			
2	1.7	3.7	1.4919		Sample Size	14
3	1.6	3.9	1.7462		Sample Mean	2.9214
4	2.8	6.7	0.0147		Sum of Squared Difference	37.9236
5	5.6	9.5	7.1747		Predicted Y (YHat)	7.6439
6	1.3	3.4	2.6290			

=COUNT(B:B)  
 =AVERAGE(A:A)  
 =SUM(C:C)  
 =TREND(B2:B15, A2:A15, CIEandPI!B4)

## Using PHStat2 Simple Linear Regression

Use the Section E10.1 instructions in “Using PHStat2 Simple Linear Regression”, but before you click **OK**, click **Confidence and Prediction Interval for X =** and enter an  $X$  value in its box (see below). Then enter a value for the **Confidence level for interval estimates** and click **OK**.



PHStat2 places the confidence interval estimate and prediction interval on a new worksheet similar to the one shown in Figure 10.21 on page 407. (PHStat2 also creates a DataCopy worksheet that is discussed in the next part of this section.)

## Using CIEandPIforSLR.xls

Open to the **CIEandPI** worksheet of the **CIEandPIforSLR.xls** workbook. This worksheet (shown in Figure 10.21 on page 407) uses the function **TINV(1-confidence level, degrees of freedom)** to determine the  $t$  value and compute the confidence interval estimate and prediction interval for the Section 10.8 Sunflower's Apparel example.

Cells B8, B11, B12, and B15 contain formulas that reference individual cells on a **DataCopy** worksheet. This worksheet, the first six rows of which are shown in Figure E10.5, contains a copy of the regression data in columns A and B and a formula in column C that squares the difference between each  $X$  and  $\bar{X}$ . The worksheet also computes the sample size, the sample mean, the sum of the squared differences [ $SSX$  in Equation (10.20) on page 404], and the predicted  $Y$  value in cells F2, F3, F4, and F5.

The cell F5 formula uses the function **TREND** (*Y variable cell range, X variable cell range, X value*) to calculate the predicted  $Y$  value. Because the formula uses the  $X$  value that has been entered on the **CIEandPI** worksheet, the  $X$  value in the cell F5 formula is set to **CIEandPI!B4**. Because the **DataCopy** and **CIEandPI** worksheets reference each other, you should consider these worksheets a matched pair that should not be broken up.

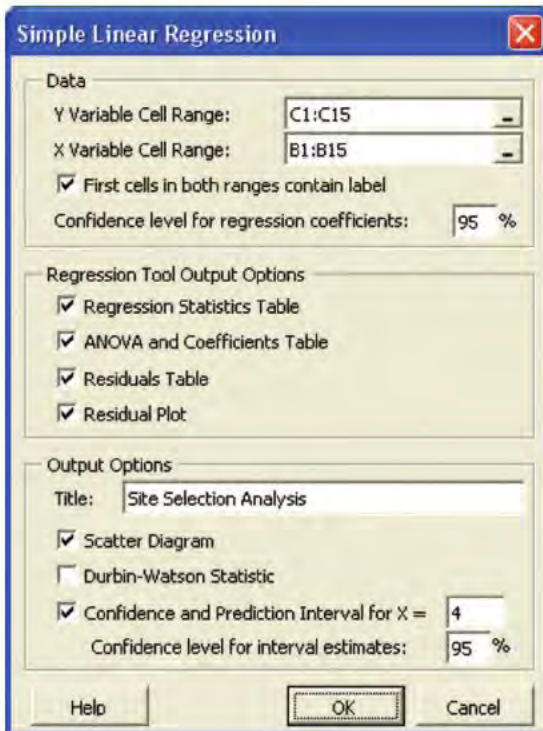
To adapt these worksheets to other problems, first create a simple linear regression results worksheet. Then, transfer the standard error value, always found in the regression results worksheet cell B7, to cell **B13** of the **CIEandPI** worksheet. Change, as is necessary, the  $X$  Value and the confidence level in cells B4 and B5 of the **CIEandPI** worksheet. Next, open to the **DataCopy** worksheet, and if your sample size is not 14, follow the instructions found in the worksheet. Enter the problem's  $X$  values in column A and  $Y$  values in column B. Finally, return to the **CIEandPI** worksheet to examine its updated results.

## E10.6 EXAMPLE: SUNFLOWERS APPAREL DATA

This section shows you how to use PHStat2 or Basic Excel to perform a regression analysis for Sunflowers Apparel using the square footage and annual sales data stored in the **site.xls** workbook.

## Using PHStat2

Open to the **Data** worksheet of the **SITE.xls** workbook. Select **PHStat** → **Regression** → **Simple Linear Regression**. In the procedure's dialog box (see Figure E10.6), enter **C1:C15** as the **Y Variable Cell Range** and **B1:B15** as the **X Variable Cell Range**. Click **First cells in both ranges**



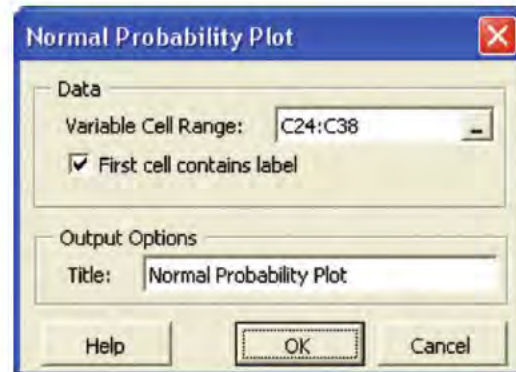
**FIGURE E10.6** Completed Simple Linear Regression dialog box

contain label and enter a value for the **Confidence level for regression coefficients**. Click the **Regression Statistics Table**, **ANOVA and Coefficients Table**, **Residuals Table**, and **Residual Plot** Regression Tool Output Options. Enter **Site Selection Analysis** as the **Title** and click **Scatter Diagram**. Click **Confidence and Prediction Interval for X=** and enter 4 in its box. Enter 95 in the **Confidence level for interval estimates** box. Click **OK** to execute the procedure.

To evaluate the assumption of linearity, you review the **Residual Plot for X1** chart sheet. Note that there is no apparent pattern or relationship between the residuals and  $X$  variable.

To evaluate the normality assumption, create a normal probability plot. With your workbook open to the **SLR** worksheet, select **PHStat** → **Probability & Prob. Distributions** → **Normal Probability Plot**. In the procedure's dialog box (see Figure E10.7), enter **C24:C38** as the **Variable Cell Range** and click **First cell contains label**. Enter **Normal Probability Plot** as the **Title** and click **OK**. In the **NormalPlot** chart sheet, observe that the data do not appear to depart substantially from a normal distribution.

To evaluate the assumption of equal variances, review the **Residual Plot for X1** chart sheet. Note that there do not appear to be major differences in the variability of the residuals.

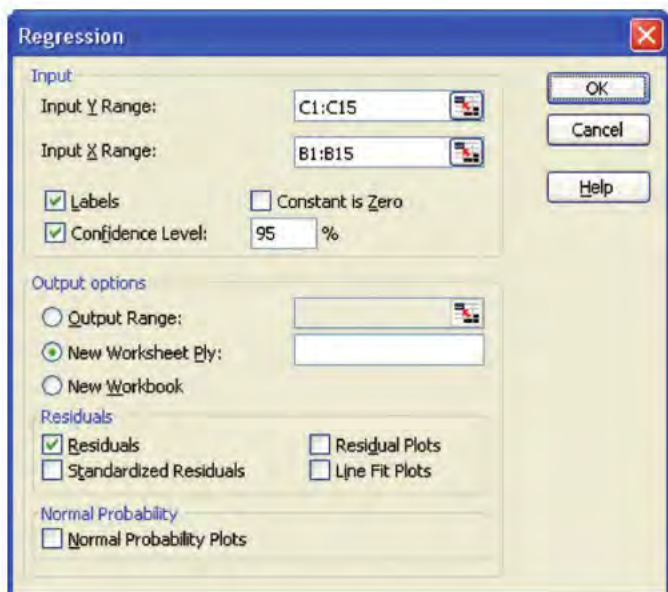


**FIGURE E10.7** Completed Normal Probability Plot dialog box

You conclude that all assumptions are valid and that you can use this simple linear regression model for the Sunflowers Apparel data. You can now open to the **SLR** worksheet to view the details of the analysis or open to the **Estimate** worksheet to make inferences about the mean of  $Y$  and the prediction of individual values of  $Y$ .

## Using Basic Excel

Open to the **Data** worksheet of the **SITE.xls** workbook. Select **Tools** → **Data Analysis (97–2003)** or **Data** → **Data Analysis (2007)**. Select **Regression** from the Data Analysis list, and click **OK**. In the procedure's dialog box (see Figure E10.8), enter **C1:C15** as the **Input Y Range** and enter **B1:B15** as the **Input X Range**. Click **Labels**, click **Confidence Level** and enter 95 in its box, and click **Residuals**. Click **OK** to execute the procedure.



**FIGURE E10.8** Completed Regression dialog box

To evaluate the assumption of linearity, you plot the residuals against the square feet (independent) variable. To simplify creating this plot, open to the Data worksheet and copy the square feet cell range B1:B15 to cell E1. Then copy the cell range of the residuals, C24:C38 on the SLR worksheet, to cell F1 of the Data worksheet. With your workbook open to the Data worksheet, use the Section E10.2 instructions on pages 422–424 to create a scatter plot. (Use **E1:F15** as the **Data range** (Excel 97–2003) or as the cell range of the  $X$  and  $Y$  variables (Excel 2007) when creating the scatter plot.) Review the scatter plot. Observe that there is no apparent pattern or relationship between the residuals and  $X$  variable. You conclude that the linearity assumption holds.

You now evaluate the normality assumption by creating a normal probability plot. Create a **Plot** worksheet, using the model worksheet in the **NPP.xls** workbook as your guide. In a new worksheet, enter **Rank** in cell A1 and then enter the series 1 through 14 in cells A2:A15. Enter **Proportion** in cell B1 and enter the formula  $=A2/15$  in cell B2. Next, enter **Z Value** in cell C1 and the formula  $=\text{NORMSINV}(B2)$  in

cell C2. Copy the residuals (including their column heading) to the cell range D1:D15. Select the formulas in cell range B2:C2 and copy them down through row 15. Open to the probability plot and observe that the data do not appear to depart substantially from a normal distribution.

To evaluate the assumption of equal variance, return to the scatter plot of the residuals and the  $X$  variable that you already developed. Observe that there do not appear to be major differences in the variability of the residuals.

You conclude that all assumptions are valid and that you can use this simple linear regression model for the Sunflowers Apparel data. You can now evaluate the details of the regression results worksheet. If you are interested in making inferences about the mean of  $Y$  and the prediction of individual values of  $Y$ , open the **CIEandPIforSLR.xls** workbook. (Usually, you would have to first make adjustments to the **DataCopy** worksheet, as discussed in Section E1 0.5, but this workbook already contains the entries for the Sunflowers Apparel analysis.) Open to the **CIEandPI** worksheet to make inferences about the mean of  $Y$  and the prediction of individual values of  $Y$ .



# CHAPTER 11

## Introduction to Multiple Regression

### USING STATISTICS @ OmniFoods

#### 11.1 DEVELOPING A MULTIPLE REGRESSION MODEL

Interpreting the Regression Coefficients  
Predicting the Dependent Variable  $Y$

#### 11.2 $r^2$ , ADJUSTED $r^2$ , AND THE OVERALL $F$ TEST

Coefficient of Multiple Determination  
Adjusted  $r^2$   
Test for the Significance of the Overall Multiple Regression Model

#### 11.3 RESIDUAL ANALYSIS FOR THE MULTIPLE REGRESSION MODEL

#### 11.4 INFERENCES CONCERNING THE POPULATION REGRESSION COEFFICIENTS

Tests of Hypothesis  
Confidence Interval Estimation

#### 11.5 TESTING PORTIONS OF THE MULTIPLE REGRESSION MODEL

Coefficients of Partial Determination

#### 11.6 USING DUMMY VARIABLES AND INTERACTION TERMS IN REGRESSION MODELS

Interactions

#### EXCEL COMPANION TO CHAPTER 11

- E11.1 Performing Multiple Regression Analyses
- E11.2 Creating Multiple Regression Residual Plots
- E11.3 Computing the Confidence Interval  
Estimate of the Mean and the Prediction Interval
- E11.4 Computing the Coefficients of Partial Determination
- E11.5 Creating Dummy Variables
- E11.6 Creating Interaction Terms

### LEARNING OBJECTIVES

In this chapter, you learn:

- How to develop a multiple regression model
- How to interpret the regression coefficients
- How to determine which independent variables to include in the regression model
- How to determine which independent variables are most important in predicting a dependent variable
- How to use categorical variables in a regression model

# Using Statistics @ OmniFoods



You are the marketing manager for OmniFoods, a large food products company. The company is planning a nationwide introduction of OmniPower, a new high-energy bar. Although originally marketed to runners, mountain climbers, and other athletes, high-energy bars are now popular with the general public. OmniFoods is anxious to capture a share of this thriving market.

Because the marketplace already contains several successful energy bars, you want to maximize the effect of your marketing plans. In particular, you need to determine the effect that price and in-store promotions will have on the sales of OmniPower before marketing the bar nationwide. You plan to use a sample of 34 stores in a supermarket chain for a test-market study of OmniPower sales. How can you extend the linear regression methods discussed in Chapter 10 to incorporate the effects of price *and* promotion into the same model? How can you use this model to improve the success of the nationwide introduction of OmniPower?

Chapter 10 focused on simple linear regression models that use *one* numerical independent variable,  $X$ , to predict the value of a numerical dependent variable,  $Y$ . Often you can make better predictions by using *more than one* independent variable. This chapter introduces you to **multiple regression models** that use two or more independent variables to predict the value of a dependent variable.

## 11.1 DEVELOPING A MULTIPLE REGRESSION MODEL

A sample of 34 stores in a supermarket chain is selected for a test-market study of OmniPower. All the stores selected have approximately the same monthly sales volume. Two independent variables are considered here—the price of an OmniPower bar, as measured in cents ( $X_1$ ), and the monthly budget for in-store promotional expenditures, measured in dollars ( $X_2$ ). In-store promotional expenditures typically include signs and displays, in-store coupons, and free samples. The dependent variable  $Y$  is the number of OmniPower bars sold in a month. Table 11.1 presents the results (stored in the file [omni.xls](#)) of the test-market study.

**TABLE 11.1**

Monthly OmniPower Sales, Price, and Promotional Expenditures

Store	Sales	Price	Promotion	Store	Sales	Price	Promotion
1	4,141	59	200	18	2,730	79	400
2	3,842	59	200	19	2,618	79	400
3	3,056	59	200	20	4,421	79	400
4	3,519	59	200	21	4,113	79	600
5	4,226	59	400	22	3,746	79	600
6	4,630	59	400	23	3,532	79	600
7	3,507	59	400	24	3,825	79	600
8	3,754	59	400	25	1,096	99	200
9	5,000	59	600	26	761	99	200
10	5,120	59	600	27	2,088	99	200
11	4,011	59	600	28	820	99	200
12	5,015	59	600	29	2,114	99	400
13	1,916	79	200	30	1,882	99	400
14	675	79	200	31	2,159	99	400
15	3,636	79	200	32	1,602	99	400
16	3,224	79	200	33	3,354	99	600
17	2,295	79	400	34	2,927	99	600

## Interpreting the Regression Coefficients

When there are several independent variables, you can extend the simple linear regression model of Equation (10.1) on page 371 by assuming a linear relationship between each independent variable and the dependent variable. For example, with  $k$  independent variables, the multiple regression model is expressed in Equation (11.1).

### MULTIPLE REGRESSION MODEL WITH $k$ INDEPENDENT VARIABLES

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \beta_3 X_{3i} + \cdots + \beta_k X_{ki} + \varepsilon_i \quad (11.1)$$

where

$\beta_0 = Y$  intercept

$\beta_1 =$  slope of  $Y$  with variable  $X_1$ , holding variables  $X_2, X_3, \dots, X_k$  constant

$\beta_2 =$  slope of  $Y$  with variable  $X_2$ , holding variables  $X_1, X_3, \dots, X_k$  constant

$\beta_3 =$  slope of  $Y$  with variable  $X_3$ , holding variables  $X_1, X_2, X_4, \dots, X_k$  constant

.

.

.

$\beta_k =$  slope of  $Y$  with variable  $X_k$ , holding variables  $X_1, X_2, X_3, \dots, X_{k-1}$  constant

$\varepsilon_i =$  random error in  $Y$  for observation  $i$

Equation (11.2) defines the multiple regression model with two independent variables.

### MULTIPLE REGRESSION MODEL WITH TWO INDEPENDENT VARIABLES

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \varepsilon_i \quad (11.2)$$

where

$\beta_0 = Y$  intercept

$\beta_1 =$  slope of  $Y$  with variable  $X_1$ , holding variable  $X_2$  constant

$\beta_2 =$  slope of  $Y$  with variable  $X_2$ , holding variable  $X_1$  constant

$\varepsilon_i =$  random error in  $Y$  for observation  $i$

Compare the multiple regression model to the simple linear regression model [Equation (10.1) on page 371]:

$$Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i$$

In the simple linear regression model, the slope,  $\beta_1$ , represents the change in the mean of  $Y$  per unit change in  $X$  and does not take into account any other variables. In the multiple regression model with two independent variables [Equation (11.2)], the slope,  $\beta_1$ , represents the change in the mean of  $Y$  per unit change in  $X_1$ , taking into account the effect of  $X_2$ .

As in the case of simple linear regression, you use the sample regression coefficients ( $b_0$ ,  $b_1$ , and  $b_2$ ) as estimates of the population parameters ( $\beta_0$ ,  $\beta_1$ , and  $\beta_2$ ). Equation (11.3) defines the regression equation for a multiple regression model with two independent variables.

### MULTIPLE REGRESSION EQUATION WITH TWO INDEPENDENT VARIABLES

$$\hat{Y}_i = b_0 + b_1 X_{1i} + b_2 X_{2i} \quad (11.3)$$

You can use Microsoft Excel to compute the values of the three regression coefficients for the OmniPower sales data (see Figure 11.1).

**FIGURE 11.1**

Partial Microsoft Excel results for OmniPower sales data



See Section E11.1 to create this.

	A	B	C	D	E	F	G
1	OmniPower Sales Analysis						
2							
3	Regression Statistics						
4	Multiple R	0.8705					
5	R Square	0.7577					
6	Adjusted R Square	0.7421					
7	Standard Error	638.0653					
8	Observations	34					
9							
10	ANOVA						
11		df	SS	MS	F	Significance F	
12	Regression	2	39472730.77	19736365.387	48.4771	2.86258E-10	
13	Residual	31	12620946.67	407127.312			
14	Total	33	52093677.44				
15							
16		Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
17	Intercept	5837.5208	628.1502	9.2932	1.791E-10	4556.3992	7118.6423
18	Price	-53.2173	6.8522	-7.7664	9.200E-09	.67.1925	-39.2421
19	Promotion	3.6131	0.6852	5.2728	9.822E-06	2.2155	5.0106

From Figure 11.1, the computed values of the regression coefficients are

$$b_0 = 5,837.5208 \quad b_1 = -53.2173 \quad b_2 = 3.6131$$

Therefore, the multiple regression equation is

$$\hat{Y}_i = 5,837.5208 - 53.2173 X_{1i} + 3.6131 X_{2i}$$

where

$\hat{Y}_i$  = predicted monthly sales of OmniPower bars for store  $i$

$X_{1i}$  = price of OmniPower bar (in cents) for store  $i$

$X_{2i}$  = monthly in-store promotional expenditures (in dollars) for store  $i$

The sample  $Y$  intercept ( $b_0 = 5,837.5208$ ) estimates the number of OmniPower bars sold in a month if the price is \$0.00 and the total amount spent on promotional expenditures is also \$0.00. Because these values of price and promotion are outside the range of price and promotion used in the test-market study, and are nonsensical, the value of  $b_0$  has no practical interpretation.

The slope of price with OmniPower sales ( $b_1 = -53.2173$ ) indicates that, for a given amount of monthly promotional expenditures, the mean sales of OmniPower are estimated to decrease by 53.2173 bars per month for each 1-cent increase in the price. The slope of monthly promotional expenditures with OmniPower sales ( $b_2 = 3.6131$ ) indicates that, for a given price, the mean sales of OmniPower are estimated to increase by 3.6131 bars for each additional \$1 spent on promotions. These estimates allow you to better understand the likely effect that price and promotion decisions will have in the marketplace. For example, a 10-cent decrease in price is estimated to increase mean sales by 532.173 bars, with a fixed amount of monthly promotional expenditures. A \$100 increase in promotional expenditures is estimated to increase mean sales by 361.31 bars, for a given price.

Regression coefficients in multiple regression are called **net regression coefficients**; they estimate the mean change in  $Y$  per unit change in a particular  $X$ , *holding constant the effect of the other  $X$  variables*. For example, in the study of OmniPower bar sales, for a store with a given amount of promotional expenditures, the mean sales are estimated to decrease by 53.2173 bars



per month for each 1-cent increase in the price of an OmniPower bar. Another way to interpret this “net effect” is to think of two stores with an equal amount of promotional expenditures. If the first store charges 1 cent more than the other store, the net effect of this difference is that the first store is predicted to sell 53.21 73 fewer bars per month than the second store. To interpret the net effect of promotional expenditures, you can consider two stores that are charging the same price. If the first store spends \$1 more on promotional expenditures, the net effect of this difference is that the first store is predicted to sell 3.61 31 more bars per month than the second store.

### Predicting the Dependent Variable Y

You can use the multiple regression equation computed by Microsoft Excel to predict values of the dependent variable. For example, what is the predicted sales for a store charging 79 cents during a month in which promotional expenditures are \$400? Using the multiple regression equation,

$$\hat{Y}_i = 5,837.5208 - 53.21\ 73 X_{1i} + 3.61\ 31 X_{2i}$$

with  $X_{1i} = 79$  and  $X_{2i} = 400$ ,

$$\begin{aligned} \hat{Y}_i &= 5,837.5208 - 53.21\ 73(79) + 3.61\ 31(400) \\ &= 3,078.57 \end{aligned}$$

Thus, your sales prediction for stores charging 79 cents and spending \$400 in promotional expenditures is 3,078.57 OmniPower bars per month.

After you have predicted  $Y$  and done a residual analysis (see Section 11.3), the next step often involves a confidence interval estimate of the mean response and a prediction interval for an individual response. The computation of these intervals is too complex to do by hand, and you should use Microsoft Excel to perform the calculations. Figure 11.2 illustrates Microsoft Excel results.

**FIGURE 11.2**  
Microsoft Excel confidence interval estimate and prediction interval for the OmniPower sales data



See Section E11.3 to create this.

	A	B	C	D
1	<b>Confidence Interval Estimate and Prediction Interval</b>			
2				
3	<b>Data</b>			
4	<b>Confidence Level</b>	95%		
5		1		
6	<b>Price given value</b>	79		
7	<b>Promotion given value</b>	400		
8				
9	XX	34	2646	13200
10		2646	214674	1018800
11		13200	1018800	6000000
12				
13	Inverse of XX	0.969163	-0.00941	-0.00053
14		-0.009408	0.000115	1.12E-06
15		-0.000535	1.12E-06	1.15E-06
16				
17	XG times Inverse of XX	0.012054	0.000149	1.49E-05
18				
19	[XG times Inverse of XX] times XG	0.029762	=MMULT(B17:D17, B5:B7)	
20	t Statistic	2.0395	=TINV(1 - B4, MR!B13)	
21	<b>Predicted Y (Yhat)</b>	3078.57	=(-MMULT(TRANPOSE(B5:B7), MR!B17:B19)) 17:B19)	
22				
23	<b>For Average Predicted Y (Yhat)</b>			
24	<b>Interval Half Width</b>	224.50	=B20 * SQRT(B19) * MR!B7	
25	<b>Confidence Interval Lower Limit</b>	2854.07	=B21 - B24	
26	<b>Confidence Interval Upper Limit</b>	3303.08	=B21 + B24	
27				
28	<b>For Individual Response Y</b>			
29	<b>Interval Half Width</b>	1320.57	=B20 * SQRT(1 + B19) * MR!B7	
30	<b>Prediction Interval Lower Limit</b>	1758.01	=B21 - B29	
31	<b>Prediction Interval Upper Limit</b>	4399.14	=B21 + B29	

The 95% confidence interval estimate of the mean OmniPower sales for all stores charging 79 cents and spending \$400 in promotional expenditures is 2,854.07 to 3,303.08 bars. The prediction interval for an individual store is 1,758.01 to 4,399.14 bars.

## PROBLEMS FOR SECTION 11.1

### Learning the Basics

**PH Grade ASSIST** **11.1** For this problem, use the following multiple regression equation:

$$\hat{Y}_i = 10 + 5X_{1i} + 3X_{2i}$$

- Interpret the meaning of the slopes.
- Interpret the meaning of the  $Y$  intercept.

**PH Grade ASSIST** **11.2** For this problem, use the following multiple regression equation:

$$\hat{Y}_i = 50 + 2X_{1i} + 7X_{2i}$$

- Interpret the meaning of the slopes.
- Interpret the meaning of the  $Y$  intercept.

### Applying the Concepts

**PH Grade ASSIST** **11.3** A marketing analyst for a shoe manufacturer is considering the development of a new brand of running shoes. The marketing analyst wants to determine which variables to use in predicting durability (that is, the effect of long-term impact). Two independent variables under consideration are  $X_1$  (FOREIMP), a measurement of the forefoot shock-absorbing capability, and  $X_2$  (MIDSOLE), a measurement of the change in impact properties over time. The dependent variable  $Y$  is LTIMP, a measure of the shoe's durability after a repeated impact test. A random sample of 15 types of currently manufactured running shoes was selected for testing, with the following results:

ANOVA	<i>df</i>	SS	MS	<i>F</i>	Significance <i>F</i>
Regression	2	12.61020	6.30510	97.69	0.0001
Error	12	0.77453	0.06454		
Total	14	13.38473			

Variable	Coefficients	Standard Error	<i>t</i> Statistic	<i>p</i> -Value
INTERCEPT	-0.02686	0.06905	-0.39	0.7034
FOREIMP	0.79116	0.06295	12.57	0.0000
MIDSOLE	0.60484	0.07174	8.43	0.0000

- State the multiple regression equation.
- Interpret the meaning of the slopes in this problem.

**SELF Test** **11.4** A mail-order catalog business selling personal computer supplies, software, and hardware maintains a centralized warehouse. Management is currently examining the process of distribution from the warehouse and wants to study the factors that affect warehouse distribution costs. Currently, a small handling fee is added to each order, regardless of the amount of the order. Data collected over the past 24 months (stored in the file

**warecost.xls**) indicate the warehouse distribution costs (in thousands of dollars), the sales (in thousands of dollars), and the number of orders received.

- State the multiple regression equation.
- Interpret the meaning of the slopes,  $b_1$  and  $b_2$ , in this problem.
- Explain why the regression coefficient,  $b_0$ , has no practical meaning in the context of this problem.
- Predict the mean monthly warehouse distribution cost when sales are \$400,000 and the number of orders is 4,500.
- Construct a 95% confidence interval estimate for the mean monthly warehouse distribution cost when sales are \$400,000 and the number of orders is 4,500.
- Construct a 95% prediction interval for the monthly warehouse distribution cost for a particular month when sales are \$400,000 and the number of orders is 4,500.

**PH Grade ASSIST** **11.5** A consumer organization wants to develop a regression model to predict gasoline mileage (as measured by miles per gallon) based on the horsepower of the car's engine and the weight of the car, in pounds. A sample of 50 recent car models was selected, with the results recorded in the file **auto.xls**.

- State the multiple regression equation.
- Interpret the meaning of the slopes,  $b_1$  and  $b_2$ , in this problem.
- Explain why the regression coefficient,  $b_0$ , has no practical meaning in the context of this problem.
- Predict the mean miles per gallon for cars that have 60 horsepower and weigh 2,000 pounds.
- Construct a 95% confidence interval estimate for the mean miles per gallon for cars that have 60 horsepower and weigh 2,000 pounds.
- Construct a 95% prediction interval for the miles per gallon for an individual car that has 60 horsepower and weighs 2,000 pounds.

**PH Grade ASSIST** **11.6** A consumer products company wants to measure the effectiveness of different types of advertising media in the promotion of its products. Specifically, the company is interested in the effectiveness of radio advertising and newspaper advertising (including the cost of discount coupons). A sample of 22 cities with approximately equal populations is selected for study during a test period of one month. Each city is allocated a specific expenditure level both for radio advertising and for newspaper advertising. The sales of the product (in thousands of dollars) and also the levels of media expenditure (in thousands of dollars) during the test month are recorded, with the following results stored in the file **advertise.xls**:

City	Sales (\$000)	Radio Advertising (\$000)	Newspaper Advertising (\$000)
1	973	0	40
2	1,119	0	40
3	875	25	25
4	625	25	25
5	910	30	30
6	971	30	30
7	931	35	35
8	1,177	35	35
9	882	40	25
10	982	40	25
11	1,628	45	45
12	1,577	45	45
13	1,044	50	0
14	914	50	0
15	1,329	55	25
16	1,330	55	25
17	1,405	60	30
18	1,436	60	30
19	1,521	65	35
20	1,741	65	35
21	1,866	70	40
22	1,717	70	40

- State the multiple regression equation.
- Interpret the meaning of the slopes,  $b_1$  and  $b_2$ , in this problem.
- Interpret the meaning of the regression coefficient,  $b_0$ .
- Predict the mean sales for a city in which radio advertising is \$20,000 and newspaper advertising is \$20,000.
- Construct a 95% confidence interval estimate for the mean sales for cities in which radio advertising is \$20,000 and newspaper advertising is \$20,000.
- Construct a 95% prediction interval for the sales for an individual city in which radio advertising is \$20,000 and newspaper advertising is \$20,000.

**11.7** The director of broadcasting operations for a television station wants to study the issue of standby hours (that is, hours in which unionized graphic artists at the station are paid but are not actually involved in any activity). The variables in the study include

Standby hours ( $Y$ )—Total number of standby hours in a week

Total staff present ( $X_1$ )—Weekly total of people-days  
 Remote hours ( $X_2$ )—Total number of hours worked by employees at locations away from the central plant  
 The results for a period of 26 weeks are in the data file [standby.xls](#).

- State the multiple regression equation.
- Interpret the meaning of the slopes,  $b_1$  and  $b_2$ , in this problem.
- Explain why the regression coefficient,  $b_0$ , has no practical meaning in the context of this problem.
- Predict the mean standby hours for a week in which the total staff present have 310 people-days and the remote hours are 400.
- Construct a 95% confidence interval estimate for the mean standby hours for weeks in which the total staff present have 310 people-days and the remote hours are 400.
- Construct a 95% prediction interval for the standby hours for a single week in which the total staff present have 310 people-days and the remote hours are 400.

**11.8** Nassau County is located approximately 25 miles east of New York City. Until all residential property was recently reappraised, property taxes were assessed based on actual value in 1938 or when the property was built, if it was constructed after 1938. Data in the file [glencove.xls](#) include the appraised value, land area of the property in acres, and age, in years, for a sample of 30 single-family homes located in Glen Cove, a small city in Nassau County. Develop a multiple linear regression model to predict appraised value based on land area of the property and age, in years.

- State the multiple regression equation.
- Interpret the meaning of the slopes,  $b_1$  and  $b_2$ , in this problem.
- Explain why the regression coefficient,  $b_0$ , has no practical meaning in the context of this problem.
- Predict the mean appraised value for a house that has a land area of 0.25 acres and is 45 years old.
- Construct a 95% confidence interval estimate for the mean appraised value for houses that have a land area of 0.25 acres and are 45 years old.
- Construct a 95% prediction interval estimate for the appraised value for an individual house that has a land area of 0.25 acres and is 45 years old.

## 11.2 $r^2$ , ADJUSTED $r^2$ , AND THE OVERALL $F$ TEST

This section discusses three methods you can use to evaluate the overall usefulness of the multiple regression model: the coefficient of multiple determination  $r^2$ , the adjusted  $r^2$ , and the overall  $F$  test.

## Coefficient of Multiple Determination

Recall from Section 10.3 that the coefficient of determination,  $r^2$ , measures the variation in  $Y$  that is explained by the independent variable  $X$  in the simple linear regression model. In multiple regression, the **coefficient of multiple determination** represents the proportion of the variation in  $Y$  that is explained by the set of independent variables. Equation (11.4) defines the coefficient of multiple determination for a multiple regression model with two or more independent variables.

### COEFFICIENT OF MULTIPLE DETERMINATION

The coefficient of multiple determination is equal to the regression sum of squares ( $SSR$ ) divided by the total sum of squares ( $SST$ ).

$$r^2 = \frac{\text{Regression sum of squares}}{\text{Total sum of squares}} = \frac{SSR}{SST} \quad (11.4)$$

where

$SSR$  = regression sum of squares

$SST$  = total sum of squares

In the OmniPower example, from Figure 11.1 on page 432,  $SSR = 39,472,730.77$  and  $SST = 52,093,677.44$ . Thus,

$$r^2 = \frac{SSR}{SST} = \frac{39,472,730.77}{52,093,677.44} = 0.7577$$

The coefficient of multiple determination ( $r^2 = 0.7577$ ) indicates that 75.77% of the variation in sales is explained by the variation in the price and in the promotional expenditures.

## Adjusted $r^2$

When considering multiple regression models, some statisticians suggest that you should use the **adjusted  $r^2$**  to reflect both the number of independent variables in the model and the sample size. Reporting the adjusted  $r^2$  is extremely important when you are comparing two or more regression models that predict the same dependent variable but have a different number of independent variables. Equation (11.5) defines the adjusted  $r^2$ .

### ADJUSTED $r^2$

$$r_{\text{adj}}^2 = 1 - \left[ (1 - r^2) \frac{n - 1}{n - k - 1} \right] \quad (11.5)$$

where  $k$  is the number of independent variables in the regression equation.

Thus, for the OmniPower data, because  $r^2 = 0.7577$ ,  $n = 34$ , and  $k = 2$ ,

$$\begin{aligned} r_{\text{adj}}^2 &= 1 - \left[ (1 - r^2) \frac{34 - 1}{(34 - 2 - 1)} \right] \\ &= 1 - \left[ (1 - 0.7577) \frac{33}{31} \right] \\ &= 1 - 0.2579 \\ &= 0.7421 \end{aligned}$$

Hence, 74.21% of the variation in sales is explained by the multiple regression model—adjusted for number of independent variables and sample size.

## Test for the Significance of the Overall Multiple Regression Model

You use the **overall  $F$  test** to test whether there is a significant relationship between the dependent variable and the entire set of independent variables (the overall multiple regression model). Because there is more than one independent variable, you use the following null and alternative hypotheses:

$H_0$ :  $\beta_1 = \beta_2 = \dots = \beta_k = 0$  (There is no linear relationship between the dependent variable and the independent variables.)

$H_1$ : At least one  $\beta_j \neq 0$ ,  $j = 1, 2, \dots, k$  (There is a linear relationship between the dependent variable and at least one of the independent variables.)

Equation (11.6) defines the statistic for the overall  $F$  test. Table 11.2 presents the associated ANOVA summary table.

### OVERALL $F$ TEST STATISTIC

The  $F$  statistic is equal to the regression mean square ( $MSR$ ) divided by the error mean square ( $MSE$ ).

$$F = \frac{MSR}{MSE} \quad (11.6)$$

where

$F$  = test statistic from an  $F$  distribution with  $k$  and  $n - k - 1$  degrees of freedom

$k$  = number of independent variables in the regression model

**TABLE 11.2**  
ANOVA Summary Table  
for the Overall  $F$  Test

Source	Degrees of Freedom	Sum of Squares	Mean Square (Variance)	$F$
Regression	$k$	$SSR$	$MSR = \frac{SSR}{k}$	$F = \frac{MSR}{MSE}$
Error	$n - k - 1$	$SSE$	$MSE = \frac{SSE}{n - k - 1}$	
Total	$n - 1$	$SST$		

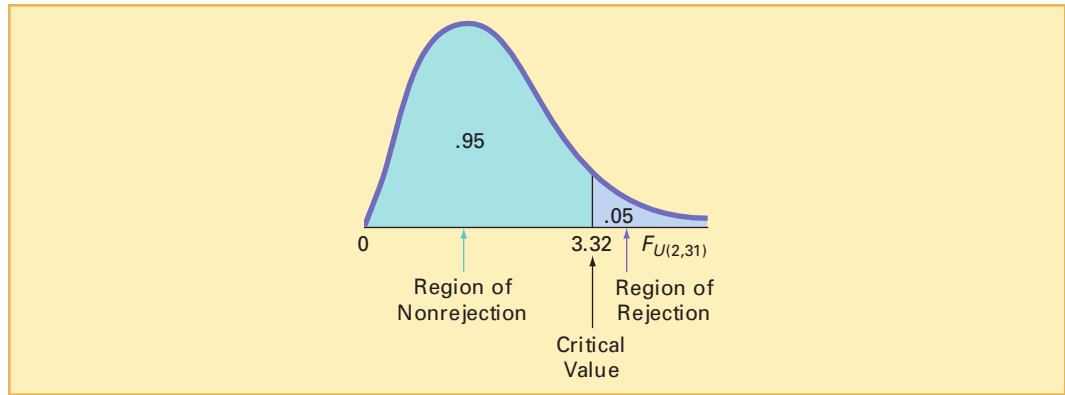
The decision rule is

Reject  $H_0$  at the  $\alpha$  level of significance if  $F > F_{U(k, n-k-1)}$ ;  
otherwise, do not reject  $H_0$ .

Using a 0.05 level of significance, the critical value of the  $F$  distribution with 2 and 31 degrees of freedom found from Table E.5 is approximately 3.32 (see Figure 11.3 on page 438). From Figure 11.1 on page 432, the  $F$  statistic given in the ANOVA summary table is 48.4771. Because  $48.4771 > 3.32$ , or because the  $p$ -value = 0.000 < 0.05, you reject  $H_0$  and conclude that at least one of the independent variables (price and/or promotional expenditures) is related to sales.

**FIGURE 11.3**

Testing for the significance of a set of regression coefficients at the 0.05 level of significance, with 2 and 31 degrees of freedom



## PROBLEMS FOR SECTION 11.2

### Learning the Basics

**PH Grade ASSIST 11.9** The following ANOVA summary table is for a multiple regression model with two independent variables:

Source	Degrees of Freedom	Sum of Squares	Mean Squares	F
Regression	2	60		
Error	18	120		
Total	20	180		

- Determine the regression mean square (*MSR*) and the mean square error (*MSE*).
- Compute the *F* statistic.
- Determine whether there is a significant relationship between *Y* and the two independent variables at the 0.05 level of significance.
- Compute the coefficient of multiple determination,  $r^2$ , and interpret its meaning.
- Compute the adjusted  $r^2$ .

**PH Grade ASSIST 11.10** The following ANOVA summary table is for a multiple regression model with two independent variables:

Source	Degrees of Freedom	Sum of Squares	Mean Squares	F
Regression	2	30		
Error	10	120		
Total	12	150		

- Determine the regression mean square (*MSR*) and the mean square error (*MSE*).
- Compute the *F* statistic.
- Determine whether there is a significant relationship between *Y* and the two independent variables at the 0.05 level of significance.
- Compute the coefficient of multiple determination,  $r^2$ , and interpret its meaning.
- Compute the adjusted  $r^2$ .

### Applying the Concepts

**11.11** Eileen M. Van Aken and Brian M. Kleiner, professors at Virginia Polytechnic Institute and State University, investigated the factors that contribute to the effectiveness of teams (“Determinants of Effectiveness for Cross-Functional Organizational Design Teams,” *Quality Management Journal*, 1997, 4, pp. 51–79). The researchers studied 34 independent variables, such as team skills, diversity, meeting frequency, and clarity in expectations. For each of the teams studied, each of the variables was given a value of 1 through 100, based on the results of interviews and survey data, where 100 represents the highest rating. The dependent variable, team performance, was also given a value of 1 through 100, with 100 representing the highest rating. Many different regression models were explored, including the following:

#### Model 1

$$\text{Team performance} = \beta_0 + \beta_1(\text{Team skills}) + \epsilon,$$

$$r_{\text{adj}}^2 = 0.68$$

#### Model 2

$$\text{Team performance} = \beta_0 + \beta_1(\text{Clarity in expectations}) + \epsilon,$$

$$r_{\text{adj}}^2 = 0.78$$

#### Model 3

$$\text{Team performance} = \beta_0 + \beta_1(\text{Team skills}) + \beta_2(\text{Clarity in expectations}) + \epsilon,$$

$$r_{\text{adj}}^2 = 0.97$$

- Interpret the adjusted  $r^2$  for each of the three models.
- Which of these three models do you think is the best predictor of team performance?

**PH Grade ASSIST** **11.12** In Problem 11.3 on page 434, you predicted the durability of a brand of running shoe, based on the forefoot shock-absorbing capability and the change in impact properties over time. The regression analysis resulted in the following ANOVA summary table:

Source	Degrees of Freedom	Sum of Squares	Mean Squares	<i>F</i>	Significance
Regression	2	12.61020	6.30510	97.69	0.0001
Error	12	0.77453	0.06454		
Total	14	13.38473			

- Determine whether there is a significant relationship between durability and the two independent variables at the 0.05 level of significance.
- Interpret the meaning of the  $p$ -value.
- Compute the coefficient of multiple determination,  $r^2$ , and interpret its meaning.
- Compute the adjusted  $r^2$ .

**PH Grade ASSIST** **11.13** In Problem 11.5 on page 434, you used horsepower and weight to predict gasoline mileage (see the [auto.xls](#) file). Using the results from that problem,

- determine whether there is a significant relationship between gasoline mileage and the two independent variables (horsepower and weight) at the 0.05 level of significance.
- interpret the meaning of the  $p$ -value.
- compute the coefficient of multiple determination,  $r^2$ , and interpret its meaning.
- compute the adjusted  $r^2$ .

**PH Grade ASSIST** **11.14** In Problem 11.4 on page 434, you used sales and number of orders to predict distribution costs at a mail-order catalog business (see the [warecost.xls](#) file). Using the results from that problem,

- determine whether there is a significant relationship between distribution costs and the two independent variables (sales and number of orders) at the 0.05 level of significance.

- interpret the meaning of the  $p$ -value.
- compute the coefficient of multiple determination,  $r^2$ , and interpret its meaning.
- compute the adjusted  $r^2$ .

**11.15** In Problem 11.7 on page 435, you used the total staff present and remote hours to predict standby hours (see the file [standby.xls](#)). Using the results from that problem,

- determine whether there is a significant relationship between standby hours and the two independent variables (total staff present and remote hours) at the 0.05 level of significance.
- interpret the meaning of the  $p$ -value.
- compute the coefficient of multiple determination,  $r^2$ , and interpret its meaning.
- compute the adjusted  $r^2$ .

**11.16** In Problem 11.6 on pages 434–435, you used radio advertising and newspaper advertising to predict sales (see the [advertise.xls](#) file). Using the results from that problem,

- determine whether there is a significant relationship between sales and the two independent variables (radio advertising and newspaper advertising) at the 0.05 level of significance.
- interpret the meaning of the  $p$ -value.
- compute the coefficient of multiple determination,  $r^2$ , and interpret its meaning.
- compute the adjusted  $r^2$ .

**11.17** In Problem 11.8 on page 435, you used the land area of a property and the age of a house to predict appraised value (see the [glencove.xls](#) file). Using the results from that problem,

- determine whether there is a significant relationship between appraised value and the two independent variables (land area of a property and age of a house) at the 0.05 level of significance.
- interpret the meaning of the  $p$ -value.
- compute the coefficient of multiple determination,  $r^2$ , and interpret its meaning.
- compute the adjusted  $r^2$ .

## 11.3 RESIDUAL ANALYSIS FOR THE MULTIPLE REGRESSION MODEL

In Section 13.5, you used residual analysis to evaluate the appropriateness of using the simple linear regression model for a set of data. For the multiple regression model with two independent variables, you need to construct and analyze the following residual plots:

- Residuals versus  $\hat{Y}_i$
- Residuals versus  $X_{1i}$
- Residuals versus  $X_{2i}$
- Residuals versus time

The first residual plot examines the pattern of residuals versus the predicted values of  $Y$ . If the residuals show a pattern for different predicted values of  $Y$ , there is evidence of a possible quadratic effect in at least one independent variable, a possible violation of the assumption of equal variance (see Figure 10.13 on page 391), and/or the need to transform the  $Y$  variable.

The second and third residual plots involve the independent variables. Patterns in the plot of the residuals versus an independent variable may indicate the existence of a quadratic effect and, therefore, indicate the need to add a quadratic independent variable to the multiple regression model. The fourth plot is used to investigate patterns in the residuals in order to validate the independence assumption when the data are collected in time order. Associated with this residual plot, as in Section 10.6, you can compute the Durbin-Watson statistic to determine the existence of positive autocorrelation among the residuals.

Figure 11.4 illustrates the Microsoft Excel residual plots for the OmniPower sales example. In Figure 11.4, there is very little or no pattern in the relationship between the residuals and the predicted value of  $Y$ , the value of  $X_1$  (price), or the value of  $X_2$  (promotional expenditures). Thus, you can conclude that the multiple regression model is appropriate for predicting sales.

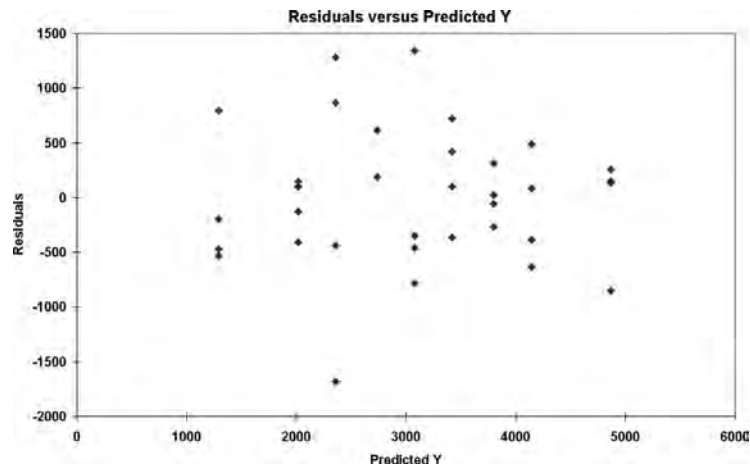
**FIGURE 11.4**

Microsoft Excel residual plots for the OmniPower sales data: Panel A, residuals versus predicted  $Y$ ; Panel B, residuals versus price; Panel C, residuals versus promotional expenditures

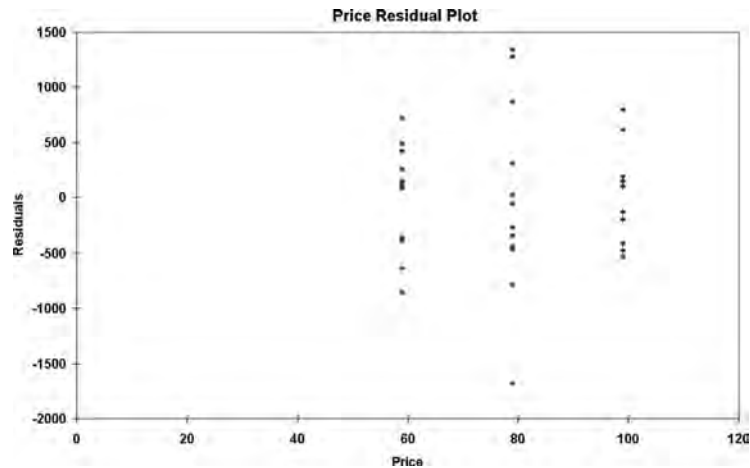


See Section E11.2 to create this.

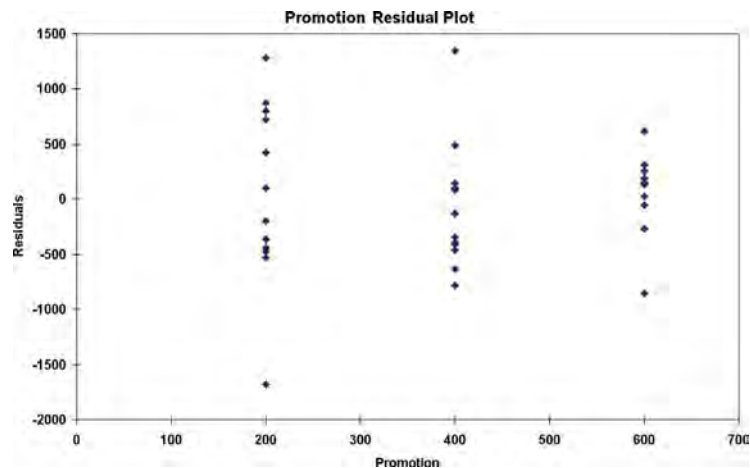
Panel A



Panel B



Panel C





## PROBLEMS FOR SECTION 11.3

### Applying the Concepts

**PH Grade ASSIST** **11.18** In Problem 11.4 on page 434, you used sales and number of orders to predict distribution costs at a mail-order catalog business (see the `warecost.xls` file).

- Perform a residual analysis on your results and determine the adequacy of the model.
- Plot the residuals against the months. Is there any evidence of a pattern in the residuals? Explain.
- Determine the Durbin-Watson statistic.
- At the 0.05 level of significance, is there evidence of positive autocorrelation in the residuals?

**PH Grade ASSIST** **11.19** In Problem 11.5 on page 434, you used horsepower and weight to predict gasoline mileage (see the `auto.xls` file). Perform a residual analysis on your results and determine the adequacy of the model.

**11.20** In Problem 11.6 on pages 434–435, you used radio advertising and newspaper advertising to predict sales (see

the `advertise.xls` file). Perform a residual analysis on your results and determine the adequacy of the model.

**11.21** In Problem 11.7 on page 435, you used the total staff present and remote hours to predict standby hours (see the `standby.xls` file).

- Perform a residual analysis on your results and determine the adequacy of the model.
- Plot the residuals against the weeks. Is there evidence of a pattern in the residuals? Explain.
- Determine the Durbin-Watson statistic.
- At the 0.05 level of significance, is there evidence of positive autocorrelation in the residuals?

**11.22** In Problem 11.8 on page 435, you used the land area of a property and the age of a house to predict appraised value (see the `glencove.xls` file). Perform a residual analysis on your results and determine the adequacy of the model.

## 11.4 INFERENCE CONCERNING THE POPULATION REGRESSION COEFFICIENTS

In Section 10.7, you tested the slope in a simple linear regression model to determine the significance of the relationship between  $X$  and  $Y$ . In addition, you constructed a confidence interval estimate of the population slope. This section extends those procedures to multiple regression.

### Tests of Hypothesis

In a simple linear regression model, to test a hypothesis concerning the population slope,  $\beta_1$ , you used Equation (10.16) on page 397:

$$t = \frac{b_1 - \beta_1}{S_{b_1}}$$

Equation (11.7) generalizes this equation for multiple regression.

#### TESTING FOR THE SLOPE IN MULTIPLE REGRESSION

$$t = \frac{b_j - \beta_j}{S_{b_j}} \quad (11.7)$$

where

$b_j$  = slope of variable  $j$  with  $Y$ , holding constant the effects of all other independent variables

$S_{b_j}$  = standard error of the regression coefficient  $b_j$

$t$  = test statistic for a  $t$  distribution with  $n - k - 1$  degrees of freedom

$k$  = number of independent variables in the regression equation

$\beta_j$  = hypothesized value of the population slope for variable  $j$  holding constant the effects of all other independent variables

To determine whether variable  $X_2$  (amount of promotional expenditures) has a significant effect on sales, taking into account the price of OmniPower bars, the null and alternative hypotheses are

$$\begin{aligned} H_0: \beta_2 &= 0 \\ H_1: \beta_2 &\neq 0 \end{aligned}$$

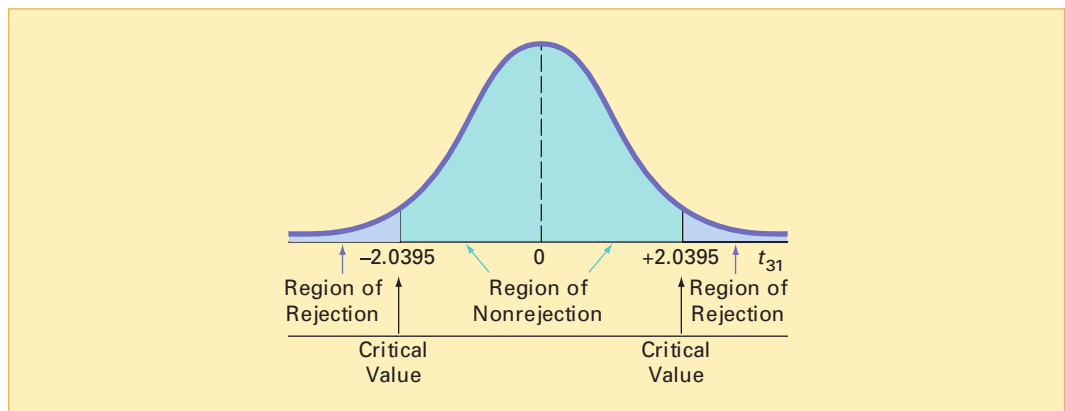
From Equation (11.7) and Figure 11.1 on page 432,

$$\begin{aligned} t &= \frac{b_2 - \beta_2}{S_{b_2}} \\ &= \frac{3.6131 - 0}{0.6852} = 5.2728 \end{aligned}$$

If you select a level of significance of 0.05, the critical values of  $t$  for 31 degrees of freedom from Table E.3 are  $-2.0395$  and  $+2.0395$  (see Figure 11.5).

**FIGURE 11.5**

Testing for significance of a regression coefficient at the 0.05 level of significance, with 31 degrees of freedom



From Figure 11.1, the  $p$ -value is 0.000009822 (or 9.822E-06 in scientific notation). Because  $t = 5.2728 > 2.0395$  or the  $p$ -value of 0.000009822  $< 0.05$ , you reject  $H_0$  and conclude that there is a significant relationship between the variable  $X_2$  (promotional expenditures) and sales, taking into account the price,  $X_1$ . This extremely small  $p$ -value allows you to strongly reject the null hypothesis that there is no linear relationship between sales and promotional expenditures. Example 11.1 presents the test for the significance of  $\beta_1$ , the slope of sales with price.

**EXAMPLE 11.1**

**TESTING FOR THE SIGNIFICANCE OF THE SLOPE OF SALES WITH PRICE**

At the 0.05 level of significance, is there evidence that the slope of sales with price is different from zero?

**SOLUTION** From Figure 11.1 on page 432,  $t = -7.7664 < -2.0395$  (the critical value for  $\alpha = 0.05$ ) or the  $p$ -value = 0.0000000092  $< 0.05$ . Thus, there is a significant relationship between price,  $X_1$ , and sales, taking into account the promotional expenditures,  $X_2$ .

As seen with each of the two  $X$  variables, the test of significance for a particular regression coefficient is actually a test for the significance of adding a particular variable into a regression model, given that the other variable is included. Therefore, the  $t$  test for the regression coefficient is equivalent to testing for the contribution of each independent variable.

## Confidence Interval Estimation

Instead of testing the significance of a population slope, you may want to estimate the value of a population slope. Equation (11.8) defines the confidence interval estimate for a population slope in multiple regression.

### CONFIDENCE INTERVAL ESTIMATE FOR THE SLOPE

$$b_j \pm t_{n-k-1} S_{b_j} \quad (11.8)$$

To construct a 95% confidence interval estimate of the population slope,  $\beta_1$  (the effect of price,  $X_1$ , on sales,  $Y$ , holding constant the effect of promotional expenditures,  $X_2$ ), the critical value of  $t$  at the 95% confidence level with 31 degrees of freedom is 2.0395 (see Table E.3). Then, using Equation (11.8) and Figure 11.1 on page 432,

$$\begin{aligned} & b_1 \pm t_{n-k-1} S_{b_1} \\ & -53.2173 \pm (2.0395)(6.8522) \\ & -53.2173 \pm 13.9752 \\ & -67.1925 \leq \beta_1 \leq -39.2421 \end{aligned}$$

Taking into account the effect of promotional expenditures, the estimated effect of a 1-cent increase in price is to reduce mean sales by approximately 39.2 to 67.2 bars. You have 95% confidence that this interval correctly estimates the relationship between these variables. From a hypothesis-testing viewpoint, because this confidence interval does not include 0, you conclude that the regression coefficient,  $\beta_1$ , has a significant effect.

Example 11.2 constructs and interprets a confidence interval estimate for the slope of sales with promotional expenditures.

#### EXAMPLE 11.2

#### CONSTRUCTING A CONFIDENCE INTERVAL ESTIMATE FOR THE SLOPE OF SALES WITH PROMOTIONAL EXPENDITURES

Construct a 95% confidence interval estimate of the population slope of sales with promotional expenditures.

**SOLUTION** The critical value of  $t$  at the 95% confidence level, with 31 degrees of freedom, is 2.0395 (see Table E.3). Using Equation (11.8) and Figure 11.1 on page 432,

$$\begin{aligned} & 3.6131 \pm (2.0395)(0.6852) \\ & 3.6131 \pm 1.3975 \\ & 2.2156 \leq \beta_2 \leq 5.0106 \end{aligned}$$

Thus, taking into account the effect of price, the estimated effect of each additional dollar of promotional expenditures is to increase mean sales by approximately 2.2 to 5.0 bars. You have 95% confidence that this interval correctly estimates the relationship between these variables. From a hypothesis-testing viewpoint, because this confidence interval does not include 0, you can conclude that the regression coefficient,  $\beta_2$ , has a significant effect.

## PROBLEMS FOR SECTION 11.4

### Learning the Basics

**PH Grade ASSIST** **11.23** Given the following information from a multiple regression analysis:

$$n = 25 \quad b_1 = 5 \quad b_2 = 10 \quad S_{b_1} = 2 \quad S_{b_2} = 8$$

- Which variable has the largest slope, in units of a  $t$  statistic?
- Construct a 95% confidence interval estimate of the population slope,  $\beta_1$ .
- At the 0.05 level of significance, determine whether each independent variable makes a significant contribution to the regression model. On the basis of these results, indicate the independent variables to include in this model.

**11.24** Given the following information from a multiple regression analysis:

$$n = 20 \quad b_1 = 4 \quad b_2 = 3 \quad S_{b_1} = 1.2 \quad S_{b_2} = 0.8$$

- Which variable has the largest slope, in units of a  $t$  statistic?
- Construct a 95% confidence interval estimate of the population slope,  $\beta_1$ .
- At the 0.05 level of significance, determine whether each independent variable makes a significant contribution to the regression model. On the basis of these results, indicate the independent variables to include in this model.

### Applying the Concepts

**PH Grade ASSIST** **11.25** In Problem 11.3 on page 434, you predicted the durability of a brand of running shoe, based on the forefoot shock-absorbing capability (FOREIMP) and the change in impact properties over time (MIDSOLE) for a sample of 15 pairs of shoes. Use the following results:

Variable	Coefficient	Standard Error	$t$ Statistic	$p$ -Value
INTERCEPT	-0.02686	0.06905	-0.39	0.7034
FOREIMP	0.79116	0.06295	12.57	0.0000
MIDSOLE	0.60484	0.07174	8.43	0.0000

- Construct a 95% confidence interval estimate of the population slope between durability and forefoot shock-absorbing capability.
- At the 0.05 level of significance, determine whether each independent variable makes a significant contribution to the regression model. On the basis of these results, indicate the independent variables to include in this model.

**PH Grade ASSIST** **11.26** In Problem 11.4 on page 434, you used sales and number of orders to predict distribution costs at a mail-order catalog business (see the **warecost.xls** file). Using the results from that problem,

- construct a 95% confidence interval estimate of the population slope between distribution cost and sales.
- at the 0.05 level of significance, determine whether each independent variable makes a significant contribution to the regression model. On the basis of these results, indicate the independent variables to include in this model.

**11.27** In Problem 11.5 on page 434, you used horsepower and weight to predict gasoline mileage (see the **auto.xls** file). Using the results from that problem,

- construct a 95% confidence interval estimate of the population slope between gasoline mileage and horsepower.
- at the 0.05 level of significance, determine whether each independent variable makes a significant contribution to the regression model. On the basis of these results, indicate the independent variables to include in this model.

**11.28** In Problem 11.6 on pages 434–435, you used radio advertising and newspaper advertising to predict sales (see the **advertise.xls** file). Using the results from that problem,

- construct a 95% confidence interval estimate of the population slope between sales and radio advertising.
- at the 0.05 level of significance, determine whether each independent variable makes a significant contribution to the regression model. On the basis of these results, indicate the independent variables to include in this model.

**11.29** In Problem 11.7 on page 435, you used the total number of staff present and remote hours to predict standby hours (see the **standby.xls** file). Using the results from that problem,

- construct a 95% confidence interval estimate of the population slope between standby hours and total number of staff present.
- at the 0.05 level of significance, determine whether each independent variable makes a significant contribution to the regression model. On the basis of these results, indicate the independent variables to include in this model.

**11.30** In Problem 11.8 on page 435, you used land area of a property and age of a house to predict appraised value (see the **glencove.xls** file). Using the results from that problem,

- construct a 95% confidence interval estimate of the population slope between appraised value and land area of a property.
- at the 0.05 level of significance, determine whether each independent variable makes a significant contribution to the regression model. On the basis of these results, indicate the independent variables to include in this model.

## 11.5 TESTING PORTIONS OF THE MULTIPLE REGRESSION MODEL

In developing a multiple regression model, you want to use only those independent variables that significantly reduce the error in predicting the value of a dependent variable. If an independent variable does not improve the prediction, you can delete it from the multiple regression model and use a model with fewer independent variables.

The **partial *F* test** is an alternative method to the *t* test discussed in Section 11.4 for determining the contribution of an independent variable. It involves determining the contribution to the regression sum of squares made by each independent variable after all the other independent variables have been included in the model. The new independent variable is included only if it significantly improves the model.

To conduct partial *F* tests for the OmniPower sales example, you need to evaluate the contribution of promotional expenditures ( $X_2$ ) after price ( $X_1$ ) has been included in the model, and also evaluate the contribution of price ( $X_1$ ) after promotional expenditures ( $X_2$ ) have been included in the model.

In general, if there are several independent variables, you determine the contribution of each independent variable by taking into account the regression sum of squares of a model that includes all independent variables except the one of interest, *SSR* (all variables except *j*). Equation (11.9) determines the contribution of variable *j* assuming that all other variables are already included.

### DETERMINING THE CONTRIBUTION OF AN INDEPENDENT VARIABLE TO THE REGRESSION MODEL

$$SSR(X_j | \text{All variables except } j) = SSR(\text{All variables including } j) - SSR(\text{All variables except } j) \quad (11.9)$$

If there are two independent variables, you use Equations (11.10a) and (11.10b) to determine the contribution of each.

### CONTRIBUTION OF VARIABLE $X_1$ , GIVEN THAT $X_2$ HAS BEEN INCLUDED

$$SSR(X_1 | X_2) = SSR(X_1 \text{ and } X_2) - SSR(X_2) \quad (11.10a)$$

### CONTRIBUTION OF VARIABLE $X_2$ , GIVEN THAT $X_1$ HAS BEEN INCLUDED

$$SSR(X_2 | X_1) = SSR(X_1 \text{ and } X_2) - SSR(X_1) \quad (11.10b)$$

The term  $SSR(X_2)$  represents the sum of squares that is due to regression for a model that includes only the independent variable  $X_2$  (promotional expenditures). Similarly,  $SSR(X_1)$  represents the sum of squares that is due to regression for a model that includes only the independent variable  $X_1$  (price). Figures 11.6 and 11.7 present Microsoft Excel results for these two models.

**FIGURE 11.6**

Microsoft Excel results of a simple linear regression analysis for sales and promotional expenditures,  $SSR(X_2)$



See Section E10.1 to create this.

	A	B	C	D	E	F	G
1	Sales & Promotional Expenses Analysis						
2							
3	<b>Regression Statistics</b>						
4	Multiple R	0.5351					
5	R Square	0.2863					
6	Adjusted R Square	0.2640					
7	Standard Error	1077.8721					
8	Observations	34					
9							
10	<b>ANOVA</b>						
11		df	SS	MS	F	Significance F	
12	Regression	1	14915814.102	14915814.102	12.8384	0.0011	
13	Residual	32	37177863.339	1161808.229			
14	Total	33	52093677.441				
15							
16		Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
17	Intercept	1496.0161	483.9789	3.0911	0.0041	510.1843	2481.8480
18	Promotion	4.1281	1.1521	3.5831	0.0011	1.7813	6.4748

**FIGURE 11.7**

Microsoft Excel results of a simple linear regression model for sales and price,  $SSR(X_1)$



See Section E10.1 to create this.

	A	B	C	D	E	F	G
1	Sales & Price Analysis						
2							
3	Regression Statistics						
4	Multiple R	0.7351					
5	R Square	0.5404					
6	Adjusted R Square	0.5261					
7	Standard Error	864.9457					
8	Observations	34					
9							
10	ANOVA						
11		df	SS	MS	F	Significance F	
12	Regression	1	28153486.15	28153486.15	37.6318	7.35855E-07	
13	Residual	32	23940191.29	748130.98			
14	Total	33	52093677.44				
15							
16		Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
17	Intercept	7512.3480	734.6189	10.2262	1.30793E-11	6015.9796	9008.7164
18	Price	-56.7138	9.2451	-6.1345	7.35855E-07	-75.5455	-37.8822

From Figure 11.6,  $SSR(X_2) = 14,915,811.102$  and from Figure 11.1 on page 432,  $SSR(X_1 \text{ and } X_2) = 39,472,730.77$ . Then, using Equation (11.10a),

$$\begin{aligned}
 SSR(X_1 | X_2) &= SSR(X_1 \text{ and } X_2) - SSR(X_2) \\
 &= 39,472,730.77 - 14,915,811.102 \\
 &= 24,556,919.667
 \end{aligned}$$

To determine whether  $X_1$  significantly improves the model after  $X_2$  has been included, you divide the regression sum of squares into two component parts, as shown in Table 11.3.

**TABLE 11.3**

ANOVA Table Dividing the Regression Sum of Squares into Components to Determine the Contribution of Variable  $X_1$

Source	Degrees of Freedom	Sum of Squares	Mean Square (Variance)	F
Regression	2	39,472,730.77	19,736,365.39	
$\left\{ \begin{array}{l} X_2 \\ X_1   X_2 \end{array} \right\}$	$\left\{ \begin{array}{l} 1 \\ 1 \end{array} \right\}$	$\left\{ \begin{array}{l} 14,915,811.10 \\ 24,556,919.667 \end{array} \right\}$	24,556,919.667	60.32
Error	31	12,620,946.67	407,127.31	
Total	33	52,093,677.44		

The null and alternative hypotheses to test for the contribution of  $X_1$  to the model are

- $H_0$ : Variable  $X_1$  does not significantly improve the model after variable  $X_2$  has been included.
- $H_1$ : Variable  $X_1$  significantly improves the model after variable  $X_2$  has been included.

Equation (11.11) defines the partial  $F$  test statistic for testing the contribution of an independent variable.

**PARTIAL F TEST STATISTIC**

$$F = \frac{SSR(X_j | \text{All variables except } j)}{MSE} \tag{11.11}$$

The partial  $F$  test statistic follows an  $F$  distribution with 1 and  $n - k - 1$  degrees of freedom.

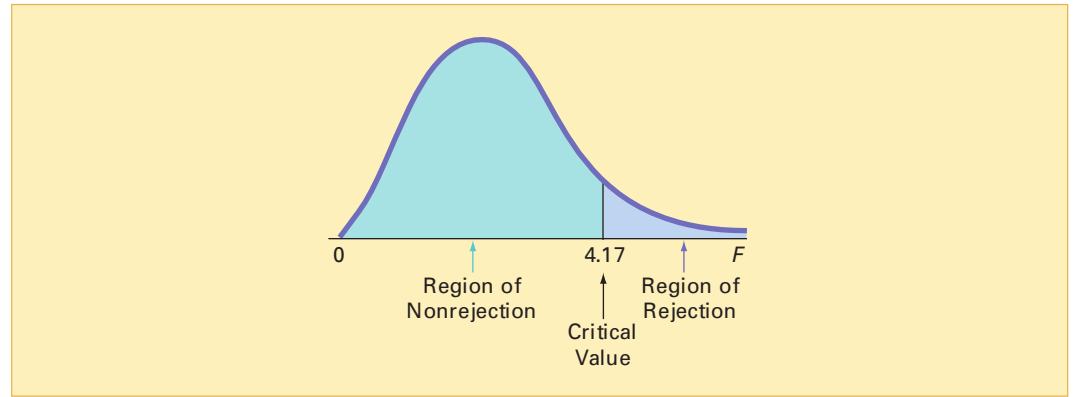
From Table 11.3,

$$F = \frac{24,556,919.667}{407,127.31} = 60.32$$

The partial  $F$  test statistic has 1 and  $n - k - 1 = 34 - 2 - 1 = 31$  degrees of freedom. Using a level of significance of 0.05, the critical value from Table E.5 is approximately 4.17 (see Figure 11.8).

**FIGURE 11.8**

Testing for contribution of a regression coefficient to a multiple regression model at the 0.05 level of significance, with 1 and 31 degrees of freedom



Because the partial  $F$  test statistic is greater than this critical  $F$  value ( $60.32 > 4.17$ ), you reject  $H_0$  and conclude that the addition of variable  $X_1$  (price) significantly improves a regression model that already contains variable  $X_2$  (promotional expenditures).

To evaluate the contribution of variable  $X_2$  (promotional expenditures) to a model in which variable  $X_1$  (price) has been included, you need to use Equation (11.10b). First, from Figure 11.7 on page 446, observe that  $SSR(X_1) = 28,153,486.15$ . Second, from Table 11.3, observe that  $SSR(X_1 \text{ and } X_2) = 39,472,730.77$ . Then, using Equation (11.10b) on page 445,

$$SSR(X_2 | X_1) = 39,472,730.77 - 28,153,486.15 = 11,319,244.62$$

To determine whether  $X_2$  significantly improves a model after  $X_1$  has been included, you can divide the regression sum of squares into two component parts, as shown in Table 11.4.

**TABLE 11.4**  
ANOVA Table Dividing the Regression Sum of Squares into Components to Determine the Contribution of Variable  $X_2$

Source	Degrees of Freedom	Sum of Squares	Mean Square (Variance)	$F$
Regression	2	39,472,730.77	19,736,365.39	
$\left\{ \begin{array}{l} X_1 \\ X_2   X_1 \end{array} \right\}$	$\left\{ \begin{array}{l} 1 \\ 1 \end{array} \right\}$	$\left\{ \begin{array}{l} 28,153,486.15 \\ 11,319,244.62 \end{array} \right\}$	11,319,244.62	27.80
Error	31	12,620,946.67	407,127.31	
Total	33	52,093,677.44		

The null and alternative hypotheses to test for the contribution of  $X_2$  to the model are

$H_0$ : Variable  $X_2$  does not significantly improve the model after variable  $X_1$  has been included.

$H_1$ : Variable  $X_2$  significantly improves the model after variable  $X_1$  has been included.

Using Equation (11.11) and Table 11.4,

$$F = \frac{11,319,244.62}{407,127.31} = 27.80$$

In Figure 11.8 above, you can see that, using a 0.05 level of significance, the critical value of  $F$ , with 1 and 31 degrees of freedom, is approximately 4.17. Because the partial  $F$  test statistic is greater than this critical value ( $27.80 > 4.17$ ), you reject  $H_0$  and conclude that the addition of variable  $X_2$  (promotional expenditures) significantly improves the multiple regression model already containing  $X_1$  (price).

Thus, by testing for the contribution of each independent variable after the other has been included in the model, you determine that each of the two independent variables significantly

improves the model. Therefore, the multiple regression model should include both price,  $X_1$ , and promotional expenditures,  $X_2$ .

The partial  $F$  test statistic developed in this section and the  $t$  test statistic of Equation (11.7) on page 441 are both used to determine the contribution of an independent variable to a multiple regression model. In fact, the hypothesis tests associated with these two statistics always result in the same decision (that is, the  $p$ -values are identical). The  $t$  values for the OmniPower regression model are  $-7.7664$  and  $+5.2728$ , and the corresponding  $F$  values are 60.32 and 27.80. Equation (11.12) illustrates the relationship between  $t$  and  $F$ .

#### RELATIONSHIP BETWEEN A $t$ STATISTIC AND AN $F$ STATISTIC

$$t_a^2 = F_{1,a} \quad (11.12)$$

where

$a$  = degrees of freedom

#### Coefficients of Partial Determination

Recall from Section 11.2 that the coefficient of multiple determination,  $r^2$ , measures the proportion of the variation in  $Y$  that is explained by variation in the independent variables. Now, the contribution of each independent variable to a multiple regression model, while holding constant the other variable, is examined. The **coefficients of partial determination** ( $r_{Y1.2}^2$  and  $r_{Y2.1}^2$ ) measure the proportion of the variation in the dependent variable that is explained by each independent variable while controlling for, or holding constant, the other independent variable. Equation (11.13) defines the coefficients of partial determination for a multiple regression model with two independent variables.

#### COEFFICIENTS OF PARTIAL DETERMINATION FOR A MULTIPLE REGRESSION MODEL CONTAINING TWO INDEPENDENT VARIABLES

$$r_{Y1.2}^2 = \frac{SSR(X_1 | X_2)}{SST - SSR(X_1 \text{ and } X_2) + SSR(X_1 | X_2)} \quad (11.13a)$$

and

$$r_{Y2.1}^2 = \frac{SSR(X_2 | X_1)}{SST - SSR(X_1 \text{ and } X_2) + SSR(X_2 | X_1)} \quad (11.13b)$$

where

$SSR(X_1 | X_2)$  = sum of squares of the contribution of variable  $X_1$  to the regression model, given that variable  $X_2$  has been included in the model

$SST$  = total sum of squares for  $Y$

$SSR(X_1 \text{ and } X_2)$  = regression sum of squares when variables  $X_1$  and  $X_2$  are both included in the multiple regression model

$SSR(X_2 | X_1)$  = sum of squares of the contribution of variable  $X_2$  to the regression model, given that variable  $X_1$  has been included in the model



For the OmniPower sales example,

$$r_{Y1.2}^2 = \frac{24,556,916.67}{52,093,677.44 - 39,472,730.77 + 24,556,916.67} = 0.6605$$

$$r_{Y2.1}^2 = \frac{11,319,244.62}{52,093,677.44 - 39,472,730.77 + 11,319,244.62} = 0.4728$$

The coefficient of partial determination of variable  $Y$  with  $X_1$  while holding  $X_2$  constant ( $r_{Y1.2}^2$ ) is 0.6605. Thus, for a given (constant) amount of promotional expenditures, 66.05% of the variation in OmniPower sales is explained by the variation in the price. The coefficient of partial determination of variable  $Y$  with  $X_2$  while holding  $X_1$  constant ( $r_{Y2.1}^2$ ) is 0.4728. Thus, for a given (constant) price, 47.28% of the variation in sales of OmniPower bars is explained by variation in the amount of promotional expenditures.

Equation (11.14) defines the coefficient of partial determination for the  $j$ th variable in a multiple regression model containing several ( $k$ ) independent variables.

**COEFFICIENT OF PARTIAL DETERMINATION FOR A MULTIPLE REGRESSION MODEL CONTAINING  $k$  INDEPENDENT VARIABLES**

$$r_{Y_j,(\text{All variables except } j)}^2 = \frac{SSR(X_j | \text{All variables except } j)}{SST - SSR(\text{All variables including } j) + SSR(X_j | \text{All variables except } j)} \tag{11.14}$$

**PROBLEMS FOR SECTION 11.5**

**Learning the Basics**

**PH Grade ASSIST** **11.31** The following is the ANOVA summary table for a multiple regression model with two independent variables:

Source	Degrees of Freedom	Sum of Squares	Mean Squares	F
Regression	2	60		
Error	18	120		
Total	20	180		

If  $SSR(X_1) = 45$  and  $SSR(X_2) = 25$ ,

- a. determine whether there is a significant relationship between  $Y$  and each of the independent variables at the 0.05 level of significance.
- b. compute the coefficients of partial determination,  $r_{Y1.2}^2$  and  $r_{Y2.1}^2$ , and interpret their meaning.

**PH Grade ASSIST** **11.32** The following is the ANOVA summary table for a multiple regression model with two independent variables:

Source	Degrees of Freedom	Sum of Squares	Mean Squares	F
Regression	2	30		
Error	10	120		
Total	12	150		

If  $SSR(X_1) = 20$  and  $SSR(X_2) = 15$ ,

- a. determine whether there is a significant relationship between  $Y$  and each of the independent variables at the 0.05 level of significance.
- b. compute the coefficients of partial determination,  $r_{Y1.2}^2$  and  $r_{Y2.1}^2$ , and interpret their meaning.

**Applying the Concepts**

**11.33** In Problem 11.5 on page 434, you used horsepower and weight to predict gasoline mileage (see the [auto.xls](#) file). Using the results from that problem,

- a. at the 0.05 level of significance, determine whether each independent variable makes a significant contribution to the regression model. On the basis of these results, indicate the most appropriate regression model for this set of data.

- b. compute the coefficients of partial determination,  $r_{Y1.2}^2$  and  $r_{Y2.1}^2$ , and interpret their meaning.



**11.34** In Problem 11.4 on page 434, you used sales and number of orders to predict distribution costs at a mail-order catalog business (see the [warecost.xls](#) file). Using the results from that problem,

- a. at the 0.05 level of significance, determine whether each independent variable makes a significant contribution to the regression model. On the basis of these results, indicate the most appropriate regression model for this set of data.
- b. compute the coefficients of partial determination,  $r_{Y1.2}^2$  and  $r_{Y2.1}^2$ , and interpret their meaning.

**11.35** In Problem 11.7 on page 434, you used the total staff present and remote hours to predict standby hours (see the [standby.xls](#) file). Using the results from that problem,

- a. at the 0.05 level of significance, determine whether each independent variable makes a significant contribution to the regression model. On the basis of these results, indicate the most appropriate regression model for this set of data.

- b. compute the coefficients of partial determination,  $r_{Y1.2}^2$  and  $r_{Y2.1}^2$ , and interpret their meaning.

**11.36** In Problem 11.6 on pages 434–435, you used radio advertising and newspaper advertising to predict sales (see the [advertise.xls](#) file). Using the results from that problem,

- a. at the 0.05 level of significance, determine whether each independent variable makes a significant contribution to the regression model. On the basis of these results, indicate the most appropriate regression model for this set of data.
- b. compute the coefficients of partial determination,  $r_{Y1.2}^2$  and  $r_{Y2.1}^2$ , and interpret their meaning.

**11.37** In Problem 11.8 on page 435, you used land area of a property and age of a house to predict appraised value (see the [glencove.xls](#) file). Using the results from that problem,

- a. at the 0.05 level of significance, determine whether each independent variable makes a significant contribution to the regression model. On the basis of these results, indicate the most appropriate regression model for this set of data.
- b. compute the coefficients of partial determination,  $r_{Y1.2}^2$  and  $r_{Y2.1}^2$ , and interpret their meaning.

## 11.6 USING DUMMY VARIABLES AND INTERACTION TERMS IN REGRESSION MODELS

The multiple regression models discussed in Sections 11.1 through 11.5 assumed that each independent variable is numerical. However, in some situations, you might want to include categorical variables as independent variables in the regression model. For example, in Section 11.1, you used price and promotional expenditures to predict the monthly sales of OmniPower high-energy bars. In addition to these numerical independent variables, you may want to include the effect of the shelf location in the store (for example, end-aisle display or no end-aisle display) when developing a model to predict OmniPower sales.

The use of **dummy variables** allows you to include categorical independent variables as part of the regression model. If a given categorical independent variable has two categories, then you need only one dummy variable to represent the two categories. A dummy variable,  $X_d$ , is defined as

$$\begin{aligned} X_d &= 0 \text{ if the observation is in category 1} \\ X_d &= 1 \text{ if the observation is in category 2} \end{aligned}$$

To illustrate the application of dummy variables in regression, consider a model for predicting the assessed value from a sample of 15 houses, based on the size of the house (in thousands of square feet) and whether the house has a fireplace. To include the categorical variable concerning the presence of a fireplace, the dummy variable  $X_2$  is defined as

$$\begin{aligned} X_2 &= 0 \text{ if the house does not have a fireplace} \\ X_2 &= 1 \text{ if the house has a fireplace} \end{aligned}$$

Table 11.5 presents the data, which are also stored in the file [house3.xls](#). In the last column of Table 11.5, you can see how the categorical data are converted to numerical values.

**TABLE 11.5**  
Predicting Assessed Value, Based on Size of House and Presence of a Fireplace

$X_1 = \text{Size of House}$				
House	$Y = \text{Assessed Value (\$000)}$	(Thousands of Square Feet)	Fireplace	$X_2 = \text{Fireplace}$
1	84.4	2.00	Yes	1
2	77.4	1.71	No	0
3	75.7	1.45	No	0
4	85.9	1.76	Yes	1
5	79.1	1.93	No	0
6	70.4	1.20	Yes	1
7	75.8	1.55	Yes	1
8	85.9	1.93	Yes	1
9	78.5	1.59	Yes	1
10	79.2	1.50	Yes	1
11	86.7	1.90	Yes	1
12	79.3	1.39	Yes	1
13	74.5	1.54	No	0
14	83.8	1.89	Yes	1
15	76.8	1.59	No	0

Assuming that the slope of assessed value with the size of the house is the same for houses that have and do not have a fireplace, the multiple regression model is

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \epsilon_i$$

where

$Y_i$  = assessed value in thousands of dollars for house  $i$

$\beta_0$  =  $Y$  intercept

$X_{1i}$  = size of the house, in thousands of square feet, for house  $i$

$\beta_1$  = slope of assessed value with size of the house, holding constant the presence or absence of a fireplace

$X_{2i}$  = dummy variable representing the presence or absence of a fireplace for house  $i$

$\beta_2$  = incremental effect of the presence of a fireplace on assessed value, holding constant the size of the house

$\epsilon_i$  = random error in  $Y$  for house  $i$

Figure 11.9 illustrates the Microsoft Excel results for this model.

**FIGURE 11.9**  
Microsoft Excel results for the regression model that includes size of the house and presence of fireplace



See Section E11.1 to create this.

	A	B	C	D	E	F	G
1	Assessed Value Analysis						
2							
3	Regression Statistics						
4	Multiple R	0.9006					
5	R Square	0.8111					
6	Adjusted R Square	0.7796					
7	Standard Error	2.2626					
8	Observations	15					
9	ANOVA						
10		df	SS	MS	F	Significance F	
11	Regression	2	263.7039	131.8520	25.7557	4.54968E-05	
12	Residual	12	61.4321	5.1193			
13	Total	14	325.1360				
14							
15							
16		Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
17	Intercept	50.0905	4.3517	11.5107	7.67943E-08	40.6090	59.5719
18	Size	16.1858	2.5744	6.2871	4.02437E-05	10.5766	21.7951
19	Fireplace	3.8530	1.2412	3.1042	0.0091	1.1486	6.5574

From Figure 11.9, the regression equation is

$$\hat{Y}_i = 50.0905 + 16.1858 X_{1i} + 3.8530 X_{2i}$$

For houses without a fireplace, you substitute  $X_2 = 0$  into the regression equation:

$$\begin{aligned}\hat{Y}_i &= 50.0905 + 16.1858 X_{1i} + 3.8530 X_{2i} \\ &= 50.0905 + 16.1858 X_{1i} + 3.8530(0) \\ &= 50.0905 + 16.1858 X_{1i}\end{aligned}$$

For houses with a fireplace, you substitute  $X_2 = 1$  into the regression equation:

$$\begin{aligned}\hat{Y}_i &= 50.0905 + 16.1858 X_{1i} + 3.8530 X_{2i} \\ &= 50.0905 + 16.1858 X_{1i} + 3.8530(1) \\ &= 53.9435 + 16.1858 X_{1i}\end{aligned}$$

In this model, the regression coefficients are interpreted as follows:

1. Holding constant whether a house has a fireplace, for each increase of 1.0 thousand square feet in the size of the house, the mean assessed value is estimated to increase by 16.1858 thousand dollars (that is, \$16,185.80).
2. Holding constant the size of the house, the presence of a fireplace is estimated to increase the mean assessed value of the house by 3.8530 thousand dollars (or \$3,853).

In Figure 11.9, the  $t$  statistic for the slope of the size of the house with assessed value is 6.2871, and the  $p$ -value is approximately 0.000; the  $t$  statistic for presence of a fireplace is 3.1042, and the  $p$ -value is 0.0091. Thus, each of the two variables makes a significant contribution to the model at a level of significance of 0.01. In addition, the coefficient of multiple determination indicates that 81.11% of the variation in assessed value is explained by variation in the size of the house and whether the house has a fireplace.

### EXAMPLE 11.3

#### MODELING A THREE-LEVEL CATEGORICAL VARIABLE

Define a multiple regression model using sales as the dependent variable and package design and price as independent variables. Package design is a three-level categorical variable with designs A, B, or C.

**SOLUTION** To model a three-level categorical variable, two dummy variables are needed:

$$\begin{aligned}X_{1i} &= 1 \text{ if package design A is used in observation } i; 0 \text{ otherwise} \\ X_{2i} &= 1 \text{ if package design B is used in observation } i; 0 \text{ otherwise}\end{aligned}$$

If observation  $i$  is for package design A, then  $X_{1i} = 1$  and  $X_{2i} = 0$ ; for package design B, then  $X_{1i} = 0$  and  $X_{2i} = 1$ ; and for package design C, then  $X_{1i} = X_{2i} = 0$ . A third independent variable is used for price:

$$X_{3i} = \text{price for observation } i$$

Thus, the regression model for this example is

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \beta_3 X_{3i} + \varepsilon_i$$

where

$Y_i$  = sales for observation  $i$

$\beta_0$  =  $Y$  intercept

$\beta_1$  = difference between the mean sales of design A and the mean sales of design C, holding the price constant

$\beta_2$  = difference between the mean sales of design B and the mean sales of design C, holding the price constant

$\beta_3$  = slope of sales with price, holding the package design constant

$\epsilon_i$  = random error in  $Y$  for observation  $i$

### Interactions

In all the regression models discussed so far, the *effect* an independent variable has on the dependent variable was assumed to be statistically independent of the other independent variables in the model. An **interaction** occurs if the effect of an independent variable on the dependent variable is affected by the *value* of a second independent variable. For example, it is possible for advertising to have a large effect on the sales of a product when the price of a product is low. However, if the price of the product is too high, increases in advertising will not dramatically change sales. In this case, price and advertising are said to interact. In other words, you cannot make general statements about the effect of advertising on sales. The effect that advertising has on sales is *dependent* on the price. You use an **interaction term** (sometimes referred to as a **cross-product term**) to model an interaction effect in a regression model.

To illustrate the concept of interaction and use of an interaction term, return to the example concerning the assessed values of homes discussed on pages 450–452. In the regression model, you assumed that the effect the size of the home has on the assessed value is independent of whether the house has a fireplace. In other words, you assumed that the slope of assessed value with size is the same for houses with fireplaces as it is for houses without fireplaces. If these two slopes are different, an interaction between size of the home and fireplace exists.

To evaluate the possibility of an interaction, you first define an interaction term that consists of the product of the independent variable  $X_1$  (size of house) and the dummy variable  $X_2$  (fireplace). You then test whether this interaction variable makes a significant contribution to the regression model. If the interaction is significant, you cannot use the original model for prediction. For the data of Table 11.5 on page 451, let

$$X_3 = X_1 \times X_2$$

Figure 11.10 illustrates Microsoft Excel results for this regression model, which includes the size of the house,  $X_1$ , the presence of a fireplace,  $X_2$ , and the interaction of  $X_1$  and  $X_2$  (which is defined as  $X_3$ ).

**FIGURE 11.10**

Microsoft Excel results for a regression model that includes size, presence of fireplace, and interaction of size and fireplace



See Sections E11.1 and E11.6 to create this.

	A	B	C	D	E	F	G
1	Assessed Value Analysis						
2							
3	Regression Statistics						
4	Multiple R	0.9179					
5	R Square	0.8426					
6	Adjusted R Square	0.7996					
7	Standard Error	2.1573					
8	Observations	15					
9							
10	ANOVA						
11		df	SS	MS	F	Significance F	
12	Regression	3	273.9441	91.3147	19.6215	0.0001	
13	Residual	11	51.1919	4.6538			
14	Total	14	325.1360				
15							
16		Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
17	Intercept	62.9522	9.6122	6.5492	4.13993E-05	41.7959	84.1085
18	Size	8.3624	5.8173	1.4375	0.1784	-4.4414	21.1662
19	Fireplace	-11.8404	10.6455	-1.1122	0.2898	-35.2710	11.5902
20	Size * Fireplace	9.5180	6.4165	1.4834	0.1661	-4.6046	23.6406

To test for the existence of an interaction, you use the null hypothesis  $H_0: \beta_3 = 0$  versus the alternative hypothesis  $H_1: \beta_3 \neq 0$ . In Figure 11.10, the  $t$  statistic for the interaction of size and fireplace is 1.4834. Because the  $p$ -value = 0.1661 > 0.05, you do not reject the null hypothesis. Therefore, the interaction does not make a significant contribution to the model, given that size and presence of a fireplace are already included.

Regression models can have several numerical independent variables. Example 11.4 illustrates a regression model in which there are two numerical independent variables as well as a categorical independent variable.

**EXAMPLE 11.4****STUDYING A REGRESSION MODEL THAT CONTAINS A DUMMY VARIABLE**

A real estate developer wants to predict heating oil consumption in single-family houses, based on atmospheric temperature,  $X_1$ , and the amount of attic insulation,  $X_2$ . Suppose that, of 15 houses selected, houses 1, 4, 6, 7, 8, 10, and 12 are ranch-style houses. The data are stored in the file [htngoil.xls](#). Develop and analyze an appropriate regression model, using these three independent variables,  $X_1$ ,  $X_2$ , and  $X_3$  (the dummy variable for ranch-style houses).

**SOLUTION** Define  $X_3$ , a dummy variable for ranch-style house, as follows:

$$\begin{aligned} X_3 &= 0 \text{ if the style is not ranch} \\ X_3 &= 1 \text{ if the style is ranch} \end{aligned}$$

Assuming that the slope between home heating oil consumption and atmospheric temperature,  $X_1$ , and between home heating oil consumption and the amount of attic insulation,  $X_2$ , is the same for both styles of houses, the regression model is

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \beta_3 X_{3i} + \varepsilon_i$$

where

$Y_i$  = monthly heating oil consumption, in gallons, for house  $i$

$\beta_0$  =  $Y$  intercept

$\beta_1$  = slope of heating oil consumption with atmospheric temperature, holding constant the effect of attic insulation and the style of the house

$\beta_2$  = slope of heating oil consumption with attic insulation, holding constant the effect of atmospheric temperature and the style of the house

$\beta_3$  = incremental effect of the presence of a ranch-style house, holding constant the effect of atmospheric temperature and attic insulation

$\varepsilon_i$  = random error in  $Y$  for house  $i$

Figure 11.11 displays Microsoft Excel results.

**FIGURE 11.11**

Microsoft Excel results for a regression model that includes temperature, insulation, and style for the heating oil data

	A	B	C	D	E	F	G
1	Heating Oil Consumption Analysis						
2							
3	Regression Statistics						
4	Multiple R	0.9942					
5	R Square	0.9884					
6	Adjusted R Square	0.9853					
7	Standard Error	15.7489					
8	Observations	15					
9							
10	ANOVA						
11		df	SS	MS	F	Significance F	
12	Regression	3	233406.9094	77802.3031	313.6822	6.21548E-11	
13	Residual	11	2728.3200	248.0291			
14	Total	14	236135.2293				
15							
16		Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
17	Intercept	592.5401	14.3370	41.3295	2.02317E-13	560.9046	624.0956
18	Temperature	-5.5251	0.2044	-27.0267	2.07188E-11	-5.9751	-5.0752
19	Insulation	-21.3761	1.4480	-14.7623	1.34816E-08	-24.5632	-18.1891
20	Ranch-style	-38.9727	8.3584	-4.6627	0.0007	-57.3695	-20.5759

From the results in Figure 11.11, the regression equation is

$$\hat{Y}_i = 592.5401 - 5.5251 X_{1i} - 21.3761 X_{2i} - 38.9727 X_{3i}$$

For houses that are not ranch style, because  $X_3 = 0$ , this reduces to

$$\hat{Y}_i = 592.5401 - 5.5251 X_{1i} - 21.3761 X_{2i}$$

For houses that are ranch style, because  $X_3 = 1$ , this reduces to

$$\hat{Y}_i = 553.5674 - 5.5251 X_{1i} - 21.3761 X_{2i}$$

The regression coefficients are interpreted as follows:

1. Holding constant the attic insulation and the house style, for each additional 1 °F increase in atmospheric temperature, you estimate that the mean oil consumption decreases by 5.5251 gallons.
2. Holding constant the atmospheric temperature and the house style, for each additional 1-inch increase in attic insulation, you estimate that the mean oil consumption decreases by 21.3761 gallons.
3.  $b_3$  measures the effect on oil consumption of having a ranch-style house ( $X_3 = 1$ ) compared with having a house that is not ranch style ( $X_3 = 0$ ). Thus, with atmospheric temperature and attic insulation held constant, you estimate that the mean oil consumption is 38.9727 gallons less for a ranch-style house than for a house that is not ranch style.

The three  $t$  statistics representing the slopes for temperature, insulation, and ranch style are  $-27.0267$ ,  $-14.7623$ , and  $-4.6627$ . Each of the corresponding  $p$ -values is extremely small, all being less than 0.001. Thus, each of the three variables makes a significant contribution to the model. In addition, the coefficient of multiple determination indicates that 98.84% of the variation in oil usage is explained by variation in the temperature, insulation, and whether the house is ranch style.

Before you can use the model in Example 11.4, you need to determine whether the independent variables interact with each other. In Example 11.5, three interaction terms are added to the model.

### EXAMPLE 11.5

### EVALUATING A REGRESSION MODEL WITH SEVERAL INTERACTIONS

For the data of Example 11.4, determine whether adding the interaction terms make a significant contribution to the regression model.

**SOLUTION** To evaluate possible interactions between the independent variables, three interaction terms are constructed. Let  $X_4 = X_1 \times X_2$ ,  $X_5 = X_1 \times X_3$ , and  $X_6 = X_2 \times X_3$ . The regression model is now

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \beta_3 X_{3i} + \beta_4 X_{4i} + \beta_5 X_{5i} + \beta_6 X_{6i} + \varepsilon_i$$

where  $X_1$  is temperature,  $X_2$  is insulation,  $X_3$  is the dummy variable ranch style,  $X_4$  is the interaction between temperature and insulation,  $X_5$  is the interaction between temperature and ranch style, and  $X_6$  is the interaction between insulation and ranch style.

To test whether the three interactions significantly improve the regression model, you use the partial  $F$  test. The null and alternative hypotheses are

$$H_0: \beta_4 = \beta_5 = \beta_6 = 0 \text{ (There are no interactions among } X_1, X_2, \text{ and } X_3.)$$

$$H_1: \beta_4 \neq 0 \text{ and/or } \beta_5 \neq 0 \text{ and/or } \beta_6 \neq 0 \text{ (} X_1 \text{ interacts with } X_2, \text{ and/or } X_1 \text{ interacts with } X_3, \text{ and/or } X_2 \text{ interacts with } X_3.)$$

From Figure 11.12,

$$SSR(X_1, X_2, X_3, X_4, X_5, X_6) = 234,510.582 \text{ with 6 degrees of freedom}$$

and from Figure 11.11 on page 454,  $SSR(X_1, X_2, X_3) = 233,406.9094$  with 3 degrees of freedom.

**FIGURE 11.12**

Microsoft Excel results for a regression model that includes temperature,  $X_1$ ; insulation,  $X_2$ ; the dummy variable ranch style,  $X_3$ ; the interaction of temperature and insulation,  $X_4$ ; the interaction of temperature and ranch style,  $X_5$ ; and the interaction of insulation and ranch style,  $X_6$

	A	B	C	D	E	F	G
1	Heating Oil Consumption Analysis						
2							
3	<b>Regression Statistics</b>						
4	Multiple R	0.9966					
5	R Square	0.9931					
6	Adjusted R Square	0.9880					
7	Standard Error	14.2506					
8	Observations	15					
9							
10	<b>ANOVA</b>						
11		df	SS	MS	F	Significance F	
12	Regression	6	234510.582	39085.097	192.4607	3.32423E-08	
13	Residual	8	1624.647	203.081			
14	Total	14	236135.229				
15							
16		Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
17	Intercept	642.8867	26.7059	24.0728	9.45284E-09	581.3027	704.4707
18	Temperature	-6.9263	0.7531	-9.1969	1.58014E-05	-8.6629	-5.1896
19	Insulation	-27.8825	3.5801	-7.7882	5.29456E-05	-36.1383	-19.6268
20	Style	-84.6088	29.9956	-2.8207	0.0225	-153.7788	-15.4389
21	Temperature * Insulation	0.1702	0.0886	1.9204	0.0911	-0.0342	0.3746
22	Temperature * Ranch-style	0.6596	0.4617	1.4286	0.1910	-0.4051	1.7242
23	Insulation * Ranch-style	4.9870	3.5137	1.4193	0.1936	-3.1156	13.0895

Thus,  $SSR(X_1, X_2, X_3, X_4, X_5, X_6) - SSR(X_1, X_2, X_3) = 234,510.582 - 233,406.9094 = 1,103.67$ . The difference in degrees of freedom is  $6 - 3 = 3$ .

To use the partial  $F$  test for the simultaneous contribution of three variables to a model, you use an extension of Equation (11.11) on page 446.<sup>1</sup> The partial  $F$  test statistic is

$$F = \frac{[SSR(X_1, X_2, X_3, X_4, X_5, X_6) - SSR(X_1, X_2, X_3)] / 3}{MSE(X_1, X_2, X_3, X_4, X_5, X_6)} = \frac{1,103.67 / 3}{203.08} = 1.81$$

You compare the  $F$  test statistic of 1.81 to the critical  $F$  value for 3 and 8 degrees of freedom. Using a level of significance of 0.05, the critical  $F$  value from Table E.5 is 4.07. Because  $1.81 < 4.07$ , you conclude that the interactions do not make a significant contribution to the model, given that the model already includes temperature,  $X_1$ ; insulation,  $X_2$ ; and whether the house is ranch style,  $X_3$ . Therefore, the multiple regression model using  $X_1$ ,  $X_2$ , and  $X_3$  but no interaction terms is the better model. If you rejected this null hypothesis, you would then test the contribution of each interaction separately in order to determine which interaction terms to include in the model.

<sup>1</sup>In general, if a model has several independent variables and you want to test whether an additional set of independent variables contribute to the model, the numerator of the  $F$  test is  $[SSR(\text{for all independent variables}) - SSR(\text{for the initial set of variables})]$  divided by the number of independent variables whose contribution is being tested.



## PROBLEMS FOR SECTION 11.6

### Learning the Basics

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ASSIST

**11.38** Suppose  $X_1$  is a numerical variable and  $X_2$  is a dummy variable and the regression equation for a sample of  $n = 20$  is

$$\hat{Y}_i = 6 + 4X_{1i} + 2X_{2i}$$

- Interpret the meaning of the slope for variable  $X_1$ .
- Interpret the meaning of the slope for variable  $X_2$ .
- Suppose that the  $t$  statistic for testing the contribution of variable  $X_2$  is 3.27. At the 0.05 level of significance, is there evidence that variable  $X_2$  makes a significant contribution to the model?

### Applying the Concepts

**11.39** The chair of the accounting department wants to develop a regression model to predict the grade point average in accounting for graduating accounting majors, based on the student's SAT score and whether the student received a grade of B or higher in the introductory statistics course (0 = no and 1 = yes).

- Explain the steps involved in developing a regression model for these data. Be sure to indicate the particular models you need to evaluate and compare.
- Suppose the regression coefficient for the variable of whether the student received a grade of B or higher in the introductory statistics course is +0.30. How do you interpret this result?

**11.40** A real estate association in a suburban community would like to study the relationship between the size of a single-family house (as measured by the number of rooms) and the selling price of the house (in thousands of dollars). Two different neighborhoods are included in the study, one on the east side of the community (=0) and the other on the west side (=1). A random sample of 20 houses was selected, with the results given in the file [neighbor.xls](#).

- State the multiple regression equation.
- Interpret the meaning of the slopes in this problem.
- Predict the mean selling price for a house with nine rooms that is located in an east-side neighborhood. Construct a 95% confidence interval estimate and a 95% prediction interval.
- Perform a residual analysis on the results and determine the adequacy of the model.
- Is there a significant relationship between selling price and the two independent variables (rooms and neighborhood) at the 0.05 level of significance?
- At the 0.05 level of significance, determine whether each independent variable makes a contribution to the

regression model. Indicate the most appropriate regression model for this set of data.

- Construct 95% confidence interval estimates of the population slope for the relationship between selling price and number of rooms and between selling price and neighborhood.
- Interpret the meaning of the coefficient of multiple determination.
- Compute the adjusted  $r^2$ .
- Compute the coefficients of partial determination and interpret their meaning.
- What assumption do you need to make about the slope of selling price with number of rooms?
- Add an interaction term to the model and, at the 0.05 level of significance, determine whether it makes a significant contribution to the model.
- On the basis of the results of (f) and (l), which model is most appropriate? Explain.

**11.41** The marketing manager of a large supermarket chain would like to determine the effect of shelf space and whether the product was placed at the front or back of the aisle on the sales of pet food. A random sample of 12 equal-sized stores is selected, with the following results (stored in the file [petfood.xls](#)):

Store	Shelf Space (Feet)	Location	Weekly Sales (Dollars)
1	5	Back	160
2	5	Front	220
3	5	Back	140
4	10	Back	190
5	10	Back	240
6	10	Front	260
7	15	Back	230
8	15	Back	270
9	15	Front	280
10	20	Back	260
11	20	Back	290
12	20	Front	310

- State the multiple regression equation.
- Interpret the meaning of the slopes in this problem.
- Predict the mean weekly sales of pet food for a store with 8 feet of shelf space situated at the back of the aisle. Construct a 95% confidence interval estimate and a 95% prediction interval.
- Perform a residual analysis on the results and determine the adequacy of the model.

- e. Is there a significant relationship between sales and the two independent variables (shelf space and aisle position) at the 0.05 level of significance?
- f. At the 0.05 level of significance, determine whether each independent variable makes a contribution to the regression model. Indicate the most appropriate regression model for this set of data.
- g. Construct 95% confidence interval estimates of the population slope for the relationship between sales and shelf space and between sales and aisle location.
- h. Compare the slope in (b) with the slope for the simple linear regression model of Problem 10.4 on page 380. Explain the difference in the results.
- i. Interpret the meaning of the coefficient of multiple determination,  $r^2$ .
- j. Compute the adjusted  $r^2$ .
- k. Compare  $r^2$  with the  $r^2$  value computed in Problem 10.16 (a) on page 387.
- l. Compute the coefficients of partial determination and interpret their meaning.
- m. What assumption about the slope of shelf space with sales do you need to make in this problem?
- n. Add an interaction term to the model and, at the 0.05 level of significance, determine whether it makes a significant contribution to the model.
- o. On the basis of the results of (f) and (n), which model is most appropriate? Explain.

**11.42** In mining engineering, holes are often drilled through rock, using drill bits. As the drill hole gets deeper, additional rods are added to the drill bit to enable additional drilling to take place. It is expected that drilling time increases with depth. This increased drilling time could be caused by several factors, including the mass of the drill rods that are strung together. A key question relates to whether drilling is faster using dry drilling holes or wet drilling holes. Using dry drilling holes involves forcing compressed air down the drill rods to flush the cuttings and drive the hammer. Using wet drilling holes involves forcing water rather than air down the hole. The data file [drill.xls](#) contains measurements for a sample of 50 drill holes of the time to drill each additional 5 feet (in minutes), the depth (in feet), and whether the hole was a dry drilling hole or a wet drilling hole. Develop a model to predict additional drilling time, based on depth and type of drilling hole (dry or wet).

Source: Extracted from R. Penner and D. G. Watts, "Mining Information," *The American Statistician*, 45, 1991, pp. 4–9.

- a. State the multiple regression equation.
- b. Interpret the meaning of the slopes in this problem.
- c. Predict the mean additional drilling time for a dry drilling hole at a depth of 100 feet. Construct a 95% confidence interval estimate and a 95% prediction interval.
- d. Perform a residual analysis on the results and determine the adequacy of the model.

- e. Is there a significant relationship between additional drilling time and the two independent variables (depth and type of drilling hole) at the 0.05 level of significance?
- f. At the 0.05 level of significance, determine whether each independent variable makes a contribution to the regression model. Indicate the most appropriate regression model for this set of data.
- g. Construct 95% confidence interval estimates of the population slope for the relationship between additional drilling time and depth and between additional drilling time and type of drilling hole.
- h. Interpret the meaning of the coefficient of multiple determination.
- i. Compute the adjusted  $r^2$ .
- j. Compute the coefficients of partial determination and interpret their meaning.
- k. What assumption do you need to make about the slope of additional drilling time with depth?
- l. Add an interaction term to the model and, at the 0.05 level of significance, determine whether it makes a significant contribution to the model.
- m. On the basis of the results of (f) and (l), which model is most appropriate? Explain.

**11.43** The owner of a moving company typically has his most experienced manager predict the total number of labor hours that will be required to complete an upcoming move. This approach has proved useful in the past, but the owner would like to be able to develop a more accurate method of predicting the labor hours by using the number of cubic feet moved and whether there is an elevator in the apartment building. In a preliminary effort to provide a more accurate method, he has collected data for 36 moves in which the origin and destination were within the borough of Manhattan in New York City and the travel time was an insignificant portion of the hours worked. The data are stored in the file [moving.xls](#).

- a. State the multiple regression equation.
- b. Interpret the meaning of the slopes in this problem.
- c. Predict the mean labor hours for moving 500 cubic feet in an apartment building that has an elevator, and construct a 95% confidence interval estimate and a 95% prediction interval.
- d. Perform a residual analysis on the results and determine the adequacy of the model.
- e. Is there a significant relationship between labor hours and the two independent variables (cubic feet moved and whether there is an elevator in the apartment building) at the 0.05 level of significance?
- f. At the 0.05 level of significance, determine whether each independent variable makes a contribution to the regression model. Indicate the most appropriate regression model for this set of data.

- g. Construct a 95% confidence interval estimate of the population slope for the relationship between labor hours and cubic feet moved, and between labor hours and whether there is an elevator in the apartment building.
- h. Interpret the meaning of the coefficient of multiple determination.
- i. Compute the adjusted  $r^2$ .
- j. Compute the coefficients of partial determination and interpret their meaning.
- k. What assumption about the slope of labor hours with cubic feet moved do you need to make?
- l. Add an interaction term to the model and, at the 0.05 level of significance, determine whether it makes a significant contribution to the model.
- m. On the basis of the results of (f) and (l), which model is most appropriate? Explain.



**11.44** In Problem 11.4 on page 434, you used sales and orders to predict distribution cost (see the file [warecost.xls](#)). Develop a regression model to predict distribution cost that includes the sales, orders, and the interaction of sales and orders.

- a. At the 0.05 level of significance, is there evidence that the interaction term makes a significant contribution to the model?
- b. Which regression model is more appropriate, the one used in (a) or the one used in Problem 11.4? Explain.

**11.45** Zagat's publishes restaurant ratings for various locations in the United States. The data file [restaurants.xls](#) contains the Zagat rating for food, décor, service, and the price per person for a sample of 50 restaurants located in an urban area (New York City) and 50 restaurants located in a suburban area (Long Island). Develop a regression model to predict the price per person based on a variable that represents the sum of the ratings for food, décor, and service and a dummy variable concerning location (New York City or Long Island).

*Source: Extracted from Zagat Survey 2002 New York City Restaurants and Zagat Survey 2001–2002 Long Island Restaurants.*

- a. State the multiple regression equation.
- b. Interpret the meaning of the slopes in this problem.
- c. Predict the mean price for a restaurant with a summated rating of 60 that is located in New York City and construct a 95% confidence interval estimate and a 95% prediction interval.
- d. Perform a residual analysis on the results and determine the adequacy of the model.
- e. Is there a significant relationship between price and the two independent variables (summated rating and location) at the 0.05 level of significance?
- f. At the 0.05 level of significance, determine whether each independent variable makes a contribution to the regression model. Indicate the most appropriate regression model for this set of data.

- g. Construct a 95% confidence interval estimate of the population slope for the relationship between price and summated rating, and between price and location.
- h. Compare the slope in (b) with the slope for the simple linear regression model of Problem 10.90 on page 419. Explain the difference in the results.
- i. Interpret the meaning of the coefficient of multiple determination.
- j. Compute the adjusted  $r^2$ .
- k. Compare  $r^2$  with the  $r^2$  value computed in Problem 10.90 (d) on page 419.
- l. Compute the coefficients of partial determination and interpret their meaning.
- m. What assumption about the slope of price with summated rating do you need to make in this problem?
- n. Add an interaction term to the model and, at the 0.05 level of significance, determine whether it makes a significant contribution to the model.
- o. On the basis of the results of (f) and (n), which model is most appropriate? Explain.

**11.46** In Problem 11.6 on pages 434–435, you used radio advertising and newspaper advertising to predict sales (see the file [advertise.xls](#)). Develop a regression model to predict sales that includes radio advertising, newspaper advertising, and the interaction of radio advertising and newspaper advertising.

- a. At the 0.05 level of significance, is there evidence that the interaction term makes a significant contribution to the model?
- b. Which regression model is more appropriate, the one used in this problem or the one used in Problem 11.6? Explain.



**11.47** In Problem 11.5 on page 434, horsepower and weight were used to predict miles per gallon (see the file [auto.xls](#)). Develop a regression model that includes horsepower, weight, and the interaction of horsepower and weight to predict miles per gallon.

- a. At the 0.05 level of significance, is there evidence that the interaction term makes a significant contribution to the model?
- b. Which regression model is more appropriate, the one used in this problem or the one used in Problem 11.5? Explain.

**11.48** In Problem 11.7 on page 435, you used total staff present and remote hours to predict standby hours (see the file [standby.xls](#)). Develop a regression model to predict standby hours that includes total staff present, remote hours, and the interaction of total staff present and remote hours.

- a. At the 0.05 level of significance, is there evidence that the interaction term makes a significant contribution to the model?
- b. Which regression model is more appropriate, the one used in this problem or the one used in Problem 11.7? Explain.

**11.49** The director of a training program for a large insurance company is evaluating three different methods of training underwriters. The three methods are traditional, CD-ROM based, and Web based. She divides 30 trainees into three randomly assigned groups of 10. Before the start of the training, each trainee is given a proficiency exam that measures mathematics and computer skills. At the end of the training, all students take the same end-of-training exam. The results, stored in the file [underwriting.xls](#) are as follows:

Proficiency Exam	End-of-Training Exam	Method
94	14	Traditional
96	19	Traditional
98	17	Traditional
100	38	Traditional
102	40	Traditional
105	26	Traditional
109	41	Traditional
110	28	Traditional
111	36	Traditional
130	66	Traditional
80	38	CD-ROM based
84	34	CD-ROM based
90	43	CD-ROM based
97	43	CD-ROM based
97	61	CD-ROM based
112	63	CD-ROM based
115	93	CD-ROM based
118	74	CD-ROM based
120	76	CD-ROM based
120	79	CD-ROM based
92	55	Web based
96	53	Web based
99	55	Web based
101	52	Web based
102	35	Web based
104	46	Web based
107	57	Web based
110	55	Web based
111	42	Web based
118	81	Web based

Develop a multiple regression model to predict the score on the end-of-training exam, based on the score on the proficiency exam and the method of training used.

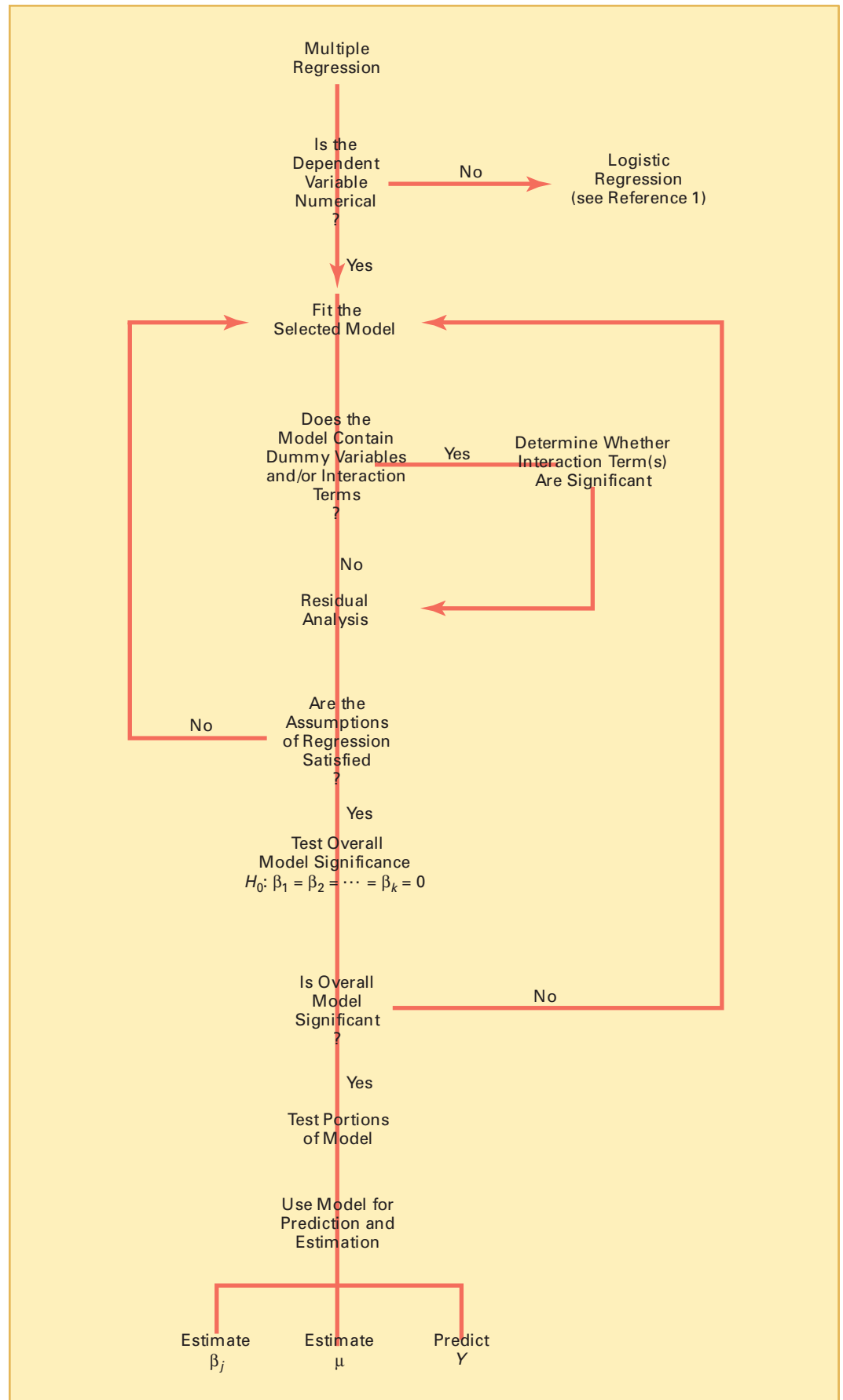
- State the multiple regression equation.
- Interpret the meaning of the slopes in this problem.
- Predict the mean end-of-training exam score for a student with a proficiency exam score of 100 who had Web-based training.
- Perform a residual analysis on your results and determine the adequacy of the model.
- Is there a significant relationship between the end-of-training exam score and the independent variables (proficiency score and training method) at the 0.05 level of significance?
- At the 0.05 level of significance, determine whether each independent variable makes a contribution to the regression model. Indicate the most appropriate regression model for this set of data.
- Construct 95% confidence interval estimates of the population slope for the relationship between end-of-training exam score and each independent variable.
- Interpret the meaning of the coefficient of multiple determination.
- Compute the adjusted  $r^2$ .
- Compute the coefficients of partial determination and interpret their meaning.
- What assumption about the slope of proficiency score with end-of-training exam score do you need to make in this problem?
- Add interaction terms to the model and, at the 0.05 level of significance, determine whether any interaction terms make a significant contribution to the model.
- On the basis of the results of (f) and (l), which model is most appropriate? Explain.

## SUMMARY

In this chapter, you learned how a marketing manager can use multiple regression analysis to understand the effects of price and promotional expenses on the sales of a new

product. You also learned how to include categorical independent variables and interaction terms in regression models. Figure 11.13 represents a roadmap of the chapter.

**FIGURE 11.13**  
Roadmap for multiple regression



## KEY EQUATIONS

### Multiple Regression Model with $k$ Independent Variables

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \beta_3 X_{3i} + \cdots + \beta_k X_{ki} + \varepsilon_i \quad (11.1)$$

### Multiple Regression Model with Two Independent Variables

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \varepsilon_i \quad (11.2)$$

### Multiple Regression Equation with Two Independent Variables

$$\hat{Y}_i = b_0 + b_1 X_{1i} + b_2 X_{2i} \quad (11.3)$$

### Coefficient of Multiple Determination

$$r^2 = \frac{\text{Regression sum of squares}}{\text{Total sum of squares}} = \frac{SSR}{SST} \quad (11.4)$$

### Adjusted $r^2$

$$r_{\text{adj}}^2 = 1 - \left[ (1 - r^2) \frac{n - 1}{n - k - 1} \right] \quad (11.5)$$

### Overall $F$ Test Statistic

$$F = \frac{MSR}{MSE} \quad (11.6)$$

### Testing for the Slope in Multiple Regression

$$t = \frac{b_j - \beta_j}{S_{b_j}} \quad (11.7)$$

### Confidence Interval Estimate for the Slope

$$b_j \pm t_{n-k-1} S_{b_j} \quad (11.8)$$

### Determining the Contribution of an Independent Variable to the Regression Model

$$SSR(X_j | \text{All variables except } j) = SSR(\text{All variables including } j) - SSR(\text{All variables except } j) \quad (11.9)$$

### Contribution of Variable $X_1$ , Given That $X_2$ Has Been Included

$$SSR(X_1 | X_2) = SSR(X_1 \text{ and } X_2) - SSR(X_2) \quad (11.10a)$$

### Contribution of Variable $X_2$ , Given That $X_1$ Has Been Included

$$SSR(X_2 | X_1) = SSR(X_1 \text{ and } X_2) - SSR(X_1) \quad (11.10b)$$

### Partial $F$ Test Statistic

$$F = \frac{SSR(X_j | \text{All variables except } j)}{MSE} \quad (11.11)$$

### Relationship Between a $t$ Statistic and an $F$ Statistic

$$t_a^2 = F_{1,a} \quad (11.12)$$

### Coefficients of Partial Determination for a Multiple Regression Model Containing Two Independent Variables

$$r_{Y1.2}^2 = \frac{SSR(X_1 | X_2)}{SST - SSR(X_1 \text{ and } X_2) + SSR(X_1 | X_2)} \quad (11.13a)$$

and

$$r_{Y2.1}^2 = \frac{SSR(X_2 | X_1)}{SST - SSR(X_1 \text{ and } X_2) + SSR(X_2 | X_1)} \quad (11.13b)$$

### Coefficient of Partial Determination for a Multiple Regression Model Containing $k$ Independent Variables

$$r_{Yj,(\text{All variables except } j)}^2 = \frac{SSR(X_j | \text{All variables except } j)}{SST - SSR(\text{All variables including } j) + SSR(X_j | \text{All variables except } j)} \quad (11.14)$$

## KEY TERMS

adjusted  $r^2$  436

coefficient of multiple determination 436

coefficient of partial determination 448

cross-product term 453

dummy variable 450

interaction 453

interaction term 453

multiple regression model 430

net regression coefficient 432

overall  $F$  test 437

partial  $F$  test 445

# CHAPTER REVIEW PROBLEMS

## Checking Your Understanding

**11.50** How does the interpretation of the regression coefficients differ in multiple regression and simple regression?

**11.51** How does testing the significance of the entire multiple regression model differ from testing the contribution of each independent variable?

**11.52** How do the coefficients of partial determination differ from the coefficient of multiple determination?

**11.53** Why and how do you use dummy variables?

**11.54** How can you evaluate whether the slope of the response variable with an independent variable is the same for each level of the dummy variable?

**11.55** Under what circumstances do you include an interaction in a regression model?

**11.56** When a dummy variable is included in a regression model that has one numerical independent variable, what assumption do you need to make concerning the slope between the response variable,  $Y$ , and the numerical independent variable,  $X$ ?

## Applying the Concepts

**11.57** Increasing customer satisfaction typically results in increased purchase behavior. For many products, there is more than one measure of customer satisfaction. In many of these instances, purchase behavior can increase dramatically with an increase in any one of the customer satisfaction measures, not necessarily all of them at the same time. Gunst and Barry (“One Way to Moderate Ceiling Effects,” *Quality Progress*, October 2003, pp. 83–85) consider a product with two satisfaction measures,  $X_1$  and  $X_2$ , that range from the lowest level of satisfaction, 1, to the highest level of satisfaction, 7. The dependent variable,  $Y$ , is a measure of purchase behavior, with the highest value generating the most sales. The following regression equation is presented:

$$\hat{Y}_i = -3.888 + 1.449 X_{1i} + 1.462 X_{2i} - 0.190 X_{1i} X_{2i}$$

Suppose that  $X_1$  is the perceived quality of the product and  $X_2$  is the perceived value of the product. (Note: If the product is overpriced in the view of the customer, he or she perceives it to be of low value and vice versa.)

- What is the predicted purchase behavior when  $X_1 = 2$  and  $X_2 = 2$ ?
- What is the predicted purchase behavior when  $X_1 = 2$  and  $X_2 = 7$ ?
- What is the predicted purchase behavior when  $X_1 = 7$  and  $X_2 = 2$ ?

- What is the predicted purchase behavior when  $X_1 = 7$  and  $X_2 = 7$ ?
- What is the regression equation when  $X_2 = 2$ ? What is the slope for  $X_1$  now?
- What is the regression equation when  $X_2 = 7$ ? What is the slope for  $X_1$  now?
- What is the regression equation when  $X_1 = 2$ ? What is the slope for  $X_2$  now?
- What is the regression equation when  $X_1 = 7$ ? What is the slope for  $X_2$  now?
- Discuss the implications of (a) through (h) within the context of increasing sales for this product with two customer satisfaction measures.

**11.58** The owner of a moving company typically has his most experienced manager predict the total number of labor hours that will be required to complete an upcoming move. This approach has proved useful in the past, but the owner would like to be able to develop a more accurate method of predicting the labor hours by using the number of cubic feet moved and the number of pieces of large furniture. In a preliminary effort to provide a more accurate method, he has collected data for 36 moves in which the origin and destination were within the borough of Manhattan in New York City and the travel time was an insignificant portion of the hours worked. The data are stored in the file [moving.xls](#).

- State the multiple regression equation.
- Interpret the meaning of the slopes in this equation.
- Predict the mean labor hours for moving 500 cubic feet with two large pieces of furniture.
- Perform a residual analysis on your results and determine the adequacy of the model.
- Determine whether there is a significant relationship between labor hours and the two independent variables (the amount of cubic feet moved and the number of pieces of large furniture) at the 0.05 level of significance.
- Determine the  $p$ -value in (e) and interpret its meaning.
- Interpret the meaning of the coefficient of multiple determination in this problem.
- Determine the adjusted  $r^2$ .
- At the 0.05 level of significance, determine whether each independent variable makes a significant contribution to the regression model. Indicate the most appropriate regression model for this set of data.
- Determine the  $p$ -values in (i) and interpret their meaning.
- Construct a 95% confidence interval estimate of the population slope between labor hours and the number of cubic feet moved. How does the interpretation of the slope here differ from that in Problem 10.44 on page 402?

- i. Compute and interpret the coefficients of partial determination.

**11.59** Professional basketball has truly become a sport that generates interest among fans around the world. More and more players come from outside the United States to play in the National Basketball Association (NBA). You want to develop a regression model to predict the number of wins achieved by each NBA team, based on field goal (shots made) percentage for the team and for the opponent. The data are stored in the file [nba2006.xls](#).

- State the multiple regression equation.
- Interpret the meaning of the slopes in this equation.
- Predict the mean number of wins for a team that has a field goal percentage of 45% and an opponent field goal percentage of 44%.
- Perform a residual analysis on your results and determine the adequacy of the fit of the model.
- Is there a significant relationship between number of wins and the two independent variables (field goal percentage for the team and for the opponent) at the 0.05 level of significance?
- Determine the  $p$ -value in (e) and interpret its meaning.
- Interpret the meaning of the coefficient of multiple determination in this problem.
- Determine the adjusted  $r^2$ .
- At the 0.05 level of significance, determine whether each independent variable makes a significant contribution to the regression model. Indicate the most appropriate regression model for this set of data.
- Determine the  $p$ -values in (i) and interpret their meaning.
- Compute and interpret the coefficients of partial determination.

**11.60** A sample of 30 recently sold single-family houses in a small city is selected. Develop a model to predict the selling price (in thousands of dollars), using the assessed value (in thousands of dollars) as well as time period (in months since reassessment). The houses in the city had been reassessed at full value one year prior to the study. The results are contained in the file [house1.xls](#).

- State the multiple regression equation.
- Interpret the meaning of the slopes in this equation.
- Predict the mean selling price for a house that has an assessed value of \$170,000 and was sold in time period 12.
- Perform a residual analysis on your results and determine the adequacy of the model.
- Determine whether there is a significant relationship between selling price and the two independent variables (assessed value and time period) at the 0.05 level of significance.
- Determine the  $p$ -value in (e) and interpret its meaning.
- Interpret the meaning of the coefficient of multiple determination in this problem.
- Determine the adjusted  $r^2$ .

- At the 0.05 level of significance, determine whether each independent variable makes a significant contribution to the regression model. Indicate the most appropriate regression model for this set of data.
- Determine the  $p$ -values in (i) and interpret their meaning.
- Construct a 95% confidence interval estimate of the population slope between selling price and assessed value. How does the interpretation of the slope here differ from that in Problem 10.76 on page 415?
- Compute and interpret the coefficients of partial determination.

**11.61** Measuring the height of a California redwood tree is a very difficult undertaking because these trees grow to heights of over 300 feet. People familiar with these trees understand that the height of a California redwood tree is related to other characteristics of the tree, including the diameter of the tree at the breast height of a person and the thickness of the bark of the tree. The data in the file [redwood.xls](#) represent the height, diameter at breast height of a person, and bark thickness for a sample of 21 California redwood trees.

- State the multiple regression equation.
- Interpret the meaning of the slopes in this equation.
- Predict the mean height for a tree that has a breast diameter of 25 inches and a bark thickness of 2 inches.
- Interpret the meaning of the coefficient of multiple determination in this problem.
- Perform a residual analysis on the results and determine the adequacy of the model.
- Determine whether there is a significant relationship between the height of redwood trees and the two independent variables (breast diameter and the bark thickness) at the 0.05 level of significance.
- Construct a 95% confidence interval estimate of the population slope between the height of the redwood trees and breast diameter and between the height of redwood trees and the bark thickness.
- At the 0.05 level of significance, determine whether each independent variable makes a significant contribution to the regression model. Indicate the independent variables to include in this model.
- Construct a 95% confidence interval estimate of the mean height for trees that have a breast diameter of 25 inches and a bark thickness of 2 inches along with a prediction interval for an individual tree.
- Compute and interpret the coefficients of partial determination.

**11.62** Develop a model to predict the assessed value (in thousands of dollars), using the size of the houses (in thousands of square feet) and the age of the houses (in years) from the following table (whose data are stored in the file [house2.xls](#)):



House	Assessed Value (\$000)	Size of House (Thousands of Square Feet)	Age (Years)
1	184.4	2.00	3.42
2	177.4	1.71	11.50
3	175.7	1.45	8.33
4	185.9	1.76	0.00
5	179.1	1.93	7.42
6	170.4	1.20	32.00
7	175.8	1.55	16.00
8	185.9	1.93	2.00
9	178.5	1.59	1.75
10	179.2	1.50	2.75
11	186.7	1.90	0.00
12	179.3	1.39	0.00
13	174.5	1.54	12.58
14	183.8	1.89	2.75
15	176.8	1.59	7.17

- State the multiple regression equation.
  - Interpret the meaning of the slopes in this equation.
  - Predict the mean assessed value for a house that has a size of 1,750 square feet and is 10 years old.
  - Perform a residual analysis on the results and determine the adequacy of the model.
  - Determine whether there is a significant relationship between assessed value and the two independent variables (size and age) at the 0.05 level of significance.
  - Determine the  $p$ -value in (e) and interpret its meaning.
  - Interpret the meaning of the coefficient of multiple determination in this problem.
  - Determine the adjusted  $r^2$ .
  - At the 0.05 level of significance, determine whether each independent variable makes a significant contribution to the regression model. Indicate the most appropriate regression model for this set of data.
  - Determine the  $p$ -values in (i) and interpret their meaning.
  - Construct a 95% confidence interval estimate of the population slope between assessed value and size. How does the interpretation of the slope here differ from that of Problem 10.77 on page 416?
  - Compute and interpret the coefficients of partial determination.
  - The real estate assessor's office has been publicly quoted as saying that the age of a house has no bearing on its assessed value. Based on your answers to (a) through (l), do you agree with this statement? Explain.
- 11.63** Crazy Dave, a well-known baseball analyst, wants to determine which variables are important in predicting a team's wins in a given season. He has collected data related to wins, earned run average (ERA), and runs scored for the 2005 season (stored in the file [bb2005.xls](#)). Develop a model to predict the number of wins based on ERA and runs scored.
- State the multiple regression equation.
  - Interpret the meaning of the slopes in this equation.
  - Predict the mean number of wins for a team that has an ERA of 4.50 and has scored 750 runs.
  - Perform a residual analysis on the results and determine the adequacy of the model.
  - Is there a significant relationship between number of wins and the two independent variables (ERA and runs scored) at the 0.05 level of significance?
  - Determine the  $p$ -value in (e) and interpret its meaning.
  - Interpret the meaning of the coefficient of multiple determination in this problem.
  - Determine the adjusted  $r^2$ .
  - At the 0.05 level of significance, determine whether each independent variable makes a significant contribution to the regression model. Indicate the most appropriate regression model for this set of data.
  - Determine the  $p$ -values in (i) and interpret their meaning.
  - Construct a 95% confidence interval estimate of the population slope between wins and ERA.
  - Compute and interpret the coefficients of partial determination.
  - Which is more important in predicting wins—pitching, as measured by ERA, or offense, as measured by runs scored? Explain.
- 11.64** Referring to Problem 11.63, suppose that in addition to using ERA to predict the number of wins, Crazy Dave wants to include the league (American vs. National) as an independent variable. Develop a model to predict wins based on ERA and league.
- State the multiple regression equation.
  - Interpret the meaning of the slopes in this problem.
  - Predict the mean number of wins for a team with an ERA of 4.50 in the American League. Construct a 95% confidence interval estimate for all teams and a 95% prediction interval for an individual team.
  - Perform a residual analysis on the results and determine the adequacy of the model.
  - Is there a significant relationship between wins and the two independent variables (ERA and league) at the 0.05 level of significance?
  - At the 0.05 level of significance, determine whether each independent variable makes a contribution to the regression model. Indicate the most appropriate regression model for this set of data.
  - Construct 95% confidence interval estimates of the population slope for the relationship between wins and ERA and between wins and league.
  - Interpret the meaning of the coefficient of multiple determination.
  - Determine the adjusted  $r^2$ .
  - Compute and interpret the coefficients of partial determination.
  - What assumption do you have to make about the slope of wins with ERA?

- l.** Add an interaction term to the model and, at the 0.05 level of significance, determine whether it makes a significant contribution to the model.
- m.** On the basis of the results of (f) and (l), which model is most appropriate? Explain.

**11.65** You are a real estate broker who wants to compare property values in Glen Cove and Roslyn (which are located

approximately 8 miles apart). In order to do so, you will analyze the data in the [gcroslyn.xls](#) file that includes samples for Glen Cove and Roslyn. Making sure to include the dummy variable for location (Glen Cove or Roslyn) in the regression model, develop a regression model to predict appraised value, based on the land area of a property, the age of a house, and location. Be sure to determine whether any interaction terms need to be included in the model.

## Managing the Springville Herald

In its continuing study of the home-delivery subscription solicitation process, a marketing department team wants to test the effects of two types of structured sales presentations (personal formal and personal informal) and the number of hours spent on telemarketing on the number of new subscriptions. The staff has recorded these data in the file [sh14.xls](#) for the past 24 weeks. You can find this data set at [www.prenhall.com/HeraldCase/EffectsData.htm](http://www.prenhall.com/HeraldCase/EffectsData.htm) and in

the **EffectsData.htm** file in the **HeraldCase** folder on the Student CD-ROM that accompanies this book.

Analyze these data and develop a multiple regression model to predict the number of new subscriptions for a week, based on the number of hours spent on telemarketing and the sales presentation type. Write a report, giving detailed findings concerning the regression model used.

## Web Case

*Apply your knowledge of multiple regression models in this Web Case, which extends the Using Statistics OmniFoods scenario from this chapter.*

To ensure a successful test marketing of its OmniPower energy bars, the OmniFoods marketing department has contracted with In-Store Placements Group (ISPG), a merchandising consultancy. ISPG will work with the grocery store chain that is conducting the test market study. Using the same 34-store sample used in the test market study, ISPG claims that the choice of shelf location and the presence of in-store OmniPower coupon dispensers each increase sale of the energy bars.

Review the ISPG claims and supporting data at the OmniFoods internal Web page, [www.prenhall.com/Springville/Omni\\_ISPGMemo.htm](http://www.prenhall.com/Springville/Omni_ISPGMemo.htm), (or open this Web

page file from the Student CD-ROM Web Case folder) and then answer the following:

1. Are the supporting data consistent with ISPG's claims? Perform an appropriate statistical analysis to confirm (or discredit) the stated relationship between sales and the two independent variables of product shelf location and the presence of in-store OmniPower coupon dispensers.
2. If you were advising OmniFoods, would you recommend using a specific shelf location and in-store coupon dispensers to sell OmniPower bars?
3. What additional data would you advise collecting in order to determine the effectiveness of the sales promotion techniques used by ISPG?

## REFERENCES

1. Hosmer, D. W., and S. Lemeshow, *Applied Logistic Regression*, 2nd ed. (New York: Wiley, 2001).
2. Kutner, M., C. Nachtsheim, J. Neter, and W. Li, *Applied Linear Statistical Models*, 5th ed. (New York: McGraw-Hill/Irwin, 2005).
3. *Microsoft Excel 2007* (Redmond, WA: Microsoft Corp., 2007).

# Excel Companion

## to Chapter 11

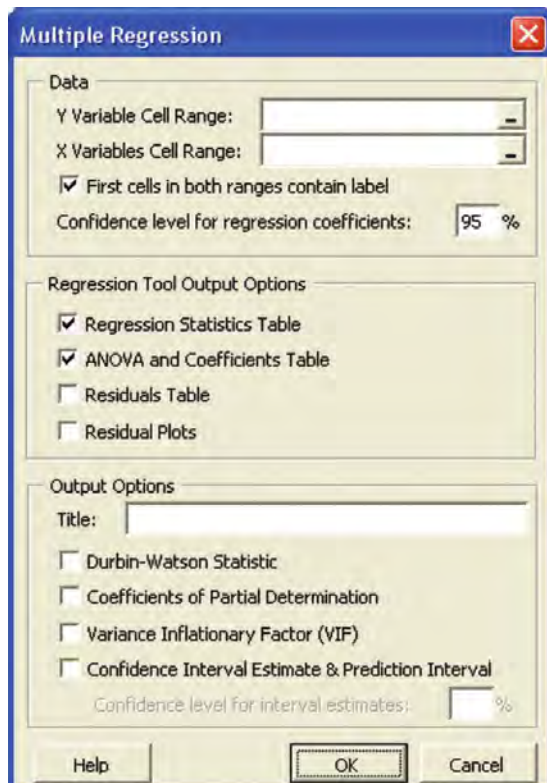
### E11.1 PERFORMING MULTIPLE REGRESSION ANALYSES

You perform a multiple regression analysis by either selecting the PHStat2 Multiple Regression procedure or the ToolPak Regression procedure.

#### Using PHStat2 Multiple Regression

Select **PHStat** → **Regression** → **Multiple Regression**. In the procedure's dialog box (shown below), enter the cell range of the *Y* variable as the **Y Variable Cell Range** and the cell range of the *X* variables as the **X Variables Cell Range**. Click **First cells in both ranges contain label** and enter a value for the **Confidence level for regression coefficients**. Click the **Regression Statistics Table** and the **ANOVA and Coefficients Table** Regression Tool Output Options, enter a title as the **Title**, and click **OK**.

PHStat2 uses the ToolPak Regression procedure to create the regression results worksheet. This worksheet does

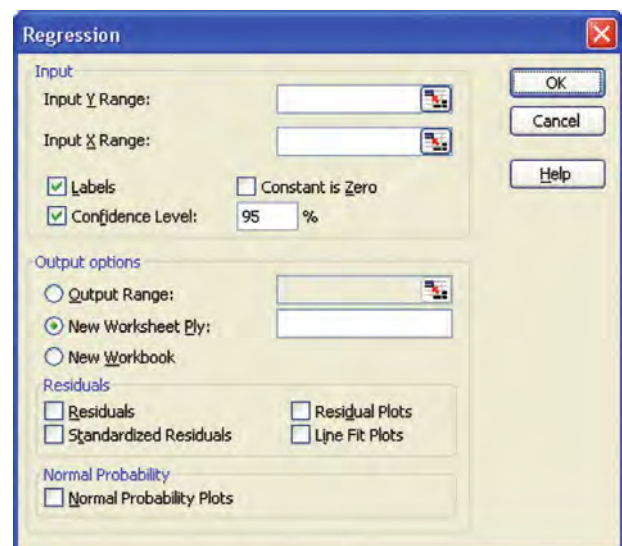


not dynamically change, and you need to rerun the PHStat2 procedure if you change your data.

The four output options found in the PHStat2 dialog box enhance the ToolPak procedure. The Durbin-Watson statistic is discussed in Section E10.4, and the other three options are explained in Sections E11.3 and E11.4.

#### Using ToolPak Regression

Open to the worksheet that contains the data for the regression analysis. Select **Tools** → **Data Analysis**, select **Regression** from the Data Analysis list, and click **OK**. In the procedure's dialog box (shown below), enter the cell range of the *Y* variable data as the **Input Y Range** and enter the cell range of the *X* variables data as the **Input X Range**. (The *X* variables cell range must be contiguous columns.) Click **Labels**. Click **Confidence Level**, and enter a value in its box, and then click **OK**. Results appear on a new worksheet.



### E11.2 CREATING MULTIPLE REGRESSION RESIDUAL PLOTS

You modify the instructions of Section E11.1 to create a table of residuals and then use Excel charting features to create the residual plots needed in multiple regression (see pages 439–440).

## Creating a Table of Residuals

You create a table of residuals by clicking additional check boxes when you perform a multiple regression analysis. If you use the PHStat2 Multiple Regression procedure, click the **Regression Statistics Table, ANOVA and Coefficients Table, Residuals Table, and Residual Plots** check boxes before you click **OK**. If you use the ToolPak Regression procedure, click the **Residuals** check box before you click **OK**.

Using either one of these modifications creates a regression results worksheet that includes a residual output table that contains columns for observation number, predicted values, and residuals. Figure E11.1 shows the residual output area for the **MR** worksheet of the **Chapter 11.xls** workbook that was used to create the residual plots in Figure 11.4 on page 440. This table of residuals, you can use Excel charting features to create the four types of residual plots needed for a multiple regression residual analysis.

	A	B	C
23	RESIDUAL OUTPUT		
24			
25	<i>Observation</i>	<i>Predicted Sales</i>	<i>Residuals</i>
26	1	3420.309524	720.6904762
27	2	3420.309524	421.6904762
28	3	3420.309524	-364.3095238
29	4	3420.309524	98.69047619
30	5	4142.921131	83.07886905
31	6	4142.921131	487.078869
32	7	4142.921131	-635.921131
33	8	4142.921131	-388.921131
34	9	4865.532738	134.4672619
35	10	4865.532738	254.4672619
36	11	4865.532738	-854.5327381
37	12	4865.532738	149.4672619
38	13	2355.962798	-439.9627976
39	14	2355.962798	-1680.962798
40	15	2355.962798	1280.037202
41	16	2355.962798	868.0372024
42	17	3078.574405	-783.5744048
43	18	3078.574405	-348.5744048
44	19	3078.574405	-460.5744048
45	20	3078.574405	1342.425595
46	21	3801.186012	311.8139881
47	22	3801.186012	-55.1860119
48	23	3801.186012	-269.1860119
49	24	3801.186012	23.8139881
50	25	1291.616071	-195.6160714
51	26	1291.616071	-530.6160714
52	27	1291.616071	796.3839286
53	28	1291.616071	-471.6160714
54	29	2014.227679	99.77232143
55	30	2014.227679	-132.2276786
56	31	2014.227679	144.7723214
57	32	2014.227679	-412.2276786
58	33	2736.839286	617.1607143
59	34	2736.839286	190.1607143

FIGURE E11.1 Creating residual plots

Use the Section E2.12 instructions, “Creating Scatter Plots,” to create residual plots. Use the cell range of your residuals as the  $Y$  variable and the cell range of either  $\hat{Y}_i$ ,

$X_{1i}$ ,  $X_{2i}$ , or *Observation* as your  $X$  variable values. (If you use PHStat2, the **Residual Plots** option created the residuals versus  $X_{1i}$  plot and the residuals versus  $X_{2i}$  plot for you, so you do not need to manually create these two plots.)

Recall from Section E2.12 that variables need to be arranged  $X$  variable first and then  $Y$  variable in order for the scatter plot to be correct. For the residuals versus  $\hat{Y}_i$  (that is, the predicted values of  $Y$ ) residual plot, this not a problem as the two exist in the proper order, in side-by-side columns, in the residual output table of the regression model worksheet. However, for the three other types of residual plots, the data for the plot is split between the regression model worksheet and the worksheet that contains the data for the model. You need to have all the data together on one worksheet (it does not matter which worksheet you use) and make sure that the column of residual values appears to the right of the column holding the  $X_{1i}$ ,  $X_{2i}$ , or *Observation* values.

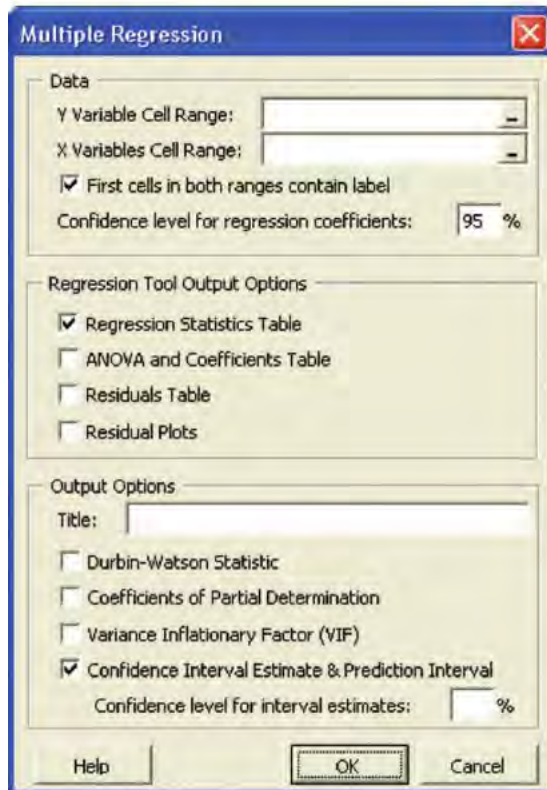
## E11.3 COMPUTING THE CONFIDENCE INTERVAL ESTIMATE OF THE MEAN AND THE PREDICTION INTERVAL

You modify the instructions for using the PHStat2 Multiple Regression procedure in Section E1.1 to compute the confidence interval estimate of the mean response and prediction interval for an individual  $Y$ .

Click **Confidence Interval Estimate & Prediction Interval** and enter a value for the **Confidence level for interval estimates** (see Figure E11.2) before you click **OK** to create the confidence interval estimate and prediction interval estimate on a separate worksheet, similar to Figure 11.2 on page 433.

Alternatives to using PHStat2 are complex and beyond the scope of this textbook. However, you can open to the **CIEandPI** worksheet of the **CIEandPIforMR.xls** workbook (shown in Figure 11.2) and review its contents. This worksheet uses the function **TINV(1-confidence level, degrees of freedom)** to compute the  $t$  statistic in cell B20. It uses the **TRANSPOSE(cell range)**, **MMULT(cell range 1, cell range 2)**, and **MINVERSE(cell range)** functions to perform advanced matrix mathematics. Some formulas in the worksheet refer to cells on the **MR** regression model worksheet, while others, not shown in Figure 11.2, refer to a **DC** worksheet that contains a copy of the regression data and a column of all 1s.

**Forum** Click on the **ADVANCED TECHNIQUES** link to learn more about the techniques used in this worksheet.



**FIGURE E11.2** Dialog box after clicking the Confidence Interval Estimate & Prediction Interval output option

## E11.4 COMPUTING THE COEFFICIENTS OF PARTIAL DETERMINATION

You modify the procedures of Section E11.1 to compute the coefficients of partial determination.

### Using PHStat2 Multiple Regression

Use the Section E11.1 instructions in “Using PHStat2 Multiple Regression,” but click the **Coefficients of Partial Determination** output option click **OK**. The coefficients of partial determination appear on a separate worksheet that is similar to Figure E11.3.

### Using ToolPak Regression

If you use the ToolPak Regression procedure, use the Section E11.1 procedures to create all necessary regression results worksheets. (For example, if you have two independent variables, you need to perform a regression analysis using  $X_1$  and  $X_2$ ,  $X_1$  alone, and  $X_2$  alone.) Then open the

	A	B	C	D	E
1	OmniPower Sales Analysis				
2	Coefficients of Partial Determination				
3					
4	Intermediate Calculations		Cell entries:		
5	SSR(X1, X2)	39472730.7730	Cell B5:	multiple regression worksheet cell C12 value	
6	SST	52093677.4412	Cell B6:	multiple regression worksheet cell C14 value	
7	SSR(X2)	14915814.1025	Cell B7:	exclude-first-X-regression worksheet cell C12 value	
8	SSR(X1)	28153486.1452	Cell B8:	exclude-second-X-regression worksheet cell C12 value	
9	SSR(X1   X2)	24558916.6706	=B5 - B7		
10	SSR(X2   X1)	11319244.6249	=B5 - B8		
11					
12	Coefficients				
13	t2 Y1.2	0.6605	=B9/(B6 - B5 * B9)		
14	t2 Y2.1	0.4728	=B10/(B6 - B5 * B10)		

**FIGURE E11.3** Completed CPD\_2 worksheet for the Omni Power sales example

**CPD.xls** workbook. Copy the worksheets for all your regression models to this workbook and then open to the CPD worksheet that matches your model. (For example, if you have two independent variables, open the **CPD\_2** worksheet.) Then follow the instructions on the worksheet to complete the worksheet. Some worksheet cells may display the #DIV/0! message until you are finished making your entries. This is not an error.

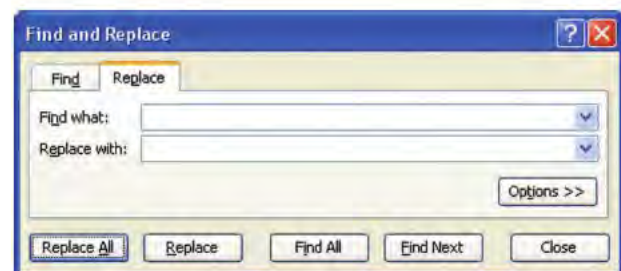
Figure E11.3 shows the CPD\_2 worksheet completed for the OmniPower sales example.

## E11.5 CREATING DUMMY VARIABLES

You use the Excel find-and-replace feature to create a dummy variable for a categorical variable. To do so, you find each categorical value and replace it with a number.

To use find-and-replace, first open the worksheet that contains your regression data. Select the (column) cell range of the categorical variable for which you are creating a dummy variable. Press **Ctrl+C** to copy these values. Select the first cell of the first empty column in your worksheet and press **Ctrl+V** to paste the values in this column.

Select all the newly pasted values and press **Ctrl+H** to display the Find and Replace dialog box (shown below).



For each unique categorical value in the column,

- Enter the categorical value as the **Find what** value.
- Enter the number that will represent that categorical value as the **Replace with** value.
- Click **Replace All**.

If a message box to confirm the replacement appears, click **OK** to continue.

Click **Close** when you are finished making all replacements. If you inadvertently click the worksheet while using this dialog box, reselect all the newly pasted values before continuing to replace. When you finish, verify your work by comparing the original column of categorical values to your new column of numerical values.

## E11.6 CREATING INTERACTION TERMS

To define the interaction term of an independent variable,  $X_1$ , and a second independent variable,  $X_2$ , create a column of formulas that multiply the first independent variable by the second independent variable. For example, if the first independent variable appeared in column B and the second independent variable appeared in column C, you would enter the formula **=B2\*C2** in the row 2 cell of an empty column and then copy the formula down through all rows of data.

# APPENDICES

## A. REVIEW OF ARITHMETIC, ALGEBRA, AND LOGARITHMS

- A.1 Rules for Arithmetic Operations
- A.2 Rules for Algebra: Exponents and Square Roots
- A.3 Rules for Logarithms

## B. SUMMATION NOTATION

## C. STATISTICAL SYMBOLS AND GREEK ALPHABET

- C.1 Statistical Symbols
- C.2 Greek Alphabet

## D. STUDENT CD-ROM CONTENTS

- D.1 CD-ROM Overview
- D.2 Data File Descriptions

## E. TABLES

- E.1 Table of Random Numbers
- E.2 The Cumulative Standardized Normal Distribution
- E.3 Critical Values of  $t$
- E.4 Critical Values of  $\chi^2$
- E.5 Critical Values of  $F$
- E.6 Table of Binomial Probabilities
- E.7 Table of Poisson Probabilities
- E.8 Lower and Upper Critical Values,  $T_1$ , of Wilcoxon Rank Sum Test
- E.9 Critical Values of the Studentized Range,  $Q$
- E.10 Critical Values  $d_L$  and  $d_U$  of the Durbin-Watson Statistic,  $D$
- E.11 Control Chart Factors

## F. MICROSOFT EXCEL AND PHSTAT2 FAQs

- F.1 Student CD-ROM FAQs
- F.2 General Computing FAQs
- F.3 Microsoft Excel FAQs
- F.4 PHStat2 FAQs
- F.5 Microsoft Excel 2007 FAQs

## SELF-TEST SOLUTIONS AND ANSWERS TO SELECTED EVEN-NUMBERED PROBLEMS

## A. REVIEW OF ARITHMETIC, ALGEBRA, AND LOGARITHMS

### A.1 RULES FOR ARITHMETIC OPERATIONS

RULE	EXAMPLE
1. $a + b = c$ and $b + a = c$	$2 + 1 = 3$ and $1 + 2 = 3$
2. $a + (b + c) = (a + b) + c$	$5 + (7 + 4) = (5 + 7) + 4 = 16$
3. $a - b = c$ but $b - a \neq c$	$9 - 7 = 2$ but $7 - 9 = -2$
4. $a \times b = b \times a$	$7 \times 6 = 6 \times 7 = 42$
5. $a \times (b + c) = (a \times b) + (a \times c)$	$2 \times (3 + 5) = (2 \times 3) + (2 \times 5) = 16$
6. $a \div b \neq b \div a$	$12 \div 3 \neq 3 \div 12$
7. $\frac{a + b}{c} = \frac{a}{c} + \frac{b}{c}$	$\frac{7 + 3}{2} = \frac{7}{2} + \frac{3}{2} = 5$
8. $\frac{a}{b + c} \neq \frac{a}{b} + \frac{a}{c}$	$\frac{3}{4 + 5} \neq \frac{3}{4} + \frac{3}{5}$
9. $\frac{1}{a} + \frac{1}{b} = \frac{b + a}{ab}$	$\frac{1}{3} + \frac{1}{5} = \frac{5 + 3}{(3)(5)} = \frac{8}{15}$
10. $\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$	$\frac{2}{3} \times \frac{6}{7} = \frac{2 \times 6}{3 \times 7} = \frac{12}{21}$
11. $\frac{a}{b} \div \frac{c}{d} = \frac{a \times d}{b \times c}$	$\frac{5}{8} \div \frac{3}{7} = \frac{5 \times 7}{8 \times 3} = \frac{35}{24}$

### A.2 RULES FOR ALGEBRA: EXPONENTS AND SQUARE ROOTS

RULE	EXAMPLE
1. $X^a \times X^b = X^{a+b}$	$4^2 \times 4^3 = 4^5$
2. $(X^a)^b = X^{ab}$	$(2^2)^3 = 2^6$
3. $(X^a/X^b) = X^{a-b}$	$\frac{3^5}{3^3} = 3^2$
4. $\frac{X^a}{X^a} = X^0 = 1$	$\frac{3^4}{3^4} = 3^0 = 1$
5. $\sqrt{XY} = \sqrt{X} \sqrt{Y}$	$\sqrt{(25)(4)} = \sqrt{25} \sqrt{4} = 10$
6. $\sqrt{\frac{X}{Y}} = \frac{\sqrt{X}}{\sqrt{Y}}$	$\sqrt{\frac{16}{100}} = \frac{\sqrt{16}}{\sqrt{100}} = 0.40$



### A.3 RULES FOR LOGARITHMS

#### Base 10

LOG is the symbol used for base-10 logarithms:

RULE	EXAMPLE
1. $\text{LOG}(10^A) = A$	$\text{LOG}(100) = \text{LOG}(10^2) = 2$
2. If $\text{LOG}(A) = B$ , then $A = 10^B$	If $\text{LOG}(A) = 2$ , then $A = 10^2 = 100$
3. $\text{LOG}(A \times B) = \text{LOG}(A) + \text{LOG}(B)$	$\text{LOG}(100) = \text{LOG}(10 \times 10)$ $= \text{LOG}(10) + \text{LOG}(10) = 1 + 1 = 2$
4. $\text{LOG}(A^B) = B \times \text{LOG}(A)$	$\text{LOG}(1000) = \text{LOG}(10^3) = 3 \times \text{LOG}(10)$ $= 3 \times 1 = 3$
5. $\text{LOG}(A/B) = \text{LOG}(A) - \text{LOG}(B)$	$\text{LOG}(100) = \text{LOG}(1000/10)$ $= \text{LOG}(1000) - \text{LOG}(10) = 3 - 1 = 2$

#### EXAMPLE

Take the base 10 logarithm of each side of the following equation:

$$Y = \beta_0 \beta_1^X \epsilon$$

SOLUTION Apply rules 3 and 4:

$$\begin{aligned} \text{LOG}(Y) &= \text{LOG}(\beta_0 \beta_1^X \epsilon) \\ &= \text{LOG}(\beta_0) + \text{LOG}(\beta_1^X) + \text{LOG}(\epsilon) \\ &= \text{LOG}(\beta_0) + X \times \text{LOG}(\beta_1) + \text{LOG}(\epsilon) \end{aligned}$$

#### Base e

LN is the symbol used for base  $e$  logarithms, commonly referred to as natural logarithms.  $e$  is Euler's number, and  $e \cong 2.718282$ :

RULE	EXAMPLE
1. $\text{LN}(e^A) = A$	$\text{LN}(7.389056) = \text{LN}(e^2) = 2$
2. If $\text{LN}(A) = B$ , then $A = e^B$	If $\text{LN}(A) = 2$ , then $A = e^2 = 7.389056$
3. $\text{LN}(A \times B) = \text{LN}(A) + \text{LN}(B)$	$\text{LN}(100) = \text{LN}(10 \times 10)$ $= \text{LN}(10) + \text{LN}(10)$ $= 2.302585 + 2.302585 = 4.605170$
4. $\text{LN}(A^B) = B \times \text{LN}(A)$	$\text{LN}(1000) = \text{LN}(10^3) = 3 \times \text{LN}(10)$ $= 3 \times 2.302585 = 6.907755$
5. $\text{LN}(A/B) = \text{LN}(A) - \text{LN}(B)$	$\text{LN}(100) = \text{LN}(1000/10)$ $= \text{LN}(1000) - \text{LN}(10)$ $= 6.907755 - 2.302585 = 4.605170$

#### EXAMPLE

Take the base  $e$  logarithm of each side of the following equation:

$$Y = \beta_0 \beta_1^X \epsilon$$

SOLUTION Apply rules 3 and 4:

$$\begin{aligned} \text{LN}(Y) &= \text{LN}(\beta_0 \beta_1^X \epsilon) \\ &= \text{LN}(\beta_0) + \text{LN}(\beta_1^X) + \text{LN}(\epsilon) \\ &= \text{LN}(\beta_0) + X \times \text{LN}(\beta_1) + \text{LN}(\epsilon) \end{aligned}$$

## B. SUMMATION NOTATION

The symbol  $\Sigma$ , the Greek capital letter sigma, is used to denote “taking the sum of.” Consider a set of  $n$  values for variable  $X$ . The expression  $\sum_{i=1}^n X_i$  means that these  $n$  values are to be added together. Thus:

$$\sum_{i=1}^n X_i = X_1 + X_2 + X_3 + \cdots + X_n$$

The following problem illustrates the use of summation notation. Consider five values of a variable  $X$ :  $X_1 = 2$ ,  $X_2 = 0$ ,  $X_3 = -1$ ,  $X_4 = 5$ , and  $X_5 = 7$ . Thus:

$$\sum_{i=1}^5 X_i = X_1 + X_2 + X_3 + X_4 + X_5 = 2 + 0 + (-1) + 5 + 7 = 13$$

In statistics, the squared values of a variable are often summed. Thus:

$$\sum_{i=1}^n X_i^2 = X_1^2 + X_2^2 + X_3^2 + \cdots + X_n^2$$

and, in the example above:

$$\begin{aligned} \sum_{i=1}^5 X_i^2 &= X_1^2 + X_2^2 + X_3^2 + X_4^2 + X_5^2 \\ &= 2^2 + 0^2 + (-1)^2 + 5^2 + 7^2 \\ &= 4 + 0 + 1 + 25 + 49 \\ &= 79 \end{aligned}$$

$\sum_{i=1}^n X_i^2$ , the summation of the squares, is *not* the same as  $\left(\sum_{i=1}^n X_i\right)^2$ , the square of the sum:

$$\sum_{i=1}^n X_i^2 \neq \left(\sum_{i=1}^n X_i\right)^2$$

In the example given earlier, the summation of squares is equal to 79. This is not equal to the square of the sum, which is  $13^2 = 169$ .

Another frequently used operation involves the summation of the product. Consider two variables,  $X$  and  $Y$ , each having  $n$  values. Then:

$$\sum_{i=1}^n X_i Y_i = X_1 Y_1 + X_2 Y_2 + X_3 Y_3 + \cdots + X_n Y_n$$

Continuing with the previous example, suppose there is a second variable,  $Y$ , whose five values are  $Y_1 = 1$ ,  $Y_2 = 3$ ,  $Y_3 = -2$ ,  $Y_4 = 4$ , and  $Y_5 = 3$ . Then,

$$\begin{aligned} \sum_{i=1}^5 X_i Y_i &= X_1 Y_1 + X_2 Y_2 + X_3 Y_3 + X_4 Y_4 + X_5 Y_5 \\ &= (2)(1) + (0)(3) + (-1)(-2) + (5)(4) + (7)(3) \\ &= 2 + 0 + 2 + 20 + 21 \\ &= 45 \end{aligned}$$

In computing  $\sum_{i=1}^n X_i Y_i$ , realize that the first value of  $X$  is multiplied by the first value of  $Y$ , the second value of  $X$  is multiplied by the second value of  $Y$ , and so on. These cross products are then summed in order to compute the desired result. However, the summation of products is *not* equal to the product of the individual sums:

$$\sum_{i=1}^n X_i Y_i \neq \left( \sum_{i=1}^n X_i \right) \left( \sum_{i=1}^n Y_i \right)$$

In this example,

$$\sum_{i=1}^5 X_i = 13$$

and

$$\sum_{i=1}^5 Y_i = 1 + 3 + (-2) + 4 + 3 = 9$$

so that

$$\left( \sum_{i=1}^5 X_i \right) \left( \sum_{i=1}^5 Y_i \right) = (13)(9) = 117$$

However,

$$\sum_{i=1}^5 X_i Y_i = 45$$

The following table summarizes these results:

VALUE	$X_i$	$Y_i$	$X_i Y_i$
1	2	1	2
2	0	3	0
3	-1	-2	2
4	5	4	20
5	7	3	21
	$\sum_{i=1}^5 X_i = 13$	$\sum_{i=1}^5 Y_i = 9$	$\sum_{i=1}^5 X_i Y_i = 45$

**RULE 1** The summation of the values of two variables is equal to the sum of the values of each summed variable:

$$\sum_{i=1}^n (X_i + Y_i) = \sum_{i=1}^n X_i + \sum_{i=1}^n Y_i$$

Thus,

$$\begin{aligned} \sum_{i=1}^5 (X_i + Y_i) &= (2 + 1) + (0 + 3) + (-1 + (-2)) + (5 + 4) + (7 + 3) \\ &= 3 + 3 + (-3) + 9 + 10 \\ &= 22 \end{aligned}$$

$$\sum_{i=1}^5 X_i + \sum_{i=1}^5 Y_i = 13 + 9 = 22$$

RULE 2 The summation of a difference between the values of two variables is equal to the difference between the summed values of the variables:

$$\sum_{i=1}^n (X_i - Y_i) = \sum_{i=1}^n X_i - \sum_{i=1}^n Y_i$$

Thus,

$$\begin{aligned} \sum_{i=1}^5 (X_i - Y_i) &= (2 - 1) + (0 - 3) + (-1 - (-2)) + (5 - 4) + (7 - 3) \\ &= 1 + (-3) + 1 + 1 + 4 \\ &= 4 \end{aligned}$$

$$\sum_{i=1}^5 X_i - \sum_{i=1}^5 Y_i = 13 - 9 = 4$$

RULE 3 The summation of a constant times a variable is equal to that constant times the summation of the values of the variable:

$$\sum_{i=1}^n cX_i = c \sum_{i=1}^n X_i$$

where  $c$  is a constant. Thus, if  $c = 2$ ,

$$\begin{aligned} \sum_{i=1}^5 cX_i &= \sum_{i=1}^5 2X_i = (2)(2) + (2)(0) + (2)(-1) + (2)(5) + (2)(7) \\ &= 4 + 0 + (-2) + 10 + 14 \\ &= 26 \end{aligned}$$

$$c \sum_{i=1}^5 X_i = 2 \sum_{i=1}^5 X_i = (2)(13) = 26$$

RULE 4 A constant summed  $n$  times will be equal to  $n$  times the value of the constant.

$$\sum_{i=1}^n c = nc$$

where  $c$  is a constant. Thus, if the constant  $c = 2$  is summed 5 times,

$$\begin{aligned} \sum_{i=1}^5 c &= 2 + 2 + 2 + 2 + 2 = 10 \\ nc &= (5)(2) = 10 \end{aligned}$$

EXAMPLE

Suppose there are six values for the variables  $X$  and  $Y$ , such that  $X_1 = 2$ ,  $X_2 = 1$ ,  $X_3 = 5$ ,  $X_4 = -3$ ,  $X_5 = 1$ ,  $X_6 = -2$ , and  $Y_1 = 4$ ,  $Y_2 = 0$ ,  $Y_3 = -1$ ,  $Y_4 = 2$ ,  $Y_5 = 7$ , and  $Y_6 = -3$ . Compute each of the following:

a.  $\sum_{i=1}^6 X_i$

d.  $\sum_{i=1}^6 Y_i^2$

b.  $\sum_{i=1}^6 Y_i$

e.  $\sum_{i=1}^6 X_i Y_i$

c.  $\sum_{i=1}^6 X_i^2$

f.  $\sum_{i=1}^6 (X_i + Y_i)$

$$\begin{array}{ll} \text{g. } \sum_{i=1}^6 (X_i - Y_i) & \text{i. } \sum_{i=1}^6 (cX_i), \text{ where } c = -1 \\ \text{h. } \sum_{i=1}^6 (X_i - 3Y_i + 2X_i^2) & \text{j. } \sum_{i=1}^6 (X_i - 3Y_i + c), \text{ where } c = +3 \end{array}$$

ANSWER

(a) 4 (b) 9 (c) 44 (d) 79 (e) 10 (f) 13 (g) -5 (h) 65 (i) -4 (j) -5

### References

1. Bashaw, W. L., *Mathematics for Statistics* (New York: Wiley, 1969).
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3. Levine, D., *The MBA Primer: Business Statistics* (Cincinnati, OH: Southwestern Publishing, 2000).
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## C. STATISTICAL SYMBOLS AND GREEK ALPHABET

### C.1 STATISTICAL SYMBOLS

+ add	× multiply
– subtract	÷ divide
= equal to	≠ not equal to
≅ approximately equal to	
> greater than	< less than
≥ greater than or equal to	≤ less than or equal to

### C.2 GREEK ALPHABET

GREEK LETTER	LETTER NAME	ENGLISH EQUIVALENT	GREEK LETTER	LETTER NAME	ENGLISH EQUIVALENT		
A	α	Alpha	a	N	ν	Nu	n
B	β	Beta	b	Ξ	ξ	Xi	x
Γ	γ	Gamma	g	O	ο	Omicron	ō
Δ	δ	Delta	d	Π	π	Pi	p
E	ε	Epsilon	ē	P	ρ	Rho	r
Z	ζ	Zeta	z	Σ	σ	Sigma	s
H	η	Eta	ē	T	τ	Tau	t
Θ	θ	Theta	th	Υ	υ	Upsilon	u
I	ι	Iota	i	Φ	φ	Phi	ph
K	κ	Kappa	k	X	χ	Chi	ch
Λ	λ	Lambda	l	Ψ	ψ	Psi	ps
M	μ	Mu	m	Ω	ω	Omega	ō

## D. STUDENT CD-ROM CONTENTS

### D.1 CD-ROM OVERVIEW

The Student CD-ROM that is packaged with this book includes five folders that contain files that support your learning of statistics (each described separately below). For your convenience, the Student CD-ROM includes a small program that will copy the contents of folders to your system. (If you choose, you can manually explore and copy files from these folders as well.) Be sure to review the **Readme.txt** file in the root folder of the Student CD-ROM for any late-breaking changes to the contents of the CD.

#### *Excel Data Files*

Located in the **Browse** folder, the Excel Data Files folder contains the Microsoft Excel workbook files (with the extension .xls) used in the examples and problems in this text. A detailed list of the files appears in Section D.2.

#### *Herald Case*

Located in the **Browse** folder, the Herald Case folder contains the files for the “Managing the *Springville Herald*” case. (These files are also available online at the URLs listed in the case.)

#### *Web Case*

Located in the **Browse** folder, the Web Case folder contains the files for all the Web Cases that appear at the end of chapters. (These files are also available online at the URLs listed in each case.)

#### *PHStat2*

Located in the **Install** folder, the PHStat2 folder contains the setup program to install PHStat2 version 2.7 on a Windows or Vista computer. You must run the setup program successfully before you can use PHStat2 inside Microsoft Excel. (Be sure to review Section F.4 in Appendix F and the PHStat2 readme file in the PHStat2 folder before you use the setup program.)

#### *Excel Companion Workbooks*

Located in the **Browse** folder, the Microsoft Excel Companion workbooks folder contains the workbooks described and used in the Excel Companion sections. Copies of these files also appear in the Excel Data Files folder.

#### *Visual Explorations*

Located in the **Browse** folder, Visual Explorations contains the files necessary to use the add-in workbook **Visual Explorations.xla**. Before using this workbook, you should review the “Macro Security Issues” part of Section E1.6 and you may also want to consult the Appendix F FAQs.

You can open and use the **Visual Explorations.xla** file directly from the CD-ROM in Microsoft Excel. If you prefer to use Visual Explorations without the CD present, copy the files **Visual Explorations.xla** and **VESHlp.hlp** to the folder of your choice. (The Veshelp.hlp file contains the orientation and help files for this add-in.)

#### *CD-ROM Topics*

Located in the **Browse** folder, the CD-ROM Topics folder contains supplemental textbook sections in Adobe PDF files. You will need the Adobe Acrobat reader program installed on your system in order to read these sections. (In the Install folder is a version of the Adobe Reader that you can install on your system, if necessary.)

### D.2 DATA FILE DESCRIPTIONS

The following presents in alphabetical order, a listing of the Excel data files that are in the Excel Data Files folder. These files are in .xls format, which can be opened in all Excel versions. Elsewhere in this book, these file names appear in this special typeface, and with the .xls extension for example as **Mutual Funds.xls**.

**AAAMILEAGE** Gasoline mileage from AAA members, and combined city-highway driving gasoline mileage according to current government standards (Chapter 10)

**ACCESS** Coded access read times (in msec), file size, programmer group, and buffer size (Chapter 11)

**ACCRES** Processing time, in seconds, and type of computer jobs (research = 0, accounting = 1) (Chapter 10)

**ACT** ACT scores for type of course (rows) and length of course (columns) (Chapter 11)

**ADPAGES** Magazine, ad pages in 2004, and ad pages in 2005 (Chapter 10)

**ADVERTISE** Sales (in thousands of dollars), radio ads (in thousands of dollars), and newspaper ads (in thousands of dollars) for 22 cities (Chapters 14, 15)

**AMPHRS** Capacity of batteries (Chapter 12)

**ANGLE** Subgroup number and angle (Chapter 18)

**ANSCOMBE** Data sets A, B, C, and D—each with 11 pairs of  $X$  and  $Y$  values (Chapter 13)

**AUTO** Miles per gallon, horsepower, and weight for a sample of 50 car models (Chapters 14, 15)

**BANKCOST1** Bank names and bounced check fees (Chapters 2, 3, 8)

**BANKTIME** Waiting times of bank customers (Chapter 18)

**BANKYIELD** Yield for money market account and yield for one-year CD (Chapters 3, 10)

**BANK1** Waiting time (in minutes) spent by a sample of 15 customers at a bank located in a commercial district (Chapters 3, 9, 10, 12)

**BANK2** Waiting time (in minutes) spent by a sample of 15 customers at a bank located in a residential area (Chapters 3, 10, 12)

**BASEBALL** Team, attendance, high temperature on game day, winning percentage of home team, opponents winning percentage, game played on Friday, Saturday, or Sunday (0 = no, 1 = yes), promotion held (0 = no, 1 = yes) (Chapter 15)

**BASKET** Year, price of bread, beef, eggs, and lettuce (Chapter 16)

**BATTERYLIFE** Life of a camera battery, in number of shots (Chapters 2, 3, 8)

**BB2001** Team; league (0 = American, 1 = National); wins; earned run average; runs scored; hits allowed; walks allowed; saves; errors; average ticket prices; fan cost index; regular season gate receipts; local television, radio, and cable revenues; other local operating revenues; player compensation and benefits; national and other local expenses; and income from baseball operations (Chapters 2, 13, 15)

**BB2005** Team, league (0 = American, 1 = National), wins, earned run average, runs scored, hits allowed, walks allowed, saves, and errors (Chapters 13, 14, and 15)

**BBREVENUE** Team, value, and revenue (Chapter 13)

**BEDBATH** Year, coded year, and number of stores opened (Chapter 16)

**BESTREST** State, city, restaurant, hotel, cost (estimated price of dinner, including one drink and tip), and rating (1 to 100, with 1 the top-rated restaurant) (Chapter 3)

**BREAKFAST** Delivery time difference, menu choice, and desired time (Chapter 11)

**BREAKFAST2** Delivery time difference, menu choice, and desired time (Chapter 11)

**BREAKSTW** Breaking strength for operators (rows) and machines (columns) (Chapter 11)

**BUBBLEGUM** Bubble diameters for four brands for six students (Chapter 11)

**BULBS** Length of life of 40 light bulbs from manufacturer A (= 1) and 40 light bulbs from manufacturer B (= 2) (Chapters 2, 10)

**CABERNET** California and Washington ratings, California and Washington rankings (Chapter 12)

**CABOT** Year and revenue for Cabot Corporation (Chapter 16)

**CANISTER** Day and number of nonconforming film canisters (Chapter 18)

**CATFOOD** Time period and weight of cat food (Chapter 18)

**CDYIELD** Yield of money market account, 6-month CD, 1-year CD, 2.5-year CD, and 5-year CD (Chapters 11, 12)

**CEO** Company and total compensation of CEOs (Chapters 2, 3)

**CHICKEN** Sandwich, calories, fat (in grams), saturated fat (in grams), carbohydrates (in grams), and sodium (in milligrams) (Chapters 2, 3, 6, 8)

**CIRCUITS** Thickness of semiconductor wafers, by batch and position (Chapter 11)

**CIRCULATION** Magazine, reported newsstand sales, and audited newsstand sales (Chapter 13)

**CITYRESTAURANTS** Location, food, décor, service, summated rating, coded location, and price (Chapter 2)

**COCACOLA** Year, coded year, and operating revenues (in billions of dollars) at Coca-Cola Company (Chapter 16)

**COFFEE** Rating of coffees, by expert and brand (Chapter 11)

**COFFEE DRINK** Product, calories, and fat in coffee drinks (Chapters 3, 13)

**COFFEEPRICE** Year and price per pound of coffee in the United States (Chapter 16)

**COLA** Sales for normal and end-aisle locations (Chapters 10, 12)

**COLASPC** Day, total number of cans filled, and number of unacceptable cans (over a 22-day period) (Chapter 18)

**COLLEGES-BASKETBALL** School, coach's salary for 2005–2006, expenses for 2004–2005, revenues for 2004–2005 (in millions of dollars), and winning percentage in 2005–2006 (Chapters 2, 3, 13)

**COMPUTERS** Download time of three brands of computers (Chapter 11)

**COMPUTERS2** Download time, brand, and browser (Chapter 11)

**CONCRETE1** Compressive strength after two days and seven days (Chapter 10)

**CONCRETE2** Compressive strength after 2 days, 7 days, and 28 days (Chapter 11)

**CONTEST2001** Returns for experts, readers, and dart throwers (Chapters 11, 12)

**COST OF LIVING** City, overall cost rating, apartment rent, and costs of a cup of coffee, a hamburger, dry cleaning a men's suit, toothpaste, and movie tickets (Chapters 2 and 3)

**CPI-U** Year, coded year, and value of CPI-U, the consumer price index (Chapter 16)

**CRACK** Type of crack and crack size (Chapters 10, 12)

**CREDIT** Month, coded month, credit charges (Chapter 16)

**CURRENCY** Year, coded year, and mean annual exchange rates (against the U.S. dollar) for the Canadian dollar, Japanese yen, and English pound (Chapter 16)

**CUSTSALE** Week number, number of customers, and sales (in thousands of dollars) over a period of 15 consecutive weeks (Chapter 13)

**DATING** Year and number of subscribers (Chapter 2)

**DEALS** Year and number of mergers and acquisitions from January 1 to January 11 (Chapters 2, 16)

**DELIVERY** Customer number, number of cases, and delivery time (Chapter 13)

**DENTAL** Annual family dental expenses for 10 employees (Chapter 8)

**DIFFTEST** Differences in the sales invoices and actual amounts from a sample of 50 vouchers (Chapter 8)

**DINNER** Time to prepare and cook dinner (Chapter 9)

**DISCOUNT** The amount of discount taken from 150 invoices (Chapter 8)

**DISPRAZ** Price, price squared, and sales of disposable razors in 15 stores (Chapter 15)

**DJIA** Year, coded year, and Dow Jones Industrial Average at the end of the year (Chapter 16)

**DOMESTICBEER** Brand, alcohol percentage, calories, and carbohydrates in U.S. domestic beers (Chapters 2, 3, 6, 15)

**DOWMC** Company, ticker symbol, and market capitalization, in billions of dollars (Chapter 3)

**DRILL** Time to drill additional 5 feet, depth, and type of hole (Chapter 14)

**DRINK** Amount of soft drink filled in a subgroup of 50 consecutive 2-liter bottles (Chapters 2, 9)

**ELECTRICITY** Year and cost of electricity (Chapter 16)

**ELECUSE** Electricity consumption (in kilowatts) and mean temperature (in degrees Fahrenheit) over a consecutive 24-month period (Chapter 13)

**ENERGY** State and per capita kilowatt hour use (Chapter 3)

**ENERGY2** Year, price of electricity, natural gas, and fuel oil (Chapter 16)

**ERRORSPC** Number of nonconforming items and number of accounts processed over 39 days (Chapter 18)

**ERWAITING** Emergency room waiting time (in minutes) at the main facility and at satellite 1, satellite 2, and satellite 3 (Chapters 11, 12)

**ESPRESSO** Tamp (the distance in inches between the espresso grounds and the top of the portafilter) and time (the number of seconds the heart, body, and crema are separated) (Chapter 13)

**FEDRECPT** Year, coded year, and federal receipts (in billions of current dollars) (Chapter 16)

**FFCHAIN** Raters and restaurant ratings (Chapter 11)

**FIFO** Historical cost (in dollars) and audited value (in dollars) for a sample of 120 inventory items (Chapter 8)

**FLYASH** Fly ash percentage, fly ash percentage squared, and strength (Chapter 15)

**FORCE** Force required to break an insulator (Chapters 2, 3, 8, 9)

**FORD-REV** Quarter, coded quarter, revenue, and three dummy variables for quarters (Chapter 16)

**FOULSPC** Number of foul shots made and number taken over 40 days (Chapter 18)

**FREEPORT** Address, appraised value, property size (acres), house size, age, number of rooms, number of bathrooms, and number of cars that can be parked in the garage located in Freeport, New York (Chapter 15)

**FRUIT** Fruit and year, price, and quantity (Chapter 16)

**FUNDTRAN** Day, number of new investigations, and number of investigations closed over a 30-day period (Chapter 18)

**FURNITURE** Days between receipt and resolution of a sample of 50 complaints regarding purchased furniture (Chapters 2, 3, 8, 9)

**GAS** Week and price per gallon, in cents (Chapter 2)

**GASOLINE** Year, gasoline price, 1980 price index, and 1995 price index (Chapter 16)

**GCFREEROSLYN** Address, appraised value, location, property size (acres), house size, age, number of rooms, number of bathrooms, and number of cars that can be parked in the garage in Glen Cove, Freeport, and Roslyn, New York (Chapter 15)

**GCROSLYN** Address, appraised value, location, property size (acres), house size, age, number of rooms, number of bathrooms, and number of cars that can be parked in the garage in Glen Cove and Roslyn, New York (Chapter 15)

**GDP** Year and real gross domestic product (in billions of constant 1996 dollars) (Chapter 16)

**GE** Year, coded year, and stock price (Chapter 16)

**GEAR** Tooth size, part positioning, and gear distortion (Chapter 11)

**GLENCOVE** Address, appraised value, property size (acres), house size, age, number of rooms, number of bathrooms, and number of cars that can be parked in the garage in Glen Cove, New York (Chapters 14, 15)

**GOLFBALL** Distance for designs 1, 2, 3, and 4 (Chapters 11, 12)

**GPIGMAT** GMAT scores and GPI for 20 students (Chapter 13)

**GRADSURVEY** Gender, age (as of last birthday), height (in inches), major, current cumulative grade point average, undergraduate area of specialization, undergraduate cumulative grade point average, GMAT score, current employment status, number of different full-time jobs held in the past 10 years, expected salary upon completion of MBA (in thousands of dollars), anticipated salary after 5 years of experience after MBA (in thousands of dollars), satisfaction with student advisement services on campus, and amount spent for books and supplies this semester (Chapters 1, 2, 3, 4, 6, 8, 10, 11, 12)

**GRANULE** Granule loss in Boston and Vermont shingles (Chapters 3, 8, 9, 10)

**HARDNESS** Tensile strength and hardness of aluminum specimens (Chapter 13)



**HARNSWELL** Day and diameter of cam rollers (in inches) for samples of five parts produced in each of 30 batches (Chapter 18)

**HEMLOCKFARMS** Asking price, hot tub, rooms, lake view, bathrooms, bedrooms, loft/den, finished basement, and number of acres (Chapter 15)

**HOMES** Price, location, condition, bedrooms, bathrooms, and other rooms (Chapter 15)

**HOSPADM** Day, number of admissions, mean processing time (in hours), range of processing times, and proportion of laboratory rework (over a 30-day period) (Chapter 18)

**HOTEL1** Day, number of rooms, number of nonconforming rooms per day over a 28-day period, and proportion of nonconforming items (Chapter 18)

**HOTEL2** Day and delivery time for subgroups of five luggage deliveries per day over a 28-day period (Chapter 18)

**HOUSE1** Selling price (in thousands of dollars), assessed value (in thousands of dollars), type (new = 0, old = 1), and time period of sale for 30 houses (Chapter 13, 14, 15)

**HOUSE2** Assessed value (in thousands of dollars), size (in thousands of square feet), and age (in years) for 15 houses (Chapters 13, 14)

**HOUSE3** Assessed value (in thousands of dollars), size (in thousands of square feet), and presence of a fireplace for 15 houses (Chapter 14)

**HTNGOIL** Monthly consumption of heating oil (in gallons), temperature (in degrees Fahrenheit), attic insulation (in inches), and style (0 = not ranch, 1 = ranch) (Chapters 14, 15)

**ICECREAM** Daily temperature (in degrees Fahrenheit) and sales (in thousands of dollars) for 21 days (Chapter 13)

**INDEXES** Year and total rate of return (in percentage) for the Dow Jones Industrial Average (DJIA), the Standards & Poor's 500 (S&P 500), and the technology-heavy NASDAQ Composite (NASDAQ) (Chapter 3)

**INSURANCE** Processing time for insurance policies (Chapters 3, 8, 9)

**INTAGLIO** Surface hardness of untreated and treated steel plates (Chapters 10, 12)

**INVOICE** Number of invoices processed and amount of time (in hours) for 30 days (Chapter 13)

**INVOICES** Amount recorded (in dollars) from a sample of 12 sales invoices (Chapter 9)

**ITEMERR** Amount of error (in dollars) from a sample of 200 items (Chapter 8)

**KEYBOARDDEFECTS** Defect and frequency (Chapter 2)

**LARGESTBONDS** Five-year return of bond funds (Chapter 3)

**LAUNDRY** Dirt (in pounds) removed for detergent brands (rows) and cycle times (columns) (Chapter 11)

**LOCATE** Sales volume (in thousands of dollars) for front, middle, and rear locations (Chapters 11, 12)

**LUGGAGE** Delivery time, in minutes, for luggage in Wing A and Wing B of a hotel (Chapters 10, 12)

**MAIL** Weight of mail and orders (Chapters 2, 13)

**MANAGERS** Sales (ratio of yearly sales divided by the target sales value for that region), score from the Wonderlic Personnel Test, score on the Strong-Campbell Interest Inventory Test, number of years of selling experience prior to becoming a sales manager, and whether the sales manager has a degree in electrical engineering (0 = no, 1 = yes) (Chapter 15)

**MCDONALD** Year, coded year, and annual total revenues (in billions of dollars) at McDonald's Corporation (Chapter 16)

**MEASUREMENT** Sample, in-line measurement, and analytical lab measurement (Chapter 10)

**MEDICARE** Difference in amount reimbursed and amount that should have been reimbursed for office visits (Chapter 8)

**MEDREC** Day, number of discharged patients, and number of records not processed for a 30-day period (Chapter 18)

**METALS** Year and the total rate of return (in percentage) for platinum, gold, and silver (Chapter 3)

**MILEAGE** Mileage of autos calculated by owner, currently, and forecasted according to government plans (Chapters 2, 3, 13)

**MOISTURE** Moisture content of Boston shingles and Vermont shingles (Chapter 9)

**MOVIEPRICES** Theatre chain and cost of two tickets, a large popcorn, and two sodas (Chapters 2, 3, 9)

**MOVIES** Year and movie attendance (Chapters 2 and 16)

**MOVING** Labor hours, cubic feet, number of large pieces of furniture, and availability of an elevator (Chapters 13 and 14)

**MUSICONLINE** Album/artist and prices at iTunes, Walmart, MusicNow, Musicmatch, and Napster (Chapter 11)

**MUTUALFUNDS** Category, objective, assets, fees, expense ratio, 2005 return, three-year return, five-year return, and risk (Chapters 2, 3, 4, 6, 8, 10, 11, 12, and 16)

**MYELOMA** Patient, measurement before transplant, and measurement after transplant (Chapter 10)

**NATURALGAS** Coded quarter, price, and three dummy variables for quarters (Chapter 16)

**NBA2006** Team, number of wins, points per game (for team, opponent, and the difference between team and opponent), field goal (shots made), percentage (for team, opponent, and the difference between team and opponent), turnovers (losing the ball before a shot is taken) per game (for team, opponent, and the difference between team and opponent), offensive rebound percentage, and defensive rebound percentage (Chapters 14, 15)

**NEIGHBOR** Selling price (in thousands of dollars), number of rooms, and neighborhood location (east = 0, west = 1) for 20 houses (Chapter 14)

**OMNI** Bars sold, price, and promotion expenses (Chapter 14)

**O-RING** Flight number, temperature, and O-ring damage index (Chapter 13)

**OYSTERS** Year, coded year, and number of bushels harvested (Chapter 16)

**PAIN-RELIEF** Temperature, brand of pain relief tablet, and time to dissolve (Chapter 11)

**PALLET** Weight of Boston and weight of Vermont shingles (Chapters 2, 8, 9, 10)

**PARACHUTE** Tensile strength of parachutes from suppliers 1, 2, 3, and 4 (Chapters 11, 12)

**PARACHUTE2** Tensile strength for looms and suppliers (Chapter 11)

**PASTA** Weight for type of pasta (rows) and cooking time (columns) (Chapter 11)

**PEN** Gender, ad, and product rating (Chapters 11, 12)

**PERFORM** Performance rating before and after motivational training (Chapter 10)

**PETFOOD** Shelf space (in feet), weekly sales (in dollars), and aisle location (back = 0, front = 1) (Chapters 13, 14)

**PHONE** Time (in minutes) to clear telephone line problems and location (I and II) for samples of 20 customer problems reported to each of the two office locations (Chapters 3, 6, 10, 12)

**PHOTO** Density for developer strength (rows) and development time (columns) (Chapter 11)

**PIZZATIME** Time period, delivery time for local restaurant, delivery time for national chain (Chapter 10)

**PLUMBINV** Difference in dollars between actual amounts recorded on sales invoices and the amounts entered into the accounting system (Chapter 8)

**POLIO** Year and incidence rates per 100,000 persons of reported poliomyelitis (Chapter 16)

**POTATO** Percentage of solids content in filter cake, acidity (in pH), lower pressure, upper pressure, cake thickness, varidrive speed, and drum speed setting for 54 measurements (Chapter 15)

**PRESCRIPTIONS** Year and average prescription drug price (Chapter 2)

**PROTEIN** Calories (in grams), protein, percentage of calories from fat, percentage of calories from saturated fat, and cholesterol (in mg) for 25 popular protein foods (Chapter 2)

**PUMPKIN** Circumference and weight of pumpkins (Chapter 13)

**RADON** Solar radiation, soil temperature, vapor pressure, wind speed, relative humidity, dew point, ambient temperature, and radon concentration (Chapter 15)

**RAISINS** Weight of packages of raisins (Chapter 12)

**REDWOOD** Height, diameter, and bark thickness (Chapter 14)

**RENT** Monthly rental cost (in dollars) and apartment size (in square footage) for a sample of 25 apartments (Chapter 13)

**RESTAURANTS** Location, food rating, decor rating, service rating, summated rating, coded location (0 = urban, 1 = suburban), and price of restaurants (Chapters 2, 10, 13, 14)

**RETURNS** Week and stock price of Microsoft, stock price of General Motors, stock price of Ford, and stock price of International Aluminum (Chapter 13)

**ROSLYN** Address, appraised value, property size (acres), house size, age, number of rooms, number of bathrooms, and number of cars that can be parked in the garage in Roslyn, New York (Chapter 15)

**ROYALS** Game, attendance, and whether there was a promotion at Kansas City Royals games (Chapters 2 and 3)

**RUDYBIRD** Day, total cases sold, and cases of Rudybird sold (Chapter 18)

**S&PSTKIN** Coded quarters, end-of-quarter values of the quarterly Standard & Poor's Composite Stock Price Index, and three quarterly dummy variables (Chapter 16)

**SAVINGS** Bank, money market rate, one-year CD rate, and five-year CD rate (Chapters 2, 3, 6)

**SCRUBBER** Airflow, water flow, recirculating water flow, orifice diameter, and NTU (Chapter 15)

**SEALANT** Sample number, sealant strength for Boston shingles, and sealant strength for Vermont shingles (Chapter 18)

**SEARS** Year, coded year, and annual total revenues (in billions of dollars) at Sears, Roebuck & Company (Chapter 16)

**SH2** Day and number of calls received at the help desk (Chapters 2, 3)

**SH8** Rate (\$) willing to pay for the newspaper (Chapter 8)

**SH9** Blackness of newsprint (Chapter 9)

**SH10** Length of early calls (in seconds), length of late calls (in seconds), and difference (in seconds) (Chapter 10)

**SH11-1** Call, presentation plan (structured = 1, semi-structured = 2, unstructured = 3), and length of call (in seconds) (Chapter 11)

**SH11-2** Gender of caller, type of greeting, and length of call (Chapter 11)

**SH13** Hours per month spent telemarketing and number of new subscriptions per month over a 24-month period (Chapter 13)

**SH14** Hours per week spent telemarketing, number of new subscriptions, and type of presentation (Chapter 14)

**SH16** Month and number of home-delivery subscriptions over the most recent 24-month period (Chapter 16)

**SH18-1** Day, number of ads with errors, number of ads, and number of errors over a 25-day period (Chapter 18)

**SH18-2** Day and newsprint blackness measures for each of five spots made over 25 consecutive weekdays (Chapter 18)

**SITE** Store number, square footage (in thousands of square feet), and sales (in millions of dollars) in 14 Sunflowers Apparel stores (Chapter 13)

**SP500** Week, weekly change in the S&P 500, weekly change in the price of Wal-Mart, weekly change in the price of Target, and weekly change in the price of Sara Lee (Chapter 13)

- SPENDING** State and per capita federal spending (\$000) in 2004 (Chapters 2, 3, 6)
- SPONGE** Day, number of sponges produced, number of nonconforming sponges, proportion of nonconforming sponges (Chapter 18)
- SPORTING** Sales, age, annual population growth, income, percentage with high school diploma, and percentage with college diploma (Chapters 13, 15)
- SPWATER** Sample number and amount of magnesium (Chapter 18)
- STANDBY** Standby hours, staff, remote hours, Dubner hours, and labor hours for 26 weeks (Chapters 14, 15)
- STATES** State, commuting time, percentage of homes with more than eight rooms, median income, and percentage of housing that costs more than 30% of family income (Chapters 2, 3)
- STEEL** Error in actual length and specified length (Chapters 2, 6, 8, 9)
- STOCKS&BONDS** Date, closing price of Vanguard Long-Term Bond Index Fund, and closing price of the Dow Jones Industrial Average (Chapter 13)
- STOCKASSETS** Fund and assets, in billions of dollars (Chapter 3)
- STOCKS2005** Week and closing weekly stock price for S&P, Sears, Target, and Sara Lee (Chapter 2)
- STRATEGIC** Year and number of barrels in U.S. strategic oil reserve (Chapter 16)
- TAX** Quarterly sales tax receipts (in thousands of dollars) for 50 business establishments (Chapter 3)
- TAXES** County taxes (in dollars) and age of house (in years) for 19 single-family houses (Chapter 15)
- TEA3** Sample number and weight of tea bags (Chapter 18)
- TEABAGS** Weight of tea bags (Chapters 3, 8, 9)
- TELESPC** Number of orders and number of corrections over 30 days (Chapter 18)
- TENSILE** Sample number and strength (Chapter 18)
- TESTRANK** Rank scores for 10 people trained using a “traditional” method (Method = 0) and 10 people trained using an “experimental” method (Method = 1) (Chapter 12)
- TEXTBOOK** Textbook, book store price, and Amazon price (Chapter 10)
- THEMEPARKS** Name of location and admission price for one-day tickets (Chapter 3)
- TIMES** Times to get ready (Chapter 3)
- TOMATOES** Year and price per pound in the United States (Chapter 16)
- TOMYLD2** Amount of fertilizer (in pounds per 100 square feet) and yield (in pounds) for 12 plots of land (Chapter 15)
- TOYS-REV** Quarter, coded quarter, revenue, and three dummy variables for quarters (Chapter 16)
- TRADE** Days, number of undesirable trades, and number of total trades made over a 30-day period (Chapter 18)
- TRADES** Day, number of incoming calls, and number of trade executions per day over a 35-day period (Chapter 13)
- TRAINING** Assembly time and training program (team-based = 0, individual-based = 1) (Chapters 10, 12)
- TRANSMIT** Day and number of errors in transmission (Chapter 18)
- TRANSPORT** Days and patient transport times (in minutes) for samples of four patients per day over a 30-day period (Chapter 18)
- TRASHBAGS** Weight required to break four brands of trash bags (Chapters 11, 12)
- TREASURY** Year and interest rate (Chapter 16)
- TROUGH** Width of trough (Chapters 2, 3, 8, 9)
- TRSNYC** Year, unit value of variable A, and unit value of variable B (Chapter 16)
- TSMODEL1** Years, coded years, and three time series (I, II, III) (Chapter 16)
- TSMODEL2** Years, coded years, and two time series (I, II) (Chapter 16)
- TUITION2006** School, in-state tuition and fees, and out-of-state tuition and fees (Chapter 3)
- UNDERGRADSURVEY** Gender, age (*as of last birthday*), height (*in inches*), class designation, major, graduate school intention, cumulative grade point average, expected starting salary (in thousands of dollars), anticipated salary after five years of experience (in thousands of dollars), current employment status, number of campus club/group/organization/team affiliations, satisfaction with student advisement services on campus, and amount spent on books and supplies this semester (Chapters 1, 2, 3, 4, 6, 8, 10, 11, 12)
- UNDERWRITING** Score on proficiency exam, score on end of training exam, and training method (Chapter 14)
- UNEMPLOY** Year, month, and monthly unemployment rates (Chapters 2, 16)
- UNLEADED** Year, month, and price (Chapter 16)
- UTILITY** Utilities charges for 50 one-bedroom apartments (Chapters 2, 6)
- VB** Time (in minutes) for nine students to write and run a Visual Basic program (Chapter 10)
- WAIT** Waiting times and seating times, in minutes (Chapter 6)
- WALMART** Quarter and quarterly revenues (Chapter 16)
- WARECOST** Distribution cost (in thousands of dollars), sales (in thousands of dollars), and number of orders for 24 months (Chapters 13, 14, 15)
- WAREHSE** Number of units handled per day and employee number (Chapter 18)
- WEEKLYNASDAQ** Week and NASDAQ value (Chapter 16)
- WHOLEFOODS1** Item, price at Whole Foods, and price at Fairway (Chapter 10)
- WHOLEFOODS2** Item, price at Whole Foods, price at Gristede’s, price at Fairway, and price at Stop & Shop (Chapter 11)

**WIP** Processing times at each of two plants ( $A = 1$ ,  $B = 2$ ) (Chapters 3, 6, 10)

**WONDERLIC** School, average Wonderlic score of football players trying out for the NFL, and graduation rate (Chapters 2, 3, 13)

**WORKFORCE** Year, population, and size of the workforce (Chapter 16)

**WRIGLEY** Year, coded year, actual revenue, consumer price index, and real revenue (Chapter 16)

**YARN** Breaking strength, pressure, yarn sample, and side-by-side aspect (nozzle = 1, opposite = 2) (Chapter 11)

**YIELD** Cleansing step, etching step, and yield (Chapter 11)

# E. TABLES

- E.1** Table of Random Numbers
- E.2** The Cumulative Standardized Normal Distribution
- E.3** Critical Values of  $t$
- E.4** Critical Values of  $\chi^2$
- E.5** Critical Values of  $F$
- E.6** Table of Binomial Probabilities
- E.7** Table of Poisson Probabilities
- E.8** Lower and Upper Critical Values,  $T_1$  of Wilcoxon Rank Sum Test
- E.9** Critical Values of the Studentized Range,  $Q$
- E.10** Critical Values  $d_L$  and  $d_U$  of the Durbin-Watson Statistic,  $D$
- E.11** Control Chart Factors

## E. TABLES

TABLE E.1

Table of Random  
Numbers

Row	Column							
	00000 12345	00001 67890	11111 12345	11112 67890	22222 12345	22223 67890	33333 12345	33334 67890
01	49280	88924	35779	00283	81163	07275	89863	02348
02	61870	41657	07468	08612	98083	97349	20775	45091
03	43898	65923	25078	86129	78496	97653	91550	08078
04	62993	93912	30454	84598	56095	20664	12872	64647
05	33850	58555	51438	85507	71865	79488	76783	31708
06	97340	03364	88472	04334	63919	36394	11095	92470
07	70543	29776	10087	10072	55980	64688	68239	20461
08	89382	93809	00796	95945	34101	81277	66090	88872
09	37818	72142	67140	50785	22380	16703	53362	44940
10	60430	22834	14130	96593	23298	56203	92671	15925
11	82975	66158	84731	19436	55790	69229	28661	13675
12	30987	71938	40355	54324	08401	26299	49420	59208
13	55700	24586	93247	32596	11865	63397	44251	43189
14	14756	23997	78643	75912	83832	32768	18928	57070
15	32166	53251	70654	92827	63491	04233	33825	69662
16	23236	73751	31888	81718	06546	83246	47651	04877
17	45794	26926	15130	82455	78305	55058	52551	47182
18	09893	20505	14225	68514	47427	56788	96297	78822
19	54382	74598	91499	14523	68479	27686	46162	83554
20	94750	89923	37089	20048	80336	94598	26940	36858
21	70297	34135	53140	33340	42050	82341	44104	82949
22	85157	47954	32979	26575	57600	40881	12250	73742
23	11100	02340	12860	74697	96644	89439	28707	25815
24	36871	50775	30592	57143	17381	68856	25853	35041
25	23913	48357	63308	16090	51690	54607	72407	55538
26	79348	36085	27973	65157	07456	22255	25626	57054
27	92074	54641	53673	54421	18130	60103	69593	49464
28	06873	21440	75593	41373	49502	17972	82578	16364
29	12478	37622	99659	31065	83613	69889	58869	29571
30	57175	55564	65411	42547	70457	03426	72937	83792
31	91616	11075	80103	07831	59309	13276	26710	73000
32	78025	73539	14621	39044	47450	03197	12787	47709
33	27587	67228	80145	10175	12822	86687	65530	49325
34	16690	20427	04251	64477	73709	73945	92396	68263
35	70183	58065	65489	31833	82093	16747	10386	59293
36	90730	35385	15679	99742	50866	78028	75573	67257
37	10934	93242	13431	24590	02770	48582	00906	58595
38	82462	30166	79613	47416	13389	80268	05085	96666
39	27463	10433	07606	16285	93699	60912	94532	95632
40	02979	52997	09079	92709	90110	47506	53693	49892
41	46888	69929	75233	52507	32097	37594	10067	67327
42	53638	83161	08289	12639	08141	12640	28437	09268
43	82433	61427	17239	89160	19666	08814	37841	12847
44	35766	31672	50082	22795	66948	65581	84393	15890
45	10853	42581	08792	13257	61973	24450	52351	16602
46	20341	27398	72906	63955	17276	10646	74692	48438
47	54458	90542	77563	51839	52901	53355	83281	19177
48	26337	66530	16687	35179	46560	00123	44546	79896
49	34314	23729	85264	05575	96855	23820	11091	79821
50	28603	10708	68933	34189	92166	15181	66628	58599

*continued*

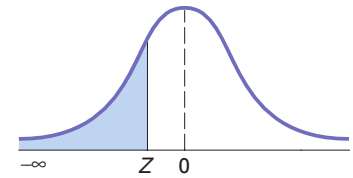
**TABLE E.1**Table of Random  
Numbers (Continued)

Row	Column							
	0000 12345	00001 67890	11111 12345	11112 67890	22222 12345	22223 67890	33333 12345	33334 67890
51	66194	28926	99547	16625	45515	67953	12108	57846
52	78240	43195	24837	32511	70880	22070	52622	61881
53	00833	88000	67299	68215	11274	55624	32991	17436
54	12111	86683	61270	58036	64192	90611	15145	01748
55	47189	99951	05755	03834	43782	90599	40282	51417
56	76396	72486	62423	27618	84184	78922	73561	52818
57	46409	17469	32483	09083	76175	19985	26309	91536
58	74626	22111	87286	46772	42243	68046	44250	42439
59	34450	81974	93723	49023	58432	67083	36876	93391
60	36327	72135	33005	28701	34710	49359	50693	89311
61	74185	77536	84825	09934	99103	09325	67389	45869
62	12296	41623	62873	37943	25584	09609	63360	47270
63	90822	60280	88925	99610	42772	60561	76873	04117
64	72121	79152	96591	90305	10189	79778	68016	13747
65	95268	41377	25684	08151	61816	58555	54305	86189
66	92603	09091	75884	93424	72586	88903	30061	14457
67	18813	90291	05275	01223	79607	95426	34900	09778
68	38840	26903	28624	67157	51986	42865	14508	49315
69	05959	33836	53758	16562	41081	38012	41230	20528
70	85141	21155	99212	32685	51403	31926	69813	58781
71	75047	59643	31074	38172	03718	32119	69506	67143
72	30752	95260	68032	62871	58781	34143	68790	69766
73	22986	82575	42187	62295	84295	30634	66562	31442
74	99439	86692	90348	66036	48399	73451	26698	39437
75	20389	93029	11881	71685	65452	89047	63669	02656
76	39249	05173	68256	36359	20250	68686	05947	09335
77	96777	33605	29481	20063	09398	01843	35139	61344
78	04860	32918	10798	50492	52655	33359	94713	28393
79	41613	42375	00403	03656	77580	87772	86877	57085
80	17930	00794	53836	53692	67135	98102	61912	11246
81	24649	31845	25736	75231	83808	98917	93829	99430
82	79899	34061	54308	59358	56462	58166	97302	86828
83	76801	49594	81002	30397	52728	15101	72070	33706
84	36239	63636	38140	65731	39788	06872	38971	53363
85	07392	64449	17886	63632	53995	17574	22247	62607
86	67133	04181	33874	98835	67453	59734	76381	63455
87	77759	31504	32832	70861	15152	29733	75371	39174
88	85992	72268	42920	20810	29361	51423	90306	73574
89	79553	75952	54116	65553	47139	60579	09165	85490
90	41101	17336	48951	53674	17880	45260	08575	49321
91	36191	17095	32123	91576	84221	78902	82010	30847
92	62329	63898	23268	74283	26091	68409	69704	82267
93	14751	13151	93115	01437	56945	89661	67680	79790
94	48462	59278	44185	29616	76537	19589	83139	28454
95	29435	88105	59651	44391	74588	55114	80834	85686
96	28340	29285	12965	14821	80425	16602	44653	70467
97	02167	58940	27149	80242	10587	79786	34959	75339
98	17864	00991	39557	54981	23588	81914	37609	13128
99	79675	80605	60059	35862	00254	36546	21545	78179
100	72335	82037	92003	34100	29879	46613	89720	13274

Source: Partially extracted from the Rand Corporation, *A Million Random Digits with 100,000 Normal Deviates* (Glencoe, IL, The Free Press, 1955).

**TABLE E.2**

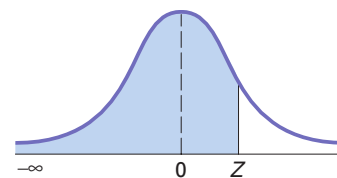
The Cumulative Standardized Normal Distribution

Entry represents area under the cumulative standardized normal distribution from  $-\infty$  to  $Z$ 

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-6.0	0.000000001									
-5.5	0.000000019									
-5.0	0.000000287									
-4.5	0.000003398									
-4.0	0.000031671									
-3.9	0.00005	0.00005	0.00004	0.00004	0.00004	0.00004	0.00004	0.00004	0.00003	0.00003
-3.8	0.00007	0.00007	0.00007	0.00006	0.00006	0.00006	0.00006	0.00005	0.00005	0.00005
-3.7	0.00011	0.00010	0.00010	0.00010	0.00009	0.00009	0.00008	0.00008	0.00008	0.00008
-3.6	0.00016	0.00015	0.00015	0.00014	0.00014	0.00013	0.00013	0.00012	0.00012	0.00011
-3.5	0.00023	0.00022	0.00022	0.00021	0.00020	0.00019	0.00019	0.00018	0.00017	0.00017
-3.4	0.00034	0.00032	0.00031	0.00030	0.00029	0.00028	0.00027	0.00026	0.00025	0.00024
-3.3	0.00048	0.00047	0.00045	0.00043	0.00042	0.00040	0.00039	0.00038	0.00036	0.00035
-3.2	0.00069	0.00066	0.00064	0.00062	0.00060	0.00058	0.00056	0.00054	0.00052	0.00050
-3.1	0.00097	0.00094	0.00090	0.00087	0.00084	0.00082	0.00079	0.00076	0.00074	0.00071
-3.0	0.00135	0.00131	0.00126	0.00122	0.00118	0.00114	0.00111	0.00107	0.00103	0.00100
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2388	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2482	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
-0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

*continued*

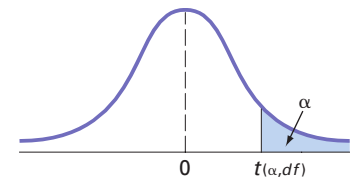


**TABLE E.2**The Cumulative Standardized Normal Distribution (*Continued*)Entry represents area under the cumulative standardized normal distribution from  $-\infty$  to  $Z$ 

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7518	0.7549
0.7	0.7580	0.7612	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.99865	0.99869	0.99874	0.99878	0.99882	0.99886	0.99889	0.99893	0.99897	0.99900
3.1	0.99903	0.99906	0.99910	0.99913	0.99916	0.99918	0.99921	0.99924	0.99926	0.99929
3.2	0.99931	0.99934	0.99936	0.99938	0.99940	0.99942	0.99944	0.99946	0.99948	0.99950
3.3	0.99952	0.99953	0.99955	0.99957	0.99958	0.99960	0.99961	0.99962	0.99964	0.99965
3.4	0.99966	0.99968	0.99969	0.99970	0.99971	0.99972	0.99973	0.99974	0.99975	0.99976
3.5	0.99977	0.99978	0.99978	0.99979	0.99980	0.99981	0.99981	0.99982	0.99983	0.99983
3.6	0.99984	0.99985	0.99985	0.99986	0.99986	0.99987	0.99987	0.99988	0.99988	0.99989
3.7	0.99989	0.99990	0.99990	0.99990	0.99991	0.99991	0.99992	0.99992	0.99992	0.99992
3.8	0.99993	0.99993	0.99993	0.99994	0.99994	0.99994	0.99994	0.99995	0.99995	0.99995
3.9	0.99995	0.99995	0.99996	0.99996	0.99996	0.99996	0.99996	0.99996	0.99997	0.99997
4.0	0.999968329									
4.5	0.999996602									
5.0	0.999999713									
5.5	0.999999981									
6.0	0.999999999									

**TABLE E.3**Critical Values of  $t$ 

For a particular number of degrees of freedom, entry represents the critical value of  $t$  corresponding to a specified upper-tail area ( $\alpha$ )



Degrees of Freedom	Upper-Tail Areas					
	0.25	0.10	0.05	0.025	0.01	0.005
1	1.0000	3.0777	6.3138	12.7062	31.8207	63.6574
2	0.8165	1.8856	2.9200	4.3027	6.9646	9.9248
3	0.7649	1.6377	2.3534	3.1824	4.5407	5.8409
4	0.7407	1.5332	2.1318	2.7764	3.7469	4.6041
5	0.7267	1.4759	2.0150	2.5706	3.3649	4.0322
6	0.7176	1.4398	1.9432	2.4469	3.1427	3.7074
7	0.7111	1.4149	1.8946	2.3646	2.9980	3.4995
8	0.7064	1.3968	1.8595	2.2060	2.8965	3.3554
9	0.7027	1.3830	1.8331	2.2622	2.8214	3.2498
10	0.6998	1.3722	1.8125	2.2281	2.7638	3.1693
11	0.6974	1.3634	1.7959	2.2010	2.7181	3.1058
12	0.6955	1.3562	1.7823	2.1788	2.6810	3.0545
13	0.6938	1.3502	1.7709	2.1604	2.6503	3.0123
14	0.6924	1.3450	1.7613	2.1448	2.6245	2.9768
15	0.6912	1.3406	1.7531	2.1315	2.6025	2.9467
16	0.6901	1.3368	1.7459	2.1199	2.5835	2.9208
17	0.6892	1.3334	1.7396	2.1098	2.5669	2.8982
18	0.6884	1.3304	1.7341	2.1009	2.5524	2.8784
19	0.6876	1.3277	1.7291	2.0930	2.5395	2.8609
20	0.6870	1.3253	1.7247	2.0860	2.5280	2.8453
21	0.6864	1.3232	1.7207	2.0796	2.5177	2.8314
22	0.6858	1.3212	1.7171	2.0739	2.5083	2.8188
23	0.6853	1.3195	1.7139	2.0687	2.4999	2.8073
24	0.6848	1.3178	1.7109	2.0639	2.4922	2.7969
25	0.6844	1.3163	1.7081	2.0595	2.4851	2.7874
26	0.6840	1.3150	1.7056	2.0555	2.4786	2.7787
27	0.6837	1.3137	1.7033	2.0518	2.4727	2.7707
28	0.6834	1.3125	1.7011	2.0484	2.4671	2.7633
29	0.6830	1.3114	1.6991	2.0452	2.4620	2.7564
30	0.6828	1.3104	1.6973	2.0423	2.4573	2.7500
31	0.6825	1.3095	1.6955	2.0395	2.4528	2.7440
32	0.6822	1.3086	1.6939	2.0369	2.4487	2.7385
33	0.6820	1.3077	1.6924	2.0345	2.4448	2.7333
34	0.6818	1.3070	1.6909	2.0322	2.4411	2.7284
35	0.6816	1.3062	1.6896	2.0301	2.4377	2.7238
36	0.6814	1.3055	1.6883	2.0281	2.4345	2.7195
37	0.6812	1.3049	1.6871	2.0262	2.4314	2.7154
38	0.6810	1.3042	1.6860	2.0244	2.4286	2.7116
39	0.6808	1.3036	1.6849	2.0227	2.4258	2.7079
40	0.6807	1.3031	1.6839	2.0211	2.4233	2.7045
41	0.6805	1.3025	1.6829	2.0195	2.4208	2.7012
42	0.6804	1.3020	1.6820	2.0181	2.4185	2.6981
43	0.6802	1.3016	1.6811	2.0167	2.4163	2.6951
44	0.6801	1.3011	1.6802	2.0154	2.4141	2.6923
45	0.6800	1.3006	1.6794	2.0141	2.4121	2.6896
46	0.6799	1.3022	1.6787	2.0129	2.4102	2.6870
47	0.6797	1.2998	1.6779	2.0117	2.4083	2.6846
48	0.6796	1.2994	1.6772	2.0106	2.4066	2.6822

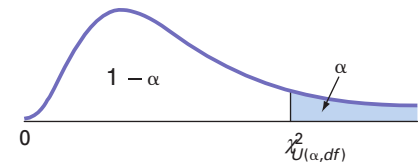
*continued*

**TABLE E.3**Critical Values of *t*  
(Continued)

Degrees of Freedom	Upper-Tail Areas					
	0.25	0.10	0.05	0.025	0.01	0.005
49	0.6795	1.2991	1.6766	2.0096	2.4049	2.6800
50	0.6794	1.2987	1.6759	2.0086	2.4033	2.6778
51	0.6793	1.2984	1.6753	2.0076	2.4017	2.6757
52	0.6792	1.2980	1.6747	2.0066	2.4002	2.6737
53	0.6791	1.2977	1.6741	2.0057	2.3988	2.6718
54	0.6791	1.2974	1.6736	2.0049	2.3974	2.6700
55	0.6790	1.2971	1.6730	2.0040	2.3961	2.6682
56	0.6789	1.2969	1.6725	2.0032	2.3948	2.6665
57	0.6788	1.2966	1.6720	2.0025	2.3936	2.6649
58	0.6787	1.2963	1.6716	2.0017	2.3924	2.6633
59	0.6787	1.2961	1.6711	2.0010	2.3912	2.6618
60	0.6786	1.2958	1.6706	2.0003	2.3901	2.6603
61	0.6785	1.2956	1.6702	1.9996	2.3890	2.6589
62	0.6785	1.2954	1.6698	1.9990	2.3880	2.6575
63	0.6784	1.2951	1.6694	1.9983	2.3870	2.6561
64	0.6783	1.2949	1.6690	1.9977	2.3860	2.6549
65	0.6783	1.2947	1.6686	1.9971	2.3851	2.6536
66	0.6782	1.2945	1.6683	1.9966	2.3842	2.6524
67	0.6782	1.2943	1.6679	1.9960	2.3833	2.6512
68	0.6781	1.2941	1.6676	1.9955	2.3824	2.6501
69	0.6781	1.2939	1.6672	1.9949	2.3816	2.6490
70	0.6780	1.2938	1.6669	1.9944	2.3808	2.6479
71	0.6780	1.2936	1.6666	1.9939	2.3800	2.6469
72	0.6779	1.2934	1.6663	1.9935	2.3793	2.6459
73	0.6779	1.2933	1.6660	1.9930	2.3785	2.6449
74	0.6778	1.2931	1.6657	1.9925	2.3778	2.6439
75	0.6778	1.2929	1.6654	1.9921	2.3771	2.6430
76	0.6777	1.2928	1.6652	1.9917	2.3764	2.6421
77	0.6777	1.2926	1.6649	1.9913	2.3758	2.6412
78	0.6776	1.2925	1.6646	1.9908	2.3751	2.6403
79	0.6776	1.2924	1.6644	1.9905	2.3745	2.6395
80	0.6776	1.2922	1.6641	1.9901	2.3739	2.6387
81	0.6775	1.2921	1.6639	1.9897	2.3733	2.6379
82	0.6775	1.2920	1.6636	1.9893	2.3727	2.6371
83	0.6775	1.2918	1.6634	1.9890	2.3721	2.6364
84	0.6774	1.2917	1.6632	1.9886	2.3716	2.6356
85	0.6774	1.2916	1.6630	1.9883	2.3710	2.6349
86	0.6774	1.2915	1.6628	1.9879	2.3705	2.6342
87	0.6773	1.2914	1.6626	1.9876	2.3700	2.6335
88	0.6773	1.2912	1.6624	1.9873	2.3695	2.6329
89	0.6773	1.2911	1.6622	1.9870	2.3690	2.6322
90	0.6772	1.2910	1.6620	1.9867	2.3685	2.6316
91	0.6772	1.2909	1.6618	1.9864	2.3680	2.6309
92	0.6772	1.2908	1.6616	1.9861	2.3676	2.6303
93	0.6771	1.2907	1.6614	1.9858	2.3671	2.6297
94	0.6771	1.2906	1.6612	1.9855	2.3667	2.6291
95	0.6771	1.2905	1.6611	1.9853	2.3662	2.6286
96	0.6771	1.2904	1.6609	1.9850	2.3658	2.6280
97	0.6770	1.2903	1.6607	1.9847	2.3654	2.6275
98	0.6770	1.2902	1.6606	1.9845	2.3650	2.6269
99	0.6770	1.2902	1.6604	1.9842	2.3646	2.6264
100	0.6770	1.2901	1.6602	1.9840	2.3642	2.6259
110	0.6767	1.2893	1.6588	1.9818	2.3607	2.6213
120	0.6765	1.2886	1.6577	1.9799	2.3578	2.6174
∞	0.6745	1.2816	1.6449	1.9600	2.3263	2.5758

**TABLE E.4**Critical Values of  $\chi^2$ 

For a particular number of degrees of freedom, entry represents the critical value of  $\chi^2$  corresponding to a specified upper-tail area ( $\alpha$ ).



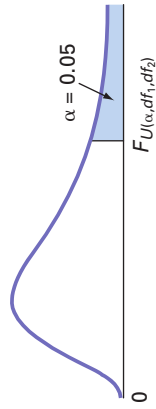
Degrees of Freedom	Upper-Tail Areas ( $\alpha$ )											
	0.995	0.99	0.975	0.95	0.90	0.75	0.25	0.10	0.05	0.025	0.01	0.005
1			0.001	0.004	0.016	0.102	1.323	2.706	3.841	5.024	6.635	7.879
2	0.010	0.020	0.051	0.103	0.211	0.575	2.773	4.605	5.991	7.378	9.210	10.597
3	0.072	0.115	0.216	0.352	0.584	1.213	4.108	6.251	7.815	9.348	11.345	12.838
4	0.207	0.297	0.484	0.711	1.064	1.923	5.385	7.779	9.488	11.143	13.277	14.860
5	0.412	0.554	0.831	1.145	1.610	2.675	6.626	9.236	11.071	12.833	15.086	16.750
6	0.676	0.872	1.237	1.635	2.204	3.455	7.841	10.645	12.592	14.449	16.812	18.458
7	0.989	1.239	1.690	2.167	2.833	4.255	9.037	12.017	14.067	16.013	18.475	20.278
8	1.344	1.646	2.180	2.733	3.490	5.071	10.219	13.362	15.507	17.535	20.090	21.955
9	1.735	2.088	2.700	3.325	4.168	5.899	11.389	14.684	16.919	19.023	21.666	23.589
10	2.156	2.558	3.247	3.940	4.865	6.737	12.549	15.987	18.307	20.483	23.209	25.188
11	2.603	3.053	3.816	4.575	5.578	7.584	13.701	17.275	19.675	21.920	24.725	26.757
12	3.074	3.571	4.404	5.226	6.304	8.438	14.845	18.549	21.026	23.337	26.217	28.299
13	3.565	4.107	5.009	5.892	7.042	9.299	15.984	19.812	22.362	24.736	27.688	29.819
14	4.075	4.660	5.629	6.571	7.790	10.165	17.117	21.064	23.685	26.119	29.141	31.319
15	4.601	5.229	6.262	7.261	8.547	11.037	18.245	22.307	24.996	27.488	30.578	32.801
16	5.142	5.812	6.908	7.962	9.312	11.912	19.369	23.542	26.296	28.845	32.000	34.267
17	5.697	6.408	7.564	8.672	10.085	12.792	20.489	24.769	27.587	30.191	33.409	35.718
18	6.265	7.015	8.231	9.390	10.865	13.675	21.605	25.989	28.869	31.526	34.805	37.156
19	6.844	7.633	8.907	10.117	11.651	14.562	22.718	27.204	30.144	32.852	36.191	38.582
20	7.434	8.260	9.591	10.851	12.443	15.452	23.828	28.412	31.410	34.170	37.566	39.997
21	8.034	8.897	10.283	11.591	13.240	16.344	24.935	29.615	32.671	35.479	38.932	41.401
22	8.643	9.542	10.982	12.338	14.042	17.240	26.039	30.813	33.924	36.781	40.289	42.796
23	9.260	10.196	11.689	13.091	14.848	18.137	27.141	32.007	35.172	38.076	41.638	44.181
24	9.886	10.856	12.401	13.848	15.659	19.037	28.241	33.196	36.415	39.364	42.980	45.559
25	10.520	11.524	13.120	14.611	16.473	19.939	29.339	34.382	37.652	40.646	44.314	46.928
26	11.160	12.198	13.844	15.379	17.292	20.843	30.435	35.563	38.885	41.923	45.642	48.290
27	11.808	12.879	14.573	16.151	18.114	21.749	31.528	36.741	40.113	43.194	46.963	49.645
28	12.461	13.565	15.308	16.928	18.939	22.657	32.620	37.916	41.337	44.461	48.278	50.993
29	13.121	14.257	16.047	17.708	19.768	23.567	33.711	39.087	42.557	45.722	49.588	52.336
30	13.787	14.954	16.791	18.493	20.599	24.478	34.800	40.256	43.773	46.979	50.892	53.672

For larger values of degrees of freedom ( $df$ ) the expression  $Z = \sqrt{2\chi^2} - \sqrt{2(df) - 1}$  may be used and the resulting upper-tail area can be found from the cumulative standardized normal distribution (Table E.2).

**TABLE E.5**

Critical Values of  $F$

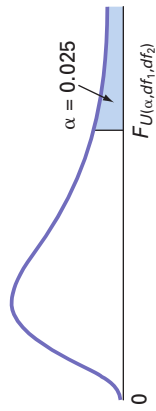
For a particular combination of numerator and denominator degrees of freedom, entry represents the critical values of  $F$  corresponding to a specified upper-tail area ( $\alpha$ ).



Denominator, $df_2$	Numerator, $df_1$																		
	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	$\infty$
1	161.40	199.50	215.70	224.60	230.20	234.00	236.80	238.90	240.50	241.90	243.90	245.90	248.00	249.10	250.10	251.10	252.20	253.30	254.30
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.43	19.45	19.45	19.46	19.47	19.48	19.49	19.50
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.36
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.40
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01	1.96
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15	2.07	2.03	1.98	1.91	1.89	1.84	1.78
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.76
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.15	2.07	1.99	1.95	1.90	1.85	1.80	1.75	1.69
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.13	2.06	1.97	1.93	1.88	1.84	1.79	1.73	1.67
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.12	2.04	1.96	1.91	1.87	1.82	1.77	1.71	1.65
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.10	2.03	1.94	1.90	1.85	1.81	1.75	1.70	1.64
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.58	1.51
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.84	1.75	1.70	1.65	1.59	1.53	1.47	1.39
120	3.92	3.07	2.68	2.45	2.29	2.17	2.09	2.02	1.96	1.91	1.83	1.75	1.66	1.61	1.55	1.50	1.43	1.35	1.25
$\infty$	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1.00

*continued*

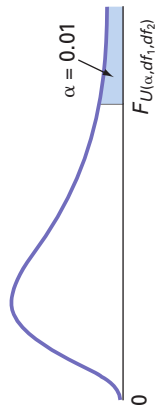
**TABLE E.5**  
Critical Values of  $F$  (Continued)



Denominator, $df_2$	Numerator, $df_1$															$\infty$			
	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30		40	60	120
1	647.80	799.50	864.20	899.60	921.80	937.10	948.20	956.70	963.30	968.60	976.70	984.90	993.10	997.20	1,001.00	1,006.00	1,010.00	1,014.00	1,018.00
2	38.51	39.00	39.17	39.25	39.30	39.33	39.36	39.39	39.39	39.40	39.41	39.43	39.45	39.46	39.46	39.47	39.48	39.49	39.50
3	17.44	16.04	15.44	15.10	14.88	14.73	14.62	14.54	14.47	14.42	14.34	14.25	14.17	14.12	14.08	14.04	13.99	13.95	13.90
4	12.22	10.65	9.98	9.60	9.36	9.20	9.07	8.98	8.90	8.84	8.75	8.66	8.56	8.51	8.46	8.41	8.36	8.31	8.26
5	10.01	8.43	7.76	7.39	7.15	6.98	6.85	6.76	6.68	6.62	6.52	6.43	6.33	6.28	6.23	6.18	6.12	6.07	6.02
6	8.81	7.26	6.60	6.23	5.99	5.82	5.70	5.60	5.52	5.46	5.37	5.27	5.17	5.12	5.07	5.01	4.96	4.90	4.85
7	8.07	6.54	5.89	5.52	5.29	5.12	4.99	4.90	4.82	4.76	4.67	4.57	4.47	4.42	4.36	4.31	4.25	4.20	4.14
8	7.57	6.06	5.42	5.05	4.82	4.65	4.53	4.43	4.36	4.30	4.20	4.10	4.00	3.95	3.89	3.84	3.78	3.73	3.67
9	7.21	5.71	5.08	4.72	4.48	4.32	4.20	4.10	4.03	3.96	3.87	3.77	3.67	3.61	3.56	3.51	3.45	3.39	3.33
10	6.94	5.46	4.83	4.47	4.24	4.07	3.95	3.85	3.78	3.72	3.62	3.52	3.42	3.37	3.31	3.26	3.20	3.14	3.08
11	6.72	5.26	4.63	4.28	4.04	3.88	3.76	3.66	3.59	3.53	3.43	3.33	3.23	3.17	3.12	3.06	3.00	2.94	2.88
12	6.55	5.10	4.47	4.12	3.89	3.73	3.61	3.51	3.44	3.37	3.28	3.18	3.07	3.02	2.96	2.91	2.85	2.79	2.72
13	6.41	4.97	4.35	4.00	3.77	3.60	3.48	3.39	3.31	3.25	3.15	3.05	2.95	2.89	2.84	2.78	2.72	2.66	2.60
14	6.30	4.86	4.24	3.89	3.66	3.50	3.38	3.29	3.21	3.15	3.05	2.95	2.84	2.79	2.73	2.67	2.61	2.55	2.49
15	6.20	4.77	4.15	3.80	3.58	3.41	3.29	3.20	3.12	3.06	2.96	2.86	2.76	2.70	2.64	2.59	2.52	2.46	2.40
16	6.12	4.69	4.08	3.73	3.50	3.34	3.22	3.12	3.05	2.99	2.89	2.79	2.68	2.63	2.57	2.51	2.45	2.38	2.32
17	6.04	4.62	4.01	3.66	3.44	3.28	3.16	3.06	2.98	2.92	2.82	2.72	2.62	2.56	2.50	2.44	2.38	2.32	2.25
18	5.98	4.56	3.95	3.61	3.38	3.22	3.10	3.01	2.93	2.87	2.77	2.67	2.56	2.50	2.44	2.38	2.32	2.26	2.19
19	5.92	4.51	3.90	3.56	3.33	3.17	3.05	2.96	2.88	2.82	2.72	2.62	2.51	2.45	2.39	2.33	2.27	2.20	2.13
20	5.87	4.46	3.86	3.51	3.29	3.13	3.01	2.91	2.84	2.77	2.68	2.57	2.46	2.41	2.35	2.29	2.22	2.16	2.09
21	5.83	4.42	3.82	3.48	3.25	3.09	2.97	2.87	2.80	2.73	2.64	2.53	2.42	2.37	2.31	2.25	2.18	2.11	2.04
22	5.79	4.38	3.78	3.44	3.22	3.05	2.93	2.84	2.76	2.70	2.60	2.50	2.39	2.33	2.27	2.21	2.14	2.08	2.00
23	5.75	4.35	3.75	3.41	3.18	3.02	2.90	2.81	2.73	2.67	2.57	2.47	2.36	2.30	2.24	2.18	2.11	2.04	1.97
24	5.72	4.32	3.72	3.38	3.15	2.99	2.87	2.78	2.70	2.64	2.54	2.44	2.33	2.27	2.21	2.15	2.08	2.01	1.94
25	5.69	4.29	3.69	3.35	3.13	2.97	2.85	2.75	2.68	2.61	2.51	2.41	2.30	2.24	2.18	2.12	2.05	1.98	1.91
26	5.66	4.27	3.67	3.33	3.10	2.94	2.82	2.73	2.65	2.59	2.49	2.39	2.28	2.22	2.16	2.09	2.03	1.95	1.88
27	5.63	4.24	3.65	3.31	3.08	2.92	2.80	2.71	2.63	2.57	2.47	2.36	2.25	2.19	2.13	2.07	2.00	1.93	1.85
28	5.61	4.22	3.63	3.29	3.06	2.90	2.78	2.69	2.61	2.55	2.45	2.34	2.23	2.17	2.11	2.05	1.98	1.91	1.83
29	5.59	4.20	3.61	3.27	3.04	2.88	2.76	2.67	2.59	2.53	2.43	2.32	2.21	2.15	2.09	2.03	1.96	1.89	1.81
30	5.57	4.18	3.59	3.25	3.03	2.87	2.75	2.65	2.57	2.51	2.41	2.31	2.20	2.14	2.07	2.01	1.94	1.87	1.79
40	5.42	4.05	3.46	3.13	2.90	2.74	2.62	2.53	2.45	2.39	2.29	2.18	2.07	2.01	1.94	1.88	1.80	1.72	1.64
60	5.29	3.93	3.34	3.01	2.79	2.63	2.51	2.41	2.33	2.27	2.17	2.06	1.94	1.88	1.82	1.74	1.67	1.58	1.48
120	5.15	3.80	3.23	2.89	2.67	2.52	2.39	2.30	2.22	2.16	2.05	1.94	1.82	1.76	1.69	1.61	1.53	1.43	1.31
$\infty$	5.02	3.69	3.12	2.79	2.57	2.41	2.29	2.19	2.11	2.05	1.94	1.83	1.71	1.64	1.57	1.48	1.39	1.27	1.00

continued

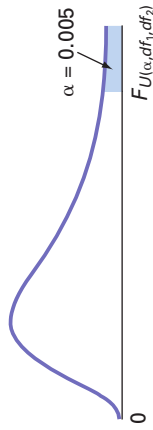
**TABLE E.5**  
Critical Values of  $F$  (Continued)



Denominator, $df_2$	Numerator, $df_1$																		
	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	$\infty$
1	4,052.00	4,999.50	5,403.00	5,625.00	5,764.00	5,859.00	5,928.00	5,982.00	6,022.00	6,056.00	6,106.00	6,157.00	6,209.00	6,235.00	6,261.00	6,287.00	6,313.00	6,339.00	6,366.00
2	98.50	99.00	99.17	99.25	99.30	99.33	99.36	99.37	99.39	99.40	99.42	99.43	99.44	99.46	99.47	99.47	99.48	99.49	99.50
3	34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.35	27.23	27.05	26.87	26.69	26.60	26.50	26.41	26.32	26.22	26.13
4	21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.66	14.55	14.37	14.20	14.02	13.93	13.84	13.75	13.65	13.56	13.46
5	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.16	10.05	9.89	9.72	9.55	9.47	9.38	9.29	9.20	9.11	9.02
6	13.75	10.92	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87	7.72	7.56	7.40	7.31	7.23	7.14	7.06	6.97	6.88
7	12.25	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.62	6.47	6.31	6.16	6.07	5.99	5.91	5.82	5.74	5.65
8	11.26	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81	5.67	5.52	5.36	5.28	5.20	5.12	5.03	4.95	4.86
9	10.56	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26	5.11	4.96	4.81	4.73	4.65	4.57	4.48	4.40	4.31
10	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85	4.71	4.56	4.41	4.33	4.25	4.17	4.08	4.00	3.91
11	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	4.54	4.40	4.25	4.10	4.02	3.94	3.86	3.78	3.69	3.60
12	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30	4.16	4.01	3.86	3.78	3.70	3.62	3.54	3.45	3.36
13	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	4.10	3.96	3.82	3.66	3.59	3.51	3.43	3.34	3.25	3.17
14	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	4.03	3.94	3.80	3.66	3.51	3.43	3.35	3.27	3.18	3.09	3.00
15	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	3.80	3.67	3.52	3.37	3.29	3.21	3.13	3.05	2.96	2.87
16	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78	3.69	3.55	3.41	3.26	3.18	3.10	3.02	2.93	2.81	2.75
17	8.40	6.11	5.18	4.67	4.34	4.10	3.93	3.79	3.68	3.59	3.46	3.31	3.16	3.08	3.00	2.92	2.83	2.75	2.65
18	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60	3.51	3.37	3.23	3.08	3.00	2.92	2.84	2.75	2.66	2.57
19	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	3.43	3.30	3.15	3.00	2.92	2.84	2.76	2.67	2.58	2.49
20	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37	3.23	3.09	2.94	2.86	2.78	2.69	2.61	2.52	2.42
21	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40	3.31	3.17	3.03	2.88	2.80	2.72	2.64	2.55	2.46	2.36
22	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26	3.12	2.98	2.83	2.75	2.67	2.58	2.50	2.40	2.31
23	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.30	3.21	3.07	2.93	2.78	2.70	2.62	2.54	2.45	2.35	2.26
24	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17	3.03	2.89	2.74	2.66	2.58	2.49	2.40	2.31	2.21
25	7.77	5.57	4.68	4.18	3.85	3.63	3.46	3.32	3.22	3.13	2.99	2.85	2.70	2.62	2.54	2.45	2.36	2.27	2.17
26	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18	3.09	2.96	2.81	2.66	2.58	2.50	2.42	2.33	2.23	2.13
27	7.68	5.49	4.60	4.11	3.78	3.56	3.39	3.26	3.15	3.06	2.93	2.78	2.63	2.55	2.47	2.38	2.29	2.20	2.10
28	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12	3.03	2.90	2.75	2.60	2.52	2.44	2.35	2.26	2.17	2.06
29	7.60	5.42	4.54	4.04	3.73	3.50	3.33	3.20	3.09	3.00	2.87	2.73	2.57	2.49	2.41	2.33	2.23	2.14	2.03
30	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98	2.84	2.70	2.55	2.47	2.39	2.30	2.21	2.11	2.01
40	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80	2.66	2.52	2.37	2.29	2.20	2.11	2.02	1.92	1.80
60	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63	2.50	2.35	2.20	2.12	2.03	1.94	1.84	1.73	1.60
120	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56	2.47	2.34	2.19	2.03	1.95	1.86	1.76	1.66	1.53	1.38
$\infty$	6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41	2.32	2.18	2.04	1.88	1.79	1.70	1.59	1.47	1.32	1.00

continued

**TABLE E.5**  
Critical Values of F (Continued)



Denominator, $df_2$	Numerator, $df_1$																			
	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	$\infty$	
1	16,211.00	20,000.00	21,615.00	22,500.00	23,056.00	23,437.00	23,715.00	23,925.00	24,091.00	24,224.00	24,426.00	24,630.00	24,836.00	24,910.00	25,044.00	25,148.00	25,253.00	25,359.00	25,465.00	25,465.00
2	198.50	199.00	199.20	199.20	199.30	199.30	199.40	199.40	199.40	199.40	199.40	199.40	199.40	199.40	199.50	199.50	199.50	199.50	199.50	199.50
3	55.55	49.80	47.47	46.19	45.39	44.84	44.43	44.13	43.88	43.69	43.39	43.08	42.78	42.62	42.47	42.31	42.15	41.99	41.83	41.83
4	31.33	26.28	24.26	23.15	22.46	21.97	21.62	21.35	21.14	20.97	20.70	20.44	20.17	20.03	19.89	19.75	19.61	19.47	19.32	19.32
5	22.78	18.31	16.53	15.56	14.94	14.51	14.20	13.96	13.77	13.62	13.38	13.15	12.90	12.78	12.66	12.53	12.40	12.27	12.11	12.11
6	18.63	14.54	12.92	12.03	11.46	11.07	10.79	10.57	10.39	10.25	10.03	9.81	9.59	9.47	9.36	9.24	9.12	9.00	8.88	8.88
7	16.24	12.40	10.88	10.05	9.52	9.16	8.89	8.68	8.51	8.38	8.18	7.97	7.75	7.65	7.53	7.42	7.31	7.19	7.08	7.08
8	14.69	11.04	9.60	8.81	8.30	7.95	7.69	7.50	7.34	7.21	7.01	6.81	6.61	6.50	6.40	6.29	6.18	6.06	5.95	5.95
9	13.61	10.11	8.72	7.96	7.47	7.13	6.88	6.69	6.54	6.42	6.23	6.03	5.83	5.73	5.62	5.52	5.41	5.30	5.19	5.19
10	12.83	9.43	8.08	7.34	6.87	6.54	6.30	6.12	5.97	5.85	5.66	5.47	5.27	5.17	5.07	4.97	4.86	4.75	4.61	4.61
11	12.23	8.91	7.60	6.88	6.42	6.10	5.86	5.68	5.54	5.42	5.24	5.05	4.86	4.75	4.65	4.55	4.44	4.34	4.23	4.23
12	11.75	8.51	7.23	6.52	6.07	5.76	5.52	5.35	5.20	5.09	4.91	4.72	4.53	4.43	4.33	4.23	4.12	4.01	3.90	3.90
13	11.37	8.19	6.93	6.23	5.79	5.48	5.25	5.08	4.94	4.82	4.64	4.46	4.27	4.17	4.07	3.97	3.87	3.76	3.65	3.65
14	11.06	7.92	6.68	6.00	5.56	5.26	5.03	4.86	4.72	4.60	4.43	4.25	4.06	3.96	3.86	3.76	3.66	3.55	3.41	3.41
15	10.80	7.70	6.48	5.80	5.37	5.07	4.85	4.67	4.54	4.42	4.25	4.07	3.88	3.79	3.69	3.58	3.48	3.37	3.26	3.26
16	10.58	7.51	6.30	5.64	5.21	4.91	4.69	4.52	4.38	4.27	4.10	3.92	3.73	3.64	3.54	3.44	3.33	3.22	3.11	3.11
17	10.38	7.35	6.16	5.50	5.07	4.78	4.56	4.39	4.25	4.14	3.97	3.79	3.61	3.51	3.41	3.31	3.21	3.10	2.98	2.98
18	10.22	7.21	6.03	5.37	4.96	4.66	4.44	4.28	4.14	4.03	3.86	3.68	3.50	3.40	3.30	3.20	3.10	2.99	2.87	2.87
19	10.07	7.09	5.92	5.27	4.85	4.56	4.34	4.18	4.04	3.93	3.76	3.59	3.40	3.31	3.21	3.11	3.00	2.89	2.78	2.78
20	9.94	6.99	5.82	5.17	4.76	4.47	4.26	4.09	3.96	3.85	3.68	3.50	3.32	3.22	3.12	3.02	2.92	2.81	2.69	2.69
21	9.83	6.89	5.73	5.09	4.68	4.39	4.18	4.02	3.88	3.77	3.60	3.43	3.24	3.15	3.05	2.95	2.84	2.73	2.61	2.61
22	9.73	6.81	5.65	5.02	4.61	4.32	4.11	3.94	3.81	3.70	3.54	3.36	3.18	3.08	2.98	2.88	2.77	2.66	2.55	2.55
23	9.63	6.73	5.58	4.95	4.54	4.26	4.05	3.88	3.75	3.64	3.47	3.30	3.12	3.02	2.92	2.82	2.71	2.60	2.48	2.48
24	9.55	6.66	5.52	4.89	4.49	4.20	3.99	3.83	3.69	3.59	3.42	3.25	3.06	2.97	2.87	2.77	2.66	2.55	2.43	2.43
25	9.48	6.60	5.46	4.84	4.43	4.15	3.94	3.78	3.64	3.54	3.37	3.20	3.01	2.92	2.82	2.72	2.61	2.50	2.38	2.38
26	9.41	6.54	5.41	4.79	4.38	4.10	3.89	3.73	3.60	3.49	3.33	3.15	2.97	2.87	2.77	2.67	2.56	2.45	2.33	2.33
27	9.34	6.49	5.36	4.74	4.34	4.06	3.85	3.69	3.56	3.45	3.28	3.11	2.93	2.83	2.73	2.63	2.52	2.41	2.29	2.29
28	9.28	6.44	5.32	4.70	4.30	4.02	3.81	3.65	3.52	3.41	3.25	3.07	2.89	2.79	2.69	2.59	2.48	2.37	2.25	2.25
29	9.23	6.40	5.28	4.66	4.26	3.98	3.77	3.61	3.48	3.38	3.21	3.04	2.86	2.76	2.66	2.56	2.45	2.33	2.21	2.21
30	9.18	6.35	5.24	4.62	4.23	3.95	3.74	3.58	3.45	3.34	3.18	3.01	2.82	2.73	2.63	2.52	2.42	2.30	2.18	2.18
40	8.83	6.07	4.98	4.37	3.99	3.71	3.51	3.35	3.22	3.12	2.95	2.78	2.60	2.50	2.40	2.30	2.18	2.06	1.93	1.93
60	8.49	5.79	4.73	4.14	3.76	3.49	3.29	3.13	3.01	2.90	2.74	2.57	2.39	2.29	2.19	2.08	1.96	1.83	1.69	1.69
120	8.18	5.54	4.50	3.92	3.55	3.28	3.09	2.93	2.81	2.71	2.54	2.37	2.19	2.09	1.98	1.87	1.75	1.61	1.43	1.43
$\infty$	7.88	5.30	4.28	3.72	3.35	3.09	2.90	2.74	2.62	2.52	2.36	2.19	2.00	1.90	1.79	1.67	1.53	1.36	1.16	1.16



# TABLE E.6

TABLE OF BINOMIAL PROBABILITIES  
(BEGINS ON THE FOLLOWING PAGE)

**TABLE E.6**  
Table of Binomial Probabilities

For a given combination of  $n$  and  $p$ , entry indicates the probability of obtaining a specified value of  $X$ . To locate entry, when  $p \leq .50$ , read  $p$  across the top heading and both  $n$  and  $X$  down the left margin; when  $p \geq .50$ , read  $p$  across the bottom heading and both  $n$  and  $X$  up the right margin.

$n$	$X$	$p$																		$X$	$n$																			
		0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50																					
2	0	0.9801	0.9604	0.9409	0.9216	0.9025	0.8836	0.8649	0.8464	0.8281	0.8100	0.7225	0.6400	0.5625	0.4900	0.4225	0.3600	0.3025	0.2500	0.2500	0.3025	0.3600	0.4225	0.4900	0.5625	0.6400	0.7225	0.8100	0.8281	0.8464	0.8649	0.8836	0.9025	0.9216	0.9409	0.9604	0.9801	2	2	
	1	0.0198	0.0392	0.0582	0.0768	0.0950	0.1128	0.1302	0.1472	0.1638	0.1800	0.2550	0.3200	0.3750	0.4200	0.4550	0.4800	0.4950	0.5000	0.5000	0.4950	0.4800	0.4550	0.4200	0.3750	0.3200	0.2550	0.1800	0.1638	0.1472	0.1302	0.1128	0.0950	0.0768	0.0582	0.0392	0.0198	1	1	
	2	0.0001	0.0004	0.0009	0.0016	0.0025	0.0036	0.0049	0.0064	0.0081	0.0100	0.0100	0.0225	0.0400	0.0625	0.0900	0.1225	0.1600	0.2025	0.2500	0.2025	0.1600	0.1225	0.0900	0.0625	0.0400	0.0225	0.0100	0.0100	0.0064	0.0049	0.0036	0.0025	0.0016	0.0009	0.0004	0.0001	0	2	
3	0	0.9703	0.9412	0.9127	0.8847	0.8574	0.8306	0.8044	0.7787	0.7536	0.7290	0.6141	0.5120	0.4219	0.3430	0.2746	0.2160	0.1664	0.1250	0.1250	0.1664	0.2160	0.2746	0.3430	0.4219	0.5120	0.6141	0.7290	0.7536	0.7787	0.8044	0.8306	0.8574	0.8847	0.9127	0.9412	0.9703	3	3	
	1	0.0294	0.0576	0.0847	0.1106	0.1354	0.1590	0.1816	0.2031	0.2236	0.2430	0.3251	0.3840	0.4219	0.4410	0.4436	0.4320	0.4084	0.3750	0.3750	0.4084	0.4320	0.4436	0.4410	0.4219	0.3840	0.3251	0.2430	0.2236	0.2031	0.1816	0.1590	0.1354	0.1106	0.0847	0.0576	0.0294	2	2	
	2	0.0003	0.0012	0.0026	0.0046	0.0071	0.0102	0.0137	0.0177	0.0221	0.0270	0.0270	0.0574	0.0960	0.1406	0.1890	0.2389	0.2880	0.3341	0.3750	0.3750	0.3341	0.2880	0.2389	0.1890	0.1406	0.0960	0.0574	0.0270	0.0270	0.0221	0.0177	0.0137	0.0102	0.0071	0.0046	0.0026	0.0012	0.0003	1
4	0	0.9606	0.9224	0.8853	0.8493	0.8145	0.7807	0.7481	0.7164	0.6857	0.6561	0.5220	0.4096	0.3164	0.2401	0.1785	0.1296	0.0915	0.0625	0.0625	0.0915	0.1296	0.1785	0.2401	0.3164	0.4096	0.5220	0.6561	0.6857	0.7164	0.7481	0.7807	0.8145	0.8493	0.8853	0.9224	0.9606	4	4	
	1	0.0388	0.0753	0.1095	0.1416	0.1715	0.1993	0.2252	0.2492	0.2713	0.2916	0.3685	0.4096	0.4219	0.4116	0.3845	0.3656	0.2995	0.2500	0.2500	0.2995	0.3656	0.4116	0.4116	0.4219	0.4096	0.3685	0.2916	0.2713	0.2492	0.2252	0.1993	0.1715	0.1416	0.1095	0.0753	0.0388	3	3	
	2	0.0006	0.0023	0.0051	0.0088	0.0135	0.0191	0.0254	0.0325	0.0402	0.0486	0.0486	0.0975	0.1536	0.2109	0.2646	0.3105	0.3456	0.3675	0.3750	0.3750	0.3675	0.3456	0.3105	0.2646	0.2109	0.1536	0.0486	0.0486	0.0402	0.0325	0.0254	0.0191	0.0135	0.0088	0.0051	0.0023	0.0006	2	2
5	0	0.9510	0.9039	0.8587	0.8154	0.7738	0.7339	0.6957	0.6591	0.6240	0.5905	0.4437	0.3277	0.2373	0.1681	0.1160	0.0778	0.0503	0.0312	0.0312	0.0503	0.0778	0.1160	0.1681	0.2373	0.3277	0.4437	0.5905	0.6240	0.6591	0.6957	0.7339	0.7738	0.8154	0.8587	0.9039	0.9510	5	5	
	1	0.0480	0.0922	0.1328	0.1699	0.2036	0.2342	0.2618	0.2866	0.3086	0.3280	0.3915	0.4096	0.3955	0.3601	0.3124	0.2592	0.2059	0.1562	0.1162	0.1162	0.2059	0.2592	0.3124	0.3601	0.3955	0.4096	0.3280	0.3086	0.2866	0.2618	0.2342	0.2036	0.1699	0.1328	0.0922	0.0480	4	4	
	2	0.0010	0.0038	0.0082	0.0142	0.0214	0.0299	0.0394	0.0498	0.0610	0.0729	0.0729	0.1382	0.2048	0.2637	0.3087	0.3364	0.3456	0.3369	0.3125	0.3125	0.3369	0.3456	0.3364	0.3087	0.2637	0.2048	0.1382	0.0729	0.0729	0.0610	0.0498	0.0394	0.0299	0.0214	0.0142	0.0082	0.0038	0.0010	3
6	0	0.9415	0.8858	0.8330	0.7828	0.7351	0.6899	0.6470	0.6064	0.5679	0.5314	0.3771	0.2621	0.1780	0.1176	0.0754	0.0467	0.0277	0.0156	0.0156	0.0277	0.0467	0.0754	0.1176	0.1780	0.2621	0.3771	0.5314	0.5679	0.6064	0.6470	0.6899	0.7351	0.7828	0.8330	0.8858	0.9415	6	6	
	1	0.0571	0.1085	0.1546	0.1957	0.2321	0.2642	0.2922	0.3164	0.3370	0.3543	0.3993	0.3932	0.3560	0.3025	0.2437	0.1866	0.1359	0.0937	0.0562	0.0562	0.0937	0.1359	0.1866	0.2437	0.3025	0.3560	0.3932	0.3543	0.3370	0.3164	0.2922	0.2642	0.2321	0.1957	0.1546	0.1085	0.0571	5	5
	2	0.0014	0.0055	0.0120	0.0204	0.0305	0.0422	0.0550	0.0688	0.0833	0.0984	0.1162	0.1248	0.1248	0.1182	0.1052	0.0879	0.0757	0.0625	0.0488	0.0488	0.0625	0.0757	0.0879	0.1052	0.1182	0.1248	0.1162	0.0984	0.0833	0.0688	0.0550	0.0422	0.0305	0.0204	0.0120	0.0055	0.0014	4	4
7	0	0.9294	0.8754	0.8230	0.7719	0.7219	0.6729	0.6249	0.5779	0.5319	0.4869	0.3439	0.2319	0.1519	0.0919	0.0519	0.0319	0.0179	0.0099	0.0099	0.0179	0.0319	0.0519	0.0919	0.1519	0.2319	0.3439	0.4869	0.5319	0.5779	0.6249	0.6729	0.7219	0.7719	0.8230	0.8754	0.9294	7	7	
	1	0.0706	0.1246	0.1726	0.2145	0.2501	0.2792	0.3018	0.3179	0.3269	0.3289	0.3229	0.3089	0.2869	0.2579	0.2219	0.1799	0.1329	0.0809	0.0339	0.0339	0.0809	0.1329	0.1799	0.2219	0.2579	0.2869	0.3089	0.3229	0.3269	0.3179	0.3018	0.2792	0.2501	0.2145	0.1726	0.1246	0.0706	6	6
	2	0.0024	0.0074	0.0144	0.0224	0.0309	0.0394	0.0479	0.0554	0.0619	0.0674	0.0719	0.0754	0.0769	0.0754	0.0719	0.0654	0.0569	0.0459	0.0329	0.0329	0.0459	0.0569	0.0654	0.0719	0.0754	0.0769	0.0754	0.0674	0.0619	0.0554	0.0479	0.0394	0.0309	0.0224	0.0144	0.0074	0.0024	5	5

7	0	0.9321	0.8681	0.8080	0.7514	0.6983	0.6485	0.6017	0.5578	0.5168	0.4783	0.3206	0.2097	0.1335	0.0824	0.0490	0.0280	0.0152	0.0078	7	
	1	0.0659	0.1240	0.1749	0.2192	0.2573	0.2897	0.3170	0.3396	0.3578	0.3720	0.3960	0.3670	0.3115	0.2471	0.1848	0.1306	0.0872	0.0547	6	
	2	0.0020	0.0076	0.0162	0.0274	0.0406	0.0555	0.0716	0.0886	0.1061	0.1240	0.2097	0.2753	0.3115	0.3177	0.2985	0.2613	0.2140	0.1641	5	
	3	0.0000	0.0003	0.0008	0.0019	0.0036	0.0059	0.0090	0.0128	0.0175	0.0230	0.0617	0.1147	0.1730	0.2269	0.2679	0.2903	0.2918	0.2734	4	
	4	—	0.0000	0.0000	0.0001	0.0002	0.0004	0.0007	0.0011	0.0017	0.0026	0.0109	0.0287	0.0577	0.0972	0.1442	0.1935	0.2388	0.2734	3	
	5	—	—	—	0.0000	0.0000	0.0000	0.0000	0.0001	0.0001	0.0002	0.0012	0.0043	0.0115	0.0250	0.0466	0.0774	0.1172	0.1641	2	
	6	—	—	—	—	—	—	—	0.0000	0.0000	0.0000	0.0001	0.0004	0.0013	0.0036	0.0084	0.0172	0.0320	0.0547	1	
	7	—	—	—	—	—	—	—	—	—	—	0.0000	0.0000	0.0001	0.0002	0.0006	0.0016	0.0037	0.0078	0	
8	0	0.9227	0.8508	0.7837	0.7214	0.6634	0.6096	0.5596	0.5132	0.4703	0.4305	0.2725	0.1678	0.1001	0.0576	0.0319	0.0168	0.0084	0.0039	8	
	1	0.0746	0.1389	0.1939	0.2405	0.2793	0.3113	0.3370	0.3570	0.3721	0.3826	0.3847	0.3355	0.2670	0.1977	0.1373	0.0896	0.0548	0.0312	7	
	2	0.0026	0.0099	0.0210	0.0351	0.0515	0.0695	0.0888	0.1087	0.1288	0.1488	0.2376	0.2936	0.3115	0.2965	0.2587	0.2090	0.1569	0.1094	6	
	3	0.0001	0.0004	0.0013	0.0029	0.0054	0.0089	0.0134	0.0189	0.0255	0.0331	0.0839	0.1468	0.2076	0.2541	0.2786	0.2787	0.2568	0.2187	5	
	4	0.0000	0.0000	0.0001	0.0002	0.0004	0.0007	0.0013	0.0021	0.0031	0.0046	0.0185	0.0459	0.0865	0.1361	0.1875	0.2322	0.2627	0.2734	4	
	5	—	—	0.0000	0.0000	0.0000	0.0000	0.0001	0.0001	0.0002	0.0004	0.0026	0.0092	0.0231	0.0467	0.0808	0.1239	0.1719	0.2187	3	
	6	—	—	—	—	—	—	0.0000	0.0000	0.0000	0.0000	0.0002	0.0011	0.0038	0.0100	0.0217	0.0413	0.0703	0.1094	2	
	7	—	—	—	—	—	—	—	—	—	—	0.0000	0.0001	0.0004	0.0012	0.0033	0.0079	0.0164	0.0312	1	
	8	—	—	—	—	—	—	—	—	—	—	—	0.0000	0.0000	0.0001	0.0002	0.0007	0.0017	0.0039	0	
9	0	0.9135	0.8337	0.7602	0.6925	0.6302	0.5730	0.5204	0.4722	0.4279	0.3874	0.2316	0.1342	0.0751	0.0404	0.0207	0.0101	0.0046	0.0020	9	
	1	0.0830	0.1531	0.2116	0.2597	0.2985	0.3292	0.3525	0.3695	0.3809	0.3874	0.3679	0.3020	0.2253	0.1556	0.1004	0.0605	0.0339	0.0176	8	
	2	0.0034	0.0125	0.0262	0.0433	0.0629	0.0840	0.1061	0.1285	0.1507	0.1722	0.2597	0.3020	0.3003	0.2668	0.2162	0.1612	0.1110	0.0703	7	
	3	0.0001	0.0006	0.0019	0.0042	0.0077	0.0125	0.0186	0.0261	0.0348	0.0446	0.1069	0.1762	0.2336	0.2668	0.2716	0.2508	0.2119	0.1641	6	
	4	0.0000	0.0000	0.0001	0.0003	0.0006	0.0012	0.0021	0.0034	0.0052	0.0074	0.0283	0.0661	0.1168	0.1715	0.2194	0.2508	0.2600	0.2461	5	
	5	—	—	0.0000	0.0000	0.0000	0.0001	0.0002	0.0003	0.0005	0.0008	0.0050	0.0165	0.0390	0.0735	0.1181	0.1672	0.2128	0.2461	4	
	6	—	—	—	—	—	—	—	—	—	0.0000	0.0006	0.0028	0.0087	0.0210	0.0424	0.0743	0.1160	0.1641	3	
	7	—	—	—	—	—	—	—	—	—	0.0000	0.0000	0.0003	0.0012	0.0039	0.0098	0.0212	0.0407	0.0703	2	
	8	—	—	—	—	—	—	—	—	—	—	—	0.0000	0.0001	0.0004	0.0013	0.0035	0.0083	0.0176	1	
	9	—	—	—	—	—	—	—	—	—	—	—	—	0.0000	0.0000	0.0001	0.0003	0.0008	0.0020	0	
10	0	0.9044	0.8171	0.7374	0.6648	0.5987	0.5386	0.4840	0.4344	0.3894	0.3487	0.1969	0.1074	0.0563	0.0282	0.0135	0.0060	0.0025	0.0010	10	
	1	0.0914	0.1667	0.2281	0.2770	0.3151	0.3438	0.3643	0.3777	0.3851	0.3874	0.3474	0.2684	0.1877	0.1211	0.0725	0.0403	0.0207	0.0098	9	
	2	0.0042	0.0153	0.0317	0.0519	0.0746	0.0988	0.1234	0.1478	0.1714	0.1937	0.2759	0.3020	0.2816	0.2335	0.1757	0.1209	0.0763	0.0439	8	
	3	0.0001	0.0008	0.0026	0.0058	0.0105	0.0168	0.0248	0.0343	0.0452	0.0574	0.1298	0.2013	0.2503	0.2668	0.2522	0.2150	0.1665	0.1172	7	
	4	0.0000	0.0000	0.0001	0.0004	0.0010	0.0019	0.0033	0.0052	0.0078	0.0112	0.0401	0.0881	0.1460	0.2001	0.2377	0.2508	0.2384	0.2051	6	
	5	—	—	0.0000	0.0000	0.0001	0.0001	0.0003	0.0005	0.0009	0.0015	0.0085	0.0264	0.0584	0.1029	0.1536	0.2007	0.2340	0.2461	5	
	6	—	—	—	—	—	—	—	—	—	0.0001	0.0012	0.0055	0.0162	0.0368	0.0689	0.1115	0.1596	0.2051	4	
	7	—	—	—	—	—	—	—	—	0.0000	0.0000	0.0001	0.0008	0.0031	0.0090	0.0212	0.0425	0.0746	0.1172	3	
	8	—	—	—	—	—	—	—	—	—	—	0.0000	0.0001	0.0004	0.0014	0.0043	0.0106	0.0229	0.0439	2	
	9	—	—	—	—	—	—	—	—	—	—	—	0.0000	0.0000	0.0001	0.0005	0.0016	0.0042	0.0098	1	
	10	—	—	—	—	—	—	—	—	—	—	—	—	—	0.0000	0.0000	0.0001	0.0003	0.0010	0	
<i>n</i>	<i>X</i>	0.99	0.98	0.97	0.96	0.95	0.94	0.93	0.92	0.91	0.90	0.85	0.80	0.75	0.70	0.65	0.60	0.55	0.50	<i>X</i>	<i>n</i>

continued

**TABLE E.6**  
Table of Binomial Probabilities (Continued)

<i>n</i>	<i>X</i>	<i>p</i>																			<i>X</i>	<i>n</i>
		0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50			
20	0	0.8179	0.6676	0.5438	0.4420	0.3585	0.2901	0.2342	0.1887	0.1516	0.1216	0.0388	0.0115	0.0032	0.0008	0.0002	0.0000	0.0000	—	20		
	1	0.1652	0.2725	0.3364	0.3683	0.3774	0.3703	0.3526	0.3282	0.3000	0.2702	0.1368	0.0576	0.0211	0.0068	0.0020	0.0005	0.0001	0.0000	19		
	2	0.0159	0.0528	0.0988	0.1458	0.1887	0.2246	0.2521	0.2711	0.2818	0.2852	0.2293	0.1369	0.0699	0.0278	0.0100	0.0031	0.0008	0.0002	18		
	3	0.0010	0.0065	0.0183	0.0364	0.0596	0.0860	0.1139	0.1414	0.1672	0.1901	0.2428	0.2054	0.1339	0.0716	0.0323	0.0123	0.0040	0.0011	17		
	4	0.0000	0.0006	0.0024	0.0065	0.0133	0.0233	0.0364	0.0523	0.0703	0.0898	0.1821	0.2182	0.1897	0.1304	0.0738	0.0350	0.0139	0.0046	16		
	5	—	0.0000	0.0002	0.0009	0.0022	0.0048	0.0088	0.0145	0.0222	0.0319	0.1028	0.1746	0.2023	0.1789	0.1272	0.0746	0.0365	0.0148	15		
	6	—	—	0.0000	0.0001	0.0003	0.0008	0.0017	0.0032	0.0055	0.0089	0.0454	0.1091	0.1686	0.1916	0.1712	0.1244	0.0746	0.0370	14		
	7	—	—	—	0.0000	0.0000	0.0001	0.0002	0.0005	0.0011	0.0020	0.0160	0.0545	0.1124	0.1643	0.1844	0.1659	0.1221	0.0739	13		
	8	—	—	—	—	—	0.0000	0.0000	0.0001	0.0002	0.0004	0.0046	0.0222	0.0609	0.1144	0.1614	0.1797	0.1623	0.1201	12		
	9	—	—	—	—	—	—	0.0000	0.0000	0.0000	0.0001	0.0011	0.0074	0.0271	0.0654	0.1158	0.1597	0.1771	0.1602	11		
	10	—	—	—	—	—	—	—	—	—	0.0000	0.0002	0.0020	0.0099	0.0308	0.0686	0.1171	0.1593	0.1762	10		
	11	—	—	—	—	—	—	—	—	—	—	0.0000	0.0005	0.0030	0.0120	0.0336	0.0710	0.1185	0.1602	9		
	12	—	—	—	—	—	—	—	—	—	—	—	0.0001	0.0008	0.0039	0.0136	0.0355	0.0727	0.1201	8		
	13	—	—	—	—	—	—	—	—	—	—	—	0.0000	0.0002	0.0010	0.0045	0.0146	0.0366	0.0739	7		
	14	—	—	—	—	—	—	—	—	—	—	—	—	0.0000	0.0002	0.0012	0.0049	0.0150	0.0370	6		
	15	—	—	—	—	—	—	—	—	—	—	—	—	—	0.0000	0.0003	0.0013	0.0049	0.0148	5		
	16	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0.0000	0.0003	0.0013	0.0046	4		
	17	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0.0000	0.0002	0.0011	3		
	18	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0.0002	0.0002	0.0002	2	
	19	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0.0000	0.0000	1	
	20	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0.0000	0	
	20	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	20	

**TABLE E.7**Table of Poisson  
ProbabilitiesFor a given value of  $\lambda$ , entry  
indicates the probability of a  
specified value of  $X$ 

$\lambda$										
$X$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	0.9048	0.8187	0.7408	0.6703	0.6065	0.5488	0.4966	0.4493	0.4066	0.3679
1	0.0905	0.1637	0.2222	0.2681	0.3033	0.3293	0.3476	0.3595	0.3659	0.3679
2	0.0045	0.0164	0.0333	0.0536	0.0758	0.0988	0.1217	0.1438	0.1647	0.1839
3	0.0002	0.0011	0.0033	0.0072	0.0126	0.0198	0.0284	0.0383	0.0494	0.0613
4	0.0000	0.0001	0.0003	0.0007	0.0016	0.0030	0.0050	0.0077	0.0111	0.0153
5	0.0000	0.0000	0.0000	0.0001	0.0002	0.0004	0.0007	0.0012	0.0020	0.0031
6	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0002	0.0003	0.0005
7	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001
$\lambda$										
$X$	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
0	0.3329	0.3012	0.2725	0.2466	0.2231	0.2019	0.1827	0.1653	0.1496	0.1353
1	0.3662	0.3614	0.3543	0.3452	0.3347	0.3230	0.3106	0.2975	0.2842	0.2707
2	0.2014	0.2169	0.2303	0.2417	0.2510	0.2584	0.2640	0.2678	0.2700	0.2707
3	0.0738	0.0867	0.0998	0.1128	0.1255	0.1378	0.1496	0.1607	0.1710	0.1804
4	0.0203	0.0260	0.0324	0.0395	0.0471	0.0551	0.0636	0.0723	0.0812	0.0902
5	0.0045	0.0062	0.0084	0.0111	0.0141	0.0176	0.0216	0.0260	0.0309	0.0361
6	0.0008	0.0012	0.0018	0.0026	0.0035	0.0047	0.0061	0.0078	0.0098	0.0120
7	0.0001	0.0002	0.0003	0.0005	0.0008	0.0011	0.0015	0.0020	0.0027	0.0034
8	0.0000	0.0000	0.0001	0.0001	0.0001	0.0002	0.0003	0.0005	0.0006	0.0009
9	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0001	0.0001	0.0002
$\lambda$										
$X$	2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9	3.0
0	0.1225	0.1108	0.1003	0.0907	0.0821	0.0743	0.0672	0.0608	0.0550	0.0498
1	0.2572	0.2438	0.2306	0.2177	0.2052	0.1931	0.1815	0.1703	0.1596	0.1494
2	0.2700	0.2681	0.2652	0.2613	0.2565	0.2510	0.2450	0.2384	0.2314	0.2240
3	0.1890	0.1966	0.2033	0.2090	0.2138	0.2176	0.2205	0.2225	0.2237	0.2240
4	0.0992	0.1082	0.1169	0.1254	0.1336	0.1414	0.1488	0.1557	0.1622	0.1680
5	0.0417	0.0476	0.0538	0.0602	0.0668	0.0735	0.0804	0.0872	0.0940	0.1008
6	0.0146	0.0174	0.0206	0.0241	0.0278	0.0319	0.0362	0.0407	0.0455	0.0504
7	0.0044	0.0055	0.0068	0.0083	0.0099	0.0118	0.0139	0.0163	0.0188	0.0216
8	0.0011	0.0015	0.0019	0.0025	0.0031	0.0038	0.0047	0.0057	0.0068	0.0081
9	0.0003	0.0004	0.0005	0.0007	0.0009	0.0011	0.0014	0.0018	0.0022	0.0027
10	0.0001	0.0001	0.0001	0.0002	0.0002	0.0003	0.0004	0.0005	0.0006	0.0008
11	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0001	0.0001	0.0002	0.0002
12	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001
$\lambda$										
$X$	3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9	4.0
0	0.0450	0.0408	0.0369	0.0334	0.0302	0.0273	0.0247	0.0224	0.0202	0.0183
1	0.1397	0.1340	0.1217	0.1135	0.1057	0.0984	0.0915	0.0850	0.0789	0.0733
2	0.2165	0.2087	0.2008	0.1929	0.1850	0.1771	0.1692	0.1615	0.1539	0.1465
3	0.2237	0.2226	0.2209	0.2186	0.2158	0.2125	0.2087	0.2046	0.2001	0.1954
4	0.1734	0.1781	0.1823	0.1858	0.1888	0.1912	0.1931	0.1944	0.1951	0.1954
5	0.1075	0.1140	0.1203	0.1264	0.1322	0.1377	0.1429	0.1477	0.1522	0.1563
6	0.0555	0.0608	0.0662	0.0716	0.0771	0.0826	0.0881	0.0936	0.0989	0.1042
7	0.0246	0.0278	0.0312	0.0348	0.0385	0.0425	0.0466	0.0508	0.0551	0.0595
8	0.0095	0.0111	0.0129	0.0148	0.0169	0.0191	0.0215	0.0241	0.0269	0.0298
9	0.0033	0.0040	0.0047	0.0056	0.0066	0.0076	0.0089	0.0102	0.0116	0.0132
10	0.0010	0.0013	0.0016	0.0019	0.0023	0.0028	0.0033	0.0039	0.0045	0.0053
11	0.0003	0.0004	0.0005	0.0006	0.0007	0.0009	0.0011	0.0013	0.0016	0.0019
12	0.0001	0.0001	0.0001	0.0002	0.0002	0.0003	0.0003	0.0004	0.0005	0.0006
13	0.0000	0.0000	0.0000	0.0000	0.0001	0.0001	0.0001	0.0001	0.0002	0.0002
14	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001

*continued*

**TABLE E.7**Table of Poisson  
Probabilities  
(Continued)

$\lambda$										
<i>X</i>	4.1	4.2	4.3	4.4	4.5	4.6	4.7	4.8	4.9	5.0
0	0.0166	0.0150	0.0136	0.0123	0.0111	0.0101	0.0091	0.0082	0.0074	0.0067
1	0.0679	0.0630	0.0583	0.0540	0.0500	0.0462	0.0427	0.0395	0.0365	0.0337
2	0.1393	0.1323	0.1254	0.1188	0.1125	0.1063	0.1005	0.0948	0.0894	0.0842
3	0.1904	0.1852	0.1798	0.1743	0.1687	0.1631	0.1574	0.1517	0.1460	0.1404
4	0.1951	0.1944	0.1933	0.1917	0.1898	0.1875	0.1849	0.1820	0.1789	0.1755
5	0.1600	0.1633	0.1662	0.1687	0.1708	0.1725	0.1738	0.1747	0.1753	0.1755
6	0.1093	0.1143	0.1191	0.1237	0.1281	0.1323	0.1362	0.1398	0.1432	0.1462
7	0.0640	0.0686	0.0732	0.0778	0.0824	0.0869	0.0914	0.0959	0.1002	0.1044
8	0.0328	0.0360	0.0393	0.0428	0.0463	0.0500	0.0537	0.0575	0.0614	0.0653
9	0.0150	0.0168	0.0188	0.0209	0.0232	0.0255	0.0280	0.0307	0.0334	0.0363
10	0.0061	0.0071	0.0081	0.0092	0.0104	0.0118	0.0132	0.0147	0.0164	0.0181
11	0.0023	0.0027	0.0032	0.0037	0.0043	0.0049	0.0056	0.0064	0.0073	0.0082
12	0.0008	0.0009	0.0011	0.0014	0.0016	0.0019	0.0022	0.0026	0.0030	0.0034
13	0.0002	0.0003	0.0004	0.0005	0.0006	0.0007	0.0008	0.0009	0.0011	0.0013
14	0.0001	0.0001	0.0001	0.0001	0.0002	0.0002	0.0003	0.0003	0.0004	0.0005
15	0.0000	0.0000	0.0000	0.0000	0.0001	0.0001	0.0001	0.0001	0.0001	0.0002
$\lambda$										
<i>X</i>	5.1	5.2	5.3	5.4	5.5	5.6	5.7	5.8	5.9	6.0
0	0.0061	0.0055	0.0050	0.0045	0.0041	0.0037	0.0033	0.0030	0.0027	0.0025
1	0.0311	0.0287	0.0265	0.0244	0.0225	0.0207	0.0191	0.0176	0.0162	0.0149
2	0.0793	0.0746	0.0701	0.0659	0.0618	0.0580	0.0544	0.0509	0.0477	0.0446
3	0.1348	0.1293	0.1239	0.1185	0.1133	0.1082	0.1033	0.0985	0.0938	0.0892
4	0.1719	0.1681	0.1641	0.1600	0.1558	0.1515	0.1472	0.1428	0.1383	0.1339
5	0.1753	0.1748	0.1740	0.1728	0.1714	0.1697	0.1678	0.1656	0.1632	0.1606
6	0.1490	0.1515	0.1537	0.1555	0.1571	0.1584	0.1594	0.1601	0.1605	0.1606
7	0.1086	0.1125	0.1163	0.1200	0.1234	0.1267	0.1298	0.1326	0.1353	0.1377
8	0.0692	0.0731	0.0771	0.0810	0.0849	0.0887	0.0925	0.0962	0.0998	0.1033
9	0.0392	0.0423	0.0454	0.0486	0.0519	0.0552	0.0586	0.0620	0.0654	0.0688
10	0.0200	0.0220	0.0241	0.0262	0.0285	0.0309	0.0334	0.0359	0.0386	0.0413
11	0.0093	0.0104	0.0116	0.0129	0.0143	0.0157	0.0173	0.0190	0.0207	0.0225
12	0.0039	0.0045	0.0051	0.0058	0.0065	0.0073	0.0082	0.0092	0.0102	0.0113
13	0.0015	0.0018	0.0021	0.0024	0.0028	0.0032	0.0036	0.0041	0.0046	0.0052
14	0.0006	0.0007	0.0008	0.0009	0.0011	0.0013	0.0015	0.0017	0.0019	0.0022
15	0.0002	0.0002	0.0003	0.0003	0.0004	0.0005	0.0006	0.0007	0.0008	0.0009
16	0.0001	0.0001	0.0001	0.0001	0.0001	0.0002	0.0002	0.0002	0.0003	0.0003
17	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0001	0.0001	0.0001
$\lambda$										
<i>X</i>	6.1	6.2	6.3	6.4	6.5	6.6	6.7	6.8	6.9	7.0
0	0.0022	0.0020	0.0018	0.0017	0.0015	0.0014	0.0012	0.0011	0.0010	0.0009
1	0.0137	0.0126	0.0116	0.0106	0.0098	0.0090	0.0082	0.0076	0.0070	0.0064
2	0.0417	0.0390	0.0364	0.0340	0.0318	0.0296	0.0276	0.0258	0.0240	0.0223
3	0.0848	0.0806	0.0765	0.0726	0.0688	0.0652	0.0617	0.0584	0.0552	0.0521
4	0.1294	0.1249	0.1205	0.1162	0.1118	0.1076	0.1034	0.0992	0.0952	0.0912
5	0.1579	0.1549	0.1519	0.1487	0.1454	0.1420	0.1385	0.1349	0.1314	0.1277
6	0.1605	0.1601	0.1595	0.1586	0.1575	0.1562	0.1546	0.1529	0.1511	0.1490
7	0.1399	0.1418	0.1435	0.1450	0.1462	0.1472	0.1480	0.1486	0.1489	0.1490
8	0.1066	0.1099	0.1130	0.1160	0.1188	0.1215	0.1240	0.1263	0.1284	0.1304
9	0.0723	0.0757	0.0791	0.0825	0.0858	0.0891	0.0923	0.0954	0.0985	0.1014
10	0.0441	0.0469	0.0498	0.0528	0.0558	0.0588	0.0618	0.0649	0.0679	0.0710
11	0.0245	0.0265	0.0285	0.0307	0.0330	0.0353	0.0377	0.0401	0.0426	0.0452
12	0.0124	0.0137	0.0150	0.0164	0.0179	0.0194	0.0210	0.0227	0.0245	0.0264
13	0.0058	0.0065	0.0073	0.0081	0.0089	0.0098	0.0108	0.0119	0.0130	0.0142
14	0.0025	0.0029	0.0033	0.0037	0.0041	0.0046	0.0052	0.0058	0.0064	0.0071

continued

**TABLE E.7**Table of Poisson  
Probabilities  
(Continued)

$\lambda$										
<i>X</i>	6.1	6.2	6.3	6.4	6.5	6.6	6.7	6.8	6.9	7.0
15	0.0010	0.0012	0.0014	0.0016	0.0018	0.0020	0.0023	0.0026	0.0029	0.0033
16	0.0004	0.0005	0.0005	0.0006	0.0007	0.0008	0.0010	0.0011	0.0013	0.0014
17	0.0001	0.0002	0.0002	0.0002	0.0003	0.0003	0.0004	0.0004	0.0005	0.0006
18	0.0000	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0002	0.0002	0.0002
19	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0001	0.0001
$\lambda$										
<i>X</i>	7.1	7.2	7.3	7.4	7.5	7.6	7.7	7.8	7.9	8.0
0	0.0008	0.0007	0.0007	0.0006	0.0006	0.0005	0.0005	0.0004	0.0004	0.0003
1	0.0059	0.0054	0.0049	0.0045	0.0041	0.0038	0.0035	0.0032	0.0029	0.0027
2	0.0208	0.0194	0.0180	0.0167	0.0156	0.0145	0.0134	0.0125	0.0116	0.0107
3	0.0492	0.0464	0.0438	0.0413	0.0389	0.0366	0.0345	0.0324	0.0305	0.0286
4	0.0874	0.0836	0.0799	0.0764	0.0729	0.0696	0.0663	0.0632	0.0602	0.0573
5	0.1241	0.1204	0.1167	0.1130	0.1094	0.1057	0.1021	0.0986	0.0951	0.0916
6	0.1468	0.1445	0.1420	0.1394	0.1367	0.1339	0.1311	0.1282	0.1252	0.1221
7	0.1489	0.1486	0.1481	0.1474	0.1465	0.1454	0.1442	0.1428	0.1413	0.1396
8	0.1321	0.1337	0.1351	0.1363	0.1373	0.1382	0.1388	0.1392	0.1395	0.1396
9	0.1042	0.1070	0.1096	0.1121	0.1144	0.1167	0.1187	0.1207	0.1224	0.1241
10	0.0740	0.0770	0.0800	0.0829	0.0858	0.0887	0.0914	0.0941	0.0967	0.0993
11	0.0478	0.0504	0.0531	0.0558	0.0585	0.0613	0.0640	0.0667	0.0695	0.0722
12	0.0283	0.0303	0.0323	0.0344	0.0366	0.0388	0.0411	0.0434	0.0457	0.0481
13	0.0154	0.0168	0.0181	0.0196	0.0211	0.0227	0.0243	0.0260	0.0278	0.0296
14	0.0078	0.0086	0.0095	0.0104	0.0113	0.0123	0.0134	0.0145	0.0157	0.0169
15	0.0037	0.0041	0.0046	0.0051	0.0057	0.0062	0.0069	0.0075	0.0083	0.0090
16	0.0016	0.0019	0.0021	0.0024	0.0026	0.0030	0.0033	0.0037	0.0041	0.0045
17	0.0007	0.0008	0.0009	0.0010	0.0012	0.0013	0.0015	0.0017	0.0019	0.0021
18	0.0003	0.0003	0.0004	0.0004	0.0005	0.0006	0.0006	0.0007	0.0008	0.0009
19	0.0001	0.0001	0.0001	0.0002	0.0002	0.0002	0.0003	0.0003	0.0003	0.0004
20	0.0000	0.0000	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0002
21	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0001
$\lambda$										
<i>X</i>	8.1	8.2	8.3	8.4	8.5	8.6	8.7	8.8	8.9	9.0
0	0.0003	0.0003	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0001	0.0001
1	0.0025	0.0023	0.0021	0.0019	0.0017	0.0016	0.0014	0.0013	0.0012	0.0011
2	0.0100	0.0092	0.0086	0.0079	0.0074	0.0068	0.0063	0.0058	0.0054	0.0050
3	0.0269	0.0252	0.0237	0.0222	0.0208	0.0195	0.0183	0.0171	0.0160	0.0150
4	0.0544	0.0517	0.0491	0.0466	0.0443	0.0420	0.0398	0.0377	0.0357	0.0337
5	0.0882	0.0849	0.0816	0.0784	0.0752	0.0722	0.0692	0.0663	0.0635	0.0607
6	0.1191	0.1160	0.1128	0.1097	0.1066	0.1034	0.1003	0.0972	0.0941	0.0911
7	0.1378	0.1358	0.1338	0.1317	0.1294	0.1271	0.1247	0.1222	0.1197	0.1171
8	0.1395	0.1392	0.1388	0.1382	0.1375	0.1366	0.1356	0.1344	0.1332	0.1318
9	0.1256	0.1269	0.1280	0.1290	0.1299	0.1306	0.1311	0.1315	0.1317	0.1318
10	0.1017	0.1040	0.1063	0.1084	0.1104	0.1123	0.1140	0.1157	0.1172	0.1186
11	0.0749	0.0776	0.0802	0.0828	0.0853	0.0878	0.0902	0.0925	0.0948	0.0970
12	0.0505	0.0530	0.0555	0.0579	0.0604	0.0629	0.0654	0.0679	0.0703	0.0728
13	0.0315	0.0334	0.0354	0.0374	0.0395	0.0416	0.0438	0.0459	0.0481	0.0504
14	0.0182	0.0196	0.0210	0.0225	0.0240	0.0256	0.0272	0.0289	0.0306	0.0324
15	0.0098	0.0107	0.0116	0.0126	0.0136	0.0147	0.0158	0.0169	0.0182	0.0194
16	0.0050	0.0055	0.0060	0.0066	0.0072	0.0079	0.0086	0.0093	0.0101	0.0109
17	0.0024	0.0026	0.0029	0.0033	0.0036	0.0040	0.0044	0.0048	0.0053	0.0058
18	0.0011	0.0012	0.0014	0.0015	0.0017	0.0019	0.0021	0.0024	0.0026	0.0029
19	0.0005	0.0005	0.0006	0.0007	0.0008	0.0009	0.0010	0.0011	0.0012	0.0014
20	0.0002	0.0002	0.0002	0.0003	0.0003	0.0004	0.0004	0.0005	0.0005	0.0006
21	0.0001	0.0001	0.0001	0.0001	0.0001	0.0002	0.0002	0.0002	0.0002	0.0003
22	0.0000	0.0000	0.0000	0.0000	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001

*continued*

**TABLE E.7**Table of Poisson  
Probabilities  
(Continued)

$\lambda$										
$X$	9.1	9.2	9.3	9.4	9.5	9.6	9.7	9.8	9.9	10
0	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0000
1	0.0010	0.0009	0.0009	0.0008	0.0007	0.0007	0.0006	0.0005	0.0005	0.0005
2	0.0046	0.0043	0.0040	0.0037	0.0034	0.0031	0.0029	0.0027	0.0025	0.0023
3	0.0140	0.0131	0.0123	0.0115	0.0107	0.0100	0.0093	0.0087	0.0081	0.0076
4	0.0319	0.0302	0.0285	0.0269	0.0254	0.0240	0.0226	0.0213	0.0201	0.0189
5	0.0581	0.0555	0.0530	0.0506	0.0483	0.0460	0.0439	0.0418	0.0398	0.0378
6	0.0881	0.0851	0.0822	0.0793	0.0764	0.0736	0.0709	0.0682	0.0656	0.0631
7	0.1145	0.1118	0.1091	0.1064	0.1037	0.1010	0.0982	0.0955	0.0928	0.0901
8	0.1302	0.1286	0.1269	0.1251	0.1232	0.1212	0.1191	0.1170	0.1148	0.1126
9	0.1317	0.1315	0.1311	0.1306	0.1300	0.1293	0.1284	0.1274	0.1263	0.1251
10	0.1198	0.1210	0.1219	0.1228	0.1235	0.1241	0.1245	0.1249	0.1250	0.1251
11	0.0991	0.1012	0.1031	0.1049	0.1067	0.1083	0.1098	0.1112	0.1125	0.1137
12	0.0752	0.0776	0.0799	0.0822	0.0844	0.0866	0.0888	0.0908	0.0928	0.0948
13	0.0526	0.0549	0.0572	0.0594	0.0617	0.0640	0.0662	0.0685	0.0707	0.0729
14	0.0342	0.0361	0.0380	0.0399	0.0419	0.0439	0.0459	0.0479	0.0500	0.0521
15	0.0208	0.0221	0.0235	0.0250	0.0265	0.0281	0.0297	0.0313	0.0330	0.0347
16	0.0118	0.0127	0.0137	0.0147	0.0157	0.0168	0.0180	0.0192	0.0204	0.0217
17	0.0063	0.0069	0.0075	0.0081	0.0088	0.0095	0.0103	0.0111	0.0119	0.0128
18	0.0032	0.0035	0.0039	0.0042	0.0046	0.0051	0.0055	0.0060	0.0065	0.0071
19	0.0015	0.0017	0.0019	0.0021	0.0023	0.0026	0.0028	0.0031	0.0034	0.0037
20	0.0007	0.0008	0.0009	0.0010	0.0011	0.0012	0.0014	0.0015	0.0017	0.0019
21	0.0003	0.0003	0.0004	0.0004	0.0005	0.0006	0.0006	0.0007	0.0008	0.0009
22	0.0001	0.0001	0.0002	0.0002	0.0002	0.0002	0.0003	0.0003	0.0004	0.0004
23	0.0000	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0002	0.0002
24	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0001	0.0001
$X$	$\lambda = 20$	$X$	$\lambda = 20$	$X$	$\lambda = 20$	$X$	$\lambda = 20$			
0	0.0000	10	0.0058	20	0.0888	30	0.0083			
1	0.0000	11	0.0106	21	0.0846	31	0.0054			
2	0.0000	12	0.0176	22	0.0769	32	0.0034			
3	0.0000	13	0.0271	23	0.0669	33	0.0020			
4	0.0000	14	0.0387	24	0.0557	34	0.0012			
5	0.0001	15	0.0516	25	0.0446	35	0.0007			
6	0.0002	16	0.0646	26	0.0343	36	0.0004			
7	0.0005	17	0.0760	27	0.0254	37	0.0002			
8	0.0013	18	0.0844	28	0.0181	38	0.0001			
9	0.0029	19	0.0888	29	0.0125	39	0.0001			



**TABLE E.8**

Lower and Upper  
Critical Values,  $T_1$ ,  
of Wilcoxon Rank  
Sum Test

$n_2$	$\alpha$		$n_1$						
	One-tail	Two-tail	4	5	6	7	8	9	10
4	0.05	0.10	11,25						
	0.025	0.05	10,26						
	0.01	0.02	—,—						
	0.005	0.01	—,—						
5	0.05	0.10	12,28	19,36					
	0.025	0.05	11,29	17,38					
	0.01	0.02	10,30	16,39					
	0.005	0.01	—,—	15,40					
6	0.05	0.10	13,31	20,40	28,50				
	0.025	0.05	12,32	18,42	26,52				
	0.01	0.02	11,33	17,43	24,54				
	0.005	0.01	10,34	16,44	23,55				
7	0.05	0.10	14,34	21,44	29,55	39,66			
	0.025	0.05	13,35	20,45	27,57	36,69			
	0.01	0.02	11,37	18,47	25,59	34,71			
	0.005	0.01	10,38	16,49	24,60	32,73			
8	0.05	0.10	15,37	23,47	31,59	41,71	51,85		
	0.025	0.05	14,38	21,49	29,61	38,74	49,87		
	0.01	0.02	12,40	19,51	27,63	35,77	45,91		
	0.005	0.01	11,41	17,53	25,65	34,78	43,93		
9	0.05	0.10	16,40	24,51	33,63	43,76	54,90	66,105	
	0.025	0.05	14,42	22,53	31,65	40,79	51,93	62,109	
	0.01	0.02	13,43	20,55	28,68	37,82	47,97	59,112	
	0.005	0.01	11,45	18,57	26,70	35,84	45,99	56,115	
10	0.05	0.10	17,43	26,54	35,67	45,81	56,96	69,111	82,128
	0.025	0.05	15,45	23,57	32,70	42,84	53,99	65,115	78,132
	0.01	0.02	13,47	21,59	29,73	39,87	49,103	61,119	74,136
	0.005	0.01	12,48	19,61	27,75	37,89	47,105	58,122	71,139

Source: Adapted from Table 1 of F. Wilcoxon and R. A. Wilcox, *Some Rapid Approximate Statistical Procedures* (Pearl River, NY: Lederle Laboratories, 1964), with permission of the American Cyanamid Company.

**TABLE E.9**  
Critical Values of the Studentized Range,  $Q$

Denominator, $df$	Upper 5% Points ( $\alpha = 0.05$ )																			
	Numerator, $df$																			
	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
1	18.00	27.00	32.80	37.10	40.40	43.10	45.40	47.40	49.10	50.60	52.00	53.20	54.30	55.40	56.30	57.20	58.00	58.80	59.60	
2	6.09	8.30	9.80	10.90	11.70	12.40	13.00	13.50	14.00	14.40	14.70	15.10	15.40	15.70	15.90	16.10	16.40	16.60	16.80	
3	4.50	5.91	6.82	7.50	8.04	8.48	8.85	9.18	9.46	9.72	9.95	10.15	10.35	10.52	10.69	10.84	10.98	11.11	11.24	
4	3.93	5.04	5.76	6.29	6.71	7.05	7.35	7.60	7.83	8.03	8.21	8.37	8.52	8.66	8.79	8.91	9.03	9.13	9.23	
5	3.64	4.60	5.22	5.67	6.03	6.33	6.58	6.80	6.99	7.17	7.32	7.47	7.60	7.72	7.83	7.93	8.03	8.12	8.21	
6	3.46	4.34	4.90	5.31	5.63	5.89	6.12	6.32	6.49	6.65	6.79	6.92	7.03	7.14	7.24	7.34	7.43	7.51	7.59	
7	3.34	4.16	4.68	5.06	5.36	5.61	5.82	6.00	6.16	6.30	6.43	6.55	6.66	6.76	6.85	6.94	7.02	7.09	7.17	
8	3.26	4.04	4.53	4.89	5.17	5.40	5.60	5.77	5.92	6.05	6.18	6.29	6.39	6.48	6.57	6.65	6.73	6.80	6.87	
9	3.20	3.95	4.42	4.76	5.02	5.24	5.43	5.60	5.74	5.87	5.98	6.09	6.19	6.28	6.36	6.44	6.51	6.58	6.64	
10	3.15	3.88	4.33	4.65	4.91	5.12	5.30	5.46	5.60	5.72	5.83	5.93	6.03	6.11	6.20	6.27	6.34	6.40	6.47	
11	3.11	3.82	4.26	4.57	4.82	5.03	5.20	5.35	5.49	5.61	5.71	5.81	5.90	5.99	6.06	6.14	6.20	6.26	6.33	
12	3.08	3.77	4.20	4.51	4.75	4.95	5.12	5.27	5.40	5.51	5.62	5.71	5.80	5.88	5.95	6.03	6.09	6.15	6.21	
13	3.06	3.73	4.15	4.45	4.69	4.88	5.05	5.19	5.32	5.43	5.53	5.63	5.71	5.79	5.86	5.93	6.00	6.05	6.11	
14	3.03	3.70	4.11	4.41	4.64	4.83	4.99	5.13	5.25	5.36	5.46	5.55	5.64	5.72	5.79	5.85	5.92	5.97	6.03	
15	3.01	3.67	4.08	4.37	4.60	4.78	4.94	5.08	5.20	5.31	5.40	5.49	5.58	5.65	5.72	5.79	5.85	5.90	5.96	
16	3.00	3.65	4.05	4.33	4.56	4.74	4.90	5.03	5.15	5.26	5.35	5.44	5.52	5.59	5.66	5.72	5.79	5.84	5.90	
17	2.98	3.63	4.02	4.30	4.52	4.71	4.86	4.99	5.11	5.21	5.31	5.39	5.47	5.55	5.61	5.68	5.74	5.79	5.84	
18	2.97	3.61	4.00	4.28	4.49	4.67	4.82	4.96	5.07	5.17	5.27	5.35	5.43	5.50	5.57	5.63	5.69	5.74	5.79	
19	2.96	3.59	3.98	4.25	4.47	4.65	4.79	4.92	5.04	5.14	5.23	5.32	5.39	5.46	5.53	5.59	5.65	5.70	5.75	
20	2.95	3.58	3.96	4.23	4.45	4.62	4.77	4.90	5.01	5.11	5.20	5.28	5.36	5.43	5.49	5.55	5.61	5.66	5.71	
24	2.92	3.53	3.90	4.17	4.37	4.54	4.68	4.81	4.92	5.01	5.10	5.18	5.25	5.32	5.38	5.44	5.50	5.54	5.59	
30	2.89	3.49	3.84	4.10	4.30	4.46	4.60	4.72	4.83	4.92	5.00	5.08	5.15	5.21	5.27	5.33	5.38	5.43	5.48	
40	2.86	3.44	3.79	4.04	4.23	4.39	4.52	4.63	4.74	4.82	4.91	4.98	5.05	5.11	5.16	5.22	5.27	5.31	5.36	
60	2.83	3.40	3.74	3.98	4.16	4.31	4.44	4.55	4.65	4.73	4.81	4.88	4.94	5.00	5.06	5.11	5.16	5.20	5.24	
120	2.80	3.36	3.69	3.92	4.10	4.24	4.36	4.48	4.56	4.64	4.72	4.78	4.84	4.90	4.95	5.00	5.05	5.09	5.13	
$\infty$	2.77	3.31	3.63	3.86	4.03	4.17	4.29	4.39	4.47	4.55	4.62	4.68	4.74	4.80	4.85	4.89	4.93	4.97	5.01	

*continued*

Upper 1% Points ( $\alpha = 0.01$ )

Denominator, <i>df</i>	Numerator, <i>df</i>																			
	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
1	90.00	135.00	164.00	186.00	202.00	216.00	227.00	237.00	246.00	253.00	260.00	266.00	272.00	277.00	282.00	286.00	290.00	294.00	298.00	
2	14.00	19.00	22.30	24.70	26.60	28.20	29.50	30.70	31.70	32.60	33.40	34.10	34.80	35.40	36.00	36.50	37.00	37.50	37.90	
3	8.26	10.60	12.20	13.30	14.20	15.00	15.60	16.20	16.70	17.10	17.50	17.90	18.20	18.50	18.80	19.10	19.30	19.50	19.80	
4	6.51	8.12	9.17	9.96	10.60	11.10	11.50	11.90	12.30	12.60	12.80	13.10	13.30	13.50	13.70	13.90	14.10	14.20	14.40	
5	5.70	6.97	7.80	8.42	8.91	9.32	9.67	9.97	10.24	10.48	10.70	10.89	11.08	11.24	11.40	11.55	11.68	11.81	11.93	
6	5.24	6.33	7.03	7.56	7.97	8.32	8.61	8.87	9.10	9.30	9.49	9.65	9.81	9.95	10.08	10.21	10.32	10.43	10.54	
7	4.95	5.92	6.54	7.01	7.37	7.68	7.94	8.17	8.37	8.55	8.71	8.86	9.00	9.12	9.24	9.35	9.46	9.55	9.65	
8	4.74	5.63	6.20	6.63	6.96	7.24	7.47	7.68	7.87	8.03	8.18	8.31	8.44	8.55	8.66	8.76	8.85	8.94	9.03	
9	4.60	5.43	5.96	6.35	6.66	6.91	7.13	7.32	7.49	7.65	7.78	7.91	8.03	8.13	8.23	8.32	8.41	8.49	8.57	
10	4.48	5.27	5.77	6.14	6.43	6.67	6.87	7.05	7.21	7.36	7.48	7.60	7.71	7.81	7.91	7.99	8.07	8.15	8.22	
11	4.39	5.14	5.62	5.97	6.26	6.48	6.67	6.84	6.99	7.13	7.25	7.36	7.46	7.56	7.65	7.73	7.81	7.88	7.95	
12	4.32	5.04	5.50	5.84	6.10	6.32	6.51	6.67	6.81	6.94	7.06	7.17	7.26	7.36	7.44	7.52	7.59	7.66	7.73	
13	4.26	4.96	5.40	5.73	5.98	6.19	6.37	6.53	6.67	6.79	6.90	7.01	7.10	7.19	7.27	7.34	7.42	7.48	7.55	
14	4.21	4.89	5.32	5.63	5.88	6.08	6.26	6.41	6.54	6.66	6.77	6.87	6.96	7.05	7.12	7.20	7.27	7.33	7.39	
15	4.17	4.83	5.25	5.56	5.80	5.99	6.16	6.31	6.44	6.55	6.66	6.76	6.84	6.93	7.00	7.07	7.14	7.20	7.26	
16	4.13	4.78	5.19	5.49	5.72	5.92	6.08	6.22	6.35	6.46	6.56	6.66	6.74	6.82	6.90	6.97	7.03	7.09	7.15	
17	4.10	4.74	5.14	5.43	5.66	5.85	6.01	6.15	6.27	6.38	6.48	6.57	6.66	6.73	6.80	6.87	6.94	7.00	7.05	
18	4.07	4.70	5.09	5.38	5.60	5.79	5.94	6.08	6.20	6.31	6.41	6.50	6.58	6.65	6.72	6.79	6.85	6.91	6.96	
19	4.05	4.67	5.05	5.33	5.55	5.73	5.89	6.02	6.14	6.25	6.34	6.43	6.51	6.58	6.65	6.72	6.78	6.84	6.89	
20	4.02	4.64	5.02	5.29	5.51	5.69	5.84	5.97	6.09	6.19	6.29	6.37	6.45	6.52	6.59	6.65	6.71	6.76	6.82	
24	3.96	4.54	4.91	5.17	5.37	5.54	5.69	5.81	5.92	6.02	6.11	6.19	6.26	6.33	6.39	6.45	6.51	6.56	6.61	
30	3.89	4.45	4.80	5.05	5.24	5.40	5.54	5.65	5.76	5.85	5.93	6.01	6.08	6.14	6.20	6.26	6.31	6.36	6.41	
40	3.82	4.37	4.70	4.93	5.11	5.27	5.39	5.50	5.60	5.69	5.77	5.84	5.90	5.96	6.02	6.07	6.12	6.17	6.21	
60	3.76	4.28	4.60	4.82	4.99	5.13	5.25	5.36	5.45	5.53	5.60	5.67	5.73	5.79	5.84	5.89	5.93	5.98	6.02	
120	3.70	4.20	4.50	4.71	4.87	5.01	5.12	5.21	5.30	5.38	5.44	5.51	5.56	5.61	5.66	5.71	5.75	5.79	5.83	
$\infty$	3.64	4.12	4.40	4.60	4.76	4.88	4.99	5.08	5.16	5.23	5.29	5.35	5.40	5.45	5.49	5.54	5.57	5.61	5.65	

Source: Reprinted from E. S. Pearson and H. O. Hartley, eds., Table 29 of *Biometrika Tables for Statisticians, Vol. 1, 3rd ed.*, 1966, by permission of the *Biometrika* Trustees, London.

**TABLE E.10**  
Critical Values  $d_L$  and  $d_U$  of the Durbin-Watson Statistic,  $D$  (Critical Values Are One Sided)<sup>a</sup>

$n$	$\alpha = 0.05$										$\alpha = 0.01$									
	$k = 1$		$k = 2$		$k = 3$		$k = 4$		$k = 5$		$k = 1$		$k = 2$		$k = 3$		$k = 4$		$k = 5$	
	$d_L$	$d_U$	$d_L$	$d_U$	$d_L$	$d_U$	$d_L$	$d_U$	$d_L$	$d_U$	$d_L$	$d_U$	$d_L$	$d_U$	$d_L$	$d_U$	$d_L$	$d_U$	$d_L$	$d_U$
15	1.08	1.36	.95	1.54	.82	1.75	.69	1.97	.56	2.21	.81	1.07	.70	1.25	.59	1.46	.49	1.70	.39	1.96
16	1.10	1.37	.98	1.54	.86	1.73	.74	1.93	.62	2.15	.84	1.09	.74	1.25	.63	1.44	.53	1.66	.44	1.90
17	1.13	1.38	1.02	1.54	.90	1.71	.78	1.90	.67	2.10	.87	1.10	.77	1.25	.67	1.43	.57	1.63	.48	1.85
18	1.16	1.39	1.05	1.53	.93	1.69	.82	1.87	.71	2.06	.90	1.12	.80	1.26	.71	1.42	.61	1.60	.52	1.80
19	1.18	1.40	1.08	1.53	.97	1.68	.86	1.85	.75	2.02	.93	1.13	.83	1.26	.74	1.41	.65	1.58	.56	1.77
20	1.20	1.41	1.10	1.54	1.00	1.68	.90	1.83	.79	1.99	.95	1.15	.86	1.27	.77	1.41	.68	1.57	.60	1.74
21	1.22	1.42	1.13	1.54	1.03	1.67	.93	1.81	.83	1.96	.97	1.16	.89	1.27	.80	1.41	.72	1.55	.63	1.71
22	1.24	1.43	1.15	1.54	1.05	1.66	.96	1.80	.86	1.94	1.00	1.17	.91	1.28	.83	1.40	.75	1.54	.66	1.69
23	1.26	1.44	1.17	1.54	1.08	1.66	.99	1.79	.90	1.92	1.02	1.19	.94	1.29	.86	1.40	.77	1.53	.70	1.67
24	1.27	1.45	1.19	1.55	1.10	1.66	1.01	1.78	.93	1.90	1.04	1.20	.96	1.30	.88	1.41	.80	1.53	.72	1.66
25	1.29	1.45	1.21	1.55	1.12	1.66	1.04	1.77	.95	1.89	1.05	1.21	.98	1.30	.90	1.41	.83	1.52	.75	1.65
26	1.30	1.46	1.22	1.55	1.14	1.65	1.06	1.76	.98	1.88	1.07	1.22	1.00	1.31	.93	1.41	.85	1.52	.78	1.64
27	1.32	1.47	1.24	1.56	1.16	1.65	1.08	1.76	1.01	1.86	1.09	1.23	1.02	1.32	.95	1.41	.88	1.51	.81	1.63
28	1.33	1.48	1.26	1.56	1.18	1.65	1.10	1.75	1.03	1.85	1.10	1.24	1.04	1.32	.97	1.41	.90	1.51	.83	1.62
29	1.34	1.48	1.27	1.56	1.20	1.65	1.12	1.74	1.05	1.84	1.12	1.25	1.05	1.33	.99	1.42	.92	1.51	.85	1.61
30	1.35	1.49	1.28	1.57	1.21	1.65	1.14	1.74	1.07	1.83	1.13	1.26	1.07	1.34	1.01	1.42	.94	1.51	.88	1.61
31	1.36	1.50	1.30	1.57	1.23	1.65	1.16	1.74	1.09	1.83	1.15	1.27	1.08	1.34	1.02	1.42	.96	1.51	.90	1.60
32	1.37	1.50	1.31	1.57	1.24	1.65	1.18	1.73	1.11	1.82	1.16	1.28	1.10	1.35	1.04	1.43	.98	1.51	.92	1.60
33	1.38	1.51	1.32	1.58	1.26	1.65	1.19	1.73	1.13	1.81	1.17	1.29	1.11	1.36	1.05	1.43	1.00	1.51	.94	1.59
34	1.39	1.51	1.33	1.58	1.27	1.65	1.21	1.73	1.15	1.81	1.18	1.30	1.13	1.36	1.07	1.43	1.01	1.51	.95	1.59
35	1.40	1.52	1.34	1.58	1.28	1.65	1.22	1.73	1.16	1.80	1.19	1.31	1.14	1.37	1.08	1.44	1.03	1.51	.97	1.59
36	1.41	1.52	1.35	1.59	1.29	1.65	1.24	1.73	1.18	1.80	1.21	1.32	1.15	1.38	1.10	1.44	1.04	1.51	.99	1.59
37	1.42	1.53	1.36	1.59	1.31	1.66	1.25	1.72	1.19	1.80	1.22	1.32	1.16	1.38	1.11	1.45	1.06	1.51	1.00	1.59
38	1.43	1.54	1.37	1.59	1.32	1.66	1.26	1.72	1.21	1.79	1.23	1.33	1.18	1.39	1.12	1.45	1.07	1.52	1.02	1.58
39	1.43	1.54	1.38	1.60	1.33	1.66	1.27	1.72	1.22	1.79	1.24	1.34	1.19	1.39	1.14	1.45	1.09	1.52	1.03	1.58
40	1.44	1.54	1.39	1.60	1.34	1.66	1.29	1.72	1.23	1.79	1.25	1.34	1.20	1.40	1.15	1.46	1.10	1.52	1.05	1.58
45	1.48	1.57	1.43	1.62	1.38	1.67	1.34	1.72	1.29	1.78	1.29	1.38	1.24	1.42	1.20	1.48	1.16	1.53	1.11	1.58
50	1.50	1.59	1.46	1.63	1.42	1.67	1.38	1.72	1.34	1.77	1.32	1.40	1.28	1.45	1.24	1.49	1.20	1.54	1.16	1.59
55	1.53	1.60	1.49	1.64	1.45	1.68	1.41	1.72	1.38	1.77	1.36	1.43	1.32	1.47	1.28	1.51	1.25	1.55	1.21	1.59
60	1.55	1.62	1.51	1.65	1.48	1.69	1.44	1.73	1.41	1.77	1.38	1.45	1.35	1.48	1.32	1.52	1.28	1.56	1.25	1.60
65	1.57	1.63	1.54	1.66	1.50	1.70	1.47	1.73	1.44	1.77	1.41	1.47	1.38	1.50	1.35	1.53	1.31	1.57	1.28	1.61
70	1.58	1.64	1.55	1.67	1.52	1.70	1.49	1.74	1.46	1.77	1.43	1.49	1.40	1.52	1.37	1.55	1.34	1.58	1.31	1.61
75	1.60	1.65	1.57	1.68	1.54	1.71	1.51	1.74	1.49	1.77	1.45	1.50	1.42	1.53	1.39	1.56	1.37	1.59	1.34	1.62
80	1.61	1.66	1.59	1.69	1.56	1.72	1.53	1.74	1.51	1.77	1.47	1.52	1.44	1.54	1.42	1.57	1.39	1.60	1.36	1.62
85	1.62	1.67	1.60	1.70	1.57	1.72	1.55	1.75	1.52	1.77	1.48	1.53	1.46	1.55	1.43	1.58	1.41	1.60	1.39	1.63
90	1.63	1.68	1.61	1.70	1.59	1.73	1.57	1.75	1.54	1.78	1.50	1.54	1.47	1.56	1.45	1.59	1.43	1.61	1.41	1.64
95	1.64	1.69	1.62	1.71	1.60	1.73	1.58	1.75	1.56	1.78	1.51	1.55	1.49	1.57	1.47	1.60	1.45	1.62	1.42	1.64
100	1.65	1.69	1.63	1.72	1.61	1.74	1.59	1.76	1.57	1.78	1.52	1.56	1.50	1.58	1.48	1.60	1.46	1.63	1.44	1.65

<sup>a</sup>  $n$  = number of observations;  $k$  = number of independent variables.

Source: This table is reproduced from *Biometrika*, 41 (1951): pp. 173 and 175, with the permission of the *Biometrika* Trustees.

**TABLE E.11**  
Control Chart Factors

Number of Observations in Sample	$d_2$	$d_3$	$D_3$	$D_4$	$A_2$
2	1.128	0.853	0	3.267	1.880
3	1.693	0.888	0	2.575	1.023
4	2.059	0.880	0	2.282	0.729
5	2.326	0.864	0	2.114	0.577
6	2.534	0.848	0	2.004	0.483
7	2.704	0.833	0.076	1.924	0.419
8	2.847	0.820	0.136	1.864	0.373
9	2.970	0.808	0.184	1.816	0.337
10	3.078	0.797	0.223	1.777	0.308
11	3.173	0.787	0.256	1.744	0.285
12	3.258	0.778	0.283	1.717	0.266
13	3.336	0.770	0.307	1.693	0.249
14	3.407	0.763	0.328	1.672	0.235
15	3.472	0.756	0.347	1.653	0.223
16	3.532	0.750	0.363	1.637	0.212
17	3.588	0.744	0.378	1.622	0.203
18	3.640	0.739	0.391	1.609	0.194
19	3.689	0.733	0.404	1.596	0.187
20	3.735	0.729	0.415	1.585	0.180
21	3.778	0.724	0.425	1.575	0.173
22	3.819	0.720	0.435	1.565	0.167
23	3.858	0.716	0.443	1.557	0.162
24	3.895	0.712	0.452	1.548	0.157
25	3.931	0.708	0.459	1.541	0.153

Source: Reprinted from ASTM-STP 15D by kind permission of the American Society for Testing and Materials.

## F. MICROSOFT EXCEL AND PHSTAT2 FAQs

Use this appendix to find answers to the most frequently asked questions about using the resources on the Student CD-ROM and using Microsoft Excel and PHStat2.

### F.1 STUDENT CD-ROM FAQs

**What textbook materials will I find on the Student CD-ROM?**

You will find the Excel workbooks (as .xls files) used in examples or named in problems, the CD-ROM sections (as .PDF files), several Excel add-in workbooks (as .xla files), and programs that set up and install the Adobe Reader and PHStat2 programs on Windows PCs.

**Does the Student CD-ROM contain Microsoft Excel?**

No, the Student CD-ROM does not contain Microsoft Excel. If your computer did not come with Microsoft Excel or Office, you can acquire a copy of Microsoft Office and install Excel on your computer.

**Can I copy the Excel workbook files to my local hard disk or other storage device?**

Yes, you can, and you are encouraged to do so. If you open and use a workbook file directly from the Student CD-ROM, Microsoft Excel labels that file as being a “read-only” file and will force you to save the file under a different name if you make changes to the file.

### F.2 GENERAL COMPUTING FAQs

**My computer came with Microsoft Works. Can I use that program instead of using Microsoft Excel?**

No, the spreadsheet component of Microsoft Works does not contain the functionality of Microsoft Excel and cannot be used with this text.

**Do I need the latest version of Microsoft Excel in order to use this book?**

No, you can use this book with all Excel versions supported by Microsoft. This includes (as of the date of the printing of this book) Excel 97, Excel 2000, Excel 2002 (sometimes identified as Excel XP), Excel 2003, and Excel 2007.

**How can I determine which version of Microsoft Excel is installed on my computer?**

Open Microsoft Excel. If your Excel window looks like Figure E1.2 on page 20, you have Excel 2007. Otherwise, select **Help** → **About Microsoft Excel**. In the dialog box

that appears, the first line states the version number and build number as well as the names of the service release (SR) or service pack (SP) updates that have been applied.

**Should I update my copy of Excel using the security updates available on the Microsoft Web site?**

Absolutely yes! You should visit the Microsoft Office Web site and apply any updates that are available for your version of Excel before you begin using this book. Then, you should periodically revisit the Web site to check for new updates that may have been added since your last visit.

**Can I use an Apple Mac running Mac OS on which Microsoft Excel has been installed with the Excel data files on the Student CD-ROM?**

Yes, you can. However, you will *not* be able to use PHStat2, which is designed for Microsoft Windows. (There is no Mac version of PHStat2 available.)

### F.3 MICROSOFT EXCEL FAQs

**Will I need access to my original Microsoft Office/Excel CDs or DVD?**

Yes, you may need to use the original program discs if the Analysis ToolPak (referred to as the “ToolPak” in this book) has not been installed on your system.

**How can I tell if the ToolPak has been installed? What Microsoft Office security settings should I use? How do I change Office security settings?**

The answers to these related questions are discussed as part of Section E1.6, “Add-ins: Making Things Easier for You,” which begins on page 28.

**Are there any special procedures for using the Visual Explorations add-in workbook (Visual Explorations.xla)?**

No, you can open and immediately use this workbook, provided that you have adjusted your Office security settings using the Section E1.6 instructions. With the wrong security settings in effect, Visual Explorations (and PHStat2, also) will not open in Excel.

**In Excel 97–2003, how can I turn toolbars such as the Standard and Formatting toolbars shown in Figure E1.1 on page 19, on and off?**

Select **View** → **Toolbars** and select the toolbars you want displayed.

### For Excel 2007, how can I configure the Excel window to look similar to Figure E1.2 on page 20?

Right-click the title bar of the Excel 2007 window and on the shortcut menu that appears, uncheck, if checked, **Minimize the Ribbon**. You can also review this setting by clicking the drop-down arrow to the immediate right of the Quick Access toolbar.

### For Excel 97–2003, how can I specify the custom settings that you recommend?

First select **Tools → Customize**. In the Customize dialog box, clear (uncheck) the **Menus show recently used commands first** check box if it is checked and click the **Close** button. Then select **Tools → Options**. In the Options dialog box, click the **Calculation** tab and verify that the **Automatic** option button of the Calculation group has been selected. Click the **Edit** tab and verify that all check boxes *except* the **Fixed decimal**, **Provide feedback with Animation**, and **Enable automatic percent entry** have been selected. (Excel 97 does not contain an Automatic percent entry check box.) Click the **General** tab and verify that the **R1C1 reference style** check box is cleared (that is, unchecked) and, if using Excel 97, check the **Macro virus protection** check box. Enter **3** as the number of **Sheets in new workbook**, select **Arial** from the **Standard font** list box, and select **10** from the **Size** drop-down list box. Click **OK** to finish the customization.

### For Excel 2007, how can I specify the custom settings that you recommend?

First, click **Office Button** and then click **Excel Options**. In the left panel of Excel Options, click **Formulas**. In the Formulas right pane, click **Automatic** under **Workbook Calculation** and verify that all check boxes are checked except **Enable iterative calculation**, **R1C1 reference style**, and **Formulas referring to cells**. Click **OK** to finish the customization.

## F.4 PHSTAT2 FAQs

### What is PHStat2?

PHStat2 is software that makes operating Microsoft Excel as distraction free as possible. As a student studying statistics, you can focus mainly on learning statistics and not worry about having to fully master Excel first. When PHStat2 is combined with the Analysis ToolPak add-in, just about all statistical methods taught in an introductory statistics course can be illustrated using Microsoft Excel.

### I do not want to use an add-in that will not be available in my business environment. Any comments?

Chapter 1 talks about these issues in detail. To summarize those pages, PHStat2 helps you learn Microsoft Excel, and using PHStat2 will not leave you any less equipped to work with Microsoft Excel in a setting where it is not available.

### What do I need to do in order to begin using PHStat2?

You need to run the PHStat2 setup program (setup.exe) that is on the Student CD-ROM. If you are using Windows 2000, Windows XP, or Windows Vista, you must have logged in to Windows using an account that has administrator or software-installing privileges. (Student and faculty accounts to log in to networked computers in academic settings typically do not have this privilege. If you have such an account, ask your network or lab technician for assistance.)

### What are the technical requirements for setting up and adding PHStat2 to my system?

If your system can run Microsoft Excel, it can also run PHStat2. You need approximately 10 MB hard disk free space during the setup process and up to 3 MB hard disk space after program setup.

### Are updates to PHStat2 available?

Yes, minor updates to resolve issues as they are identified are available for free download from the PHStat2 Web site ([www.prenhall.com/phstat](http://www.prenhall.com/phstat)). When you visit that Web site, link to the page that applies to your version of PHStat2 (the Student CD-ROM contains Version 2.7).

### How can I identify which version of PHStat2 I have?

Open Microsoft Excel with PHStat2 and select **Help for PHStat** from the PHStat menu. A dialog box will display your current XLA and DLL version numbers. The XLA version number identifies the version of PHStat2 you have.

### Where can I get help setting up PHStat2?

First, carefully review the Student CD-ROM readme file for PHStat2. If your problem is unresolved, visit the PHStat2 Web site for further information. If your problem is still unresolved, contact Pearson Education technical support by following the appropriate link on the PHStat2 Web site.

## F.5 EXCEL 2007 ISSUES FAQs

### I do not see the menu for an add-in workbook that I opened, where is it?

Unlike earlier versions of Excel that allowed add-ins to add menus to the menu bar, Excel 2007 places all add-in menus under the Add-ins tab. If you click Add-ins, you find the menus of all properly loaded add-ins.

### What does “Compatibility Mode” mean?

When you see “Compatibility Mode” in the title bar, Excel 2007 is telling you that you are using a workbook compati-

ble with earlier Excel versions. When you save such a workbook, Excel 2007 will automatically use the .xls file format of earlier versions.

### How can I update an older workbook to the Excel 2007 .xlsx format?

The simplest way is to open the workbook file and click **Office Button → Convert**. Then save your file (click **Office Button → Save**).

You can also open the workbook file and click **Office Button → Save As** and select **Excel Workbook (\*.xlsx)** from the **Save as type** list in the Save As dialog box.



# Self-Test Solutions and Answers to Selected Even-Numbered Problems

The following represent worked-out solutions to Self-Test Problems and brief answers to most of the even-numbered problems in the text. For more detailed solutions, including explanations, interpretations, and Excel output, see the *Student Solutions Manual*.

## CHAPTER 1

**1.2** Small, medium, and large sizes imply order but do not specify how much more soft drink is added at increasing levels.

**1.4 (a)** The number of telephones is a numerical variable that is discrete because the variable is counted. It is ratio scaled because it has a true zero point. **(b)** The length of the longest long-distance call is a numerical variable that is continuous because any value within a range of values can occur. It is ratio scaled because it has a true zero point. **(c)** Whether there is a cell phone in the household is a categorical variable because the answer can be only yes or no. This also makes it a nominal-scaled variable. **(d)** Same answer as in (c).

**1.6 (a)** Categorical, nominal scale. **(b)** Numerical, continuous, ratio scale. **(c)** Numerical, discrete, ratio scale. **(d)** Numerical, discrete, ratio scale.

**1.8 (a)** Numerical, continuous, ratio scale. **(b)** Numerical, discrete, ratio scale. **(c)** Numerical, continuous, ratio scale. **(d)** Categorical, nominal scale.

**1.10** The underlying variable, ability of the students, may be continuous, but the measuring device, the test, does not have enough precision to distinguish between the two students.

**1.24 (a)** A primary data source collected through a survey was used in this study. **(b)** Deciding what to make for dinner at home at the last minute. **(c)** The amount of time to prepare dinner.

**1.26 (a)** Cat owner households in the United States. **(b)** 1. Categorical. 2. Categorical. 3. Numerical, discrete. 4. Categorical.

## CHAPTER 2

**2.4 (b)** The Pareto diagram is best for portraying these data because it not only sorts the frequencies in descending order, it also provides the cumulative polygon on the same scale. **(c)** You can conclude that “labor” and “lost business/revenue” make up about half of the costs.

**2.6 (b)** 88%. **(d)** The Pareto diagram allows you to see which sources account for most of the electricity.

**2.8 (b)** The bar chart is more suitable if the purpose is to compare the categories. The pie chart is more suitable if the main objective is to investigate the portion of the whole that is in a particular category.

**2.10 (b)** Rooms dirty, rooms not stocked, and rooms need maintenance have the largest number of complaints, so focusing on these categories can reduce the number of complaints the most.

**2.12** Stem-and-leaf display of finance scores:

```

5 | 34
6 | 9
7 | 4
8 | 0
9 | 38
  | n = 7
    
```

**2.14** 50 74 74 76 81 89 92

**2.16 (a)** Ordered array:

```

$15 $15 $18 $18 $20 $20 $20 $20 $21 $22 $22
$25 $25 $25 $25 $25 $26 $28 $29 $30 $30 $30
    
```

**(b) Stem-and-leaf display:**

```

1 | 5 5 8 8
2 | 0 0 0 0 1 2 2 5 5 5 5 6 8 9
3 | 0 0 0
    
```

**(c)** The stem-and-leaf display provides more information because it not only orders values from the smallest to the largest into stems and leaves, it also conveys information on how the values distribute and cluster in the data set. **(d)** The bounced check fees seem to be concentrated around \$20 and \$25 because there are five occurrences of each of the two values in the sample of 23 banks.

**2.18 (a)** Ordered array:

4 5 7 8 16 19 19 20 20 23 24 25 29 29 30 30 30 30 40 56

**(b) Stem-and-leaf display for fat stem, 10 unit:**

```

0 | 4 5 7 8
1 | 6 9 9
2 | 0 0 3 4 5 9 9
3 | 0 0 0 0
4 | 0
5 | 6
    
```

**(c)** The stem-and-leaf display conveys more information than the ordered array. You can also obtain a sense of the distribution of the data from the stem-and-leaf display.

**2.20 (a)** 10 but less than 20, 20 but less than 30, 30 but less than 40, 40 but less than 50, 50 but less than 60, 60 but less than 70, 70 but less than 80, 80 but less than 90, 90 but less than 100. **(b)** 10. **(c)** 15, 25, 35, 45, 55, 65, 75, 85, 95.

<b>2.22 (a)</b>	<b>Electricity Costs</b>	<b>Frequency</b>	<b>Percentage</b>
	\$80 up to \$100	4	8%
	\$100 up to \$120	7	14
	\$120 up to \$140	9	18
	\$140 up to \$160	13	26
	\$160 up to \$180	9	18
	\$180 up to \$200	5	10
	\$200 up to \$220	3	6

(c) **Electricity**

Costs	Frequency	Percentage	Cumulative %
\$ 99	4	8.00%	8.00%
\$119	7	14.00	22.00
\$139	9	18.00	40.00
\$159	13	26.00	66.00
\$179	9	18.00	84.00
\$199	5	10.00	94.00
\$219	3	6.00	100.00

(d) The majority of utility charges are clustered between \$120 and \$180.

2.24 (a)

Width	Frequency	Percentage
8.310–8.329	3	6.12%
8.330–8.349	2	4.08
8.350–8.369	1	2.04
8.370–8.389	4	8.16
8.390–8.409	5	10.20
8.410–8.429	16	31.65
8.430–8.449	5	10.20
8.450–8.469	5	10.20
8.470–8.489	6	12.24
8.490–8.509	2	4.08

(d) All the troughs will meet the company's requirements of between 8.31 and 8.61 inches wide.

2.26 (a)

Bulb Life (hrs)	Percentage, Mfgr A	Percentage, Mfgr B
650–749	7.5%	0.0%
750–849	12.5	5.0
850–949	50.0	20.0
950–1049	22.5	40.0
1050–1149	7.5	22.5
1150–1249	0.0	12.5

(c)

Bulb Life (hrs)	Percentage Less Than, Mfgr A	Percentage Less Than, Mfgr B
650–749	7.5%	0.0%
750–849	20.0	5.0
850–949	70.0	25.0
950–1049	92.5	65.0
1050–1149	100.0	87.5
1150–1249	100.0	100.0

(d) Manufacturer B produces bulbs with longer lives than Manufacturer A. The cumulative percentage for Manufacturer B shows 65% of its bulbs lasted less than 1,050 hours, contrasted with 70% of Manufacturer A's bulbs, which lasted less than 950 hours. None of Manufacturer A's bulbs lasted more than 1,149 hours, but 12.5% of Manufacturer B's bulbs lasted between 1,150 and 1,249 hours. At the same time, 7.5% of Manufacturer A's bulbs lasted less than 750 hours, while all of Manufacturer B's bulbs lasted at least 750 hours.

2.28 (a) Table of frequencies for all student responses:

GENDER	STUDENT MAJOR CATEGORIES			Totals
	A	C	M	
Male	14	9	2	25
Female	<u>6</u>	<u>6</u>	<u>3</u>	<u>15</u>
Totals	20	15	5	40

(b) Table of percentages based on overall student responses:

GENDER	STUDENT MAJOR CATEGORIES			Totals
	A	C	M	
Male	35.0%	22.5%	5.0%	62.5%
Female	<u>15.0</u>	<u>15.0</u>	<u>7.5</u>	<u>37.5</u>
Totals	50.0	37.5	12.5	100.0

(c) Table based on row percentages:

GENDER	STUDENT MAJOR CATEGORIES			Totals
	A	C	M	
Male	56.0%	36.0%	8.0%	100.0%
Female	<u>40.0</u>	<u>40.0</u>	<u>20.0</u>	<u>100.0</u>
Totals	50.0	37.5	12.5	100.0

(d) Table based on column percentages:

GENDER	STUDENT MAJOR CATEGORIES			Totals
	A	C	M	
Male	70.0%	60.0%	40.0%	62.5%
Female	<u>30.0</u>	<u>40.0</u>	<u>60.0</u>	<u>37.5</u>
Totals	100.0	100.0	100.0	100.0

2.30 (a) Contingency table:

QUALITY	CONDITION OF DIE		Totals
	No Particles	Particles	
Good	320	14	334
Bad	<u>80</u>	<u>36</u>	<u>116</u>
Totals	400	50	450

Table of total percentages:

QUALITY	CONDITION OF DIE		Totals
	No Particles	Particles	
Good	71%	3%	74%
Bad	<u>18</u>	<u>8</u>	<u>26</u>
Totals	89	11	100

Table of row percentages:

QUALITY	CONDITION OF DIE		Totals
	No Particles	Particles	
Good	96%	4%	100%
Bad	<u>69</u>	<u>31</u>	<u>100</u>
Totals	89	11	100

Table of column percentages:

QUALITY	CONDITION OF DIE		Totals
	No Particles	Particles	
Good	80%	28%	74%
Bad	$\frac{20}{100}$	$\frac{72}{100}$	$\frac{26}{100}$
Totals	100	100	100

(c) The data suggest that there is some association between condition of the die and the quality of wafer because more good wafers are produced when no particles are found in the die, and more bad wafers are produced when there are particles found in the die.

2.32 (a) Table of row percentages:

ENJOY SHOPPING FOR CLOTHING	GENDER		Total
	Male	Female	
Yes	38%	62%	100%
No	$\frac{74}{48}$	$\frac{26}{52}$	$\frac{100}{100}$
Total	48	52	100

Table of column percentages:

ENJOY SHOPPING FOR CLOTHING	GENDER		Total
	Male	Female	
Yes	57%	86%	72%
No	$\frac{43}{100}$	$\frac{14}{100}$	$\frac{28}{100}$
Total	100	100	100

Table of total percentages:

ENJOY SHOPPING FOR CLOTHING	GENDER		Total
	Male	Female	
Yes	27%	45%	72%
No	$\frac{21}{48}$	$\frac{7}{52}$	$\frac{28}{100}$
Total	48	52	100

(c) The percentage that enjoys shopping for clothing is higher among females than among males.

2.34 (b) The number of MBA and undergraduate students who choose the lowest-cost fund and the second-lowest-cost fund is about the same. More MBA students chose the third-lowest cost fund while more undergraduate students chose the highest cost fund.

2.36 (b) Yes, there is a strong positive relationship between  $X$  and  $Y$ . As  $X$  increases, so does  $Y$ .

2.38 (b) There is a positive relationship between owner mileage and current government standard mileage.

2.40 (b) There appears to be a positive relationship between the coach's salary and revenue. (c) Yes, this is borne out by the data.

2.42 (b) The unemployment rate appeared to be trending upward from January 2001 to around January 2002. It stayed at around 6% until October 2003 and then trended downward until December 2005.

2.44 (b) The average price of prescription drugs has been rising since 2000. (c) The predicted average price for 2005 was around \$70.

2.62 (c) The publisher gets the largest portion (64.8%) of the revenue. About half (32.2%) of the revenue received by the publisher covers manufacturing costs. The publisher's marketing and promotion account for the next largest share of the revenue, at 15.4%. Author, bookstore employee salaries and benefits, and publisher administrative costs and taxes each accounts for around 10% of the revenue, while the publisher after-tax profit, bookstore operations, bookstore pretax profit, and freight constitute the "trivial few" allocations of the revenue. Yes, the bookstore gets twice the revenue of the authors.

2.64 (b) In 2005, the United States relied on petroleum heavily, followed by coal and natural gas as major sources of energy, while renewable fuels accounted for less than 4% of the total consumption. Wood accounted for more than half of the renewable energy consumption.

2.66 (a) There is no particular pattern to the deaths due to terrorism on U.S. soil between 1990 and 2001. There are exceptionally high death counts in 1995 and 2001 due to the Oklahoma City bombing and the September 11 attacks. (c) The Pareto diagram is best to portray these data because it not only sorts the frequencies in descending order, it also provides the cumulative polygon on the same scale. The labels in the pie chart become cluttered because there are too many categories in the causes of death. (d) The major causes of death in the United States in 2000 were heart disease followed by cancer. These two accounted for more than 70% of the total deaths.

2.68 (a)

DESSERT ORDERED	GENDER		Total
	Male	Female	
Yes	71%	29%	100%
No	$\frac{48}{53}$	$\frac{52}{47}$	$\frac{100}{100}$
Total	53	47	100

DESSERT ORDERED	GENDER		Total
	Male	Female	
Yes	30%	14%	23%
No	$\frac{70}{100}$	$\frac{86}{100}$	$\frac{77}{100}$
Total	100	100	100

DESSERT ORDERED	BEEF ENTRÉE		Total
	Yes	No	
Yes	52%	48%	100%
No	$\frac{25}{31}$	$\frac{75}{69}$	$\frac{100}{100}$
Total	31	69	100

DESSERT ORDERED	BEEF ENTRÉE		Total
	Yes	No	
Yes	38%	16%	23%
No	$\frac{62}{100}$	$\frac{84}{100}$	$\frac{77}{100}$
Total	100	100	100

DESSERT ORDERED	BEEF ENTRÉE		Total
	Yes	No	
Yes	12%	11%	23%
No	$\frac{19}{31}$	$\frac{58}{69}$	$\frac{77}{100}$
Total	31	69	100

(b) If the owner is interested in finding out the percentage of males and females who order dessert or the percentage of those who order a beef entrée and a dessert among all patrons, the table of total percentages is most

informative. If the owner is interested in the effect of gender on ordering of dessert or the effect of ordering a beef entrée on the ordering of dessert, the table of column percentages will be most informative. Because dessert is usually ordered after the main entrée, and the owner has no direct control over the gender of patrons, the table of row percentages is not very useful here. **(c)** 30% of the men ordered desserts, compared to 14% of the women; men are more than twice as likely to order desserts as women. Almost 38% of the patrons ordering a beef entrée ordered dessert, compared to 16% of patrons ordering all other entrees. Patrons ordering beef are more than 2.3 times as likely to order dessert as patrons ordering any other entrée.

**2.70 (a)** 23575R15 accounts for over 80% of the warranty claims. **(b)** Tread separation accounts for the majority (70%) of the warranty claims. **(c)** Tread separation accounts for more than 70% of the warranty claims among the ATX model. **(d)** The number of claims is evenly distributed among the three incidents; other/unknown incidents account for almost 40% of the claims, tread separation accounts for about 35% of the claims, and blowout accounts for about 25% of the claims.

**2.72 (c)** More than 80% of the 58 domestic beers have less than 5.2% alcohol content, with their alcohol content scattered around 4.7%, while fewer than 10% have more than 5.7% alcohol content. The number of calories scatters around 149, and about half (27 out of 58) of the domestic beers have between 10 and 14 grams of carbohydrates. There appears to be a positive relationship between the alcohol content and the number of calories and between calories and carbohydrates.

**2.74 (c)** In general, five-year CDs have the highest yields, followed by one-year CDs and then money market accounts. The yields from five-year CDs are approximately distributed around 4%; there are a few one-year CDs with very low yields of around 2.4%, and the rest have higher yields, in the neighborhood of 3.4%; a few (2 out of 40) money market accounts have yields higher than 2%, but most of them have yields lower than 1.8%.

The money market accounts and one-year CDs have yields that are positively related. One-year CDs and five-year CDs also have yields that are positively related. The yields of money market accounts are also positively related to the yields of five-year CDs. So banks that have a high yield in one of the money market accounts, one-year CDs, or five-year CDs also tend to have high yields on the others.

**2.76 (a)**

Frequencies (Boston)		
Weight (Boston)	Frequency	Percentage
3,015 but less than 3,050	2	0.54%
3,050 but less than 3,085	44	11.96
3,085 but less than 3,120	122	33.15
3,120 but less than 3,155	131	35.60
3,155 but less than 3,190	58	15.76
3,190 but less than 3,225	7	1.90
3,225 but less than 3,260	3	0.82
3,260 but less than 3,295	1	0.27

**(b)**

Frequencies (Vermont)		
Weight (Vermont)	Frequency	Percentage
3,550 but less than 3,600	4	1.21%
3,600 but less than 3,650	31	9.39
3,650 but less than 3,700	115	34.85
3,700 but less than 3,750	131	39.70
3,750 but less than 3,800	36	10.91
3,800 but less than 3,850	12	3.64
3,850 but less than 3,900	1	0.30

**(d)** 0.54% of the Boston shingles pallets are underweight, while 0.27% are overweight. 1.21% of the Vermont shingles pallets are underweight, while 3.94% are overweight.

**2.78 (a), (c)**

Calories	Frequency	Percentage	Percentage Less Than
50 but less than 100	3	12%	12%
100 but less than 150	3	12	24
150 but less than 200	9	36	60
200 but less than 250	6	24	84
250 but less than 300	3	12	96
300 but less than 350	0	0	96
350 but less than 400	1	4	100

Protein	Frequency	Percentage	Percentage Less Than
16 but less than 20	1	4%	4%
20 but less than 24	5	20	24
24 but less than 28	8	32	56
28 but less than 32	9	36	92
32 but less than 36	2	8	100

Calories from Fat	Frequency	Percentage	Percentage Less Than
0% but less than 10%	3	12%	12%
10% but less than 20%	4	16	28
20% but less than 30%	2	8	36
30% but less than 40%	5	20	56
40% but less than 50%	3	12	68
50% but less than 60%	5	20	88
60% but less than 70%	2	8	96
70% but less than 80%	1	4	100

Calories from Saturated Fat	Frequency	Percentage	Percentage Less Than
0% but less than 5%	6	24%	24%
5% but less than 10%	2	8	32
10% but less than 15%	5	20	52
15% but less than 20%	5	20	72
20% but less than 25%	5	20	92
25% but less than 30%	2	8	100

Cholesterol	Frequency	Percentage	Percentage Less Than
0 but less than 50	2	8%	8%
50 but less than 100	17	68	76
100 but less than 150	4	16	92
150 but less than 200	1	4	96
200 but less than 250	0	0	96
250 but less than 300	0	0	96
300 but less than 350	0	0	96
350 but less than 400	0	0	96
400 but less than 450	0	0	96
450 but less than 500	1	4	100

(d) The sampled fresh red meats, poultry, and fish vary from 98 to 397 calories per serving, with the highest concentration between 150 to 200 calories. One protein source, spareribs, with 397 calories, is more than 100 calories beyond the next highest caloric food. The protein content of the sampled foods varies from 16 to 33 grams, with 68% of the data values falling between 24 and 32 grams. Spareribs and fried liver are both very different from other foods sampled—the former on calories and the latter on cholesterol content.

2.80 (a), (c)

Average Ticket \$	Frequency	Percentage	Cumulative %
6 but less than 12	3	10.00%	10.00%
12 but less than 18	12	40.00	50.00
18 but less than 24	11	36.67	86.67
24 but less than 30	3	10.00	96.67
30 but less than 36	0	0.00	96.67
36 but less than 42	1	3.33	100.00

Fan Cost Index	Frequency	Percentage	Cumulative %
80 but less than 105	2	6.67%	6.67%
105 but less than 130	7	23.33	30.00
130 but less than 155	10	33.33	63.33
155 but less than 180	9	30.00	93.33
180 but less than 205	1	3.33	96.67
205 but less than 230	1	3.33	100.00

Regular Season Game Receipts (\$millions)	Frequency	Percentage	Cumulative %
5 but less than 20	5	16.67%	16.67%
20 but less than 35	7	23.33	40.00
35 but less than 50	5	16.67	56.67
50 but less than 65	6	20.00	76.67
65 but less than 80	5	16.67	93.33
80 but less than 95	1	3.33	96.67
95 but less than 110	1	3.33	100.00

Local TV, Radio, and Cable (\$millions)	Frequency	Percentage	Cumulative %
0 but less than 10	7	23.33%	23.33%
10 but less than 20	12	40.00	63.33
20 but less than 30	6	20.00	83.33
30 but less than 40	3	10.00	93.33
40 but less than 50	1	3.33	96.67
50 but less than 60	1	3.33	100.00

Other Local Operating Revenue (\$millions)	Frequency	Percentage	Cumulative %
0 but less than 10	6	20.00%	20.00%
10 but less than 20	3	10.00	30.00
20 but less than 30	8	26.67	56.67
30 but less than 40	8	26.67	83.33
40 but less than 50	3	10.00	93.33
50 but less than 60	1	3.33	96.67
60 but less than 70	1	3.33	100.00

Player Compensation and Benefits (\$millions)	Frequency	Percentage	Cumulative %
30 but less than 45	5	16.67%	16.67%
45 but less than 60	8	26.67	43.33
60 but less than 75	4	13.33	56.67
75 but less than 90	5	16.67	73.33
90 but less than 105	5	16.67	90.00
105 but less than 120	3	10.00	100.00

National and Other Local Expenses (\$millions)	Frequency	Percentage	Cumulative %
30 but less than 40	4	13.33%	13.33%
40 but less than 50	10	33.33	46.67
50 but less than 60	9	30.00	76.67
60 but less than 70	2	6.67	83.33
70 but less than 80	3	10.00	93.33
80 but less than 90	2	6.67	100.00

Income from Baseball Operations (\$millions)	Frequency	Percentage	Cumulative %
-60 but less than -45	2	6.67%	6.67%
-45 but less than -30	2	6.67	13.33
-30 but less than -15	8	26.67	40.00
-15 but less than 0	8	26.67	66.67
0 but less than 15	7	23.33	90.00
15 but less than 30	1	3.33	93.33
30 but less than 45	2	6.67	100.00

(d) There appears to be a weak positive linear relationship between number of wins and player compensation and benefits.

2.82 (c) Total fat seems to be most closely related to calories because the points in the scatter plot are closer to the imaginary line that passes through the data points.

2.84 (b) There is a downward trend in the amount filled. (c) The amount filled in the next bottle will most likely be below 1.894 liter.

(d) The scatter plot of the amount of soft drink filled against time reveals the trend of the data, while a histogram only provides information on the distribution of the data.

CHAPTER 3

3.2 (a) Mean = 7, median = 7, mode = 7. (b) Range = 9, interquartile range = 5,  $S^2 = 10.8$ ,  $S = 3.286$ ,  $CV = 46.943\%$ . (c) Z scores: 0, -0.913, 0.609, 0, -1.217, 1.521. None of the Z scores is larger than 3.0 or smaller than -3.0. There is no outlier. (d) Symmetric because mean = median.

3.4 (a) Mean = 2, median = 7, mode = 7. (b) Range = 17, interquartile range = 14.5,  $S^2 = 62$ ,  $S = 7.874$ ,  $CV = 393.7\%$ . (d) Left skewed because mean < median.

	Grade X	Grade Y
Mean	575	575.4
Median	575	575
Standard deviation	6.40	2.07

(b) If quality is measured by central tendency, Grade X tires provide slightly better quality because X's mean and median are both equal to the expected value, 575 mm. If, however, quality is measured by consistency, Grade Y provides better quality because, even though Y's mean is only slightly larger than the mean for Grade X, Y's standard deviation is much

smaller. The range in values for Grade  $Y$  is 5 mm compared to the range in values for Grade  $X$ , which is 16 mm.

	Grade $X$	Grade $Y$ , Altered
Mean	575	577.4
Median	575	575
Standard deviation	6.40	6.11

When the fifth  $Y$  tire measures 588 mm rather than 578 mm,  $Y$ 's mean inner diameter becomes 577.4 mm, which is larger than  $X$ 's mean inner diameter, and  $Y$ 's standard deviation increases from 2.07 mm to 6.11 mm. In this case,  $X$ 's tires are providing better quality in terms of the mean inner diameter, with only slightly more variation among the tires than  $Y$ 's.

**3.8** The article reports the median home price and not the mean home price because the median is a better measure of central tendency in the presence of some extremely expensive homes that drive the mean home price upward.

**3.10 (a)** Calories: mean = 380, median = 350, 1st quartile = 260, 3rd quartile = 510. Fat: mean = 15.79, median = 19, 1st quartile = 8, 3rd quartile = 22. **(b)** Calories: variance = 12,800, standard deviation = 113.14, range = 290, interquartile range = 250,  $CV = 29.77\%$ . None of the  $Z$  scores are less than  $-3$  or greater than 3. There is no outlier in calories. Fat: variance = 52.82, standard deviation = 7.27, range = 18.5, interquartile range = 14,  $CV = 46.04\%$ . None of the  $Z$  scores are less than  $-3$  or greater than 3. There is no outlier in fat. **(c)** Calories are slightly right-skewed, while fat is slightly left-skewed. **(d)** The mean calories are 380, while the middle ranked calories is 350. The average scatter of calories around the mean is 113.14. The middle 50% of the calories are scattered over 250, while the difference between the highest and the lowest calories is 290. The mean fat is 15.79 grams, while the middle-ranked fat is 19 grams. The average scatter of fat around the mean is 7.27 grams. 50% of the values are scattered over 14 grams, while the difference between the highest and the lowest fat is 18.5 grams.

**3.12 (a)** Mean = 226.67, median = 210, first quartile = 110, third quartile = 380. **(b)** Variance = 17756.06, standard deviation = 133.25, range = 425, interquartile range = 270,  $CV = 58.79\%$ . There is no outlier because none of the  $Z$  scores has an absolute value that is greater than 3.0. **(c)** The data appear to be skewed to the right because the mean is greater than the median.

**3.14 (a)** Mean = 46.8, median = 42.5, first quartile = 40, third quartile = 58. **(b)** Range = 34, variance = 123.29, standard deviation = 11.10. **(c)** The admission price for one-day tickets is slightly skewed to the right because the mean is slightly greater than the median. **(d)** **(a)** Mean = 50.8, median = 42.5, first quartile = 40, third quartile = 62. **(b)** Range = 69, variance = 382.84, standard deviation = 19.57. **(c)** The admission price for one-day tickets is skewed to the right because the mean is much greater than the median due to the much higher price of the first observation, at \$98.

**3.16 (a)** Mean = 7.11, median = 6.68,  $Q_1 = 5.64$ ,  $Q_3 = 8.73$ . **(b)** Variance = 4.336, standard deviation = 2.082, range = 6.67, interquartile range = 3.09,  $CV = 29.27\%$ . **(c)** Because the mean is greater than the median, the distribution is right-skewed. **(d)** The mean and median are both greater than 5 minutes. The distribution is right-skewed, meaning that

there are some unusually high values. Further, 13 of the 15 bank customers sampled (or 86.7%) had waiting times greater than 5 minutes. So the customer is likely to experience a waiting time in excess of 5 minutes. The manager overstated the bank's service record in responding that the customer would "almost certainly" not wait longer than 5 minutes for service.

**3.18 (a)**  $\bar{R}_G = ((1 + 3.614)(1 - 0.78))^{1/2} - 1 = 0.751\%$

**(b)**  $P_{2004} = \$1,000$ ;  $P_{2005} = 3.614(P_{2004}) + P_{2004} = \$4,614$ ;

$P_{2006} = -0.78(P_{2005}) + P_{2005} = \$1,015.08$

Its value at the end of 2005 will be \$1,015.08. **(c)** The geometric mean rate of return is 0.751%, which is lower than the 9.047% in Problem 3.17 (c), and, hence, the value of the stock at the end of 2005 is lower than that in Problem 3.17 (c).

<b>3.20 (a)</b>	Year	Platinum	Gold	Silver
	2005	12.3	17.8	29.5
	2004	5.7	4.6	14.2
	2003	36.0	19.9	27.8
	2002	24.6	25.6	3.3
	<b>Geometric mean</b>	19.09%	16.71%	18.21%

**(b)** Platinum has the highest rate of return over the four years, while gold has the lowest rate of return. **(c)** In general, investing in precious metals yielded much higher rate of returns than investing in the stock market over the four-year period.

**3.22 (a)** Population mean,  $\mu = 6$ . **(b)** Population standard deviation,  $\sigma = 1.673$ , population variance,  $\sigma^2 = 2.8$ .

**3.24 (a)** 68%. **(b)** 95%. **(c)** Not calculable, 75%, 88.89%. **(d)**  $\mu - 4\sigma$  to  $\mu + 4\sigma$  or  $-2.8$  to 19.2.

**3.26 (a)** Mean = 12,999.2158, variance = 14,959,700.52, standard deviation = 3,867.7772. **(b)** 64.71%, 98.04%, and 100% of these states have mean per capita energy consumption within 1, 2, and 3 standard deviation of the mean, respectively. **(c)** This is consistent with the 68%, 95%, and 99.7%, according to the empirical rule. **(d)** **(a)** Mean = 12,857.7402, variance = 14,238,110.67, standard deviation = 3,773.3421. **(b)** 66%, 98%, and 100% of these states have a mean per capita energy consumption within 1, 2, and 3 standard deviations of the mean, respectively. **(c)** This is consistent with the 68%, 95%, and 99.7% according to the empirical rule.

**3.28 (a)** 3, 4, 7, 9, 12. **(b)** The distances between the median and the extremes are close, 4 and 5, but the differences in the sizes of the whiskers are different (1 on the left and 3 on the right), so this distribution is slightly right-skewed. **(c)** In 3.2 (d), because mean = median, the distribution was said to be symmetric. The box part of the graph is symmetric, but the whiskers show right skewness.

**3.30 (a)**  $-8$ ,  $-6.5$ , 7, 8, 9. **(b)** The shape is left-skewed. **(c)** This is consistent with the answer in 3.4 (d).

**3.32 (a)** Five-number summary: 35 110 210 380 460. **(b)** The distribution is slightly skewed to the right.

**3.34 (a)** Five-number summary for calories: 240 260 350 510 530. Five-number summary for fat: 3.5 8 19 22 22.

**3.36 (a)** Commercial district five-number summary: 0.38 3.2 4.5 5.55 6.46. Residential area five-number summary: 3.82 5.64 6.68 8.73 10.49. **(b)** Commercial district: The distribution is skewed to the left. Residential area: The distribution is skewed slightly to the right. **(c)** The central tendency of the waiting times for the bank branch located in the commercial district of a city is lower than that of the branch located in the residential area. There are a few longer than normal waiting times for the branch located in the residential area whereas there are a few exceptionally short waiting times for the branch located in the commercial area.

**3.38 (a)** You can say that there is a strong positive linear relationship between the return on investment of U.S. stocks and the international large cap stocks, U.S. stocks, and emerging market stocks; a moderate positive linear relationship between U.S. stocks and international small cap stocks, U.S. stocks, and emerging market debt stocks; and a very weak positive linear relationship between U.S. stocks and international bonds. **(b)** In general, there is a positive linear relationship between the return on investment of U.S. stocks and international stocks, U.S. bonds and international bonds, U.S. stocks and emerging market debt, and there is a very weak negative linear relationship, if any, between the return on investment of U.S. bonds and international stocks.

**3.40 (a)**  $\text{cov}(X, Y) = 591.667$ . **(b)**  $r = 0.7196$ . **(c)** The correlation coefficient is more valuable for expressing the relationship between calories and fat because it does not depend on the units used to measure calories and fat. **(d)** There is a strong positive linear relationship between calories and fat.

**3.42 (a)**  $\text{cov}(X, Y) = 1.2132$  **(b)**  $S_X^2 = 0.1944$ ,  $S_Y^2 = 20.4054$

$$r = \frac{\text{cov}(X, Y)}{S_X S_Y} = \frac{1.2132}{(0.4409)(4.5172)} = 0.6092$$

**(c)** There is a moderate positive linear relationship between the coach's salary and revenue.

**3.56 (a)** Mean = 43.89, median = 45, 1st quartile = 18, 3rd quartile = 63. **(b)** Range = 76, interquartile range = 45, variance = 639.2564, standard deviation = 25.28,  $CV = 57.61\%$ . **(c)** The distribution is skewed to the right because there are a few policies that require an exceptionally long period to be approved. **(d)** The mean approval process takes 43.89 days, with 50% of the policies being approved in less than 45 days. 50% of the applications are approved in between 18 and 63 days. About 67% of the applications are approved in between 18.6 and 69.2 days.

**3.58 (a)** Mean = 8.421, median = 8.42, range = 0.186,  $S = 0.0461$ . The mean and median width are both 8.42 inches. The range of the widths is 0.186 inches, and the average scatter around the mean is 0.0461 inches. **(b)** 8.312, 8.404, 8.42, 8.459, 8.498. **(c)** Even though mean = median, the left whisker is slightly longer so the distribution is slightly left-skewed. **(d)** All the troughs in this sample meet the specifications.

**3.60 (a)** Office I: mean = 2.214, median = 1.54; Office II: mean = 2.011, median = 1.505; Office I:  $Q_1 = 0.93$ ,  $Q_3 = 3.93$ ; Office II:  $Q_1 = 0.6$ ,  $Q_3 = 3.75$ . **(b)** Office I: range = 5.80, Interquartile range = 3.00,  $S^2 = 2.952$ ,  $S = 1.718$ ,  $CV = 77.597\%$ ; Office II: range = 7.47, Interquartile range = 3.15,  $S^2 = 3.579$ ,  $S = 1.892$ ,  $CV = 94.04\%$ . **(c)** Yes, they are both right-skewed. **(d)** Office II has more variability in times to clear

problems, with a wider range and a larger standard deviation. Office II has a lower mean time to clear problems.

**3.62 (a), (b)**

Excel output:

	In-State Tuition/Fees	Out-of State Tuition/Fees
Mean	6841.815534	17167.71
Median	6340	16340
Mode	7062	14901
Standard Deviation	2212.221516	4274.009
Sample Variance	4893924.034	18267150
Kurtosis	4.295740452	-0.33509
Skewness	1.350377978	0.524429
Range	14466	20007
Minimum	3094	8965
Maximum	17560	28972
Sum	704707	1768274
Count	103	103
First Quartile	5378	13928
Third Quartile	8143	20134
Interquartile Range	2765	6206
Coefficient of Variation	32.33%	24.90%

**(c)** Both in-state and out-of-state tuition and fees are right skewed.

**(d)**  $r = \frac{\text{cov}(X, Y)}{S_X S_Y} = 0.4911$  **(e)** Both in-state and out-of-state tuition and fees are right-skewed due to the outliers in the right tails. There is a moderate positive linear relationship between in-state and out-of-state tuition and fees. Those schools with high in-state tuition and fees tend to also have high out-of-state tuition and fees.

**3.64 (a)** With promotion: mean = 20,748.93, standard deviation = 8,109.50; without promotion: mean = 13,935.70, standard deviation = 4,437.92. **(b)** With promotion: minimum = 10,470, 1st quartile = 14,905, median = 19,775, 3rd quartile = 24,456, maximum = 40,605. Without promotion: minimum = 9,555, 1st quartile = 11,779, median = 12,952, 3rd quartile = 14,367, maximum = 28,834. **(d)** The mean attendance is 6,813 more when there is a promotion than when there is not, and the variation in attendance when there is a promotion is larger than that when there is no promotion. Many factors can cause variation in the paid attendance. Some of them are weather condition, times, day of the game, and the visiting team.

**3.66 (a)** Boston: 0.04, 0.17, 0.23, 0.32, 0.98; Vermont: 0.02, 0.13, 0.20, 0.28, 0.83. **(b)** Both distributions are right-skewed. **(c)** Both sets of shingles did quite well in achieving a granule loss of 0.8 grams or less. The Boston shingles had only two data points greater than 0.8 grams. The next highest to these was 0.6 grams. These two data points can be considered outliers. Only 1.176% of the shingles failed the specification. In the Vermont shingles, only one data point was greater than 0.8 grams. The next highest was 0.58 grams. Thus, only 0.714% of the shingles failed to meet the specification.

3.68 (a), (b)

	Mean	Median	$Q_1$	$Q_3$	
Average Ticket \$	18.1333	17.83	15.20	20.84	
Fan Cost Index	144.5737	143.475	124.25	160.76	
Reg. Season \$	46.1367	47.55	30.20	62.10	
Local TV, Radio, Cable	19.0467	16.35	10.90	23.60	
Other Local Revenue	27.5933	29.05	13.90	37.00	
Player Comp.	71.3567	70.80	49.40	92.80	
National and local expenses	54.6467	50.50	46.90	58.50	
Income from baseball operations	-8.3733	-8.35	-18.50	1.90	
	Variance	Standard Deviation	Interquartile Range	CV	
Average Ticket \$	35.9797	5.9983	33.07	5.64	33.08%
Fan Cost Index	843.4552	29.0423	143.84	36.51	20.09%
Reg. Season \$	512.5445	22.6394	91.60	31.90	49.07%
Local TV, Radio, Cable	151.0184	12.2890	56.30	12.70	64.52%
Other Local Revenue	234.6186	15.3173	58.70	23.10	55.51%
Player Comp.	663.8405	25.7651	88.00	43.40	36.11%
National and local expenses	176.4081	13.2819	49.20	11.60	24.3%
Income from baseball operations	428.1531	20.6919	93.80	20.40	-247.12%

(c) Average ticket prices, local TV, radio and cable receipts, and national and other local expenses are skewed to the right; fan cost index is slightly skewed to the right; all other variables are approximately symmetrical. (d)  $r = 0.3985$ . There is a moderate positive linear relationship between the number of wins and player compensation and benefits.

3.70 (a) 0.80. (b) 0.53. (c) 0.92. (d) Total fat seems to be most closely related to calories because it has the highest correlation coefficient with calories.

3.72 (a) Mean = 7.5273, median = 7.263, first quartile = 6.353, third quartile = 8.248. (b) Range = 7.416, interquartile range = 1.895, variance = 2.6609, standard deviation = 1.6312, CV = 21.67%. (c) The data are skewed to the right. (d) The per capita spending by the 50 states is right-skewed because a few states spend a lot more than the rest.

CHAPTER 4

4.2 (a) Simple events include selecting a red ball. (b) Selecting a white ball.

4.4 (a)  $60/100 = 3/5 = 0.6$ . (b)  $10/100 = 1/10 = 0.1$ . (c)  $35/100 = 7/20 = 0.35$ . (d)  $9/10 = 0.9$ .

4.6 (a) Mutually exclusive, not collectively exhaustive. (b) Not mutually exclusive, not collectively exhaustive. (c) Mutually exclusive, not collectively exhaustive. (d) Mutually exclusive, collectively exhaustive.

4.8 (a) "Makes less than \$50,000." (b) "Makes less than \$50,000 and tax code is unfair." (c) The complement of "tax code is fair" is "tax code is

unfair." (d) "Tax code is fair and makes less than \$50,000" is a joint event because it consists of two characteristics or attributes.

4.10 (a) "A wafer is good." (b) "A wafer is good and no particle was found on the die." (c) "Bad wafer." (d) A wafer that is a "good wafer" and was produced by a die "with particles" is a joint event because it consists of two characteristics.

4.12 (a)  $P(\text{Selected the highest-cost fund}) = (27+18)/200 = 0.225$ . (b)  $P(\text{Selected the highest-cost fund and is an undergraduate}) = 27/200 = 0.135$ . (c)  $P(\text{Selected the highest-cost fund or is an undergraduate}) = (45+100-27)/200 = 0.59$ . (d) The probability of "selected the highest cost fund or is an undergraduate" includes the probability of "selected the highest cost fund," plus the probability of "undergraduate minus the joint probability of highest cost fund and undergraduate."

4.14 (a)  $360/500 = 18/25 = 0.72$ . (b)  $224/500 = 56/125 = 0.448$ . (c)  $396/500 = 99/125 = 0.792$ . (d)  $500/500 = 1.00$ .

4.16 (a)  $10/30 = 1/3 = 0.33$ . (b)  $20/60 = 1/3 = 0.33$ . (c)  $40/60 = 2/3 = 0.67$ . (d) Since  $P(A|B) = P(A) = 1/3$ , events  $A$  and  $B$  are statistically independent.

4.18  $\frac{1}{2} = 0.5$

4.20 Since  $P(A \text{ and } B) = 0.20$  and  $P(A)P(B) = 0.12$ , events  $A$  and  $B$  are not statistically independent.

4.22 (a)  $36/116 = 0.3103$ . (b)  $14/334 = 0.0419$ . (c)  $320/334 = 0.9581$ .  $P(\text{No particles}) = 400/450 = 0.8889$ . Since  $P(\text{No particles} | \text{Good}) \neq P(\text{No particles})$ , "a good wafer" and "a die with no particle" are not statistically independent.

4.24 (a)  $P(\text{Selected the highest cost fund} | \text{Is an undergraduate}) = 27/100 = 0.27$ . (b)  $P(\text{Is an undergraduate} | \text{Selected the highest cost fund}) = 27/(27+18) = 0.6$ . (c) The conditional events are reversed. (d) Since  $P(\text{Selected the highest-cost fund} | \text{Is an undergraduate}) = 0.27$  is not equal to  $P(\text{Selected the highest-cost fund}) = 0.225$ , the two events "student group" and "fund selected" are not statistically independent.

4.26 (a)  $0.025/0.6 = 0.0417$ . (b)  $0.015/0.4 = 0.0375$ . (c) Since  $P(\text{Needs warranty repair} | \text{Manufacturer based in U.S.}) = 0.0417$  and  $P(\text{Needs warranty repair}) = 0.04$ , the two events are not statistically independent.

4.28 (a) 0.0045. (b) 0.012. (c) 0.0059. (d) 0.0483.

4.30 0.095.

4.32 (a) 0.736. (b) 0.997.

4.34 (a) 0.4615. (b) 0.325.

4.36 (a)  $P(\text{Huge success} | \text{Favorable review}) = 0.099/0.459 = 0.2157$ ;  $P(\text{Moderate success} | \text{Favorable review}) = 0.14/0.459 = 0.3050$ ;  $P(\text{Break even} | \text{Favorable review}) = 0.16/0.459 = 0.3486$ ;  $P(\text{Loser} | \text{Favorable review}) = 0.06/0.459 = 0.1307$ . (b)  $P(\text{Favorable review}) = 0.459$ .

4.48 (a) 0.015. (b) 0.49. (c) 0.995. (d) 0.02. (e) 0.6667. (f) The conditions are switched. Part (d) answers  $P(A|B)$  and part (e) answers  $P(B|A)$ .

4.50 (a) A simple event can be "a firm that has a transactional public Web site" and a joint event can be "a firm that has a transactional public Web site and has sales greater than \$10 billion." (b) 0.3469. (c) 0.1449. (d) Since  $P(\text{Transactional public Web site}) \times P(\text{Sales in excess of } \$10 \text{ billion}) \neq P(\text{Transactional public Web site and sales in excess of } \$10 \text{ billion})$ , the two events "sales in excess of ten billion dollars" and "has a transactional public Web site" are not independent.



**4.52 (a)** 0.0225. **(b)** 3,937.5  $\approx$  3,938 can be expected to read the advertisement and place an order. **(c)** 0.03. **(d)** 5,250 can be expected to read the advertisement and place an order.

**4.54 (a)** 0.4712. **(b)** Since the probability that a fatality involved a rollover, given that the fatality involved an SUV, a van, or a pickup is 0.4712, which is almost twice the probability that a fatality involved a rollover with any vehicle type, at 0.24, SUVs, vans, and pickups are generally more prone to rollover accidents.

**CHAPTER 5**

**5.2 (a)** C:  $\mu = 2$ , D:  $\mu = 2$ . **(b)** C:  $\sigma = 1.414$ , D:  $\sigma = 1.095$ . **(c)** Distribution C is uniform and symmetric; Distribution D is symmetric and has a single mode.

**5.4 (a)**  $\mu = 2$ . **(b)**  $\sigma = 1.183$ .

<b>5.6 (a)</b>	$X$	$P(X)$
	\$ -1	21/36
	\$ +1	15/36

<b>(b)</b>	$X$	$P(X)$
	\$ -1	21/36
	\$ +1	15/36

<b>(c)</b>	$X$	$P(X)$
	\$ -1	30/36
	\$ +4	6/36

**(d)** \$-0.167 for each method of play.

**5.8 (a)** 90; 30. **(b)** 126.10, 10.95. **(c)** -1, 300. **(d)** 120.

**5.10 (a)** 9.5 minutes. **(b)** 1.9209 minutes.

**5.12**

$X^*P(X)$	$Y^*P(Y)$	$(X - \mu_X)^2 * P(X)$	$(Y - \mu_Y)^2 * P(Y)$	$(X - \mu_X)(Y - \mu_Y) * P(XY)$
-10	5	2,528.1	129.6	-572.4
0	45	1,044.3	5,548.8	-2,407.2
24	-6	132.3	346.8	-214.2
45	-30	2,484.3	3,898.8	-3,112.2

**(a)**  $E(X) = \mu_X = \sum_{i=1}^N X_i P(X_i) = 59$ ,  $E(Y) = \mu_Y = \sum_{i=1}^N Y_i P(Y_i) = 14$

**(b)**  $\sigma_X = \sqrt{\sum_{i=1}^N [X_i - E(X)]^2 P(X_i)} = 78.6702$

$\sigma_Y = \sqrt{\sum_{i=1}^N [Y_i - E(Y)]^2 P(Y_i)} = 99.62$

**(c)**  $\sigma_{XY} = \sum_{i=1}^N [X_i - E(X)][Y_i - E(Y)]P(X_i Y_i) = -6,306$

**(d)** Stock X gives the investor a lower standard deviation while yielding a higher expected return, so the investor should select stock X

**5.14 (a)** \$71; \$97. **(b)** 61.88, 84.27. **(c)** 5,113. **(d)** Risk-averse investors would invest in stock X, while risk takers would invest in stock Y.

**5.16 (a)**  $E(X) = \$77$ ,  $E(Y) = \$97$ . **(b)**  $\sigma_X = 39.76$ ,  $\sigma_Y = 108.95$ . **(c)**  $\sigma_{XY} = 41.61$ . **(d)** Common stock fund gives the investor a higher expected return than corporate bond fund but also has a standard deviation more than 2.5 times higher than that for corporate bond fund. An investor should carefully weigh the increased risk.

**5.18 (a)** 0.5997. **(b)** 0.0016. **(c)** 0.0439. **(d)** 0.4018.

<b>5.20</b>	Mean	Standard Deviation
<b>(a)</b>	0.40	0.600
<b>(b)</b>	1.60	0.980
<b>(c)</b>	4.00	0.894
<b>(d)</b>	1.50	0.866

**5.22 (a)** 0.0778. **(b)** 0.6826. **(c)** 0.0870. **(d)(a)**  $P(X = 5) = 0.3277$ . **(b)**  $P(X \geq 3) = 0.9421$ . **(c)**  $P(X < 2) = 0.0067$ .

**5.24** Given  $p = 0.90$  and  $n = 3$ ,

**(a)**  $P(X = 3) = \frac{n!}{X!(n-x)!} p^X (1-p)^{n-X} = \frac{3!}{3!0!} (0.9)^3 (0.1)^0 = 0.729$ .

**(b)**  $P(X = 0) = \frac{n!}{X!(n-x)!} p^X (1-p)^{n-X} = \frac{3!}{0!3!} (0.9)^0 (0.1)^3 = 0.001$ .

**(c)**  $P(X \geq 2) = P(X = 2) + P(X = 3) = \frac{3!}{2!1!} (0.9)^2 (0.1)^1 + \frac{3!}{3!0!} (0.9)^3 (0.1)^0 = 0.972$

**(d)**  $E(X) = np = 3(0.9) = 2.7$ ,  $\sigma_X = \sqrt{np(1-p)} = \sqrt{3(0.9)(0.1)} = 0.5196$ .

**5.26 (a)**  $P(X = 0) = 0.0563$ . **(b)**  $P(X = 5) = 0.0584$ . **(c)**  $P(X \leq 5) = 0.9803$ . **(d)**  $P(X \geq 6) = 0.0197$ . **(e)** You could infer that Californians' view of the economy is very likely to be different from the United States as a whole because the probability of seeing all 6 in the random sample of 10 having a positive outlook is only 1.622%, which is not a very likely event, if their view is the same as the United States as a whole.

**5.28 (a)**  $P(X \leq 5) = .00009919$ . **(b)**  $P(X \leq 10) = 0.0719$ . **(c)**  $P(X \leq 15) = 0.8173$ .

**5.30 (a)** 0.2565. **(b)** 0.1396. **(c)** 0.3033. **(d)** 0.0247.

**5.32 (a)** 0.0337. **(b)** 0.0067. **(c)** 0.9596. **(d)** 0.0404.

**5.34 (a)**

$P(X < 5) = P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4)$   
 $= \frac{e^{-6}(6)^0}{0!} + \frac{e^{-6}(6)^1}{1!} + \frac{e^{-6}(6)^2}{2!} + \frac{e^{-6}(6)^3}{3!} + \frac{e^{-6}(6)^4}{4!}$   
 $= 0.002479 + 0.014873 + 0.044618 + 0.089235 + 0.133853$   
 $= 0.2851$ .

**(b)**  $P(X = 5) = \frac{e^{-6}(6)^5}{5!} = 0.1606$ .

**(c)**  $P(X \geq 5) = 1 - P(X < 5) = 1 - 0.2851 = 0.7149$ .

**(d)**  $P(X = 4 \text{ or } X = 5) = P(X = 4) + P(X = 5) = \frac{e^{-6}(6)^4}{4!} + \frac{e^{-6}(6)^5}{5!} = 0.2945$ .

**5.36**  $\lambda = 4.06$ . (a)  $P(X=0) = 0.0172$ . (b)  $P(X \geq 1) = 0.9828$ .  
(c)  $P(X \geq 2) = 0.9127$ .

**5.38** (a) 0.0176. (b) 0.9093. (c) 0.9220.

**5.40** (a)  $\lambda = 1.53$ ,  $P(X=0) = 0.2165$ . (b)  $P(X \leq 2) = 0.8013$ . (c) Because Kia had a higher mean rate of problems per car, the probability of a randomly selected Kia having no more than two problems is lower than that with a randomly chosen Lexus. Likewise, the probability of a randomly selected Kia having zero problems is lower than that with a randomly chosen Lexus.

**5.42** (a)  $\lambda = 1.40$ ,  $P(X=0) = 0.2466$ . (b)  $P(X \leq 2) = 0.8335$ . (c) Because Kia had a lower mean rate of problems per car in 2005 compared to 2004, the probability of a randomly selected Kia having zero problems and the probability of no more than two problems are both higher than their values in 2004.

**5.44** (a) 0.238. (b) 0.2. (c) 0.1591. (d) 0.0083.

**5.46** (a) If  $n = 6$ ,  $A = 25$ , and  $N = 100$ ,

$$\begin{aligned} P(X \geq 2) &= 1 - [P(X=0) + P(X=1)] \\ &= 1 - \left[ \frac{\binom{25}{0} \binom{100-25}{6-0}}{\binom{100}{6}} + \frac{\binom{25}{1} \binom{100-25}{6-1}}{\binom{100}{6}} \right] \\ &= 1 - [0.1689 + 0.3620] = 0.4691 \end{aligned}$$

(b) If  $n = 6$ ,  $A = 30$ , and  $N = 100$ ,

$$\begin{aligned} P(X \geq 2) &= 1 - [P(X=0) + P(X=1)] \\ &= 1 - \left[ \frac{\binom{30}{0} \binom{100-30}{6-0}}{\binom{100}{6}} + \frac{\binom{30}{1} \binom{100-30}{6-1}}{\binom{100}{6}} \right] \\ &= 1 - [0.1100 + 0.3046] = 0.5854 \end{aligned}$$

(c) If  $n = 6$ ,  $A = 5$ , and  $N = 100$ ,

$$\begin{aligned} P(X \geq 2) &= 1 - [P(X=0) + P(X=1)] \\ &= 1 - \left[ \frac{\binom{5}{0} \binom{100-5}{6-0}}{\binom{100}{6}} + \frac{\binom{5}{1} \binom{100-5}{6-1}}{\binom{100}{6}} \right] \\ &= 1 - [0.7291 + 0.2430] = 0.0279 \end{aligned}$$

(d) If  $n = 6$ ,  $A = 10$ , and  $N = 100$ ,

$$\begin{aligned} P(X \geq 2) &= 1 - [P(X=0) + P(X=1)] \\ &= 1 - \left[ \frac{\binom{10}{0} \binom{100-10}{6-0}}{\binom{100}{6}} + \frac{\binom{10}{1} \binom{100-10}{6-1}}{\binom{100}{6}} \right] \\ &= 1 - [0.5223 + 0.3687] = 0.1090 \end{aligned}$$

(e) The probability that the entire group will be audited is very sensitive to the true number of improper returns in the population. If the true number is very low ( $A = 5$ ), the probability is very low (0.0279). When the true number is increased by a factor of 6 ( $A = 30$ ), the probability the group will be audited increases by a factor of almost 21 (0.5854).

**5.48** (a)  $P(X=4) = 0.00003649$ . (b)  $P(X=0) = 0.5455$ .  
(c)  $P(X \geq 1) = 0.4545$ . (d)  $n = 6$ . (a)  $P(X=4) = 0.0005$ .  
(b)  $P(X=0) = 0.3877$ . (c)  $P(X \geq 1) = 0.6123$ .

**5.50** (a)  $P(X=1) = 0.4835$ . (b)  $P(X \geq 1) = 0.6374$ . (c)  $P(X \leq 2) = 0.9912$ .  
(d)  $\mu = n \times (A/N) = 0.8$ .

**5.56** (a) 0.74. (b) 0.74. (c) 0.3898. (d) 0.0012. (e) The assumption of independence may not be true.

**5.58** (a) If  $p = 0.50$  and  $n = 11$ ,  $P(X \geq 9) = 0.0327$ . (b) If  $p = 0.74$  and  $n = 11$ ,  $P(X=9) = 0.2474$  and  $P(X \geq 9) = 0.4247$ .

**5.60** (a) 0.018228. (b) 0.089782. (c) 0.89199. (d) Mean = 3.3, standard deviation = 1.486943.

**5.62** (a) 0.0000. (b) 0.04924. (c) 0.909646. (d) 0.49578.

**5.64** (a) 0.0003. (b) 0.2289. (c) 0.4696. (d) 0.5304. (e) 0.469581.  
(f) 4.4, so about 4 people on average will refuse to participate.

**5.66** (a)  $\mu = np = 2.6$ . (b)  $\sigma = \sqrt{np(1-p)} = 1.5040$ .  
(c)  $P(X=0) = 0.0617$ . (d)  $P(X \leq 2) = 0.5080$ . (e)  $P(X \geq 3) = 0.4920$ .

**5.68** (a) If  $p = 0.50$  and  $n = 35$ ,  $P(X \geq 30) = 0.0000112$ .  
(b) If  $p = 0.70$  and  $n = 35$ ,  $P(X \geq 30) = 0.0269$ . (c) If  $p = 0.90$  and  $n = 35$ ,  $P(X \geq 30) = 0.8684$ . (d) Based on the results in (a)–(c), the probability that the Standard & Poor's 500 index will increase if there is an early gain in the first five trading days of the year is very likely to be close to 0.90 because that yields a probability of 86.84% that at least 30 of the 35 years the Standard & Poor's 500 index will increase the entire year.

**5.70** (a) The assumptions needed are (i) the probability that a golfer loses a golf ball in a given interval is constant, (ii) the probability that a golfer loses more than one golf ball approaches 0 as the interval gets smaller, (iii) the probability that a golfer loses a golf ball is independent from interval to interval. (b) 0.0111. (c) 0.70293. (d) 0.29707.

**5.72** (a) Virtually zero. (b) 0.00000037737. (c) 0.00000173886.  
(d) 0.000168669. (e) 0.0011998. (f) 0.00407937. (g) 0.006598978.  
(h) 0.0113502. (i) 0.976601.

## CHAPTER 6

**6.2** (a) 0.9089. (b) 0.0911. (c) +1.96. (d) -1.00 and +1.00.

**6.4** (a) 0.1401. (b) 0.4168. (c) 0.3918. (d) +1.00.

**6.6** (a) 0.9599. (b) 0.0228. (c) 43.42. (d) 46.64 and 53.36.

**6.8** (a)  $P(34 < X < 50) = P(-1.33 < Z < 0) = 0.4082$ . (b)  $P(X < 30) + P(X > 60) = P(Z < -1.67) + P(Z > 0.83) = 0.0475 + (1.0 - 0.7967) = 0.2508$ .

(c)  $P(Z < -0.84) \cong 0.20$ ,  $Z = -0.84 = \frac{X - 50}{12}$ .  $X = 50 - 0.84(12) = 39.92$  thousand miles, or 39,920 miles.

(d) The smaller standard deviation makes the  $Z$  values larger.

(a)  $P(34 < X < 50) = P(-1.60 < Z < 0) = 0.4452$ .

(b)  $P(X < 30) + P(X > 60) = P(Z < -2.00) + P(Z > 1.00) = 0.0228 + (1.0 - 0.8413) = 0.1815$ .

(c)  $X = 50 - 0.84(10) = 41.6$  thousand miles, or 41,600 miles.

**6.10 (a)** 0.9878. **(b)** 0.8185. **(c)** 86.16%. **(d)** Option 1: Since your score of 81% on this exam represents a  $Z$  score of 1.00, which is below the minimum  $Z$  score of 1.28, you will not earn an A grade on the exam under this grading option. Option 2: Since your score of 68% on this exam represents a  $Z$  score of 2.00, which is well above the minimum  $Z$  score of 1.28, you will earn an A grade on the exam under this grading option. You should prefer Option 2.

**6.12 (a)**  $P(X < 1.5) = P(Z < -2) = 0.0228$ . **(b)**  $P(X < 1.7) = P(Z < -0.6667) = 0.2525$ . **(c)**  $P(X > 2.0) = P(Z > 1.3333) = 0.0912$ . **(d)**  $P(X > 2.3) = P(Z > 3.3333) = 0.0004$ .

**6.14** With 39 values, the smallest of the standard normal quantile values covers an area under the normal curve of 0.025. The corresponding  $Z$  value is  $-1.96$ . The middle (20th) value has a cumulative area of 0.50 and a corresponding  $Z$  value of 0.0. The largest of the standard normal quantile values covers an area under the normal curve of 0.975, and its corresponding  $Z$  value is  $+1.96$ .

**6.16 (a)** Mean = 23.2, median = 23.5, range = 52, standard deviation = 12.3868,  $6(S_X) = 6(12.3868) = 74.3205$ , interquartile range = 14,  $1.33(S_X) = 1.33(12.3868) = 16.4744$ . The mean is almost equal to the median; the range is smaller than 6 times the standard deviation, and the interquartile range is slightly smaller than 1.33 times the standard deviation. The data appear to be approximately normally distributed but slightly skewed to the right. **(b)** The normal probability plot suggests that the data are skewed to the right.

**6.18 (a) Plant A:**  $\bar{X} = 9.382$   $S = 3.998$   
**Five-number summary:** 4.42 7.29 8.515 11.42 21.62

The distribution is right-skewed since the mean is greater than the median.

**Plant B:**  $\bar{X} = 11.354$   $S = 5.126$   
**Five-number summary:** 2.33 6.25 11.96 14.25 25.75

Although the results are inconsistent due to an extreme value in the sample, since the mean is less than the median, we can say that the data for Plant B is left-skewed. **(b)** The normal probability plot for Plant A is right-skewed. Except for the extreme value, the normal probability plot for Plant B is left-skewed.

**6.20 (a)** Interquartile range = 0.0025;  $S_X = 0.0017$ ; range = 0.008;  $1.33(S_X) = 0.0023$ ;  $6(S_X) = 0.0102$ . Since the interquartile range is close to  $1.33(S_X)$  and the range is also close to  $6(S_X)$ , the data appear to be approximately normally distributed. **(b)** The normal probability plot suggests that the data appear to be approximately normally distributed.

**6.22 (a)** Five-number summary: 82 127 148.5 168 213; mean = 147.06; mode = 130; range = 131; interquartile range = 41; standard deviation = 31.69. The mean is very close to the median. The five-number summary suggests that the distribution is approximately symmetrical around the median. The interquartile range is very close to 1.33 times the standard deviation. The range is about \$50 below 6 times the standard deviation. In general, the distribution of the data appears to closely resemble a normal distribution. **(b)** The normal probability plot confirms that the data appear to be approximately normally distributed.

**6.24 (a)** 0.1667. **(b)** 0.1667. **(c)** 0.7083. **(d)** Mean = 60, standard deviation = 34.641.

**6.26 (a)**  $P(5:55 \text{ a.m.} < X < 7:38 \text{ p.m.}) = P(355 < X < 1,178) = (1178 - 355)/(1,440) = 0.5715$ . **(b)**  $P(10 \text{ p.m.} < X < 5 \text{ a.m.}) = P(1,320 < X < 1,440) + P(0 < X < 300) = (1,440 - 1,320)/1,440 + (300)/1,440 = 0.2917$ . **(c)** Let  $X$  be duration between the occurrence of a failure and its detection,  $a = 0$ ,  $b = 60$ ,  $P(0 < X < 10) = 10/60 = 0.1667$ . **(d)**  $P(40 < X < 60) = (60 - 40)/60 = 0.3333$ .

**6.28 (a)** 0.6321. **(b)** 0.3679. **(c)** 0.2326. **(d)** 0.7674.

**6.30 (a)** Approximately 1.0. **(b)** 0.0003. **(c)** 0.00029. **(d)** 0.99971.

**6.32 (a)** 0.864665. **(b)** 0.99996. **(c)** 0.6321, 0.9933.

**6.34 (a)** 0.6321. **(b)** 0.3935. **(c)** 0.0952.

**6.36 (a)** 0.8647. **(b)** 0.3297. **(c)** 0.9765, 0.5276.

**6.46 (a)** 0.4772. **(b)** 0.9544. **(c)** 0.0456. **(d)** 1.8835. **(e)** 1.8710 and 2.1290.

**6.48 (a)** 0.2734. **(b)** 0.2038. **(c)** 4.404 ounces. **(d)** 4.188 ounces and 5.212 ounces.

**6.50 (a)** Waiting time will more closely resemble an exponential distribution. **(b)** Seating time will more closely resemble a normal distribution. **(c)** Both the histogram and normal probability plot suggest that waiting time more closely resembles an exponential distribution. **(d)** Both the histogram and normal probability plot suggest that seating time more closely resembles a normal distribution.

**6.52 (a)** 0.8413. **(b)** 0.9330. **(c)** 0.9332. **(d)** 0.3347. **(e)** 0.4080 and 1.1920.

## CHAPTER 7

**7.2** Sample without replacement: Read from left to right in three-digit sequences and continue unfinished sequences from the end of the row to the beginning of the next row:

**Row 05:** 338 505 855 551 438 855 077 186 579 488 767 833 170

**Rows 05–06:** 897

**Row 06:** 340 033 648 847 204 334 639 193 639 411 095 924

**Rows 06–07:** 707

**Row 07:** 054 329 776 100 871 007 255 980 646 886 823 920 461

**Row 08:** 893 829 380 900 796 959 453 410 181 277 660 908 887

**Rows 08–09:** 237

**Row 09:** 818 721 426 714 050 785 223 801 670 353 362 449

**Rows 09–10:** 406

*Note:* All sequences above 902 and duplicates are discarded.

**7.4** A simple random sample would be less practical for personal interviews because of travel costs (unless interviewees are paid to go to a central interviewing location).

**7.6** Here all members of the population are equally likely to be selected, and the sample selection mechanism is based on chance. But selection of two elements is not independent; for example, if  $A$  is in the sample, we know that  $B$  is also and that  $C$  and  $D$  are not.

**7.8 (a)**

**Row 16:** 2323 6737 5131 8888 1718 0654 6832 4647 6510 4877

**Row 17:** 4579 4269 2615 1308 2455 7830 5550 5852 5514 7182

**Row 18:** 0989 3205 0514 2256 8514 4642 7567 8896 2977 8822

**Row 19:** 5438 2745 9891 4991 4523 6847 9276 8646 1628 3554

**Row 20:** 9475 0899 2337 0892 0048 8033 6945 9826 9403 6858

**Row 21:** 7029 7341 3553 1403 3340 4205 0823 4144 1048 2949

**Row 22:** 8515 7479 5432 9792 6575 5760 0408 8112 2507 3742

**Row 23:** 1110 0023 4012 8607 4697 9664 4894 3928 7072 5815

**Row 24:** 3687 1507 7530 5925 7143 1738 1688 5625 8533 5041

**Row 25:** 2391 3483 5763 3081 6090 5169 0546

*Note:* All sequences above 5,000 are discarded. There were no repeating sequences.

(b)

189 189 289 389 489 589 689 789 889 989  
 1089 1189 1289 1389 1489 1589 1689 1789 1889 1989  
 2089 2189 2289 2389 2489 2589 2689 2789 2889 2989  
 3089 3189 3289 3389 3489 3589 3689 3789 3889 3989  
 4089 4189 4289 4389 4489 4589 4689 4789 4889 4989

(c) With the single exception of invoice #0989, the invoices selected in the simple random sample are not the same as those selected in the systematic sample. It would be highly unlikely that a simple random sample would select the same units as a systematic sample.

7.10 Before accepting the results of a survey of college students, you might want to know, for example:

Who funded the survey? Why was it conducted?

What was the population from which the sample was selected?

What sampling design was used?

What mode of response was used: a personal interview, a telephone interview, or a mail survey? Were interviewers trained? Were survey questions field-tested?

What questions were asked? Were they clear, accurate, unbiased, and valid?

What operational definition of “vast majority” was used?

What was the response rate?

What was the sample size?

7.12 (a) The four types of survey errors are: coverage error, nonresponse error, sampling error, and measurement error. (b) When people who answer the survey tell you what they think you want to hear, rather than what they really believe, it introduces the halo effect, which is a source of measurement error. Also, every survey will have sampling error that reflects the chance differences from sample to sample, based on the probability of particular individuals being selected in the particular sample.

7.14 Before accepting the results of the survey, you might want to know, for example:

Who funded the study? Why was it conducted?

What was the population from which the sample was selected?

What sampling design was used?

What mode of response was used: a personal interview, a telephone interview, or a mail survey? Were interviewers trained? Were survey questions field-tested?

What other questions were asked? Were they clear, accurate, unbiased, and valid?

What was the response rate?

What was the margin of error?

What was the sample size?

What was the frame being used?

7.16 Before accepting the results of the survey, you might want to know, for example:

Who funded the study? Why was it conducted?

What was the population from which the sample was selected?

What was the frame being used?

What sampling design was used?

What mode of response was used: a personal interview, a telephone interview, or a mail survey? Were interviewers trained? Were survey questions field-tested?

What other questions were asked? Were they clear, accurate, unbiased, and valid?

What was the response rate?

What was the margin of error?

What was the sample size?

7.18 (a) Virtually zero. (b) 0.1587. (c) 0.0139. (d) 50.195.

7.20 (a) Both means are equal to 6. This property is called unbiasedness.

(c) The distribution for  $n = 3$  has less variability. The larger sample size has resulted in sample means being closer to  $\mu$ .

7.22 (a) When  $n = 2$ , the shape of the sampling distribution of  $\bar{X}$  should closely resemble the shape of the distribution of the population from which the sample is selected. Since the mean is larger than the median, the distribution of the sales price of new houses is skewed to the right, and so is the sampling distribution of  $\bar{X}$ . (b) When  $n = 100$ , the sampling distribution of  $\bar{X}$  should be very close to a normal distribution due to the Central Limit Theorem. (c) When  $n = 100$ , the sample mean should be close to the population mean.  $P(\bar{X} < 250,000) = P(Z < (250,000 - 279,100)/(90,000/\sqrt{100})) = P(Z < -3.2333) = 0.0062$ .

7.24 (a)  $P(\bar{X} > 3) = P(Z > -1.00) = 1.0 - 0.1587 = 0.8413$ .

(b)  $P(Z < 1.04) = 0.85$   $\bar{X} = 3.10 + 1.04(0.1) = 3.204$ .

(c) To be able to use the standardized normal distribution as an approximation for the area under the curve, you must assume that the population is approximately symmetrical. (d)  $P(Z < 1.04) = 0.85$   $\bar{X} = 3.10 + 1.04(0.05) = 3.152$ .

7.26 (a) 0.9969. (b) 0.0142. (c) 2.3830 and 2.6170. (d) 2.6170.

Note: These answers are computed using Microsoft Excel. They may be slightly different when Table E.2 is used.

7.28 (a) 0.30. (b) 0.0693.

7.30 (a)  $\pi = 0.501$ ,  $\sigma_p = \sqrt{\frac{\pi(1-\pi)}{n}} = \sqrt{\frac{0.501(1-0.501)}{100}} = 0.05$

$P(p > 0.55) = P(Z > 0.98) = 1.0 - 0.8365 = 0.1635$

(b)  $\pi = 0.60$ ,  $\sigma_p = \sqrt{\frac{\pi(1-\pi)}{n}} = \sqrt{\frac{0.6(1-0.6)}{100}} = 0.04899$

$P(p > 0.55) = P(Z > -1.021) = 1.0 - 0.1539 = 0.8461$

(c)  $\pi = 0.49$ ,  $\sigma_p = \sqrt{\frac{\pi(1-\pi)}{n}} = \sqrt{\frac{0.49(1-0.49)}{100}} = 0.05$

$P(p > 0.55) = P(Z > 1.20) = 1.0 - 0.8849 = 0.1151$ .

(d) Increasing the sample size by a factor of 4 decreases the standard error by a factor of 2.

(a)  $P(p > 0.55) = P(Z > 1.96) = 1.0 - 0.9750 = 0.0250$ .

(b)  $P(p > 0.55) = P(Z > -2.04) = 1.0 - 0.0207 = 0.9793$ .

(c)  $P(p > 0.55) = P(Z > 2.40) = 1.0 - 0.9918 = 0.0082$ .

7.32 (a) 0.50. (b) 0.5717. (c) 0.9523. (d) (a) 0.50. (b) 0.4246. (c) 0.8386.

7.34 (a) Since  $n = 200$ , which is quite large, we use the sample proportion to approximate the population proportion and, hence,  $\pi = 0.50$ .

$$\mu_p = \pi = 0.5, \sigma_p = \sqrt{\frac{\pi(1-\pi)}{n}} = \sqrt{\frac{0.5(0.5)}{200}} = .0354$$

$P(0.45 < p < 0.55) = P(-1.4142 < Z < 1.4142) = 0.8427$ .

(b)  $P(A < p < B) = P(-1.6449 < Z < 1.6449) = 0.90$ .  $A = 0.50 - 1.6449(0.0354) = 0.4418$ .  $B = 0.50 + 1.6449(0.0354) = 0.5582$ .

The probability is 90% that the sample percentage will be contained within 5.8% symmetrically around the population percentage.

(c)  $P(A < p < B) = P(-1.96 < Z < +1.96) = 0.95$ .  $A = 0.50 - 1.96(0.0354) = 0.4307$ .  $B = 0.50 + 1.96(0.0354) = 0.5694$ .

The probability is 95% that the sample percentage will be contained within 6.94% symmetrically around the population percentage.

7.36 (a) 0.6314. (b) 0.0041. (c)  $P(p > .35) = P(Z > 1.3223) = 0.0930$ .

If the population proportion is 29%, the proportion of the samples with 35% or more who do not intend to work for pay at all is 9.3%, an unlikely occurrence. Hence, the population estimate of 29% is likely to be an underestimation. (d) When the sample size is smaller in (c) compared to (b), the standard error of the sampling distribution of the sample proportion is larger.

7.38 (a) 0.3626. (b) 0.9816. (c) 0.0092. *Note:* These answers are computed using Microsoft Excel. They may be slightly different when Table E.2 is used.

7.50 (a) 0.4999. (b) 0.00009. (c) 0. (d) 0. (e) 0.7518.

7.52 (a) 0.8944. (b) 4.617, 4.783. (c) 4.641.

7.54 (a)  $\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{20}{\sqrt{10}} = 6.3246$   $P(\bar{X} < 0) = P(Z < 2.514) = 0.00597$ .

(b)  $P(0 < \bar{X} < 6) = P(-2.514 < Z < -1.565) = 0.0528$

(c)  $P(\bar{X} > 10) = P(Z > -0.9329) = 0.8246$ .

7.56 Even though Internet polling is less expensive and faster and offers higher response rates than telephone surveys, it may lead to more coverage error since a greater proportion of the population may have telephones than have Internet access. It may also lead to nonresponse bias since a certain class and/or age group of people may not use the Internet or may use the Internet less frequently than others. Due to these errors, the data collected are not appropriate for making inferences about the general population.

7.58 (a) With a response rate of only 15.5%, nonresponse error should be the major cause of concern in this study. Measurement error is a possibility also. (b) The researchers should follow up with the nonrespondents. (c) The step mentioned in (b) could have been followed to increase the response rate to the survey, thus increasing its worthiness.

7.60 (a) What was the comparison group of "other workers"? Were they another sample? Where did they come from? Were they truly comparable? What was the sampling scheme? What was the population from which the sample was selected? How was salary measured? What was the mode of response? What was the response rate? (b) Various answers are possible.

## CHAPTER 8

8.2  $114.68 \leq \mu \leq 135.32$

8.4 In order to have 100% certainty, the entire population would have to be sampled.

8.6 Yes, it is true since 5% of intervals will not include the true mean.

8.8 (a)  $\bar{X} \pm Z \frac{\sigma}{\sqrt{n}} = 350 \pm 1.96 \frac{100}{\sqrt{64}}$ ;  $325.50 \leq \mu \leq 374.50$ .

(b) No. The manufacturer cannot support a claim that the bulbs have a mean of 400 hours. Based on the data from the sample, a mean of 400 hours would represent a distance of 4 standard deviations above the sample mean of 350 hours. (c) No. Since  $\sigma$  is known and  $n = 64$ , from the

Central Limit Theorem, you know that the sampling distribution of  $\bar{X}$  is approximately normal. (d) The confidence interval is narrower, based on a population standard deviation of 80 hours rather than the original standard deviation of 100 hours.

(a)  $\bar{X} \pm Z \frac{\sigma}{\sqrt{n}} = 350 \pm 1.96 \frac{80}{\sqrt{64}}$ ,  $330.4 \leq \mu \leq 369.6$ . (b) Based

on the smaller standard deviation, a mean of 400 hours would represent a distance of 5 standard deviations above the sample mean of 350 hours.

No, the manufacturer cannot support a claim that the bulbs have a mean life of 400 hours.

8.10 (a) 2.2622. (b) 3.2498. (c) 2.0395. (d) 1.9977. (e) 1.7531.

8.12  $38.95 \leq \mu \leq 61.05$ .

8.14  $-0.12 \leq \mu \leq 11.84$ ,  $2.00 \leq \mu \leq 6.00$ . The presence of the outlier increases the sample mean and greatly inflates the sample standard deviation.

8.16 (a)  $29.44 \leq \mu \leq 34.56$ . (b) The quality improvement team can be 95% confident that the population mean turnaround time is between 29.44 hours and 34.56 hours. (c) The project was a success because the initial turnaround time of 68 hours does not fall into the interval.

8.18 (a)  $\$21.01 \leq \mu \leq \$24.99$ . (b) You can be 95% confident that the population mean bounced-check fee is between \$21.01 and \$24.99.

8.20 (a)  $31.12 \leq \mu \leq 54.96$ . (b) The number of days is approximately normally distributed. (c) Yes, the outliers skew the data. (d) Since the sample size is fairly large, at  $n = 50$ , the use of the  $t$  distribution is appropriate.

8.22 (a)  $142.00 \leq \mu \leq 311.33$ . (b) The population distribution needs to be normally distributed. (c) Both the normal probability plot and the box-and-whisker plot show that the distribution for battery life is approximately normally distributed.

8.24  $0.19 \leq \pi \leq 0.31$ .

8.26 (a)  $p = \frac{X}{n} = \frac{135}{500} = 0.27$

$p \pm Z \sqrt{\frac{p(1-p)}{n}} = 0.27 \pm 2.58 \sqrt{\frac{0.27(0.73)}{500}}$   $0.2189 \leq \pi \leq 0.3211$ .

(b) The manager in charge of promotional programs concerning residential customers can infer that the proportion of households that would purchase an additional telephone line if it were made available at a substantially reduced installation cost is somewhere between 0.22 and 0.32, with 99% confidence.

8.28 (a)  $0.2311 \leq \pi \leq 0.3089$ . (b)  $0.2425 \leq \pi \leq 0.2975$ . (c) The larger the sample size, the narrower is the confidence interval, holding everything else constant.

8.30 (a)  $0.5638 \leq \pi \leq 0.6362$ . (b)  $0.2272 \leq \pi \leq 0.2920$ .

8.32 (a)  $0.3783 \leq \pi \leq 0.4427$ . (b) You can be 95% confident that the population proportion of all workers whose primary reason for staying on their job is interesting job responsibilities is somewhere between 0.3783 and 0.4427.

8.34  $n = 35$ .

8.36  $n = 1,041$ .

8.38 (a)  $n = \frac{Z^2 \sigma^2}{e^2} = \frac{(1.96^2)(400^2)}{50^2} = 245.86$

Use  $n = 246$ .

(b)  $n = \frac{Z^2 \sigma^2}{e^2} = \frac{(1.96^2)(400^2)}{25^2} = 983.41$

Use  $n = 984$ .

**8.40**  $n = 97$ .

**8.42 (a)**  $n = 167$ . **(b)**  $n = 97$ .

**8.44 (a)**  $n = 246$ . **(b)**  $n = 385$ . **(c)**  $n = 554$ .

**8.46 (a)**  $n = 2,156$ . **(b)**  $n = 2,239$ . **(c)** The sample size is larger in (b) than in (a) because the estimate of the true proportion is closer to 0.5 in (b) than in (a). **(d)** If you were to design the follow-up study, you would use one sample and ask the respondents both questions rather than selecting two separate samples because it costs more to select two samples than one.

**8.48 (a)**  $p = 0.5498$ ;  $0.5226 \leq \pi \leq 0.5770$ . **(b)**  $p = 0.4697$ ;  $0.4424 \leq \pi \leq 0.4970$ . **(c)**  $p = 0.2799$ ;  $0.2554 \leq \pi \leq 0.3045$  **(d)** **(a)**  $n = 2,378$  **(b)**  $n = 2,393$  **(c)**  $n = 1,936$

**8.50**  $\$10,721.53 \leq \text{Total} \leq \$14,978.47$ .

**8.52 (a)** 0.054. **(b)** 0.0586. **(c)** 0.066.

**8.54**  $\$543,176.96 \leq \text{Total} \leq \$1,025,224.04$ .

**8.56**  $\$5,443 \leq \text{Total difference} \leq \$54,229$ .

**8.58 (a)** 0.0542. **(b)** Since the upper bound is higher than the tolerable exception rate of 0.04, the auditor should request a larger sample.

**8.66**  $940.50 \leq \mu \leq 1007.50$ . Based on the evidence gathered from the sample of 34 stores, the 95% confidence interval for the mean per-store count in all of the franchise's stores is from 940.50 to 1,007.50. With a 95% level of confidence, the franchise can conclude that the mean per-store count in all its stores is somewhere between 940.50 and 1,007.50, which is larger than the original average of 900 mean per-store count before the price reduction. Hence, reducing coffee prices is a good strategy to increase the mean customer count.

**8.68 (a)**  $p = 0.80$ ,  $0.7765 \leq \pi \leq 0.8235$ . **(b)**  $p = 0.88$ ,  $0.8609 \leq \pi \leq 0.8991$ . **(c)**  $p = 0.56$ ,  $0.5309 \leq \pi \leq 0.5891$ . **(d)** **(a)**  $n = 1,537$ . **(b)**  $n = 1,015$ . **(c)**  $n = 2,366$ .

**8.70 (a)**  $14.085 \leq \mu \leq 16.515$ . **(b)**  $0.530 \leq \pi \leq 0.820$ . **(c)**  $n = 25$ .

**(d)**  $n = 784$ . **(e)** If a single sample were to be selected for both purposes, the larger of the two sample sizes ( $n = 784$ ) should be used.

**8.72 (a)**  $8.049 \leq \mu \leq 11.351$ . **(b)**  $0.284 \leq \pi \leq 0.676$ . **(c)**  $n = 35$ .

**(d)**  $n = 121$ . **(e)** If a single sample were to be selected for both purposes, the larger of the two sample sizes ( $n = 121$ ) should be used.

**8.74 (a)**  $\$25.80 \leq \mu \leq \$31.24$ . **(b)**  $0.3037 \leq \pi \leq 0.4963$ . **(c)**  $n = 97$ .

**(d)**  $n = 423$ . **(e)** If a single sample were to be selected for both purposes, the larger of the two sample sizes ( $n = 423$ ) should be used.

**8.76 (a)**  $\$36.66 \leq \mu \leq \$40.42$ . **(b)**  $0.2027 \leq \pi \leq 0.3973$ . **(c)**  $n = 110$ .

**(d)**  $n = 423$ . **(e)** If a single sample were to be selected for both purposes, the larger of the two sample sizes ( $n = 423$ ) should be used.

**8.78 (a)**  $\pi \leq 0.2013$ . **(b)** Since the upper bound is higher than the tolerable exception rate of 0.15, the auditor should request a larger sample.

**8.80 (a)**  $n = 27$ . **(b)**  $\$402,652.53 \leq \text{Population total} \leq \$450,950.79$ .

**8.82 (a)**  $8.41 \leq \mu \leq 8.43$ . **(b)** With 95% confidence, the population mean width of troughs is somewhere between 8.41 and 8.43 inches.

**8.84 (a)**  $0.2425 \leq \mu \leq 0.2856$ . **(b)**  $0.1975 \leq \mu \leq 0.2385$ . **(c)** The amounts of granule loss for both brands are skewed to the right. **(d)** Since the two confidence intervals do not overlap, you can conclude that the mean granule loss of Boston shingles is higher than that of Vermont shingles.

## CHAPTER 9

**9.2**  $H_1$  denotes the alternative hypothesis.

**9.4**  $\beta$ .

**9.6**  $\alpha$  is the probability of making a Type I error.

**9.8** The power of the test is  $1 - \beta$ .

**9.10** It is possible to not reject a null hypothesis when it is false since it is possible for a sample mean to fall in the nonrejection region even if the null hypothesis is false.

**9.12** All else being equal, the closer the population mean is to the hypothesized mean, the larger  $\beta$  will be.

**9.14**  $H_0$ : Defendant is guilty;  $H_1$ : Defendant is innocent. A Type I error would be not convicting a guilty person. A Type II error would be convicting an innocent person.

**9.16**  $H_0$ :  $\mu = 20$  minutes. 20 minutes is adequate travel time between classes.

$H_1$ :  $\mu \neq 20$  minutes. 20 minutes is not adequate travel time between classes.

**9.18**  $H_0$ :  $\mu = 1.00$ . The mean amount of paint per one-gallon can is one gallon.

$H_1$ :  $\mu \neq 1.00$ . The mean amount of paint per one-gallon can differs from one gallon.

**9.20** Since  $Z = +2.21 > 1.96$ , reject  $H_0$ .

**9.22** Reject  $H_0$  if  $Z < -2.58$  or if  $Z > 2.58$ .

**9.24**  $p$ -value = 0.0456.

**9.26**  $p$ -value = 0.1676.

**9.28 (a)**  $H_0$ :  $\mu = 70$  pounds;  $H_1$ :  $\mu \neq 70$  pounds.

Decision rule: Reject  $H_0$  if  $Z < -1.96$  or  $Z > +1.96$ .

Test statistic:  $Z = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}} = \frac{69.1 - 70}{3.5 / \sqrt{49}} = -1.80$

Decision: Since  $-1.96 < Z = -1.80 < 1.96$ , do not reject  $H_0$ . There is insufficient evidence to conclude that the cloth has a mean breaking strength that differs from 70 pounds. **(b)**  $p$ -value =  $2(0.0359) = 0.0718$ . Interpretation: The probability of getting a sample of 49 pieces that yield a mean strength that is farther away from the hypothesized population mean than this sample is 0.0718, or 7.18%.

**(c)** Decision rule: Reject  $H_0$  if  $Z < -1.96$  or  $Z > +1.96$ .

Test statistic:  $Z = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}} = \frac{69.1 - 70}{1.75 / \sqrt{49}} = -3.60$ .

Decision: Since  $Z = -3.60 < -1.96$ , reject  $H_0$ . There is enough evidence to conclude that the cloth has a mean breaking strength that differs from 70 pounds. **(d)** Decision rule: Reject  $H_0$  if  $Z < -1.96$  or

$Z > +1.96$ . Test statistic:  $Z = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}} = \frac{69 - 70}{3.5 / \sqrt{49}} = -2.00$ .

Decision: Since  $Z = -2.00 < -1.96$ , reject  $H_0$ . There is enough evidence to conclude that the cloth has a mean breaking strength that differs from 70 pounds.

**9.30 (a)** Since  $Z = -2.00 < -1.96$ , reject  $H_0$ . **(b)**  $p$ -value = 0.0456. **(c)**  $325.5 \leq \mu \leq 374.5$ . **(d)** The conclusions are the same.

**9.32 (a)** Since  $-1.96 < Z = -0.80 < 1.96$ , do not reject  $H_0$ . **(b)**  $p$ -value = 0.4238. **(c)** Since  $Z = -2.40 < -1.96$ , reject  $H_0$ . **(d)** Since  $Z = -2.26 < -1.96$ , reject  $H_0$ .

**9.34**  $Z = +2.33$ .

**9.36**  $Z = -2.33$ .

**9.38**  $p$ -value = 0.0228.

**9.40**  $p$ -value = 0.0838.

**9.42**  $p$ -value = 0.9162.

**9.44 (a)** Since  $Z = -1.75 < -1.645$ , reject  $H_0$ . **(b)**  $p$ -value = 0.0401 < 0.05, reject  $H_0$ . **(c)** The probability of getting a sample mean of 2.73 feet or less if the population mean is 2.8 feet is 0.0401. **(d)** They are the same.

**9.46 (a)**  $H_0: \mu \leq 5$ ;  $H_1: \mu > 5$ . **(b)** A Type I error occurs when you conclude that children take a mean of more than five trips a week to the store when in fact they take a mean of no more than five trips a week to the store. A Type II error occurs when you conclude that children take a mean of no more than five trips a week to the store when in fact they take a mean of more than five trips a week to the store. **(c)** Since  $Z_{\text{calc}} = 2.9375 > 2.3263$  or the  $p$ -value of 0.0017 is less than 0.01, reject  $H_0$ . There is enough evidence to conclude the population mean number of trips to the store is greater than five per week. **(d)** The probability that the sample mean is 5.47 trips or more when the null hypothesis is true is 0.0017.

**9.48**  $t = 2.00$ .

**9.50 (a)**  $t = \pm 2.1315$ . **(b)**  $t = +1.7531$ .

**9.52** No, you should not use a  $t$  test since the original population is left-skewed and the sample size is not large enough for the  $t$  test to be valid.

**9.54 (a)** Since  $t = -2.4324 < -1.9842$ , reject  $H_0$ . There is enough evidence to conclude that the population mean has changed from 41.4 days. **(b)** Since  $t = -2.4324 > -2.6264$ , do not reject  $H_0$ . There is not enough evidence to conclude that the population mean has changed from 41.4 days.

**(c)** Since  $t = -2.9907 < -1.9842$ , reject  $H_0$ . There is enough evidence to conclude that the population mean has changed from 41.4 days.

**9.56** Since  $t = -1.30 > -1.6694$  and the  $p$ -value of 0.0992 > 0.05, do not reject  $H_0$ . There is not enough evidence to conclude that the mean waiting time is less than 3.7 minutes.

**9.58** Since  $t = 0.8556 < 2.5706$ , do not reject  $H_0$ . There is not enough evidence to conclude that the mean price for two tickets, with online service charges, large popcorn, and two medium soft drinks, is different from \$35. **(b)** The  $p$ -value is 0.4313. If the population mean is \$35, the probability of observing a sample of six theater chains that will result in a sample mean farther away from the hypothesized value than this sample is 0.4313. **(c)** That the distribution of prices is normally distributed. **(d)** With a small sample size, it is difficult to evaluate the assumption of normality. However, the distribution may be symmetric since the mean and the median are close in value.

**9.60 (a)** Since  $-2.0096 < t = 0.114 < 2.0096$ , do not reject  $H_0$ . **(b)**  $p$ -value = 0.9095. **(c)** Yes, the data appear to have met the normality assumption. **(d)** The amount of fill is decreasing over time. Therefore, the  $t$  test is invalid.

**9.62 (a)** Since  $t = -5.9355 < -2.0106$ , reject  $H_0$ . There is enough evidence to conclude that mean widths of the troughs is different from 8.46 inches.

**(b)** That the population distribution is normal. **(c)** Although the distribution of the widths is left-skewed, the large sample size means that the validity of the  $t$  test is not seriously affected.

**9.64 (a)** Since  $-2.68 < t = 0.094 < 2.68$ , do not reject  $H_0$ .

**(b)**  $5.462 \leq \mu \leq 5.542$ . **(c)** The conclusions are the same.

**9.66**  $p = 0.22$ .

**9.68** Do not reject  $H_0$ .

**9.70 (a)**  $H_0: \pi \leq 0.5$   $H_1: \pi > 0.5$  Decision rule: If  $Z > 1.6449$ , reject  $H_0$ .

$$\text{Test statistic: } Z = \frac{p - \pi}{\sqrt{\frac{\pi(1 - \pi)}{n}}} = \frac{0.5496 - 0.5}{\sqrt{\frac{0.5(1 - 0.5)}{464}}} = 2.1355$$

Decision: Since  $Z = 2.1355 > 1.6449$ , reject  $H_0$ . There is enough evidence to show that the majority of Cincinnati-area adults were planning on modifying or canceling their summer travel plans because of high gas prices.

**(b)**  $H_0: \pi \leq 0.5$   $H_1: \pi > 0.5$  Decision rule: If  $p$ -value < 0.05, reject  $H_0$ .  $p$ -value = 0.0164 Decision: Since  $p$ -value = 0.0164 < 0.05, reject  $H_0$ . There is enough evidence to show that the majority of Cincinnati-area adults were planning on modifying or canceling their summer travel plans because of high gas prices. **(c)** The conclusions from (a) and (b) are the same as they should be.

**9.72 (a)** Since  $-1.96 < Z = 0.6381 < 1.96$ , do not reject  $H_0$  and conclude that there is not enough evidence to show that the percentage of people who trust energy-efficiency ratings differs from 50%. **(b)**  $p$ -value = 0.5234. Since the  $p$ -value of 0.5234 > 0.05, do not reject  $H_0$ .

**9.74 (a)**  $p = 0.7112$ . **(b)** Since  $Z = 5.7771 > 1.6449$ , reject  $H_0$ . There is enough evidence to conclude that more than half of all successful women executives have children. **(c)** Since  $Z_{\text{calc}} = 1.2927 < 1.6449$ , do not reject  $H_0$ . There is not enough evidence to conclude that more than two-thirds of all successful women executives have children. **(d)** The random sample assumption is not likely to be valid because the criteria used in defining "successful women executives" is very likely to be quite different than those used in defining the "most powerful women in business" who attended the summit.

**9.84 (a)** Buying a site that is not profitable. **(b)** Not buying a profitable site. **(c)** Type I. **(d)** If the executives adopt a less stringent rejection criterion by buying sites for which the computer model predicts moderate or large profit, the probability of committing a Type I error will increase. Many more of the sites the computer model predicts that will generate moderate profit may end up not being profitable at all. On the other hand, the less stringent rejection criterion will lower the probability of committing a Type II error since more potentially profitable sites will be purchased.

**9.86 (a)** Since  $t = 3.248 > 2.0010$ , reject  $H_0$ . **(b)**  $p$ -value = 0.0019. **(c)** Since  $Z = -0.32 > -1.645$ , do not reject  $H_0$ . **(d)** Since  $-2.0010 < t = 0.75 < 2.0010$ , do not reject  $H_0$ . **(e)** Since  $t = -1.61 > -1.645$ , do not reject  $H_0$ .

**9.88 (a)** Since  $t = -1.69 > -1.7613$ , do not reject  $H_0$ . **(b)** The data are from a population that is normally distributed.

**9.90 (a)** Since  $t = -1.47 > -1.6896$ , do not reject  $H_0$ . **(b)**  $p$ -value = 0.0748. **(c)** Since  $t = -3.10 < -1.6973$ , reject  $H_0$ . **(d)**  $p$ -value = 0.0021. **(e)** The data in the population are assumed to be normally distributed.

**9.92 (a)**  $t = -21.61$ , reject  $H_0$ . **(b)**  $p$ -value = 0.0000. **(c)**  $t = -27.19$ , reject  $H_0$ . **(d)**  $p$ -value = 0.0000.

## CHAPTER 10

10.2 (a) Yes. (b) No. (c) No. (d) Yes.

$$10.4 (b) \quad b_1 = \frac{SSXY}{SSX} = \frac{2,775}{375} = 7.4$$

$$b_0 = \bar{Y} - b_1 \bar{X} = 237.5 - 7.4(12.5) = 145$$

For each increase in shelf space of an additional foot, weekly sales are estimated to increase by \$7.40.

(c)  $\hat{Y} = 145 + 7.4X = 145 + 7.4(8) = 204.2$ , or \$204.20.

10.6 (b)  $b_0 = -2.37$ ,  $b_1 = 0.0501$ . (c) For every cubic foot increase in the amount moved, mean labor hours are estimated to increase by 0.0501. (d) 22.67 labor hours.

10.8 (b)  $b_0 = -368.2846$ ,  $b_1 = 4.7306$ . (c) For each additional million-dollar increase in revenue, the mean annual value will increase by an estimated \$4.7306 million. Literal interpretation of  $b_0$  is not meaningful because an operating franchise cannot have zero revenue. (d) \$341.3027 million.

10.10 (b)  $b_0 = 6.048$ ,  $b_1 = 2.019$ . (c) For every one Rockwell E unit increase in hardness, the mean tensile strength is estimated to increase by 2,019 psi. (d) 66.62 or 66,620 psi.

10.12  $r^2 = 0.90$ . 90% of the variation in the dependent variable can be explained by the variation in the independent variable.

10.14  $r^2 = 0.75$ . 75% of the variation in the dependent variable can be explained by the variation in the independent variable.

10.16 (a)  $r^2 = \frac{SSR}{SST} = \frac{20,535}{30,025} = 0.684$ . 68.4% of the variation in sales can be explained by the variation in shelf space.

$$(b) \quad S_{YX} = \sqrt{\frac{SSE}{n-2}} = \sqrt{\frac{\sum_{i=1}^n (Y_i - \hat{Y}_i)^2}{n-2}} = \sqrt{\frac{9490}{10}} = 30.8058.$$

(c) Based on (a) and (b), the model should be very useful for predicting sales.

10.18 (a)  $r^2 = 0.8892$ . 88.92% of the variation in labor hours can be explained by the variation in cubic feet moved. (b)  $S_{YX} = 5.0314$ .

(c) Based on (a) and (b), the model should be very useful for predicting the labor hours.

10.20 (a)  $r^2 = 0.9334$ . 93.34% of the variation in value of a baseball franchise can be explained by the variation in its annual revenue.

(b)  $S_{YX} = 42.4335$ . (c) Based on (a) and (b), the model should be very useful for predicting the value of a baseball franchise.

10.22 (a)  $r^2 = 0.4613$ . 46.13% of the variation in the tensile strength can be explained by the variation in the hardness. (b)  $S_{YX} = 9.0616$ . (c) Based on (a) and (b), the model is only marginally useful for predicting tensile strength.

10.24 A residual analysis of the data indicates a pattern, with sizable clusters of consecutive residuals that are either all positive or all negative. This pattern indicates a violation of the assumption of linearity. A quadratic model should be investigated.

10.26 (a) There does not appear to be a pattern in the residual plot. (b) The assumptions of regression do not appear to be seriously violated.

10.28 (a) Based on the residual plot, there appears to be a nonlinear pattern in the residuals. A quadratic model should be investigated. (b) The assumptions of normality and equal variance do not appear to be seriously violated.

10.30 (a) Based on the residual plot, there appears to be a nonlinear pattern in the residuals. A quadratic model should be investigated.

(b) There is some right-skewness in the residuals and some violation of the equal-variance assumption.

10.32 (a) An increasing linear relationship exists. (b) There is evidence of a strong positive autocorrelation among the residuals.

10.34 (a) No, because the data were not collected over time. (b) If a single store had been selected, then studied over a period of time, you would compute the Durbin-Watson statistic.

$$10.36 (a) \quad b_1 = \frac{SSXY}{SSX} = \frac{201,399.05}{12,495,626} = 0.0161.$$

$b_0 = \bar{Y} - b_1 \bar{X} = 71.2621 - 0.0161(4393) = 0.458$ . (b)  $\hat{Y} = 0.458 + 0.0161X = 0.458 + 0.0161(4500) = 72.908$ , or \$72,908. (c) There is no evidence of a pattern in the residuals over time.

$$(d) \quad D = \frac{\sum_{i=2}^n (e_i - e_{i-1})^2}{\sum_{i=1}^n e_i^2} = \frac{1,243.2244}{599.0683} = 2.08 > 1.45. \text{ There is no}$$

evidence of positive autocorrelation among the residuals. (e) Based on a residual analysis, the model appears to be adequate.

10.38 (a)  $b_0 = -2.535$ ,  $b_1 = .06073$ . (b) \$2,505.40. (d)  $D = 1.64 > d_U = 1.42$ , so there is no evidence of positive autocorrelation among the residuals.

(e) The plot shows some nonlinear pattern, suggesting that a nonlinear model might be better. Otherwise, the model appears to be adequate.

10.40 (a) 3.00. (b)  $t_{16} = \pm 2.1199$ . (c) Reject  $H_0$ . There is evidence that the fitted linear regression model is useful. (d)  $1.32 \leq \beta_1 \leq 7.68$ .

10.42 (a)  $t = \frac{b_1 - \beta_1}{S_{b_1}} = \frac{7.4}{1.59} = 4.65 > t_{10} = 2.2281$  with 10 degrees of freedom for  $\alpha = 0.05$ . Reject  $H_0$ . There is evidence that the fitted linear regression model is useful. (b)  $b_1 \pm t_{n-2} S_{b_1} = 7.4 \pm 2.2281(1.59)$   
 $3.86 \leq \beta_1 \leq 10.94$

10.44 (a)  $t = 16.52 > 2.0322$ ; reject  $H_0$ . (b)  $0.0439 \leq \beta_1 \leq 0.0562$ .

10.46 (a) Since the  $p$ -value is approximately zero, reject  $H_0$  at the 5% level of significance. There is evidence of a linear relationship between annual revenue and franchise value. (b)  $3.7888 \leq \beta_1 \leq 4.5906$ .

10.48 (a) The  $p$ -value is virtually  $0 < 0.05$ ; reject  $H_0$ . (b)  $1.246 \leq \beta_1 \leq 2.792$ .

10.50 (b) If the S&P gains 30% in a year, the UOPIX is expected to gain an estimated 60%. (c) If the S&P loses 35% in a year, the UOPIX is expected to lose an estimated 70%.

10.52 (a)  $r = 0.8935$ . There appears to be a strong positive linear relationship between the mileage as calculated by owners and by current government standards. (b)  $t = 5.2639 > 2.3646$ ,  $p$ -value =  $0.0012 < 0.05$ . Reject  $H_0$ . At the 0.05 level of significance, there is a significant linear relationship between the mileage as calculated by owners and by current government standards.

10.54 (a)  $r = 0.5497$ . There appears to be a moderate positive linear relationship between the average Wonderlic score of football players trying out for the NFL and the graduation rate for football players at selected schools. (b)  $t = 3.9485$ ,  $p$ -value =  $0.0004 < 0.05$ . Reject  $H_0$ . At the 0.05 level of significance, there is a significant linear relationship between the average Wonderlic score of football players trying out for the NFL and the graduation rate for football players at selected schools.



(c) There is a significant linear relationship between the average Wonderlic score of football players trying out for the NFL and the graduation rate for football players at selected schools, but the positive linear relationship is only moderate.

$$10.56 \text{ (a) } 15.95 \leq \mu_{Y|X=4} \leq 18.05. \text{ (b) } 14.651 \leq Y_{X=4} \leq 19.349.$$

$$10.58 \text{ (a) } \hat{Y} \pm t_{n-2} S_{YX} \sqrt{h_i} \\ = 204.2 \pm 2.2281 (30.81) \sqrt{0.1373} \\ 178.76 \leq \mu_{Y|X=8} \leq 229.64$$

$$\text{(b) } \hat{Y} \pm t_{n-2} S_{YX} \sqrt{1 + h_i} \\ = 204.2 \pm 2.2281 (30.81) \sqrt{1 + 0.1373} \\ 131.00 \leq Y_{X=8} \leq 277.40$$

(c) Part (b) provides a prediction interval for the individual response given a specific value of the independent variable, and part (a) provides an interval estimate for the mean value, given a specific value of the independent variable. Since there is much more variation in predicting an individual value than in estimating a mean value, a prediction interval is wider than a confidence interval estimate.

$$10.60 \text{ (a) } 20.799 \leq \mu_{Y|X=500} \leq 24.542. \text{ (b) } 12.276 \leq Y_{X=500} \leq 33.065.$$

$$10.62 \text{ (a) } 367.0757 \leq \mu_{Y|X=150} \leq 397.3254. \text{ (b) } 311.3562 \leq Y_{X=150} \leq 453.0448.$$

10.74 (a)  $b_0 = 24.84$ ,  $b_1 = 0.14$ . (b) For each additional case, the predicted mean delivery time is estimated to increase by 0.14 minutes. (c) 45.84. (d) No, 500 is outside the relevant range of the data used to fit the regression equation. (e)  $r^2 = 0.972$ . (f) There is no obvious pattern in the residuals, so the assumptions of regression are met. The model appears to be adequate. (g)  $t = 24.88 > 2.1009$ ; reject  $H_0$ . (h)  $44.88 \leq \mu_{Y|X=150} \leq 46.80$ . (i)  $41.56 \leq Y_{X=150} \leq 50.12$ . (j)  $0.128 \leq \beta_1 \leq 0.152$ .

10.76 (a)  $b_0 = -122.3439$ ,  $b_1 = 1.7817$ . (b) For each additional thousand dollars in assessed value, the estimated mean selling price of a house increases by \$1.7817 thousand. The estimated mean selling price of a house with a 0 assessed value is \$-122.3439 thousand. However, this interpretation is not meaningful in the current setting since the assessed value is very unlikely to be 0 for a house. (c)  $\hat{Y} = -122.3439 + 1.78171X = -122.3439 + 1.78171(170) = 180.5475$  thousand dollars. (d)  $r^2 = 0.9256$ . So 92.56% of the variation in selling price can be explained by the variation in assessed value. (e) Neither the residual plot nor the normal probability plot reveal any potential violation of the linearity, equal variance and normality assumptions. (f)  $t = 18.6648 > 2.0484$ ,  $p$ -value is virtually zero. Since  $p$ -value  $< 0.05$ , reject  $H_0$ . There is evidence of a linear relationship between selling price and assessed value. (g)  $178.7066 \leq \mu_{Y|X=170} \leq 182.3884$ . (h)  $173.1953 \leq Y_{X=170} \leq 187.8998$ . (i)  $1.5862 \leq \beta_1 \leq 1.9773$ .

10.78 (a)  $b_0 = 0.30$ ,  $b_1 = 0.00487$ . (b) For each additional point on the GMAT score, the predicted mean GPI is estimated to increase by 0.00487. (c) 3.2225. (d)  $r^2 = 0.798$ . (e) There is no obvious pattern in the residuals, so the assumptions of regression are met. The model appears to be adequate. (f)  $t = 8.43 > 2.1009$ ; reject  $H_0$ . (g)  $3.144 \leq \mu_{Y|X=600} \leq 3.301$ . (h)  $2.886 \leq Y_{X=600} \leq 3.559$ . (i)  $.00366 \leq \beta_1 \leq .00608$ .

10.80 (a) There is no clear relationship shown on the scatterplot.

(c) Looking at all 23 flights, when the temperature is lower, there is likely to be some O-ring damage, particularly if the temperature is below 60 degrees. (d) 31 degrees is outside the relevant range, so a prediction should not be made. (e) Predicted  $Y = 18.036 - 0.240X$ , where  $X$  = temperature and  $Y$  = O-ring damage. (g) A nonlinear model would be more appropriate. (h) The appearance on the residual plot of a nonlinear pattern indicates a nonlinear model would be better.

10.82 (a)  $b_0 = 14.6816$ ,  $b_1 = 0.1135$ . (b) For each additional percentage increase in graduation rate, the estimated mean average Wonderlic score increases by 0.1135. The estimated mean average Wonderlic score is 14.6816 for a school that has a 0% graduation rate. However, this interpretation is not meaningful in the current setting since graduation rate is very unlikely to be 0% for any school. (c)  $\hat{Y} = 14.6816 + 0.11347X = 14.6816 + 0.11347(50) = 20.4$ . (d)  $r^2 = 0.3022$ . So 30.22% of the variation in average Wonderlic score can be explained by the variation in graduation rate. (e) Neither the residual plot nor the normal probability plot reveal any potential violation of the linearity, equal variance, and normality assumptions. (f)  $t = 3.9485 > 2.0281$ ,  $p$ -value = 0.0004. Since  $p$ -value  $< 0.05$ , reject  $H_0$ . There is evidence of a linear relationship between the average Wonderlic score for football players trying out for the NFL from a school and the graduation rate. (g)  $19.6 \leq \mu_{Y|X=50} \leq 21.1$ . (h)  $15.9 \leq Y_{X=50} \leq 24.8$ . (i)  $0.0552 \leq \beta_1 \leq 0.1718$ .

10.84 (a)  $b_0 = -2629.222$ ,  $b_1 = 82.472$ . (b) For each additional centimeter in circumference, the mean weight is estimated to increase by 82.472 grams. (c) 2,319.08 grams. (e)  $r^2 = 0.937$ . (f) There appears to be a nonlinear relationship between circumference and weight. (g)  $p$ -value is virtually 0  $< 0.05$ ; reject  $H_0$ . (h)  $72.7875 \leq \beta_1 \leq 92.156$ . (i)  $2186.959 \leq \mu_{Y|X=60} \leq 2451.202$ . (j)  $1726.551 \leq Y_{X=60} \leq 2911.610$ .

10.86 (b)  $\hat{Y} = 931,626.16 + 21,782.76X$  (c)  $b_1 = 21,782.76$  means that as the median age of the customer base increases by one year, the latest one-month mean sales total is estimated to increase by \$21,782.76. (d)  $r^2 = 0.0017$ . Only 0.17% of the total variation in the franchise's latest one-month sales total can be explained by using the median age of customer base. (e) The residuals are very evenly spread out across different range of median age. (f) Since  $-2.4926 < t = 0.2482 < 2.4926$ , do not reject  $H_0$ . There is not enough evidence to conclude that there is a linear relationship between the one-month sales total and the median age of the customer base. (g)  $-156,181.50 \leq \beta_1 \leq 199,747.02$ .

10.88 (a) There is a positive linear relationship between total sales and the percentage of customer base with a college diploma. (b)  $\hat{Y} = 789,847.38 + 35,854.15X$  (c)  $b_1 = 35,854.15$  means that for each increase of one percent of the customer base having received a college diploma, the latest one-month mean sales total is estimated to increase by \$35,854.15. (d)  $r^2 = 0.1036$ . So 10.36% of the total variation in the franchise's latest one-month sales total can be explained by the percentage of the customer base with a college diploma. (e) The residuals are quite evenly spread out around zero. (f) Since  $t = 2.0392 > 2.0281$ , reject  $H_0$ . There is enough evidence to conclude that there is a linear relationship between one-month sales total and percentage of customer base with a college diploma. (g)  $b_1 \pm t_{n-2} S_{b_1} = 35,854.15 \pm 2.0281(17,582.269) 195.75 \leq \beta_1 \leq 71,512.60$ .

10.90 (a)  $b_0 = -13.6561$ ,  $b_1 = 0.8923$ . (b) For each additional unit increase in summated rating, the mean price per person is estimated to increase by \$0.89. Since no restaurant will receive a summated rating of 0, it is inappropriate to interpret the  $Y$  intercept. (c) \$31.01. (d)  $r^2 = 0.4246$ .

(e) There is no obvious pattern in the residuals so the assumptions of regression are met. The model appears to be adequate. (f) The  $p$ -value is virtually  $0 < 0.05$ ; reject  $H_0$ . (g)  $\$29.07 \leq \mu_{Y|X=50} \leq \$32.94$ . (h)  $\$16.95 \leq Y_{X=50} \leq \$45.06$ . (i)  $0.6848 \leq \beta_1 \leq 1.1017$ .

**10.92 (a)** Correlation of Microsoft and Ford is  $-0.07176$ , Microsoft and GM is  $-0.39235$ , Microsoft and IAL is  $0.06686$ , Ford and GM is  $0.860418$ , Ford and IAL is  $-0.91585$ , and GM and IAL is  $-0.83584$ . (b) There is a strong negative linear relationship between the stock price of Ford Motor Company and International Aluminum, a strong negative linear relationship between the stock price of General Motors and International Aluminum, a strong positive linear relationship between Ford Motor Company and General Motors, Inc., a moderately weak negative linear relationship between the stock price of General Motors and Microsoft, Inc., and almost no linear relationship between the stock price of Microsoft and Ford Motor Company and between Microsoft and International Aluminum. (c) It is not a good idea to have all the stocks in an individual's portfolio be strongly positively correlated among each other because the portfolio risk can be reduced if some of the stocks are negatively correlated.

**CHAPTER 11**

**11.2 (a)** For each one-unit increase in  $X_1$ , you estimate that  $Y$  will decrease 2 units, holding  $X_2$  constant. For each one-unit increase in  $X_2$ , you estimate that  $Y$  will increase 7 units, holding  $X_1$  constant. (b) The  $Y$ -intercept equal to 50 estimates the predicted value of  $Y$  when both  $X_1$  and  $X_2$  are zero.

**11.4 (a)**  $\hat{Y} = -2.72825 + 0.047114X_1 + 0.011947X_2$ . (b) For a given number of orders, for each increase of \$1,000 in sales, mean distribution cost is estimated to increase by \$47.114. For a given amount of sales, for each increase of one order, mean distribution cost is estimated to increase by \$11.95. (c) The interpretation of  $b_0$  has no practical meaning here because it would represent the estimated distribution cost when there were no sales and no orders. (d)  $\hat{Y} = -2.72825 + 0.047114(400) + 0.011947(4500) = 69.878$  or \$69,878. (e)  $\$66,419.93 \leq \mu_{Y|X} \leq \$73,337.01$ . (f)  $\$59,380.61 \leq Y_X \leq \$80,376.33$ .

**11.6 (a)**  $\hat{Y} = 156.4 + 13.081X_1 + 16.795X_2$ . (b) For a given amount of newspaper advertising, each increase by \$1,000 in radio advertising is estimated to result in a mean increase in sales of \$13,081. For a given amount of radio advertising, each increase by \$1,000 in newspaper advertising is estimated to result in a mean increase in sales of \$16,795. (c) When there is no money spent on radio advertising and newspaper advertising, the estimated mean sales is \$156,430.44. (d)  $\hat{Y} = 156.4 + 13.081(20) + 16.795(20) = 753.95$  or \$753,950. (e)  $\$623,038.31 \leq \mu_{Y|X} \leq \$884,860.93$ . (f)  $\$396,522.63 \leq Y_X \leq \$1,111,376.60$ .

**11.8 (a)**  $\hat{Y} = 400.8057 + 456.4485X_1 - 2.4708X_2$ , where  $X_1 = \text{land}$ ,  $X_2 = \text{age}$ . (b) For a given age, each increase by one acre in land area is estimated to result in a mean increase in appraised value by \$456.45 thousands. For a given acreage, each increase of one year in age is estimated to result in the mean decrease in appraised value by \$2.47 thousands. (c) The interpretation of  $b_0$  has no practical meaning here because it would represent the estimated appraised value of a new house that has no land area. (d)  $\hat{Y} = 400.8057 + 456.4485(0.25) - 2.4708(45) = \$403.73$  thousands. (e)  $372.7370 \leq \mu_{Y|X} \leq 434.7243$ . (f)  $235.1964 \leq Y_X \leq 572.2649$ .

**11.10 (a)**  $MSR = 15$ ,  $MSE = 12$ . (b) 1.25. (c)  $F = 1.25 < 4.10$ ; do not reject  $H_0$ . (d) 0.20. (e) 0.04.

**11.12 (a)**  $F = 97.69 > F_{U(2,15-2-1)} = 3.89$ . Reject  $H_0$ . There is evidence of a significant linear relationship with at least one of the independent variables. (b) The  $p$ -value is 0.0001. (c)  $r^2 = 0.9421$ . 94.21% of the variation in the long-term ability to absorb shock can be explained by variation in forefoot absorbing capability and variation in midsole impact. (d)  $r_{adj}^2 = 0.93245$ .

**11.14 (a)**  $F = 74.13 > 3.467$ ; reject  $H_0$ . (b)  $p$ -value = 0. (c)  $r^2 = 0.8759$ . 87.59% of the variation in distribution cost can be explained by variation in sales and variation in number of orders. (d)  $r_{adj}^2 = 0.8641$ .

**11.16 (a)**  $F = 40.16 > F_{U(2,22-2-1)} = 3.522$ . Reject  $H_0$ . There is evidence of a significant linear relationship. (b) The  $p$ -value is less than 0.001. (c)  $r^2 = 0.8087$ . 80.87% of the variation in sales can be explained by variation in radio advertising and variation in newspaper advertising. (d)  $r_{adj}^2 = 0.7886$ .

**11.18 (a)** Based on a residual analysis, the model appears to be adequate. (b) There is no evidence of a pattern in the residuals versus time.

(c)  $D = \frac{1,077.0956}{477.0430} = 2.26$ . (d)  $D = 2.26 > 1.55$ . There is no evidence of positive autocorrelation in the residuals.

**11.20** There appears to be a quadratic relationship in the plot of the residuals against both radio and newspaper advertising. Thus, quadratic terms for each of these explanatory variables should be considered for inclusion in the model.

**11.22** There is no particular pattern in the residual plots, and the model appears to be adequate.

**11.24 (a)** Variable  $X_2$  has a larger slope in terms of the  $t$  statistic of 3.75 than variable  $X_1$ , which has a smaller slope in terms of the  $t$  statistic of 3.33. (b)  $1.46824 \leq \beta_1 \leq 6.53176$ . (c) For  $X_1$ :  $t = 4/1.2 = 3.33 > 2.1098$ , with 17 degrees of freedom for  $\alpha = 0.05$ . Reject  $H_0$ . There is evidence that  $X_1$  contributes to a model already containing  $X_2$ . For  $X_2$ :  $t = 3/0.8 = 3.75 > 2.1098$ , with 17 degrees of freedom for  $\alpha = 0.05$ . Reject  $H_0$ . There is evidence that  $X_2$  contributes to a model already containing  $X_1$ . Both  $X_1$  and  $X_2$  should be included in the model.

**11.26 (a)** 95% confidence interval on  $\beta_1$ :  $b_1 \pm t_{n-k-1} S_{b_1}$ ,  $0.0471 \pm 2.0796(0.0203)$ ,  $0.00488 \leq \beta_1 \leq 0.08932$ . (b) For  $X_1$ :  $t = b_1/S_{b_1} = 0.0471/0.0203 = 2.32 > 2.0796$ . Reject  $H_0$ . There is evidence that  $X_1$  contributes to a model already containing  $X_2$ . For  $X_2$ :  $t = b_2/S_{b_2} = 0.01195/0.00225 = 5.31 > 2.0796$ . Reject  $H_0$ . There is evidence that  $X_2$  contributes to a model already containing  $X_1$ . Both  $X_1$  (sales) and  $X_2$  (orders) should be included in the model.

**11.28 (a)**  $9.398 \leq \beta_1 \leq 16.763$ . (b) For  $X_1$ :  $t = 7.43 > 2.093$ . Reject  $H_0$ . There is evidence that  $X_1$  contributes to a model already containing  $X_2$ . For  $X_2$ :  $t = 5.67 > 2.093$ . Reject  $H_0$ . There is evidence that  $X_2$  contributes to a model already containing  $X_1$ . Both  $X_1$  (radio advertising) and  $X_2$  (newspaper advertising) should be included in the model.

**11.30 (a)**  $227.5865 \leq \beta_1 \leq 685.3104$ . (b) For  $X_1$ :  $t = 4.0922$  and  $p$ -value = 0.0003. Since  $p$ -value  $< 0.05$ , reject  $H_0$ . There is evidence that  $X_1$  contributes to a model already containing  $X_2$ . For  $X_2$ :  $t = -3.6295$  and  $p$ -value = 0.0012. Since  $p$ -value  $< 0.05$ , reject  $H_0$ . There is evidence that  $X_2$  contributes to a model already containing  $X_1$ . Both  $X_1$  (land area) and  $X_2$  (age) should be included in the model.

**11.32 (a)** For  $X_1$ :  $F = 1.25 < 4.96$ ; do not reject  $H_0$ . For  $X_2$ :  $F = 0.833 < 4.96$ ; do not reject  $H_0$ . **(b)** 0.1111, 0.0769.

**11.34 (a)** For  $X_1$ :  $SSR(X_1|X_2) = SSR(X_1 \text{ and } X_2) - SSR(X_2) = 3,368.087 - 3,246.062 = 122.025$ ,

$$F = \frac{SSR(X_2 | X_1)}{MSE} = \frac{122.025}{477.043 / 21} = 5.37 > 4.325.$$

Reject  $H_0$ . There is evidence that  $X_1$  contributes to a model already containing  $X_2$ . For  $X_2$ :  $SSR(X_2|X_1) = SSR(X_1 \text{ and } X_2) - SSR(X_1) = 3,368.087 - 2,726.822 = 641.265$ ,

$$F = \frac{SSR(X_2 | X_1)}{MSE} = \frac{641.265}{477.043 / 21} = 28.23 > 4.325. \text{ Reject } H_0.$$

There is evidence that  $X_2$  contributes to a model already containing  $X_1$ . Since both  $X_1$  and  $X_2$  make a significant contribution to the model in the presence of the other variable, both variables should be included in the

model. **(b)**  $\eta_{1,2}^2 = \frac{SSR(X_1 | X_2)}{SST - SSR(X_1 \text{ and } X_2) + SSR(X_1 | X_2)}$   
 $= \frac{122.025}{3,845.13 - 3,368.087 + 122.025} = 0.2037$ .

Holding constant the effect of the number of orders, 20.37% of the variation in distribution cost can be explained by the variation in sales.

$$\eta_{2,1}^2 = \frac{SSR(X_2 | X_1)}{SST - SSR(X_1 \text{ and } X_2) + SSR(X_2 | X_1)}$$

$$= \frac{641.265}{3,845.13 - 3,368.087 + 641.265} = 0.5734$$

Holding constant the effect of sales, 57.34% of the variation in distribution cost can be explained by the variation in the number of orders.

**11.36 (a)** For  $X_1$ :  $F = 55.28 > 4.381$ . Reject  $H_0$ . There is evidence that  $X_1$  contributes to a model already containing  $X_2$ . For  $X_2$ :  $F = 32.12 > 4.381$ . Reject  $H_0$ . There is evidence that  $X_2$  contributes to a model already containing  $X_1$ . Since both  $X_1$  and  $X_2$  make a significant contribution to the model in the presence of the other variable, both variables should be included in the model. **(b)**  $\eta_{1,2}^2 = 0.7442$ . Holding constant the effect of newspaper advertising, 74.42% of the variation in sales can be explained by the variation in radio advertising.  $\eta_{2,1}^2 = 0.6283$ . Holding constant the effect of radio advertising, 62.83% of the variation in sales can be explained by the variation in newspaper advertising.

**11.40 (a)**  $\hat{Y} = 243.7371 + 9.2189X_1 + 12.6967X_2$ , where  $X_1$  = number of rooms and  $X_2$  = neighborhood (east = 0). **(b)** Holding constant the effect of neighborhood, for each additional room, the selling price is estimated to increase by a mean of 9.2189 thousands of dollars, or \$9218.9. For a given number of rooms, a west neighborhood is estimated to increase the mean selling price over an east neighborhood by 12.6967 thousands of dollars, or \$12,696.7. **(c)**  $\hat{Y} = 243.7371 + 9.2189(9) + 12.6967(0) = 326.7076$ , or \$326,707.6.  $\$309,560.04 \leq Y_X \leq \$343,855.1$ .  $\$321,471.44 \leq \mu_{Y|X} \leq \$331,943.71$ . **(d)** Based on a residual analysis, the model appears to be adequate. **(e)**  $F = 55.39$ , the  $p$ -value is virtually 0. Since  $p$ -value  $< 0.05$ , reject  $H_0$ . There is evidence of a significant relationship between selling price and the two independent variables (rooms and neighborhood). **(f)** For  $X_1$ :  $t = 8.9537$ , the  $p$ -value is virtually 0. Reject  $H_0$ . Number of rooms makes a significant contribution and

should be included in the model. For  $X_2$ :  $t = 3.5913$ ,  $p$ -value = 0.0023  $< 0.05$ . Reject  $H_0$ . Neighborhood makes a significant contribution and should be included in the model. Based on these results, the regression model with the two independent variables should be used. **(g)**  $7.0466 \leq \beta_1 \leq 11.3913$ ,  $5.2378 \leq \beta_2 \leq 20.1557$ .

**(h)**  $r^2 = 0.867$ . 86.7% of the variation in selling price can be explained by variation in number of rooms and variation in neighborhood. **(i)**  $r_{adj}^2 = 0.851$ . **(j)**  $\eta_{1,2}^2 = 0.825$ . Holding constant the effect of neighborhood, 82.5% of the variation in selling price can be explained by variation in number of rooms.  $\eta_{2,1}^2 = 0.431$ . Holding constant the effect of number of rooms, 43.1% of the variation in selling price can be explained by variation in neighborhood. **(k)** The slope of selling price with number of rooms is the same, regardless of whether the house is located in an east or west neighborhood.

**(l)**  $\hat{Y} = 253.95 + 8.032X_1 - 5.90X_2 + 2.089X_1X_2$ . For  $X_1X_2$ ,  $p$ -value = 0.330. Do not reject  $H_0$ . There is no evidence that the interaction term makes a contribution to the model.

**(m)** The model in (a) should be used.

**11.42 (a)** Predicted time =  $8.01 + 0.00523 \text{ Depth} - 2.105 \text{ Dry}$ . **(b)** Holding constant the effect of type of drilling, for each foot increase in depth of the hole, the mean drilling time is estimated to increase by 0.0052 minutes. For a given depth, a dry drilling hole is estimated to reduce the mean drilling time over wet drilling by 2.1052 minutes. **(c)** 6.428 minutes,  $6.210 \leq \mu_{Y|X} \leq 6.646$ ,  $4.923 \leq Y_X \leq 7.932$ . **(d)** The model appears to be adequate.

**(e)**  $F = 111.11 > 3.09$ ; reject  $H_0$ . **(f)**  $t = 5.03 > 1.9847$ ; reject  $H_0$ .  $t = -1.403 < -1.9847$ ; reject  $H_0$ . Include both variables. **(g)**  $0.0032 \leq \beta_1 \leq 0.0073$ ,  $-2.403 \leq \beta_2 \leq -1.808$ . **(h)** 69.6% of the variation in drill time is explained by the variation of depth and variation in type of drilling. **(i)** 69.0%. **(j)** 0.207, 0.670. **(k)** The slope of the additional drilling time with the depth of the hole is the same, regardless of the type of drilling method used. **(l)** The  $p$ -value of the interaction term = 0.462  $> 0.05$ , so the term is not significant and should not be included in the model. **(m)** The model in part (a) should be used.

**11.44 (a)**  $\hat{Y} = 31.5594 + 0.0296X_1 + 0.0041X_2 + 0.000017159X_1X_2$ , where  $X_1$  = sales,  $X_2$  = orders,  $p$ -value = 0.3249  $> 0.05$ . Do not reject  $H_0$ . There is not enough evidence that the interaction term makes a contribution to the model. **(b)** Since there is not enough evidence of any interaction effect between sales and orders, the model in Problem 11.4 should be used.

**11.46 (a)** The  $p$ -value of the interaction term = 0.002  $< .05$ , so the term is significant and should be included in the model. **(b)** Use the model developed in this problem.

**11.48 (a)** For  $X_1X_2$ ,  $p$ -value = 0.2353  $> 0.05$ . Do not reject  $H_0$ . There is not enough evidence that the interaction term makes a contribution to the model. **(b)** Since there is not enough evidence of an interaction effect between total staff present and remote hours, the model in Problem 11.7 should be used.

**11.58 (a)**  $\hat{Y} = -3.9152 + 0.0319X_1 + 4.2228X_2$ , where  $X_1$  = number of cubic feet moved and  $X_2$  = number of pieces of large furniture. **(b)** Holding constant the number of pieces of large furniture, for each additional cubic feet moved, the mean labor hours are estimated to increase by 0.0319. Holding constant the amount of cubic feet moved, for each additional piece of large furniture, the mean labor hours are estimated to increase by 4.2228. **(c)**

$$\hat{Y} = -3.9152 + 0.0319(500) + 4.2228(2) = 20.4926$$

(d) Based on a residual analysis, the errors appear to be normally distributed. The equal-variance assumption might be violated because the variances appear to be larger around the center region of both independent variables. There might also be violation of the linearity assumption. A model with quadratic terms for both independent variables might be fitted. (e)  $F = 228.80$ ,  $p$ -value is virtually 0. Since  $p$ -value  $< 0.05$ , reject  $H_0$ . There is evidence of a significant relationship between labor hours and the two independent variables (the amount of cubic feet moved and the number of pieces of large furniture). (f) The  $p$ -value is virtually 0. The probability of obtaining a test statistic of 228.80 or greater is virtually 0 if there is no significant relationship between labor hours and the two independent variables (the amount of cubic feet moved and the number of pieces of large furniture). (g)  $r^2 = 0.9327$ . 93.27% of the variation in labor hours can be explained by variation in the amount of cubic feet moved and the number of pieces of large furniture. (h)  $r_{adj}^2 = 0.9287$ . (i) For  $X_1: t = 6.9339$ , the  $p$ -value is virtually 0. Reject  $H_0$ . The amount of cubic feet moved makes a significant contribution and should be included in the model. For  $X_2: t = 4.6192$ , the  $p$ -value is virtually 0. Reject  $H_0$ . The number of pieces of large furniture makes a significant contribution and should be included in the model. Based on these results, the regression model with the two independent variables should be used. (j) For  $X_1: t = 6.9339$ , the  $p$ -value is virtually 0. The probability of obtaining a sample that will yield a test statistic farther away than 6.9339 is virtually 0 if the number of cubic feet moved does not make a significant contribution holding the effect of the number of pieces of large furniture constant. For  $X_2: t = 4.6192$ , the  $p$ -value is virtually 0. The probability of obtaining a sample that will yield a test statistic farther away than 4.6192 is virtually 0 if the number of pieces of large furniture does not make a significant contribution holding the effect of the amount of cubic feet moved constant. (k)  $0.0226 \leq \beta_1 \leq 0.0413$ . We are 95% confident that the mean labor hours will increase by somewhere between 0.0226 and 0.0413 for each additional cubic foot moved, holding constant the number of pieces of large furniture. In Problem 13.44, we are 95% confident that the mean labor hours will increase by somewhere between 0.0439 and 0.0562 for each additional cubic foot moved regardless of the number of pieces of large furniture. (l)  $\eta_{1,2}^2 = 0.5930$ . Holding constant the effect of the number of pieces of large furniture, 59.3% of the variation in labor hours can be explained by variation in the amount of cubic feet moved.  $\eta_{2,1}^2 = 0.3927$ . Holding constant the effect of the amount of cubic feet moved, 39.27% of the variation in labor hours can be explained by variation in the number of pieces of large furniture.

**11.60 (a)**  $\hat{Y} = -120.0483 + 1.7506X_1 + 0.3680X_2$ , where  $X_1$  = assessed value and  $X_2$  = time period. (b) Holding constant the time period, for each additional thousand dollars of assessed value, the mean selling price is estimated to increase by 1.7507 thousand dollars. Holding constant the assessed value, for each additional month since assessment, the mean selling price is estimated to increase by 0.3680 thousand dollars. (c)  $\hat{Y} = -120.0483 + 1.7506(170) + 0.3680(12) = 181.9692$  thousand dollars. (d) Based on a residual analysis, the model appears to be adequate. (e)  $F = 223.46$ , the  $p$ -value is virtually 0. Since  $p$ -value  $< 0.05$ , reject  $H_0$ . There is evidence of a significant relationship between selling price and the two independent variables (assessed value and time period). (f) The  $p$ -value is virtually 0. The probability of obtaining a test statistic of 223.46 or greater is virtually 0 if there is no significant relationship between selling price and the two independent variables (assessed value and time period). (g)  $r^2 = 0.9430$ . 94.30% of the variation in selling price can be explained by variation in assessed value and time period. (h)  $r_{adj}^2 = 0.9388$ . (i) For  $X_1: t = 20.4137$ , the  $p$ -value is virtually 0. Reject  $H_0$ . The assessed value makes a significant contribution and should be included in the model. For  $X_2: t = 2.8734$ ,

$p$ -value = 0.0078  $< 0.05$ . Reject  $H_0$ . The time period makes a significant contribution and should be included in the model. Based on these results, the regression model with the two independent variables should be used.

(j) For  $X_1: t = 20.4137$ , the  $p$ -value is virtually 0. The probability of obtaining a sample that will yield a test statistic farther away than 20.4137 is virtually 0 if the assessed value does not make a significant contribution holding time period constant. For  $X_2: t = 2.8734$ , the  $p$ -value is virtually 0. The probability of obtaining a sample that will yield a test statistic farther away than 2.8734 is virtually 0 if the time period does not make a significant contribution holding the effect of the assessed value constant. (k)  $1.5746 \leq \beta_1 \leq 1.9266$ . We are 95% confident that the mean selling price will increase by an amount somewhere between \$1.5746 thousand and \$1.9266 thousand for each additional thousand-dollar increase in assessed value, holding constant the time period. In Problem 13.76, we are 95% confident that the mean selling price will increase by an amount somewhere between \$1.5862 thousand and \$1.9773 thousand for each additional thousand-dollar increase in assessed value regardless of the time period. (l)  $\eta_{1,2}^2 = 0.9392$ . Holding constant the effect of the time period, 93.92% of the variation in selling price can be explained by variation in the assessed value.  $\eta_{2,1}^2 = 0.2342$ . Holding constant the effect of the assessed value, 23.42% of the variation in selling price can be explained by variation in the time period.

**11.62 (a)**  $\hat{Y} = 163.7751 + 10.7252X_1 - 0.2843X_2$ , where  $X_1$  = size and  $X_2$  = age. (b) Holding age constant for each additional thousand square feet, the mean assessed value is estimated to increase by \$10.7252 thousand. Holding constant the size, for each additional year, the mean assessed value is estimated to decrease by \$0.2843 thousand. (c)  $\hat{Y} = 163.7751 + 10.7252(1.75) - 0.2843(10) = 179.7017$  thousand dollars. (d) Based on a residual analysis, the errors appear to be normally distributed. The equal-variance assumption appears to be valid. There might also be violation of the linearity assumption for age. You may want to include a quadratic term for age in the model. (e)  $F = 28.58$ ,  $p$ -value =  $2.72776 \times 10^{-5}$ . Since  $p$ -value  $< 0.05$ , reject  $H_0$ . There is evidence of a significant relationship between assessed value and the two independent variables (size and age). (f)  $p$ -value = 0.0000272776. The probability of obtaining a test statistic of 28.58 or greater is virtually 0 if there is no significant relationship between assessed value and the two independent variables (size and age). (g)  $r^2 = 0.8265$ . 82.65% of the variation in assessed value can be explained by variation in size and age. (h)  $r_{adj}^2 = 0.7976$ . (i) For  $X_1: t = 3.5581$ ,  $p$ -value = 0.0039  $< 0.05$ . Reject  $H_0$ . The size of a house makes a significant contribution and should be included in the model. For  $X_2: t = -3.4002$ ,  $p$ -value = 0.0053  $< 0.05$ . Reject  $H_0$ . The age of a house makes a significant contribution and should be included in the model. Based on these results, the regression model with the two independent variables should be used. (j) For  $X_1: p$ -value = 0.0039. The probability of obtaining a sample that will yield a test statistic farther away than 3.5581 is 0.0039 if the size of a house does not make a significant contribution holding age constant. For  $X_2: p$ -value = 0.0053. The probability of obtaining a sample that will yield a test statistic farther away than  $-3.4002$  is 0.0053 if the age of a house does not make a significant contribution, holding the effect of the size constant. (k)  $4.1575 \leq \beta_1 \leq 17.2928$ . We are 95% confident that the mean assessed value will increase by an amount somewhere between \$4.1575 thousand and \$17.2928 thousand for each additional thousand-square-foot increase in the size of a house, holding constant the age. In Problem 13.77, we are 95% confident that the mean assessed value will increase by an amount somewhere between \$9.4695 thousand and \$23.7972 thousand for each additional thousand-square-foot increase in heating area regardless of the age. (l)  $\eta_{1,2}^2 = 0.5134$ . Holding constant the effect of age, 51.34% of the variation in assessed value can be explained by variation in the size.

$r_{Y_2,1}^2 = 0.4907$ . Holding constant the effect of the size, 49.07% of the variation in assessed value can be explained by variation in the age.

**(m)** Based on your answers to (a) through (l), the age of a house does have an effect on its assessed value.

**11.64 (a)**  $\hat{Y} = 146.0959 - 14.1276X_1 - 5.7491X_2$ , where  $X_1 = \text{ERA}$  and  $X_2 = \text{League (American = 0)}$ . **(b)** Holding constant the effect of the league, for each additional ERA, the mean number of wins is estimated to decrease by 14.1276. For a given ERA, a team in the National League is estimated to have a mean of 5.7491 fewer wins than a team in the American League. **(c)**  $\hat{Y} = 146.0959 - 14.1276(4.5) - 5.7491(0) = 82.5216$  wins. **(d)** Based on a residual analysis, the errors appear to be right-skewed. The equal-variance and linearity assumptions appear to be valid. **(e)**  $F = 26.37$ ,  $p\text{-value} = 24.47667 \times 10^{-7}$ . Since  $p\text{-value} < 0.05$ , reject  $H_0$ . There is evidence of a significant relationship between wins and the two independent variables (ERA and league). **(f)** For  $X_1$ :  $t = -7.2404$ , the  $p\text{-value}$  is virtually 0. Reject  $H_0$ . ERA makes a significant

contribution and should be included in the model. For  $X_2$ :  $t = -2.33$ ,  $p\text{-value} = 0.0275 < 0.05$ . Reject  $H_0$ . The league makes a significant contribution and should be included in the model. Based on these results, the regression model with the two independent variables should be used. **(g)**  $-18.1312 \leq \beta_1 \leq -10.1241$ .  $-10.8119 \leq \beta_2 \leq -0.6863$ . **(h)**  $r^2 = 0.6614$ . So 66.14% of the variation in the number of wins can be explained by ERA and league. **(i)**  $r_{adj}^2 = 0.6363$ . **(j)**  $r_{Y_{1,2}}^2 = 0.6601$ . Holding constant the effect of league, 66.01% of the variation in the number of wins can be explained by variation in ERA.  $r_{Y_{2,1}}^2 = 0.1674$ . Holding constant the effect of ERA, 16.74% of the variation in the number of wins can be explained by the league a team plays in. **(k)** The slope of the number of wins with ERA is the same, regardless of whether the team belongs to the American or the National League. **(l)**  $\hat{Y} = 152.0064 - 15.4246X_1 - 19.1124X_2 + 3.0526X_1X_2$ . For  $X_1X_2$ , the  $p\text{-value}$  is 0.4497. Do not reject  $H_0$ . There is no evidence that the interaction term makes a contribution to the model. **(m)** The model in (a) should be used.



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